Topological order in quantum matter

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1. Classical XY model in 2 and 3 dimensions

2. Topological order in the classical XY model in 3 dimensions

3. Topological order in the quantum XY model in 2+1 dimensions

4. Topological order in the Hubbard model

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$$\mathcal{Z}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-H/T\right)$$

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

Describes non-zero *T* phase transitions of superfluids, magnets with `easy-plane' spins, ...

In spatial dimension $d = 3$, in the low T phase, the symmetry $\theta_i \to \theta_i + c$ is "spontaneously broken". There is (off-diagonal) long-range order (LRO) characterized by ($\Psi_i \equiv e^{i\theta_i}$)

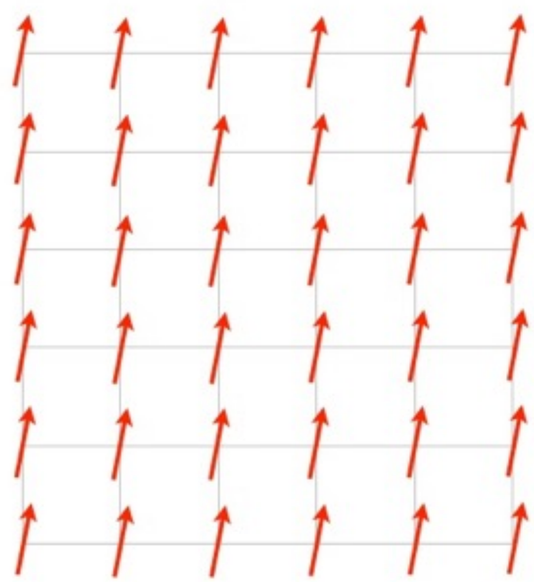
$$\lim_{|r_i - r_j| \to \infty} \left\langle \Psi_i \Psi_j^* \right\rangle = |\Psi_0|^2 \neq 0 \,.$$

We break the symmetry by choosing an overall phase so that

$$\langle \Psi_i \rangle = \Psi_0 \neq 0$$

$\langle \Psi_i \rangle = \Psi_0 \neq 0$

LRO



$\langle \Psi_i \rangle = 0$

$\left\langle \Psi_i \Psi_j^* \right\rangle \sim \exp(-|r_i - r_j|/\xi)$

SRO

T_c T

In spatial dimension $d = 3$, in the low T phase, the symmetry $\theta_i \to \theta_i + c$ is "spontaneously broken". There is (off-diagonal) long-range order (LRO) characterized by ($\Psi_i \equiv e^{i\theta_i}$)

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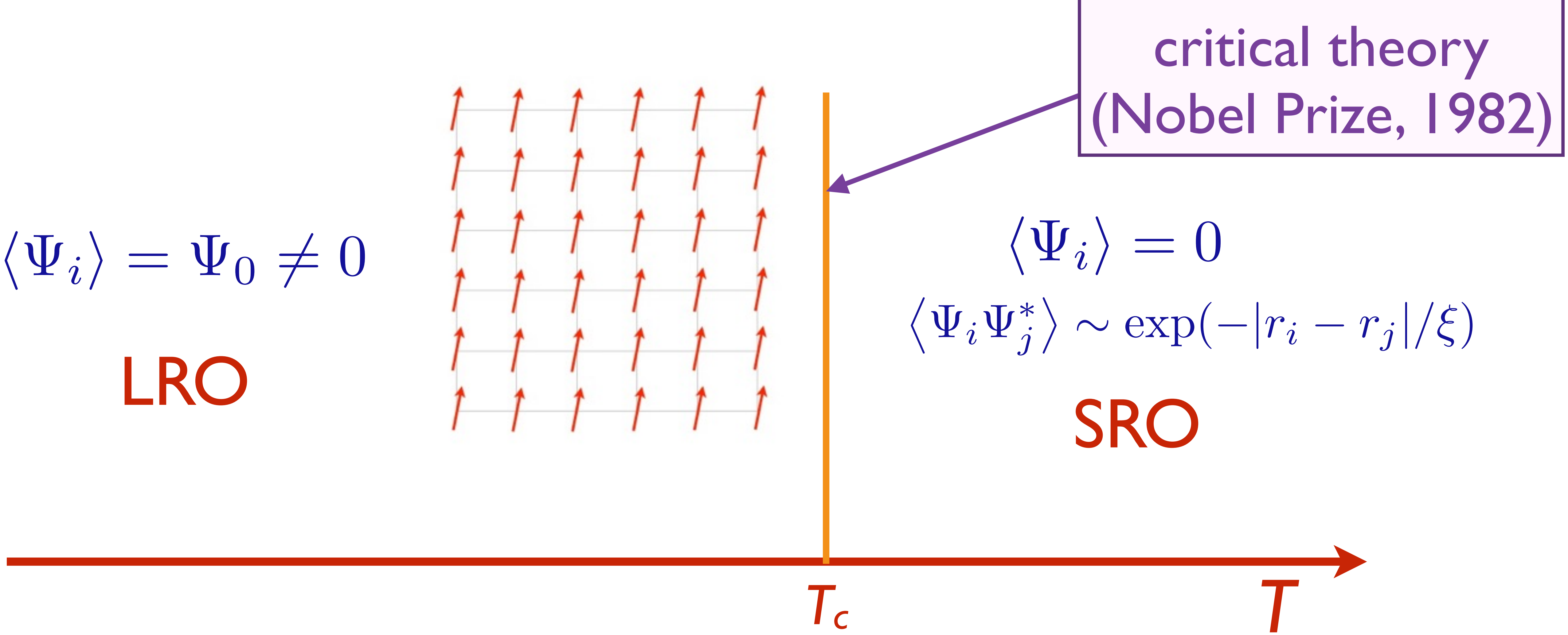


In spatial dimension $d = 2$, the symmetry $\theta_i \to \theta_i + c$ is preserved at all non-zero T. There is no LRO, and

$$\langle \Psi_i \rangle = 0 \text{ for all } T > 0.$$

Nevertheless, there is a phase transition at $T = T_{KT}$, where the nature of the correlations changes

$$\lim_{|r_i - r_j| \to \infty} \langle \Psi_i \Psi_j^* \rangle \sim \begin{cases} |r_i - r_j|^{-\alpha}, & \text{for } T < T_{KT}, \text{ (QLRO)} \\ \exp(-|r_i - r_j|/\xi), & \text{for } T > T_{KT}, \text{ (SRO)} \end{cases}$$

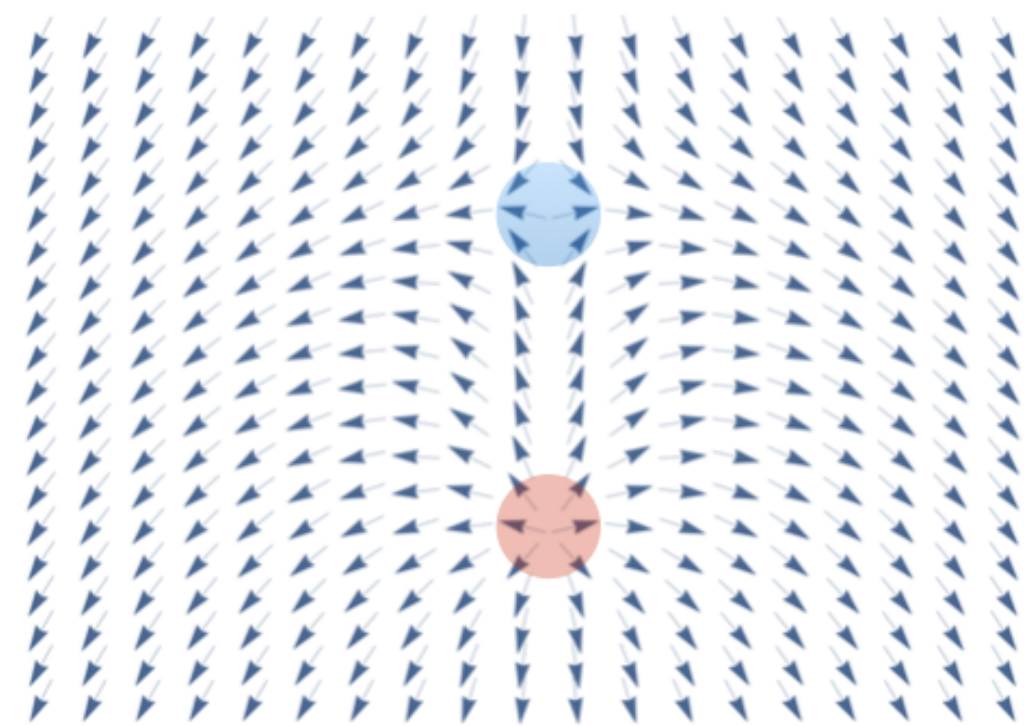
Kosterlitz-Thouless theory in d=2

KT theory
(Nobel Prize, 2016)

QLRO
Topological order

Vortices suppressed

SRO



Vortices proliferate

T_{KT} T

The low T phase also has **topological order** associated with the suppression of vortices.

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Can we modify the XY model Hamiltonian to obtain a phase with "topological order" in d=3 ?

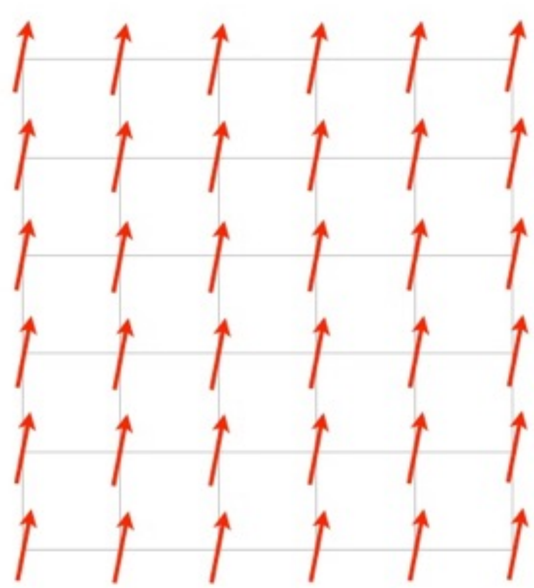
$$\langle \Psi_i \rangle = 0$$

$$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$$

SRO

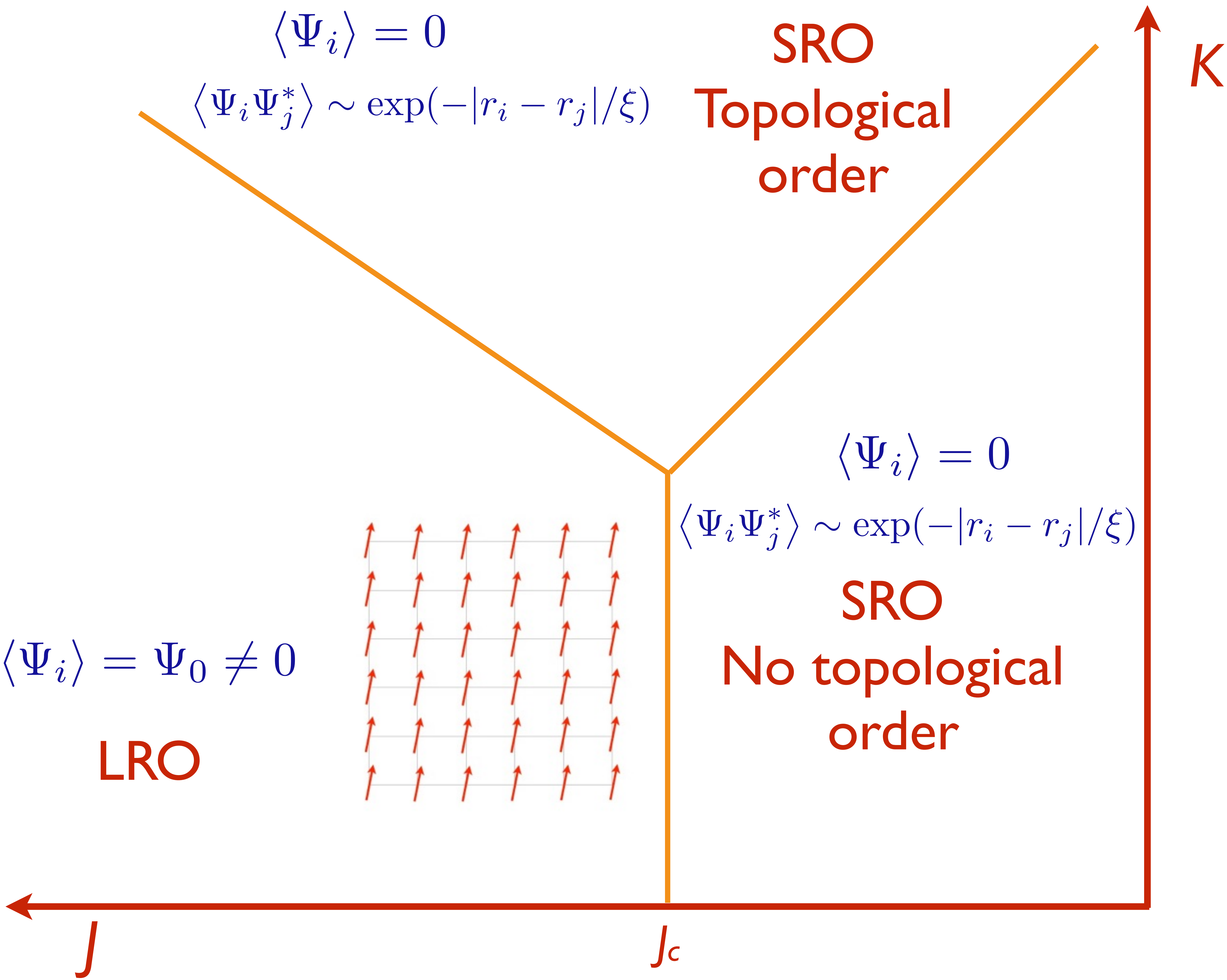
$$\langle \Psi_i \rangle = \Psi_0 \neq 0$$

LRO



J

J_c


$\langle \Psi_i \rangle = 0$
$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$
SRO
Topological order
K
$\langle \Psi_i \rangle = 0$
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SRO
No topological order
$\langle \Psi_i \rangle = \Psi_0 \neq 0$
LRO
J
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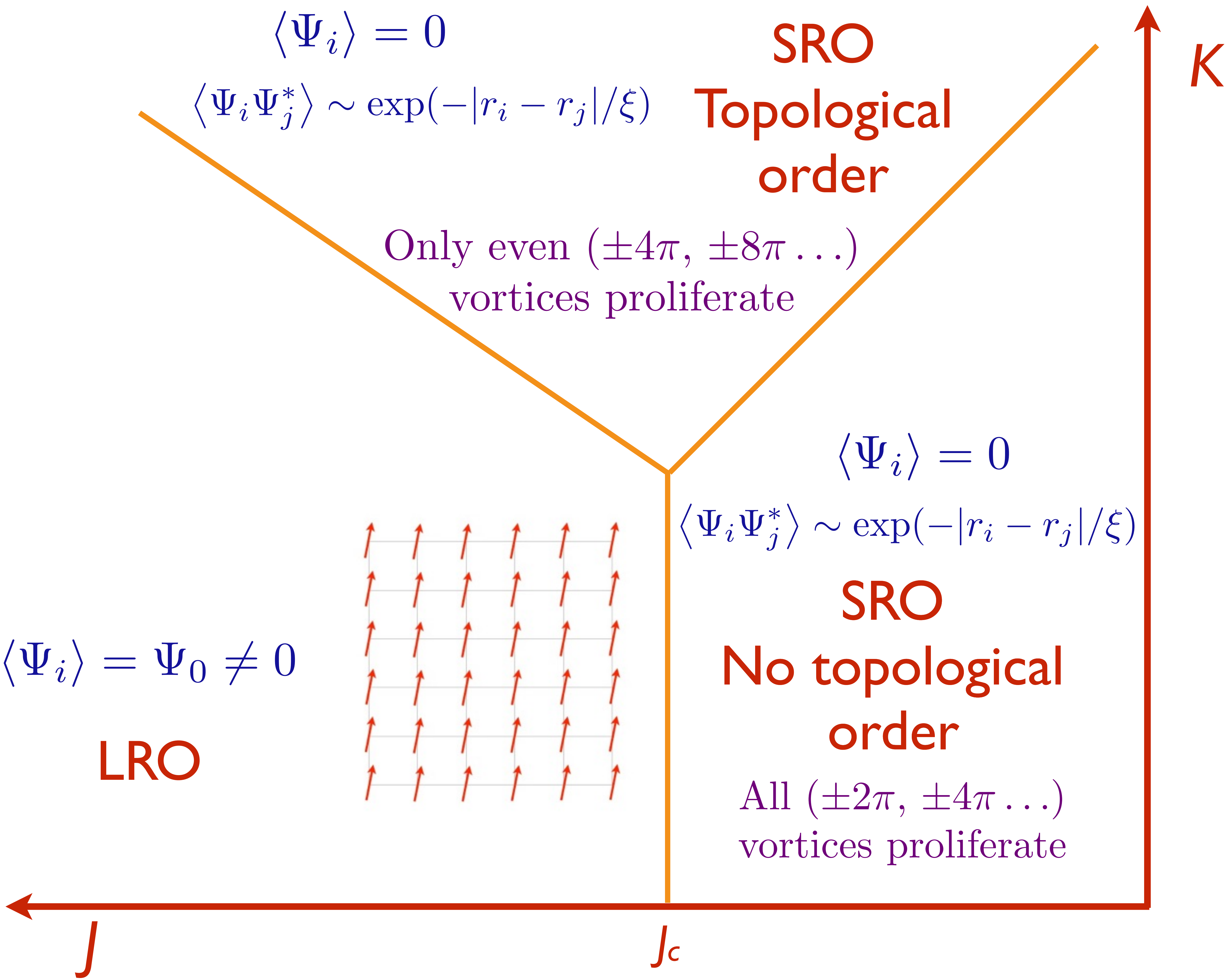


$\langle \Psi_i \rangle = 0$
$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$
SRO
Topological
order
K
Only even $(\pm 4\pi, \pm 8\pi \ldots)$
vortices proliferate
$\langle \Psi_i \rangle = 0$
$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$
SRO
No topological
order
All $(\pm 2\pi, \pm 4\pi \ldots)$
vortices proliferate
$\langle \Psi_i \rangle = \Psi_0 \neq 0$
LRO
J
J_c

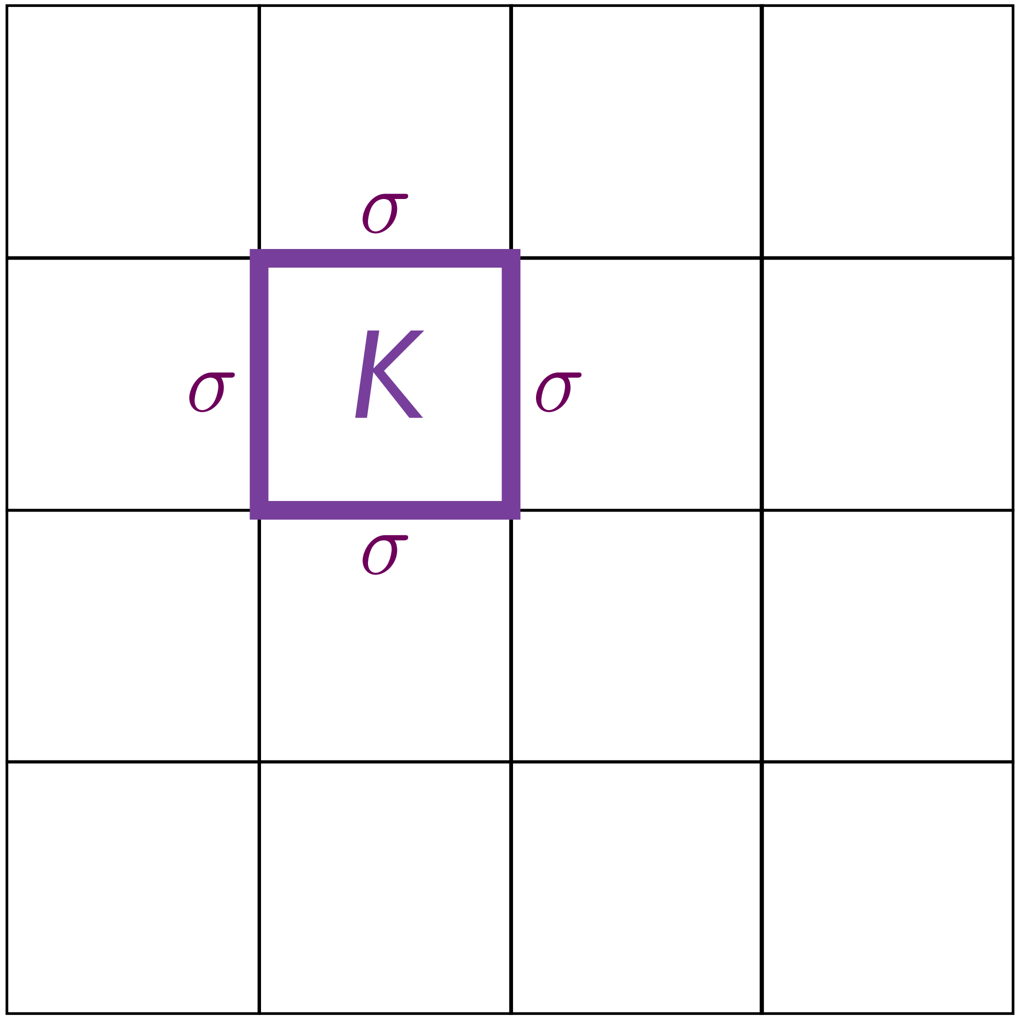
$$\widetilde{\mathcal{Z}}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) + \sum_{ijk\ell} J_{ijk\ell} \cos(\theta_i + \theta_j - \theta_k - \theta_\ell) + \ldots\ldots$$

Add terms which suppress single but not double vortices…..

$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij}$$



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$$
\begin{aligned}
\widetilde{\mathcal{Z}}_{XY} &= \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right) \\
\widetilde{H} &= -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij}
\end{aligned}
$$

- At small K, we can explicitly sum over σ_{ij}, order-by-order in K, and the theory reduces to an ordinary XY model with multi-site interactions. The resulting effective action of the XY model is periodic in $\theta_i \rightarrow \theta_i + 2\pi$ (for any site i), and preserves the symmetry $\theta_i \rightarrow \theta_i + c$ (for all sites i).

$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij}$$

- The theory has a $\mathbb{Z}_2$ gauge invariance: we can change

$$\theta_i \rightarrow \theta_i + \pi(1 - \eta_i)$$
$$\sigma_{ij} \rightarrow \eta_i \sigma_{ij} \eta_j \ ,$$

  with $\eta_i = \pm 1$, and the energy remains unchanged.

- The XY order parameter $\Psi_i = e^{i\theta_i}$ is gauge invariant, as are all physical observables. So this is an XY model with a modified Hamiltonian, and no additional degrees of freedom have been introduced.

$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij}$$

- A single (odd) 2π vortex in θ_i has $\prod_{(ij)\in\square} \cos\left[(\theta_i - \theta_j)/2\right] < 0$.
- So for $J > 0$, such a vortex will prefer $\prod_{(ij)\in\square} \sigma_{ij} = -1$, *i.e.* a 2π vortex has $\mathbb{Z}_2$ flux $= -1$ in its core.
- So a large $K > 0$ will suppress (odd) 2π vortices.
- There is no analogous suppression of (even) 4π vortices.

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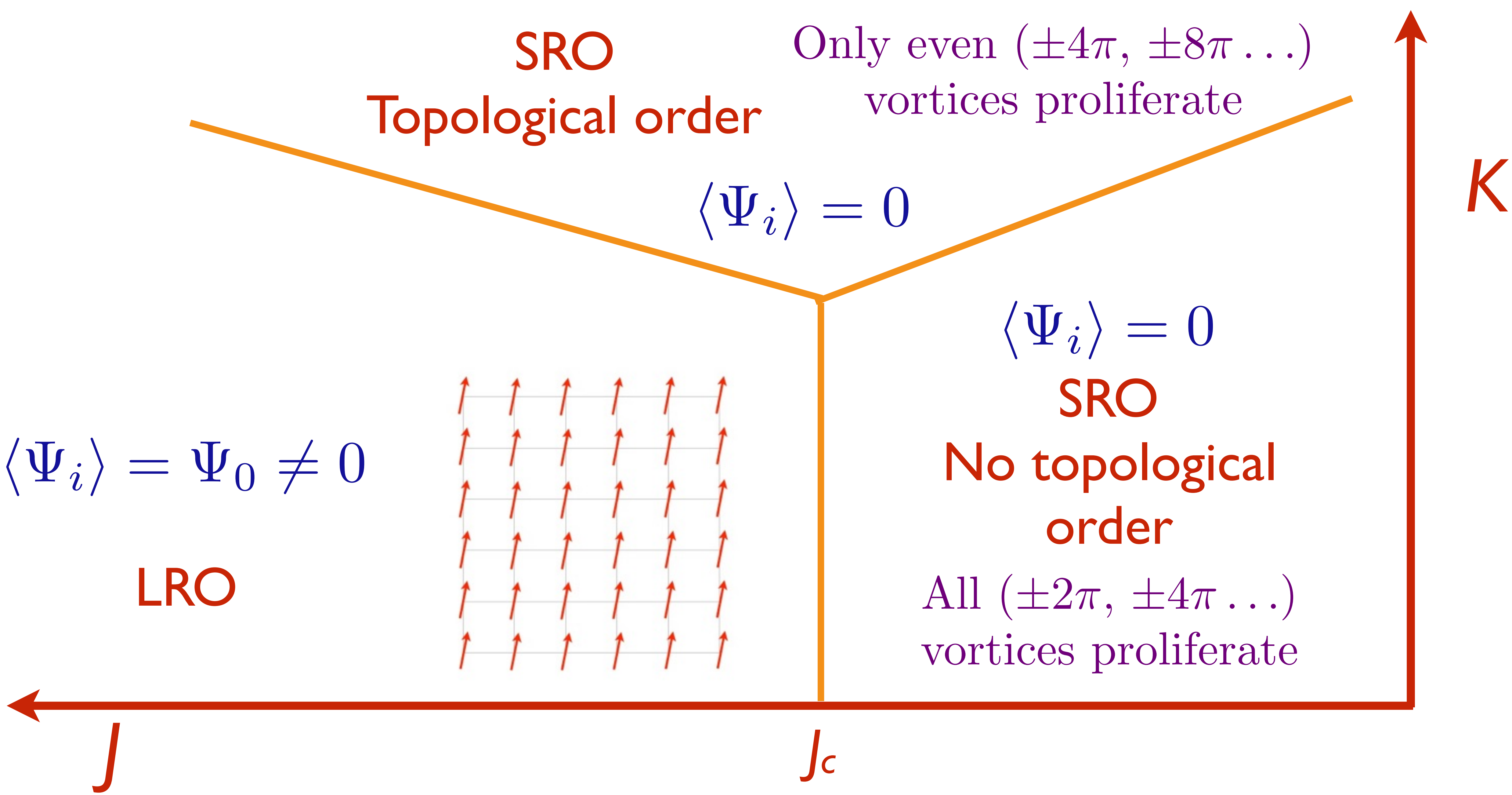

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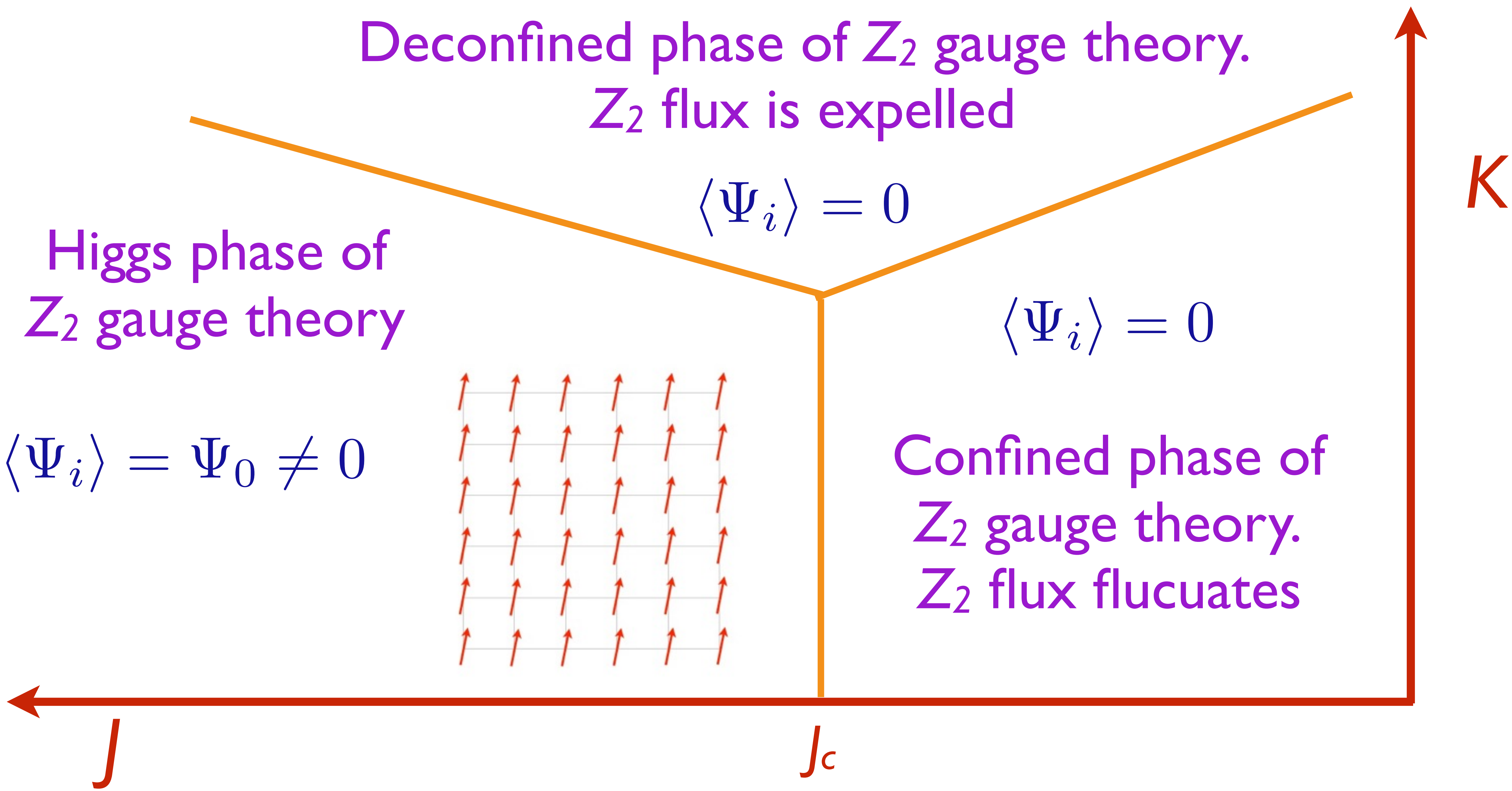

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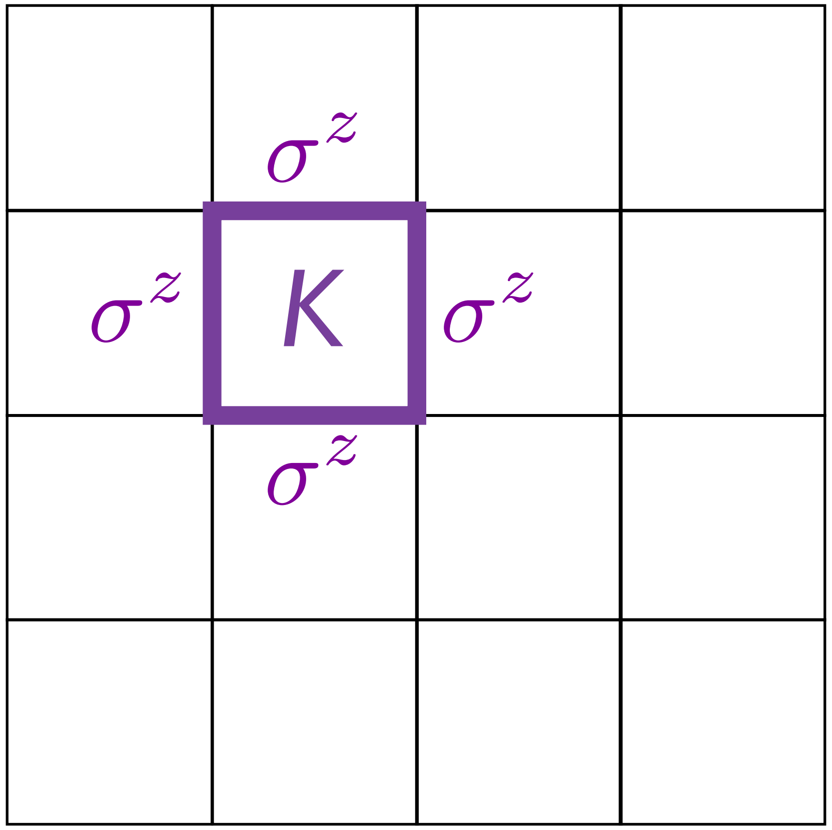
A quantum Hamiltonian in 2+1 dimensions

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma^z_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma^z_{ij}$$

$$+U \sum_i (\hat{n}_i)^2 - g \sum_{\langle ij \rangle} \sigma^x_{ij} \quad ; \quad [\theta_i, \hat{n}_j] = i\delta_{ij}$$



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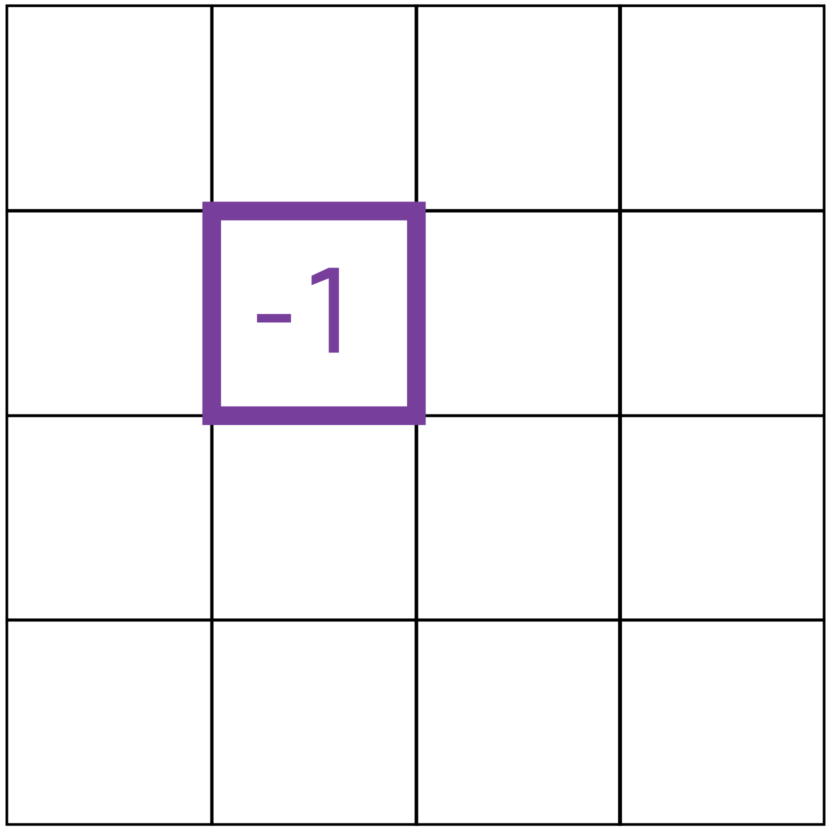
- In the topological phase, the suppressed Z_2 fluxes of -1 become well-defined gapped quasiparticle excitations ('visons') above the ground state.



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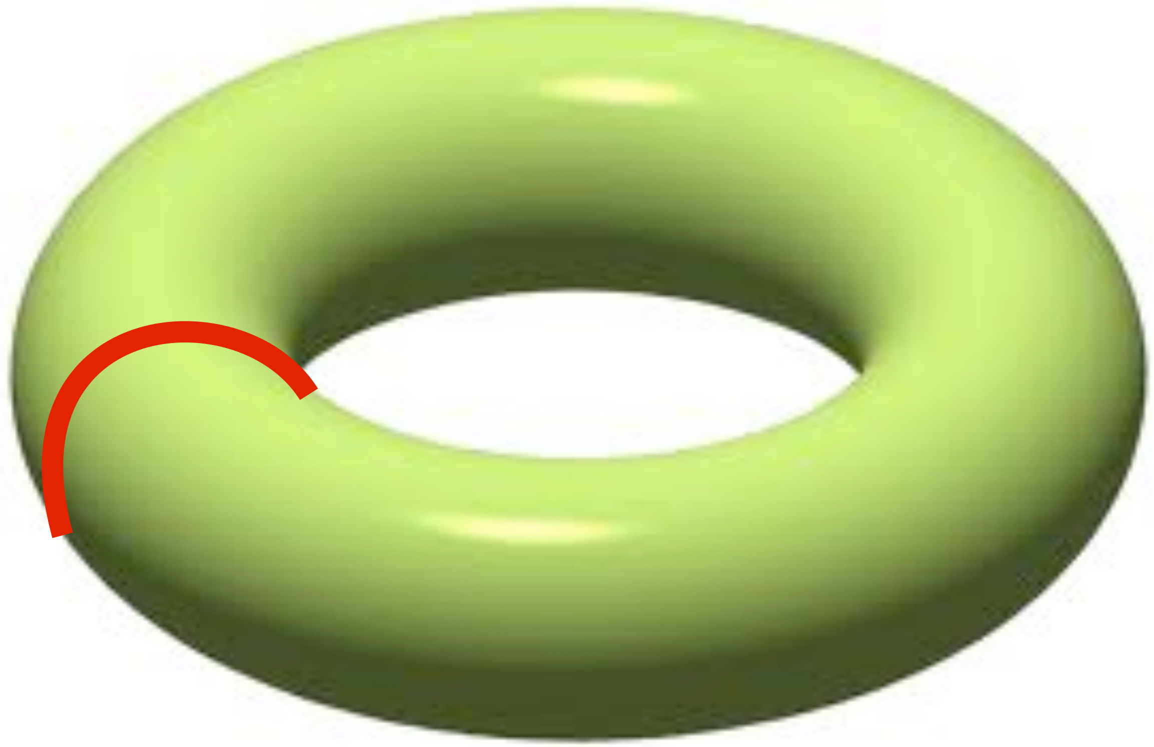
- In the topological phase, on a torus, inserting the Z_2 flux of -1 into one of the cycles of the torus leads to an orthogonal state whose energy cost vanishes exponentially in the size of the torus: there are 4 degenerate ground states on a large torus.



A quantum Hamiltonian in 2+1 dimensions

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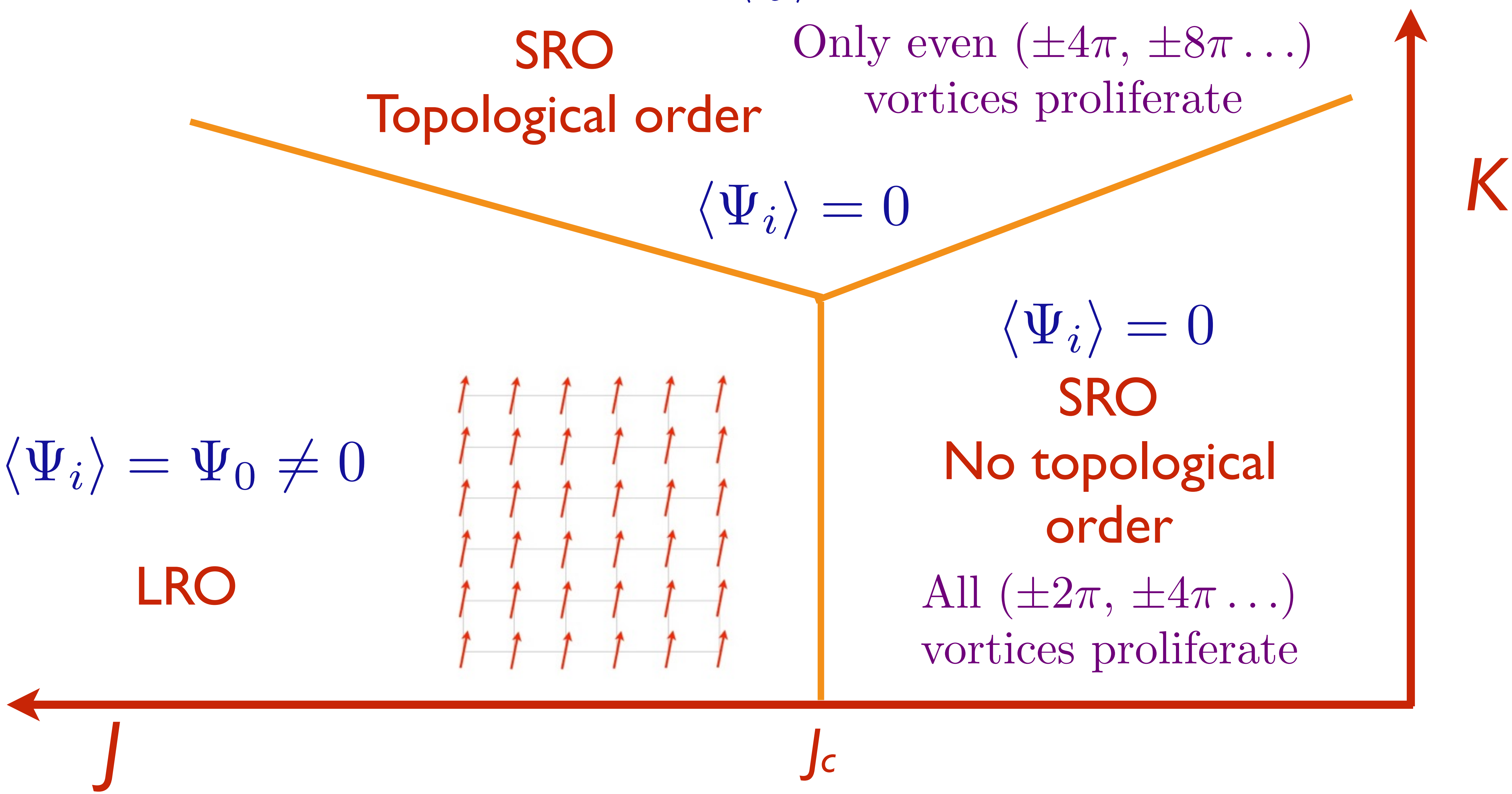


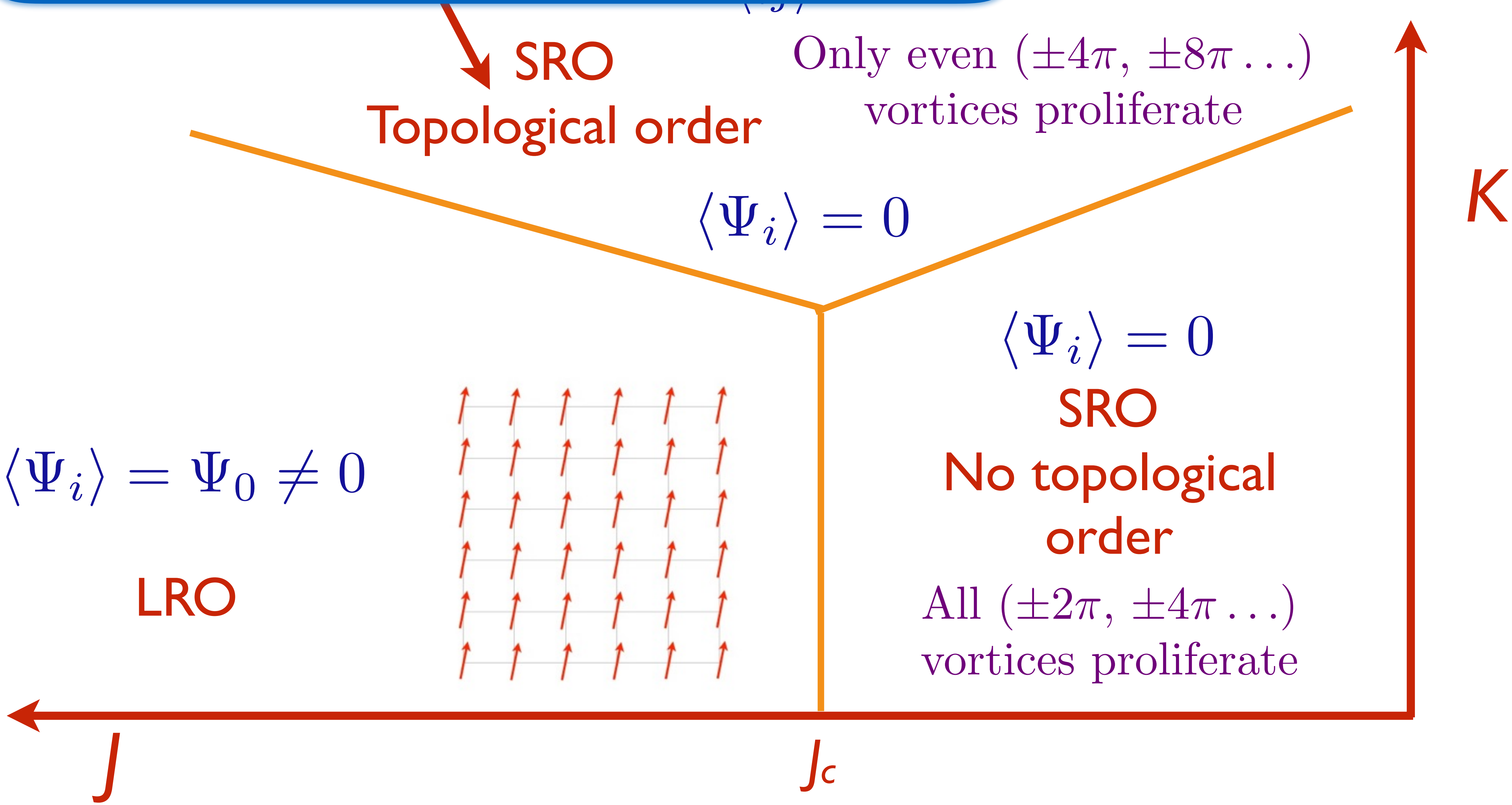
A quantum Hamiltonian in 2+1 dimensions

$$- K \sum_{\square} \prod_{(ij)\in\square} \sigma^z_{ij}$$

$$[\theta_i, \hat{n}_j] = i\delta_{ij}$$

The topological order is the same as that of the 'toric code', or of the U(1)×U(1) Chern-Simons theory

$$\mathcal{L}_{cs} = \frac{1}{\pi}\epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda$$



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The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_{i} c_{i\alpha}^{\dagger} c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Spin index $\alpha = \uparrow, \downarrow$

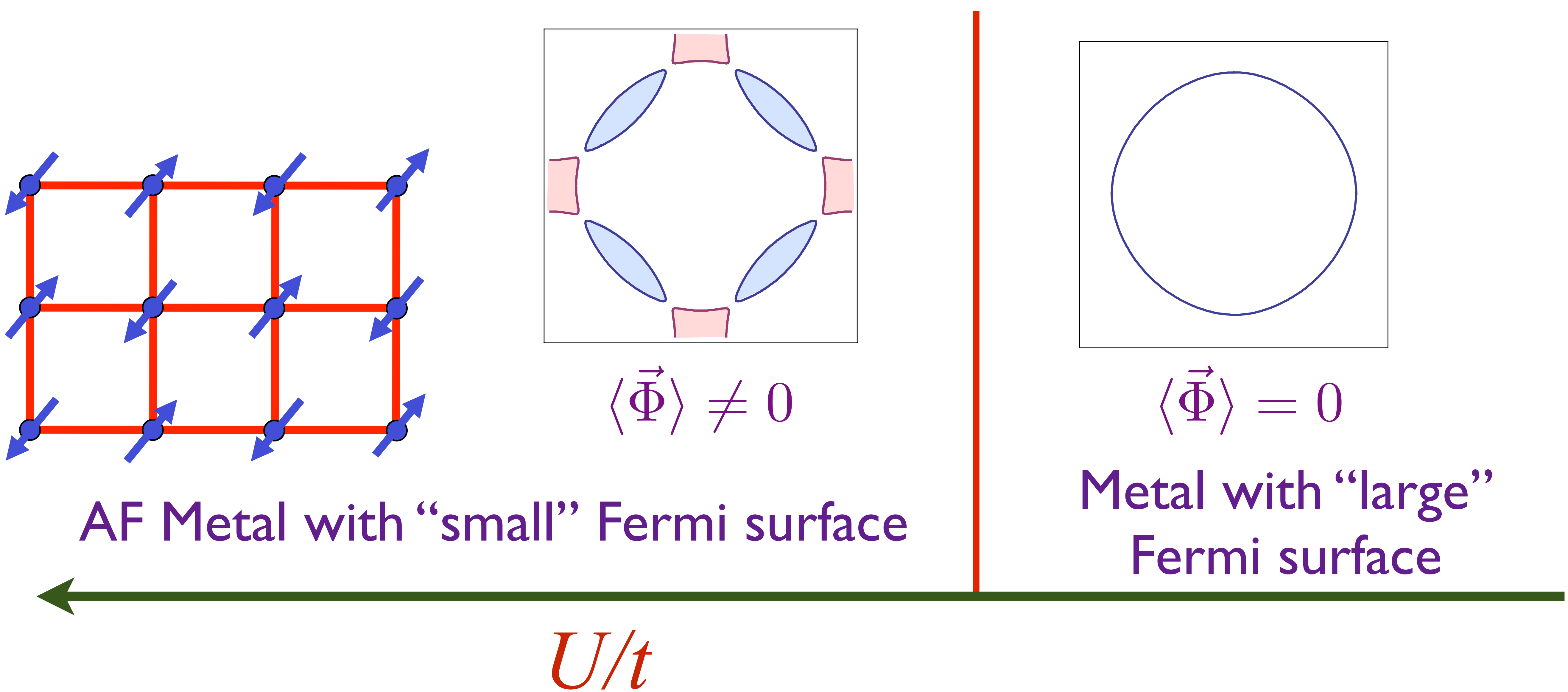
$$n_{i\alpha} = c_{i\alpha}^{\dagger} c_{i\alpha}$$

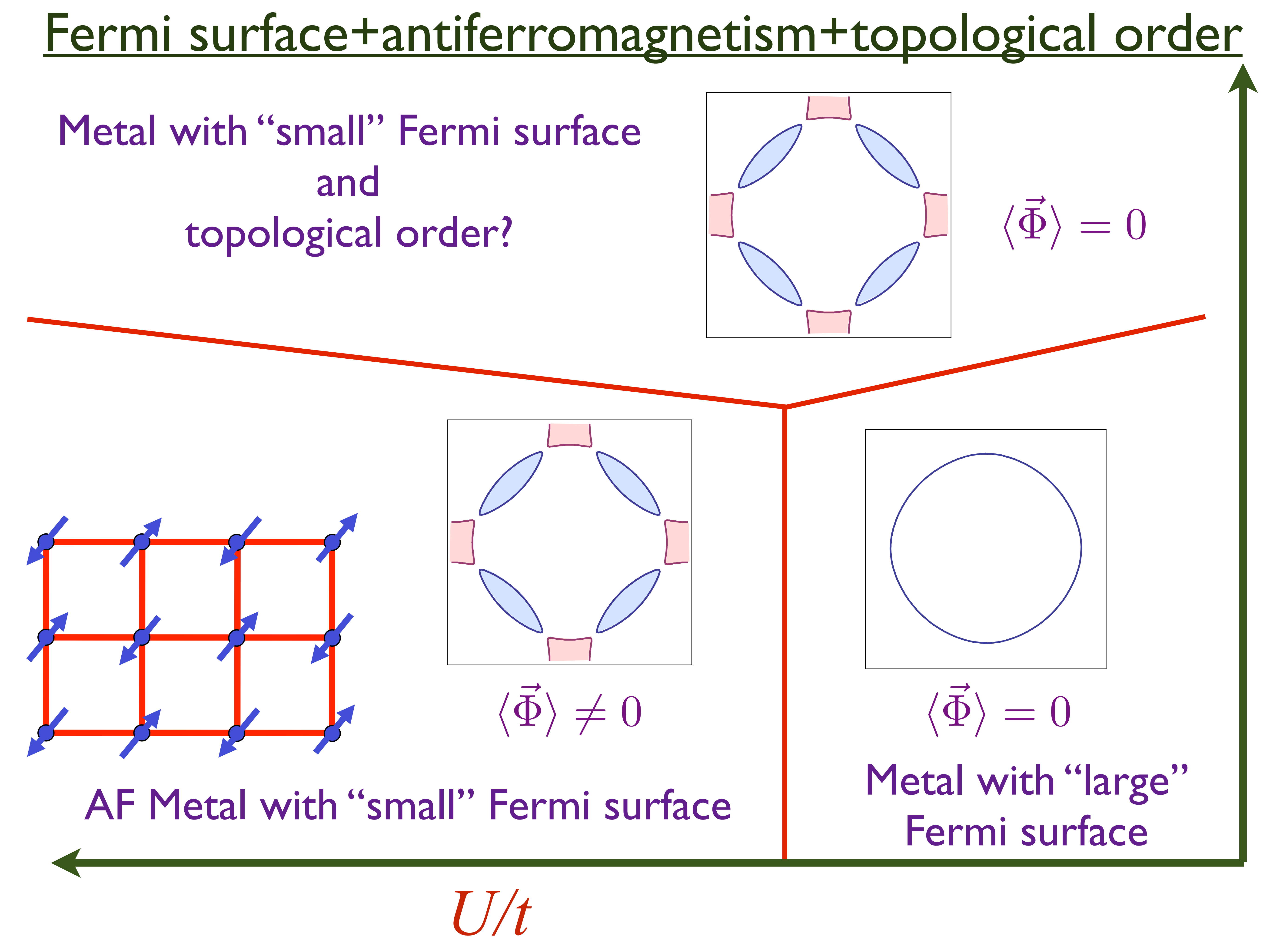
$$c_{i\alpha}^{\dagger} c_{j\beta} + c_{j\beta} c_{i\alpha}^{\dagger} = \delta_{ij} \delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

Will study on the square lattice

Fermi surface+antiferromagnetism

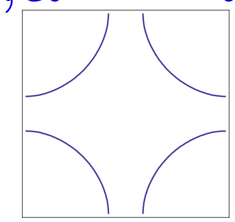
Mean-field theory with an antiferromagnetic order parameter $\vec{\Phi}_i = (-1)^{i_x+i_y} \left\langle \vec{S}_i \right\rangle$




Fermi surface+antiferromagnetism+topological order
Metal with "small" Fermi surface
and
topological order?
$\langle \vec{\Phi} \rangle = 0$
$\langle \vec{\Phi} \rangle \neq 0$
$\langle \vec{\Phi} \rangle = 0$
Metal with "large"
Fermi surface
AF Metal with "small" Fermi surface
U/t

We can (exactly) transform the Hubbard model to the "spin-fermion" model: **electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\text{int}}$$



are coupled to an **antiferromagnetic order parameter $\Phi^\ell(i)$,** $\ell = x, y, z$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i \Phi^\ell(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^\ell c_{i,\beta} + V_\Phi$$

where $\eta_i = \pm 1$ on the two sublattices. (For suitable V_Φ, integrating out the Φ yields back the Hubbard model).

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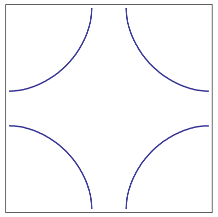

are coupled to an **antiferromagnetic order parameter** $\Phi^\ell(i)$**,** $\ell = x, y, z$

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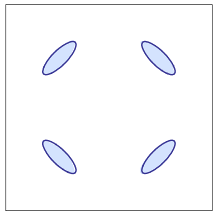
where $\eta_i = \pm 1$ on the two sublattices. (For suitable V_Φ, integrating out the Φ yields back the Hubbard model).

When $\Phi^\ell(i) =$ (non-zero constant) independent of i, we have long-range AF order, which transforms the Fermi surfaces from large to small.



For fluctuating antiferromagnetism, we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^\ell \Phi^\ell(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the AFM order in the rotating reference frame.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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The Higgs field is the AFM order in the rotating reference frame. Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right) \rightarrow V_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right)$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Fluctuating antiferromagnetism

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the AFM order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho,s} + \psi_{i+\boldsymbol{v}_\rho,s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'} + V_H$$

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$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'} + V_H$$

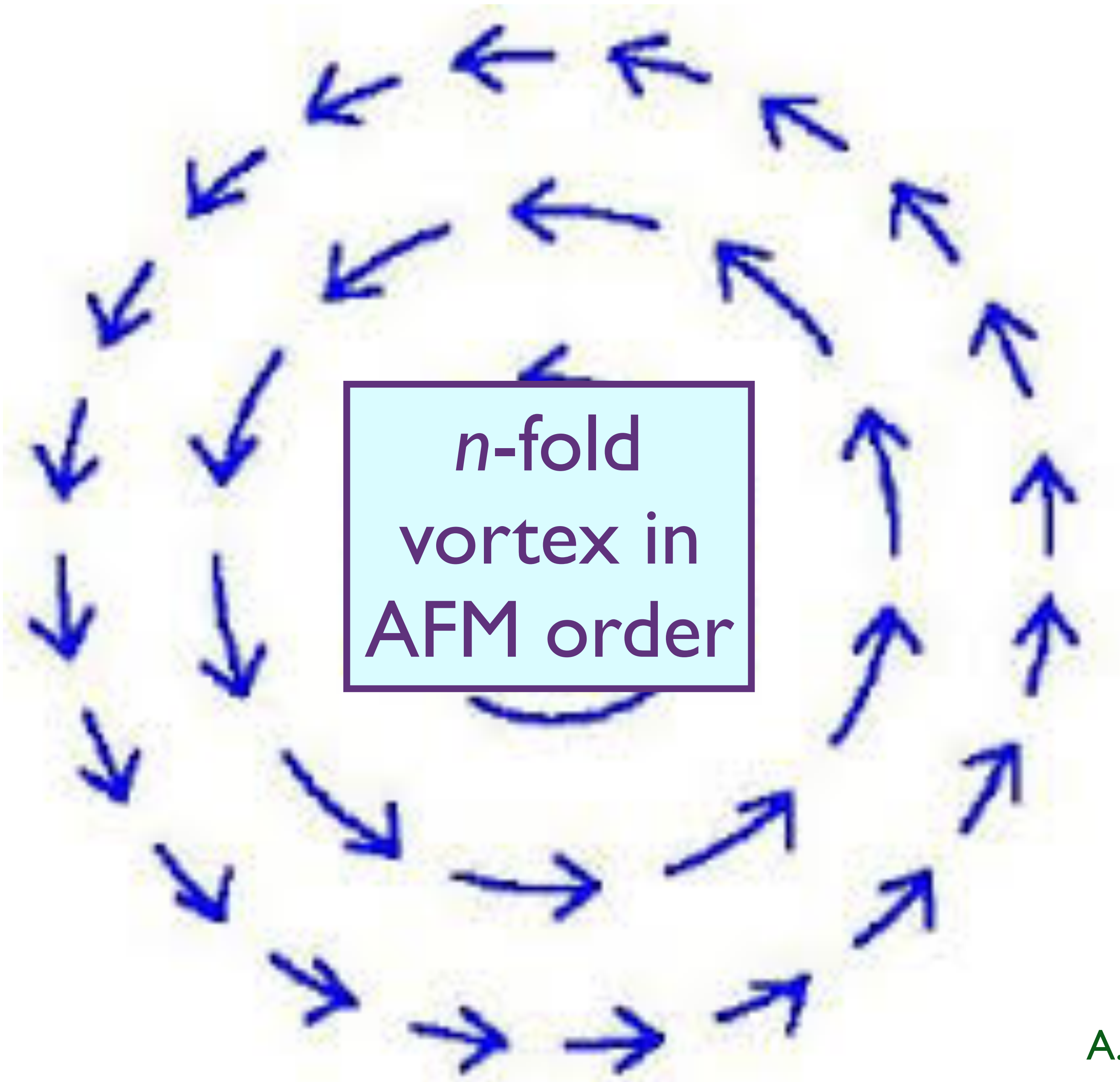
IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of fluctuating AFM will inherit the small Fermi surfaces of the electrons in the presence of static AFM.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Fluctuating antiferromagnetism

We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !

(assume easy-plane AFM for simplicity)



A.V. Chubukov, T. Senthil and S. Sachdev, PRL **72**, 2089 (1994);
S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)

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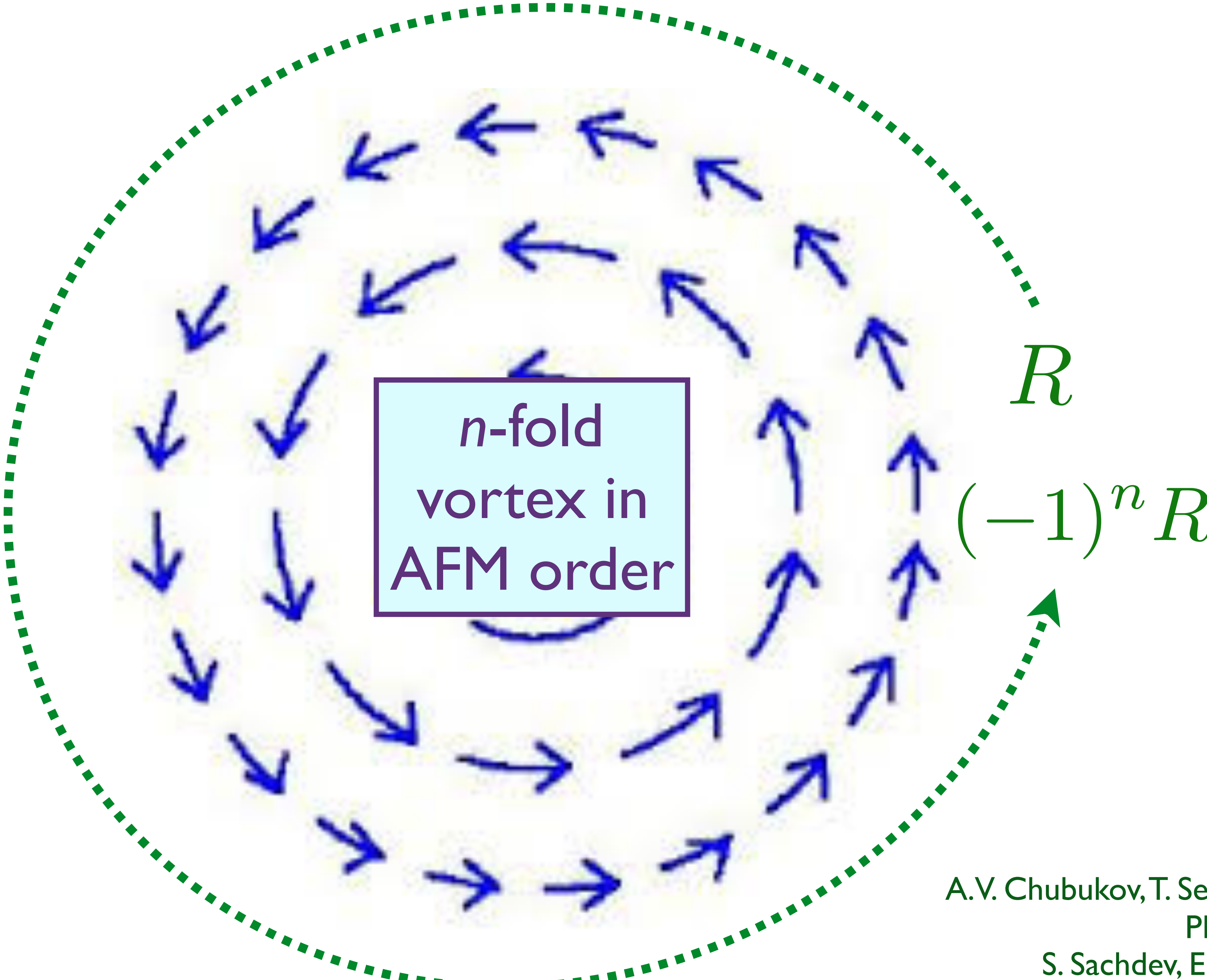

A.V. Chubukov, T. Senthil and S. Sachdev, PRL **72**, 2089 (1994);
S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)

Topological order

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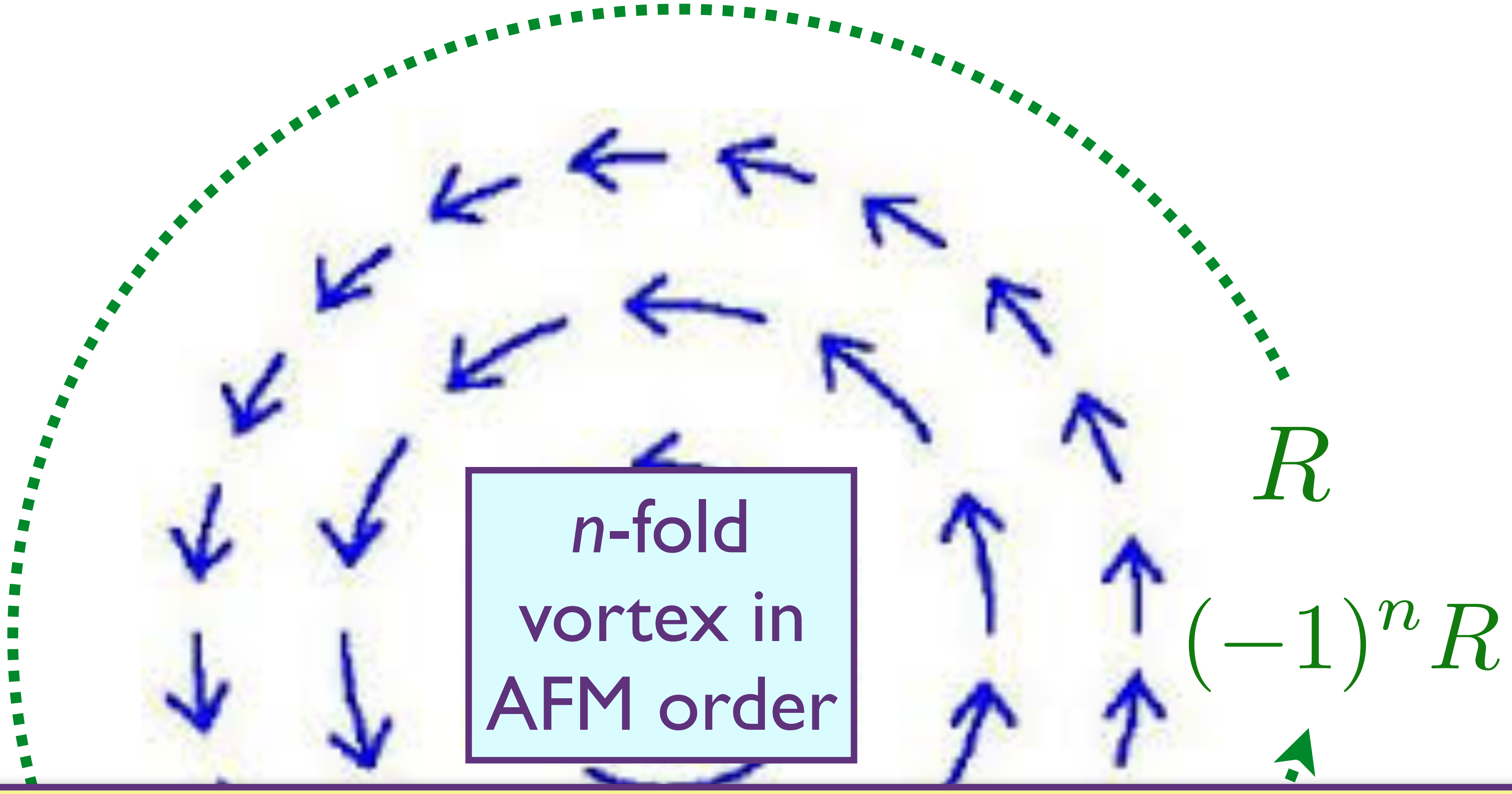
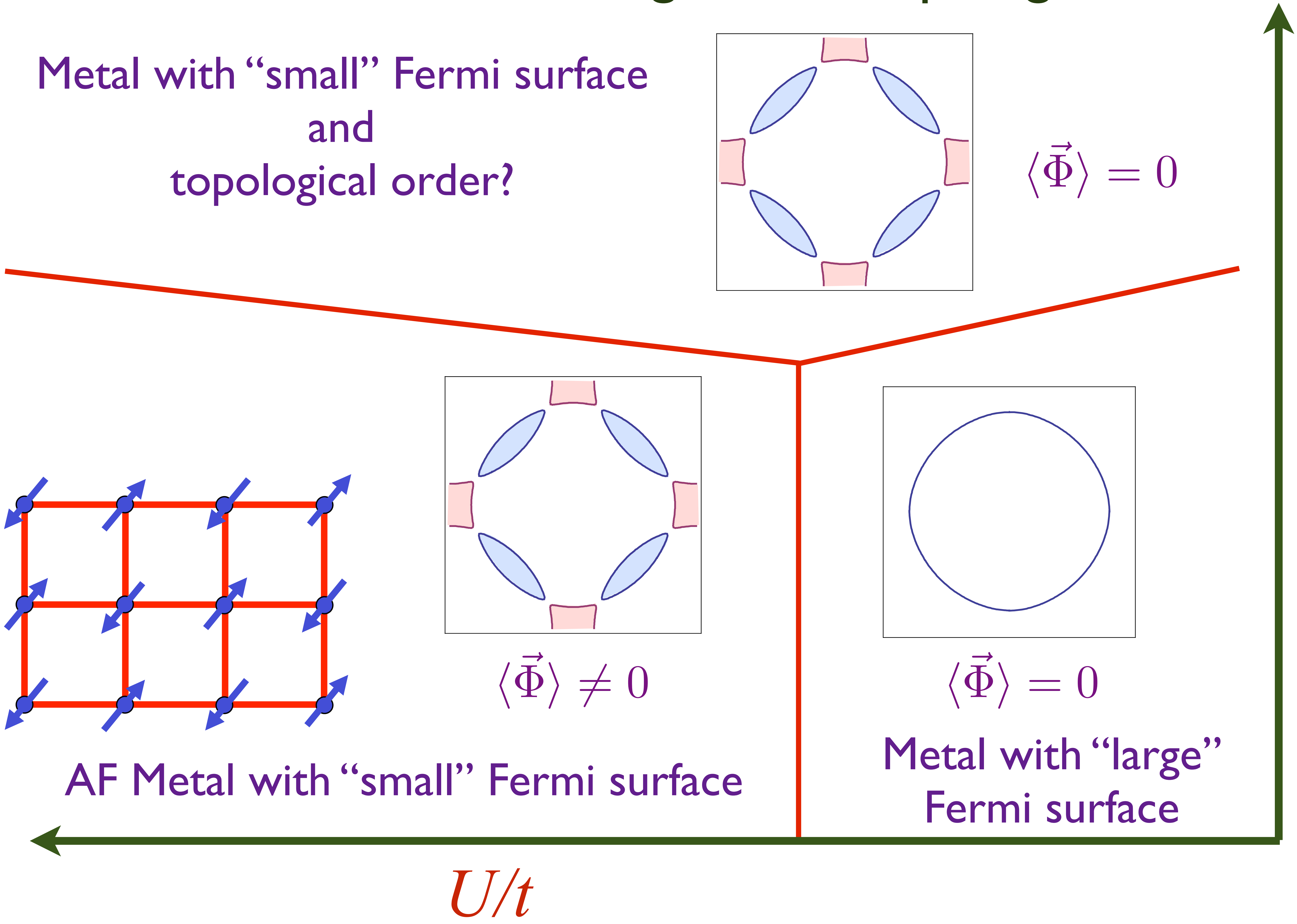
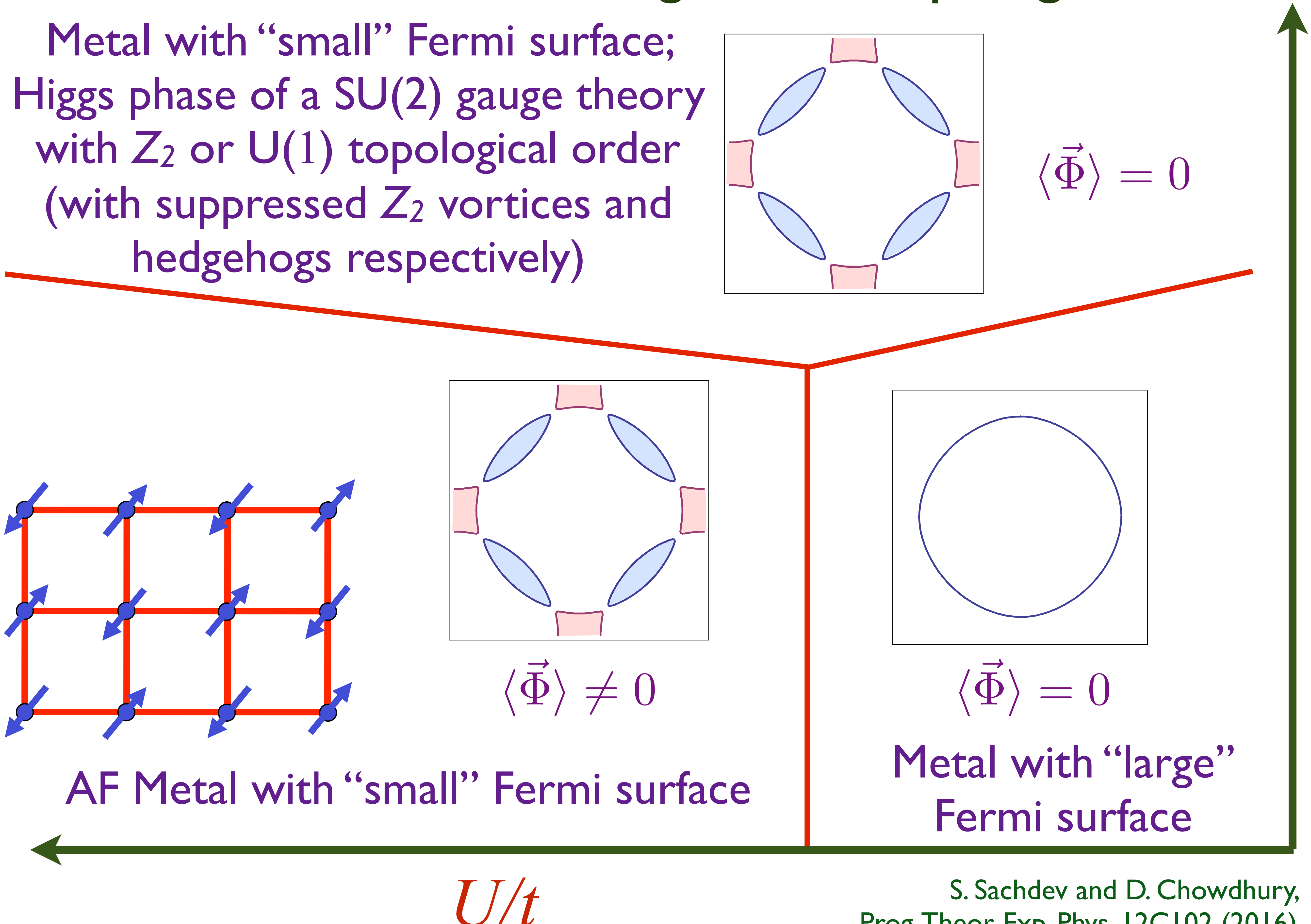


Vortices with n odd must be suppressed: such a metal with "fluctuating antiferromagnetism" has BULK $\mathbb{Z}_2$ TOPOLOGICAL ORDER and fermions which inherit the small Fermi surfaces of the antiferromagnetic metal.

Fermi surface+antiferromagnetism+topological order



Fermi surface+antiferromagnetism+topological order



S. Sachdev and D. Chowdhury,
Prog. Theor. Exp. Phys. 12C102 (2016)

Topological order in the pseudogap metal

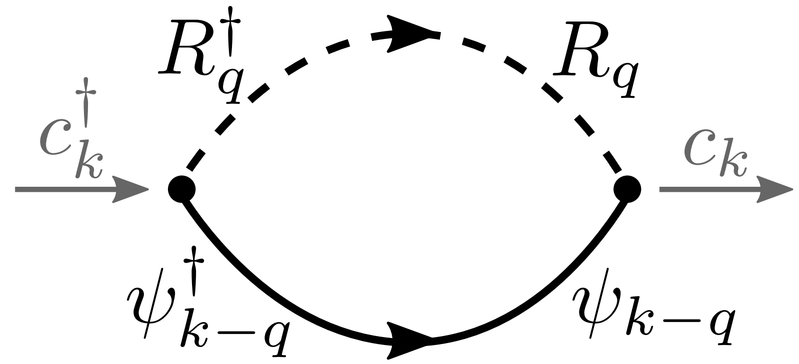
Mathias S. Scheurer,[1] Shubhayu Chatterjee,[1] Wei Wu,[2,3] Michel Ferrero,[2,3] Antoine Georges,[2,4,3,5] and Subir Sachdev[1,6,7]

We compute the electronic Green's function of the topologically ordered Higgs phase of a SU(2) gauge theory of fluctuating antiferromagnetism on the square lattice. The results are compared with cluster extensions of dynamical mean field theory, and quantum Monte Carlo calculations, on the pseudogap phase of the strongly interacting hole-doped Hubbard model. Good agreement is found in the momentum, frequency, hopping, and doping dependencies of the spectral function and electronic self-energy. We show that lines of (approximate) zeros of the zero-frequency electronic Green's function are signs of the underlying topological order of the gauge theory, and describe how these lines of zeros appear in our theory of the Hubbard model. We also derive a modified, non-perturbative version of the Luttinger theorem that holds in the Higgs phase.

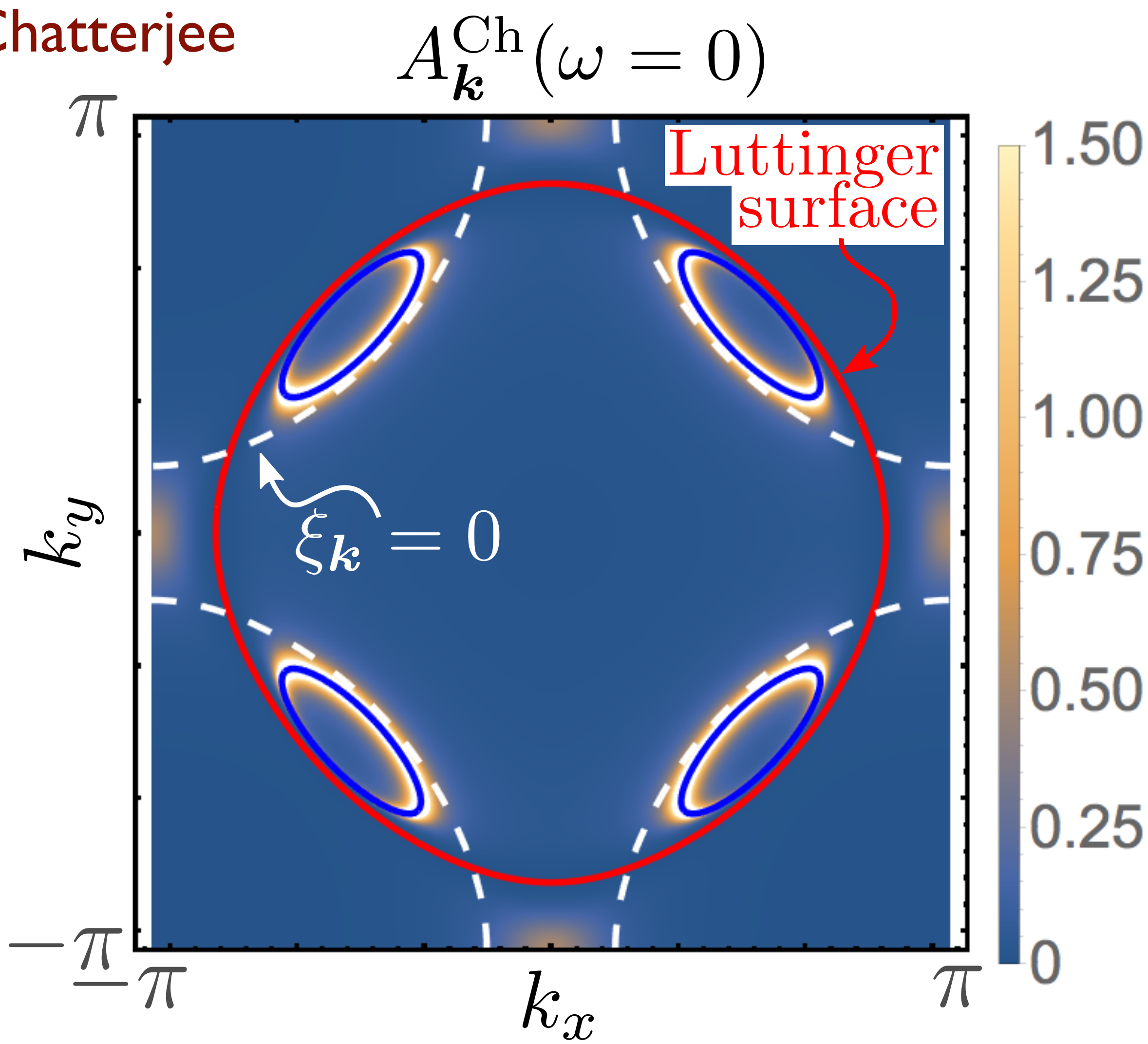
to appear.....

Shubhayu Chatterjee

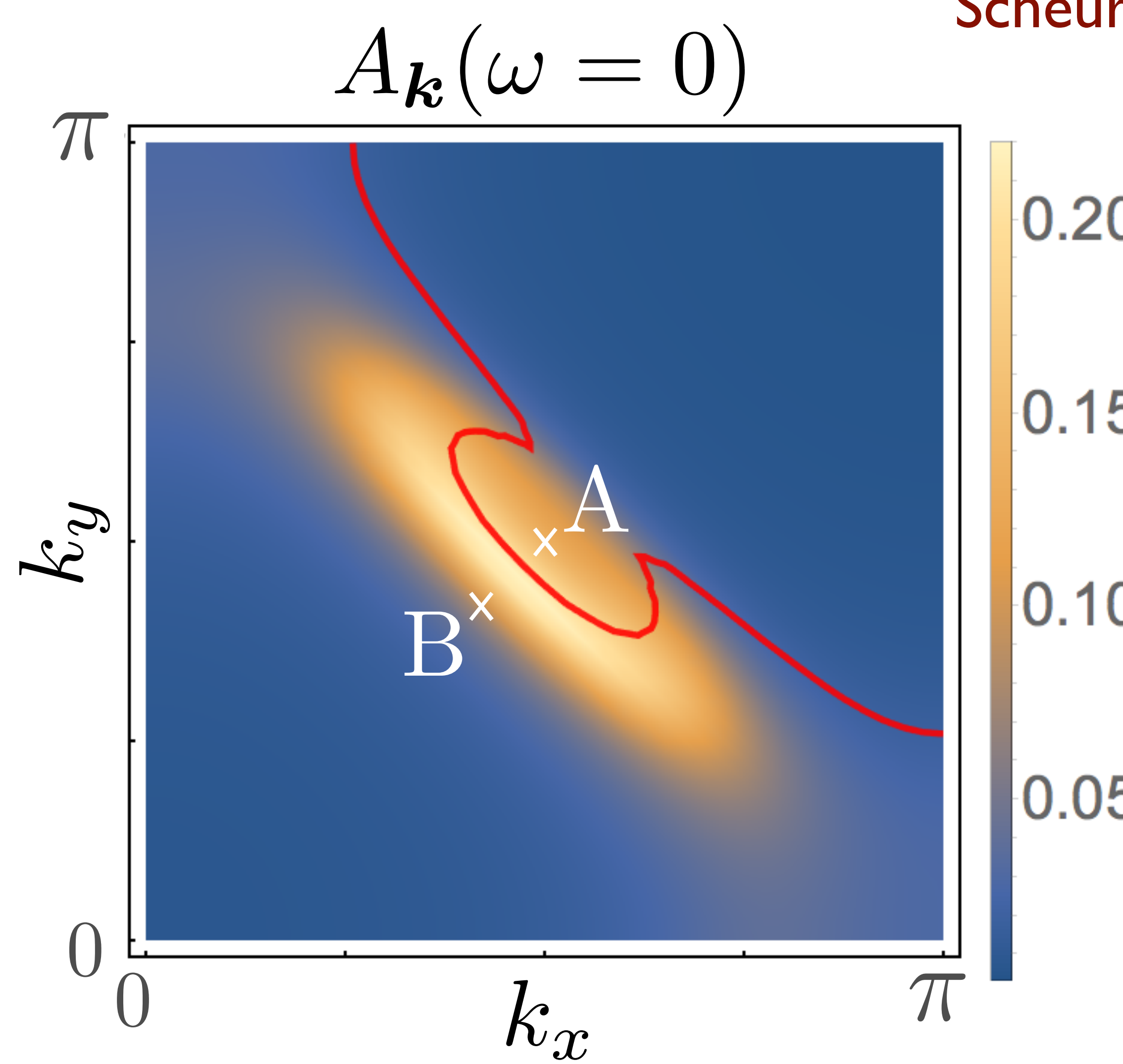
One loop computation of SU(2) gauge theory higgsed down to U(1)



Mathias Scheurer



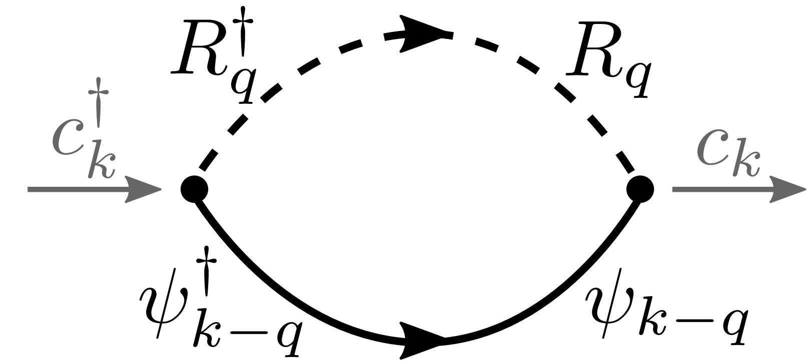
Red line indicates the locus of $G(\mathbf{k}, \omega = 0) = 0$

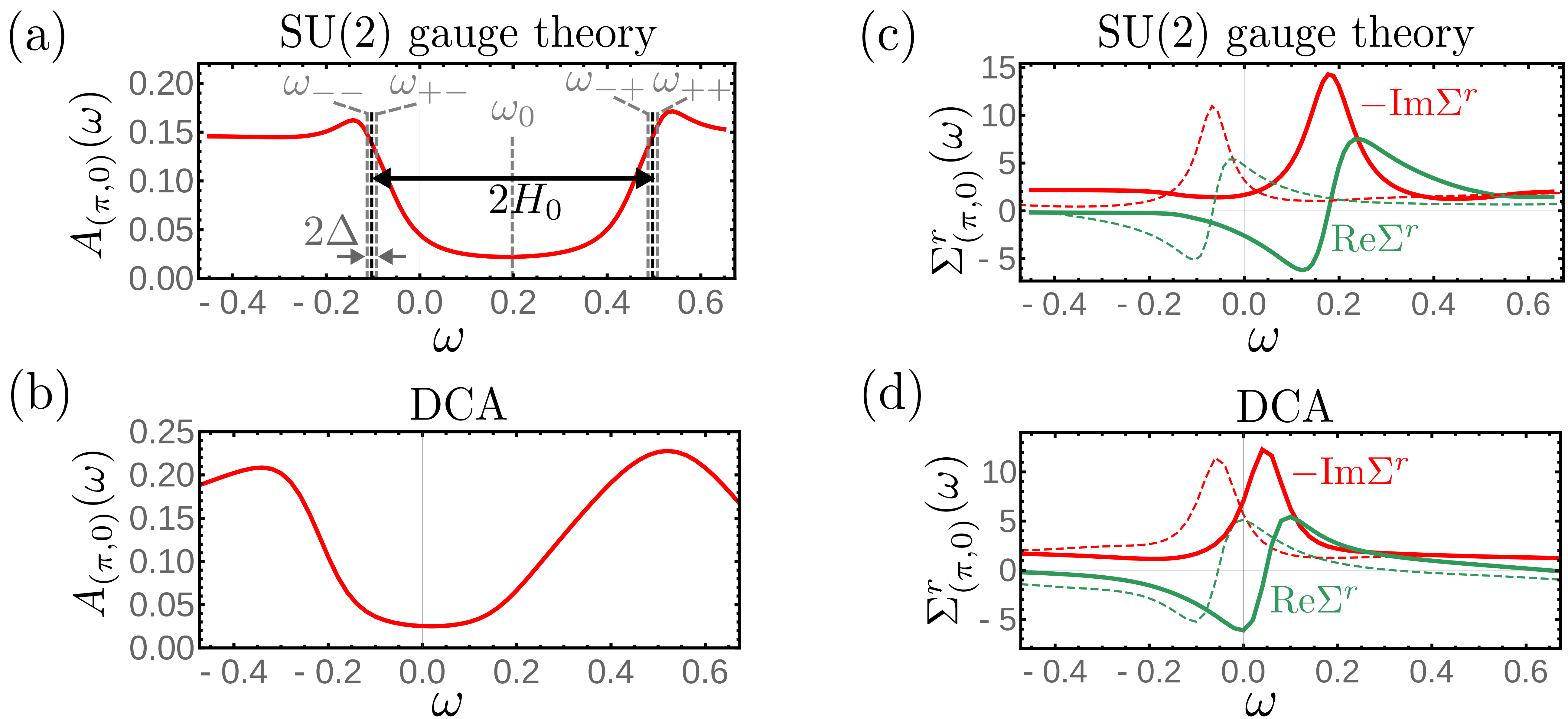


Red line indicates the locus of $\mathrm{Re}\, G(\mathbf{k}, \omega = 0) = 0$

Full Brillouin zone spectra of chargons (ψ) and electrons (c)

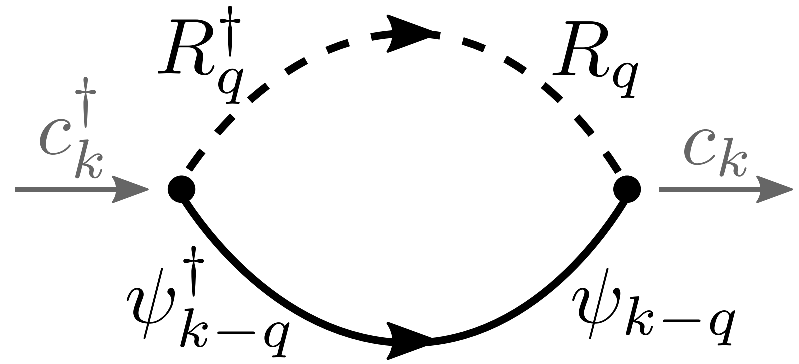
One loop computation of SU(2) gauge theory higgsed down to U(1)

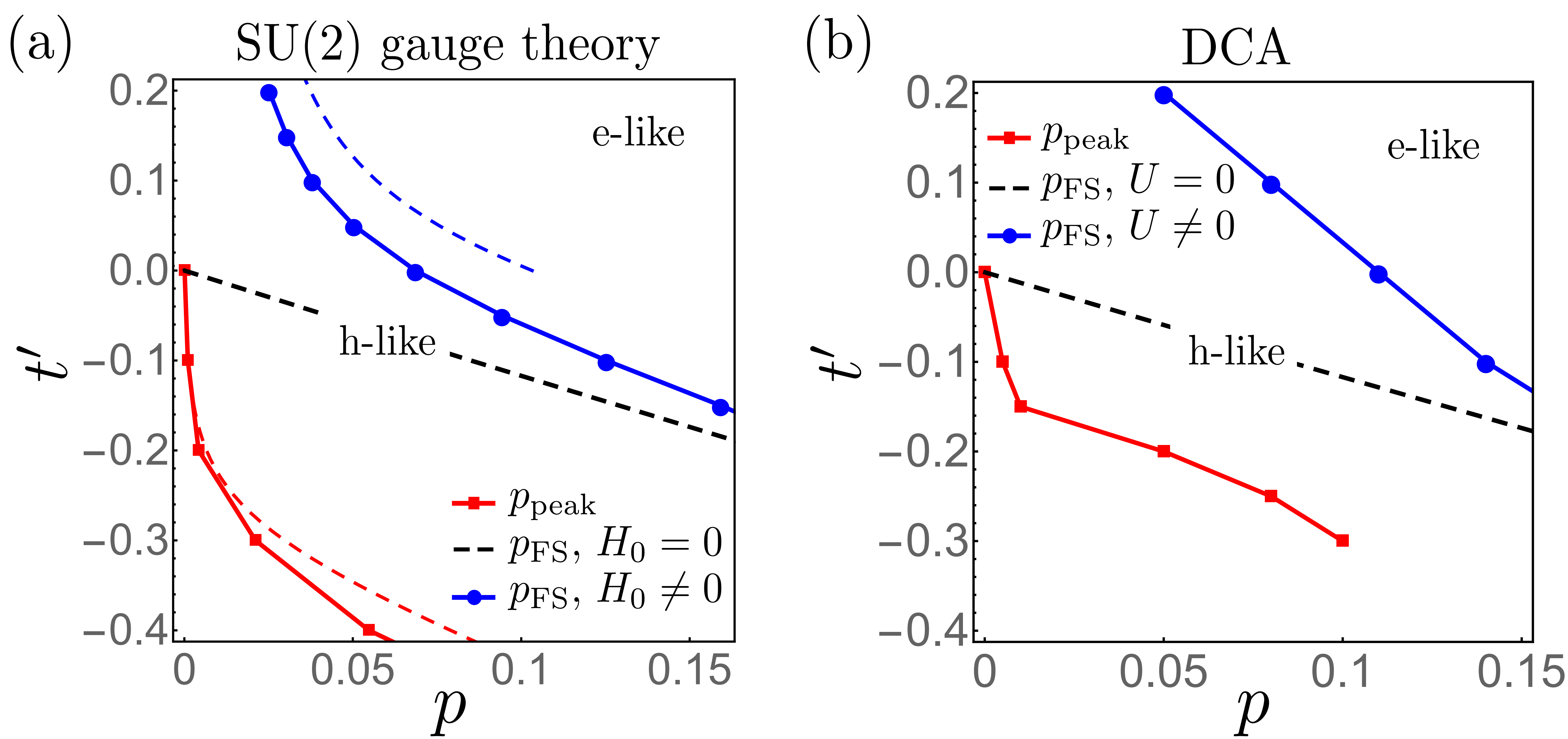




Anti-nodal spectra compared to cluster DMFT

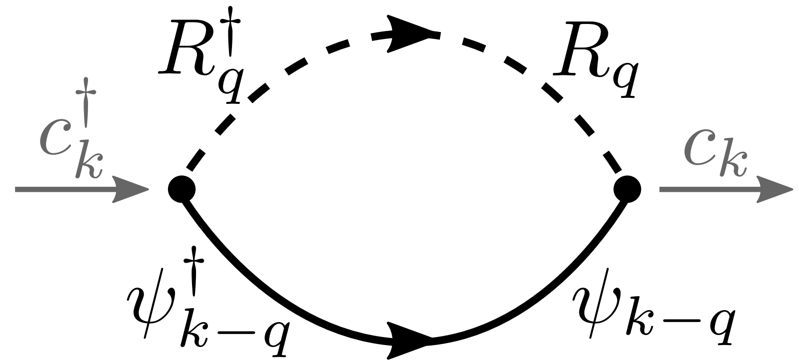
One loop computation of SU(2) gauge theory higgsed down to U(1)

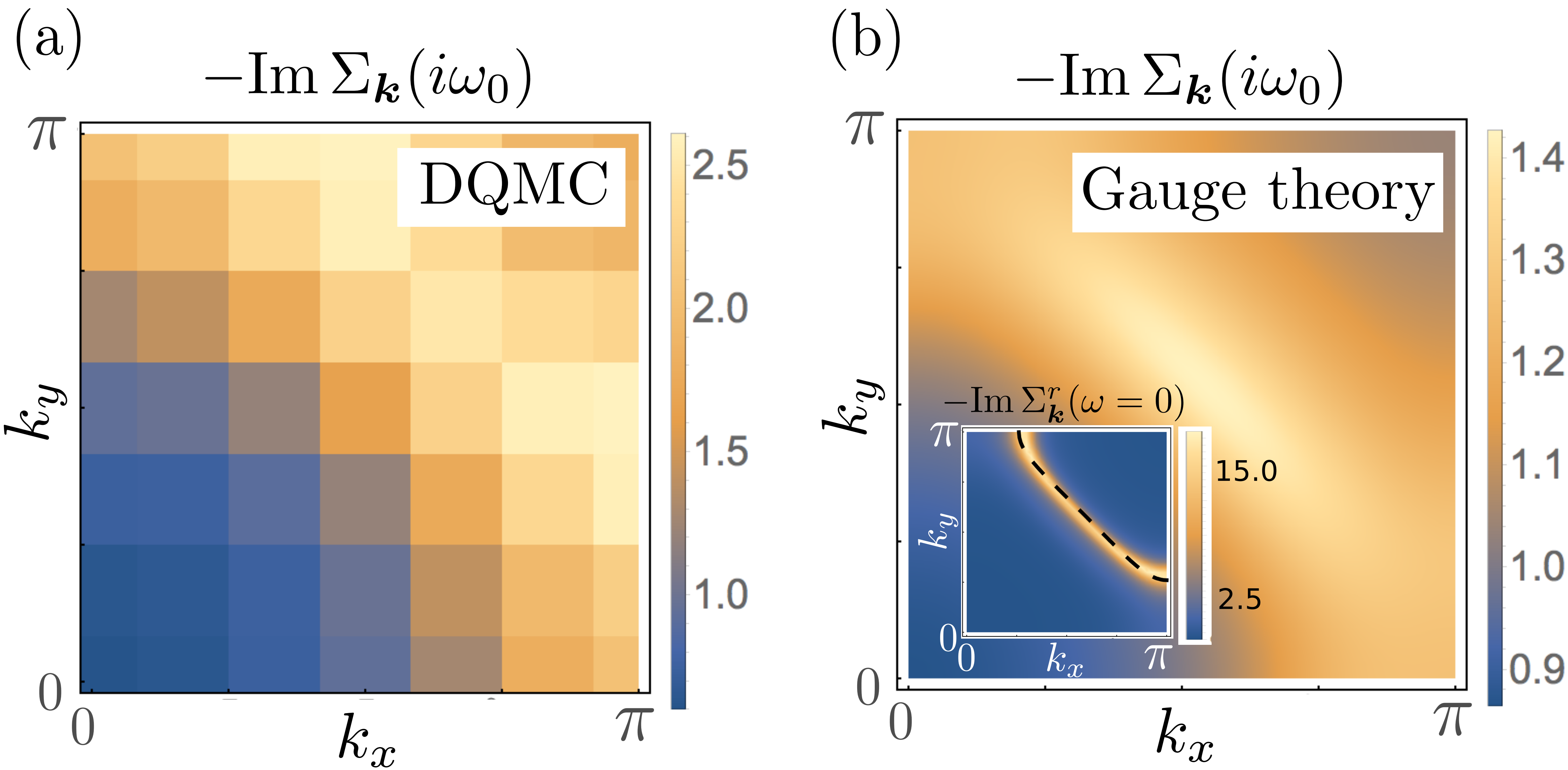




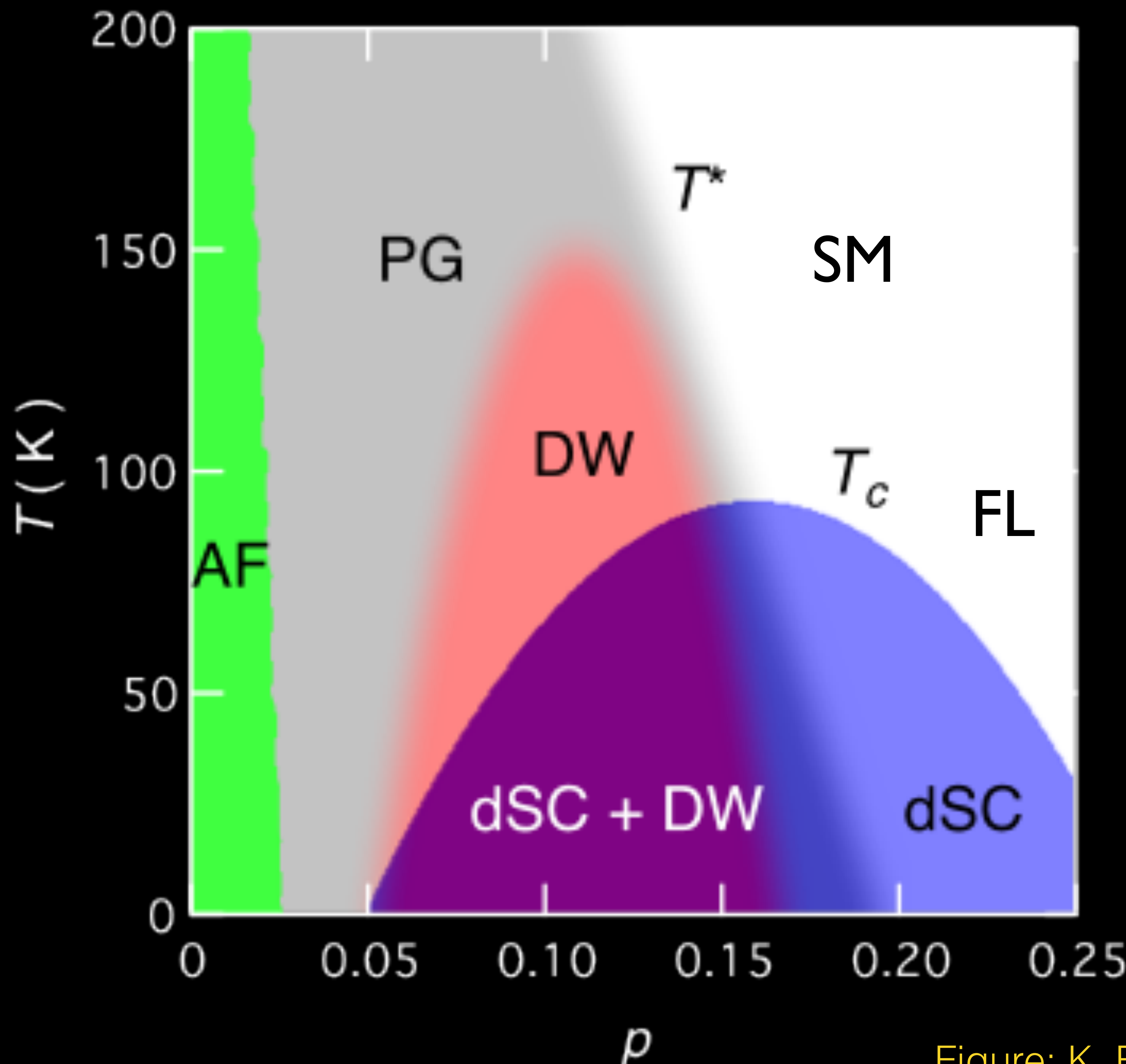
Lifshitz transition compared to cluster DMFT

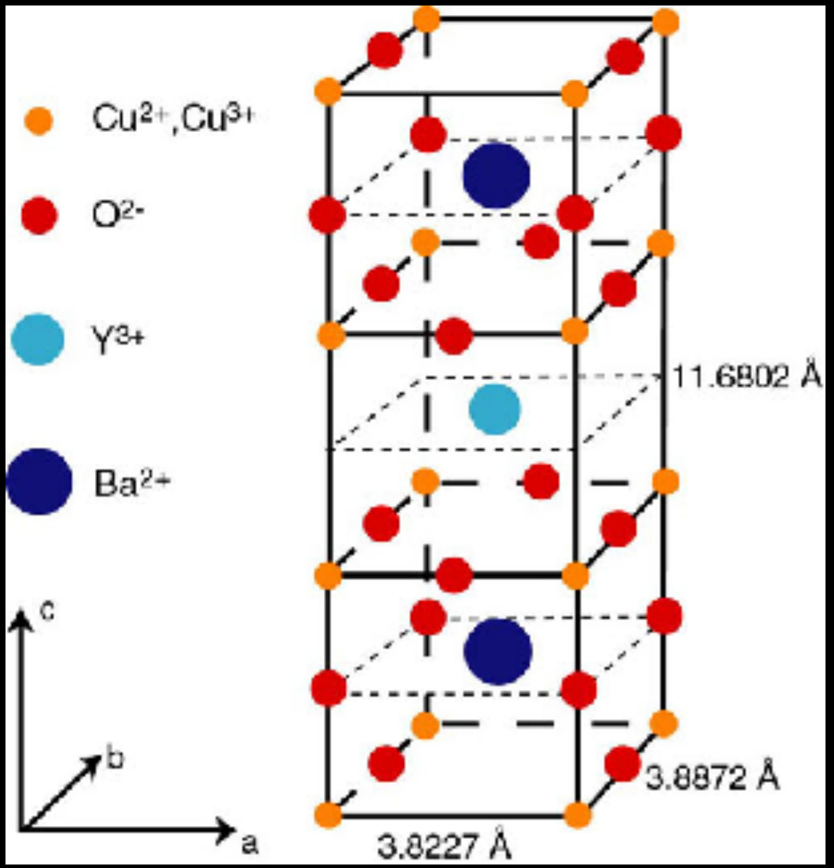
One loop computation of SU(2) gauge theory higgsed down to U(1)





Self energy compared to cluster DMFT





$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

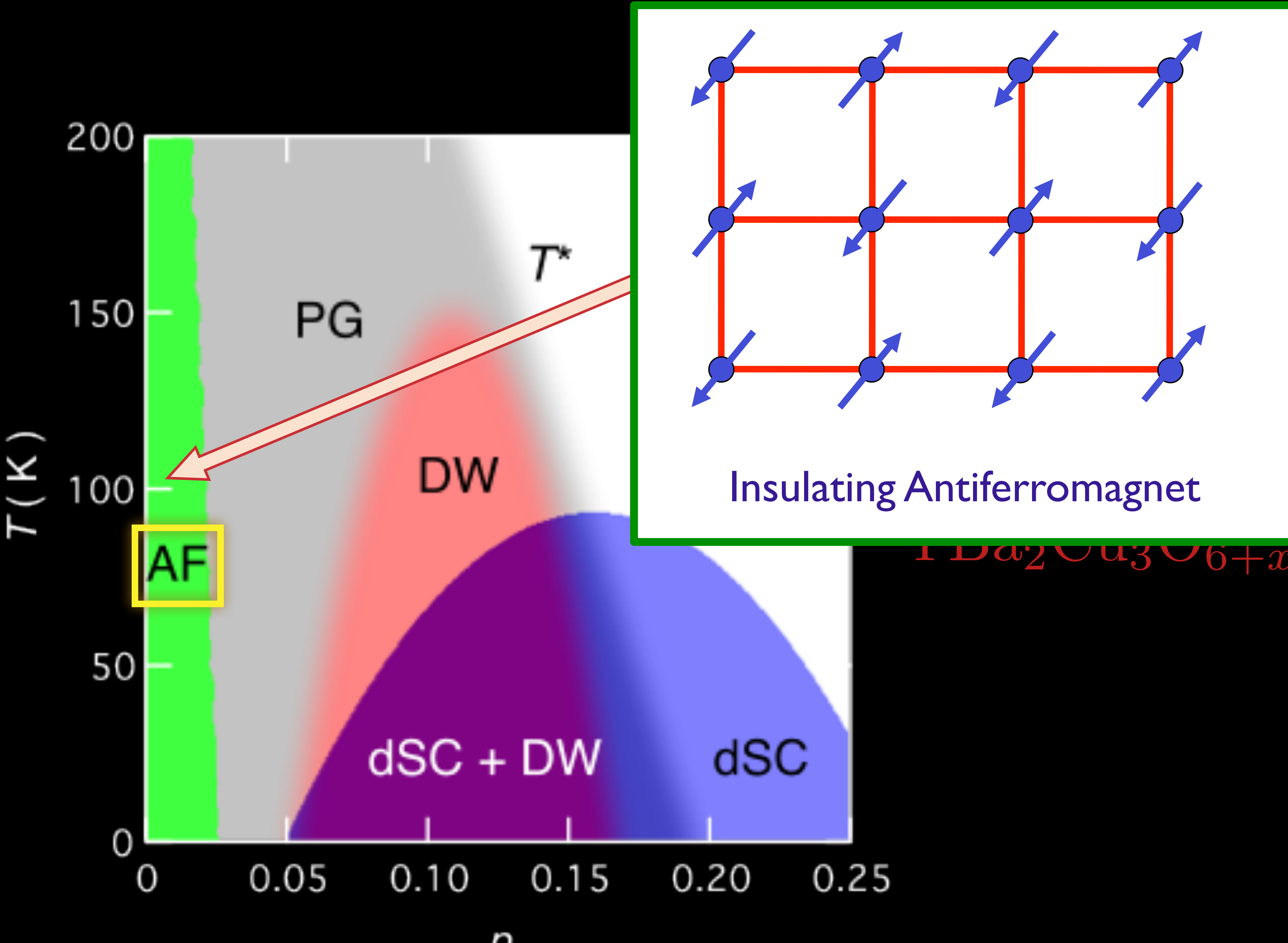
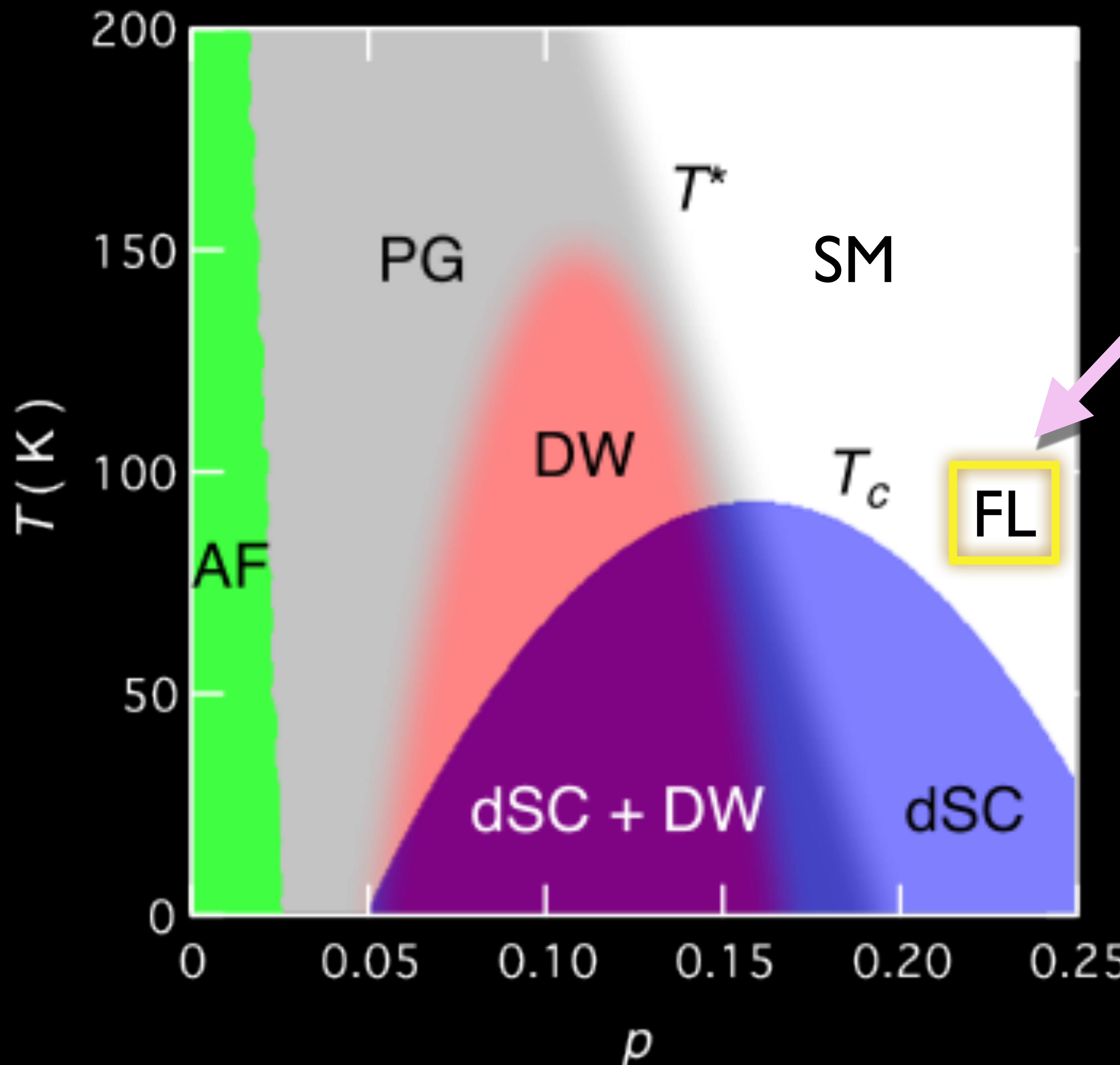
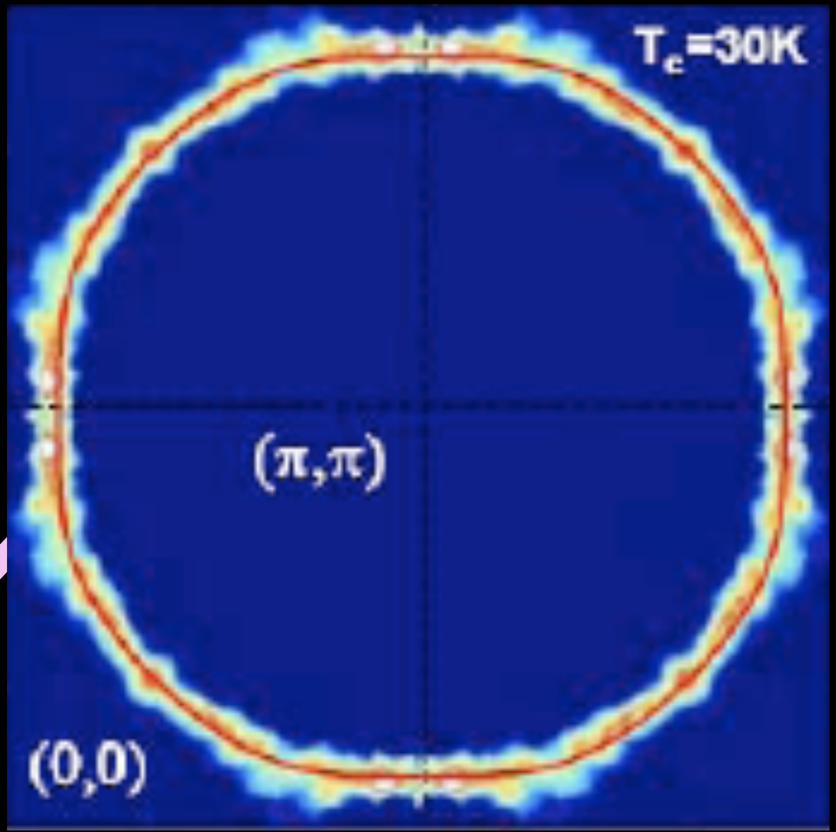


Figure: K. Fujita and J. C. Seamus Davis

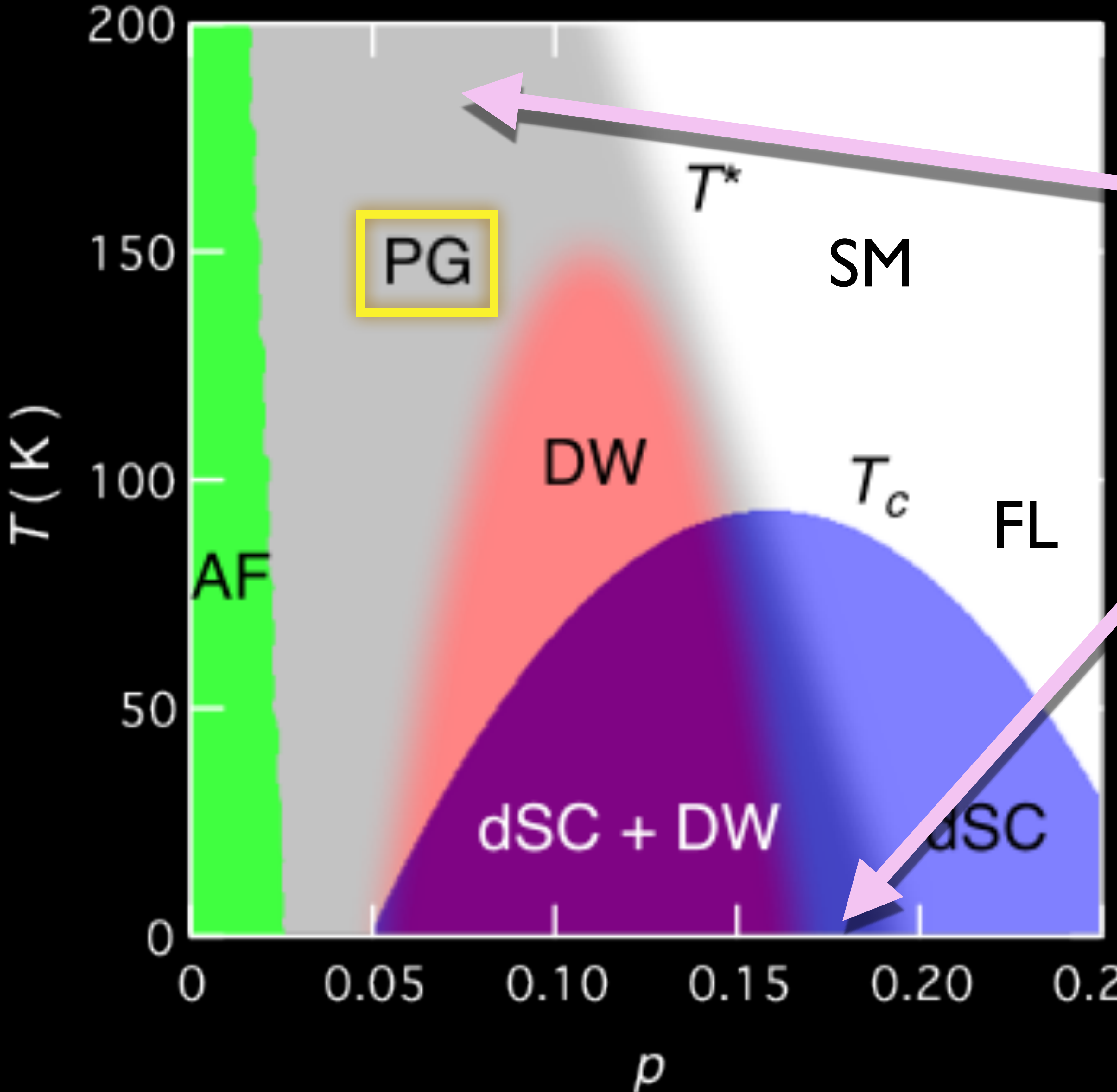
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





A conventional metal: the Fermi liquid with Fermi surface of size *1+p*

T. Senthil, M. Vojta and S. Sachdev, PRB **69**, 035111 (2004)

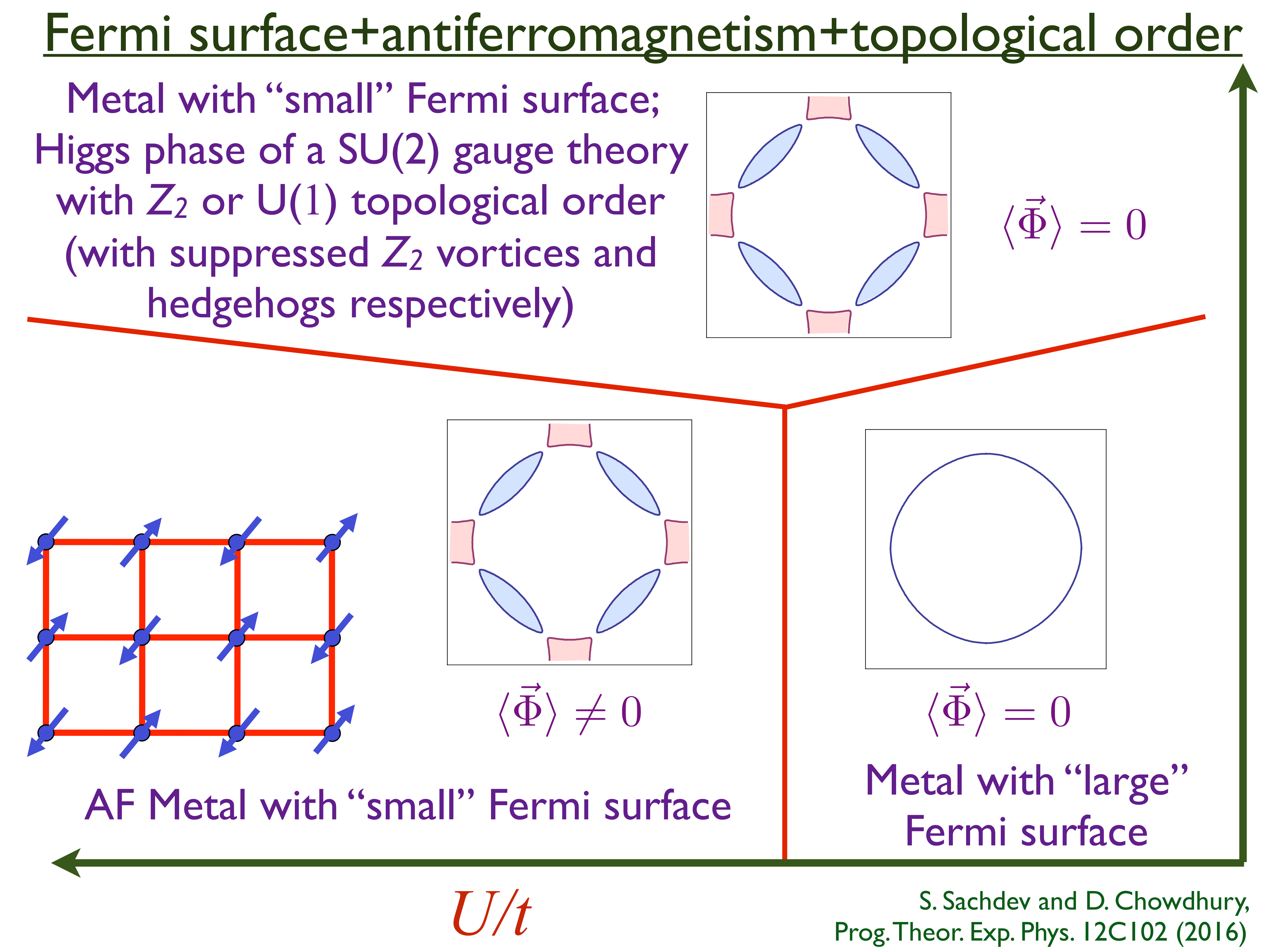


Pseudogap metal

at low p

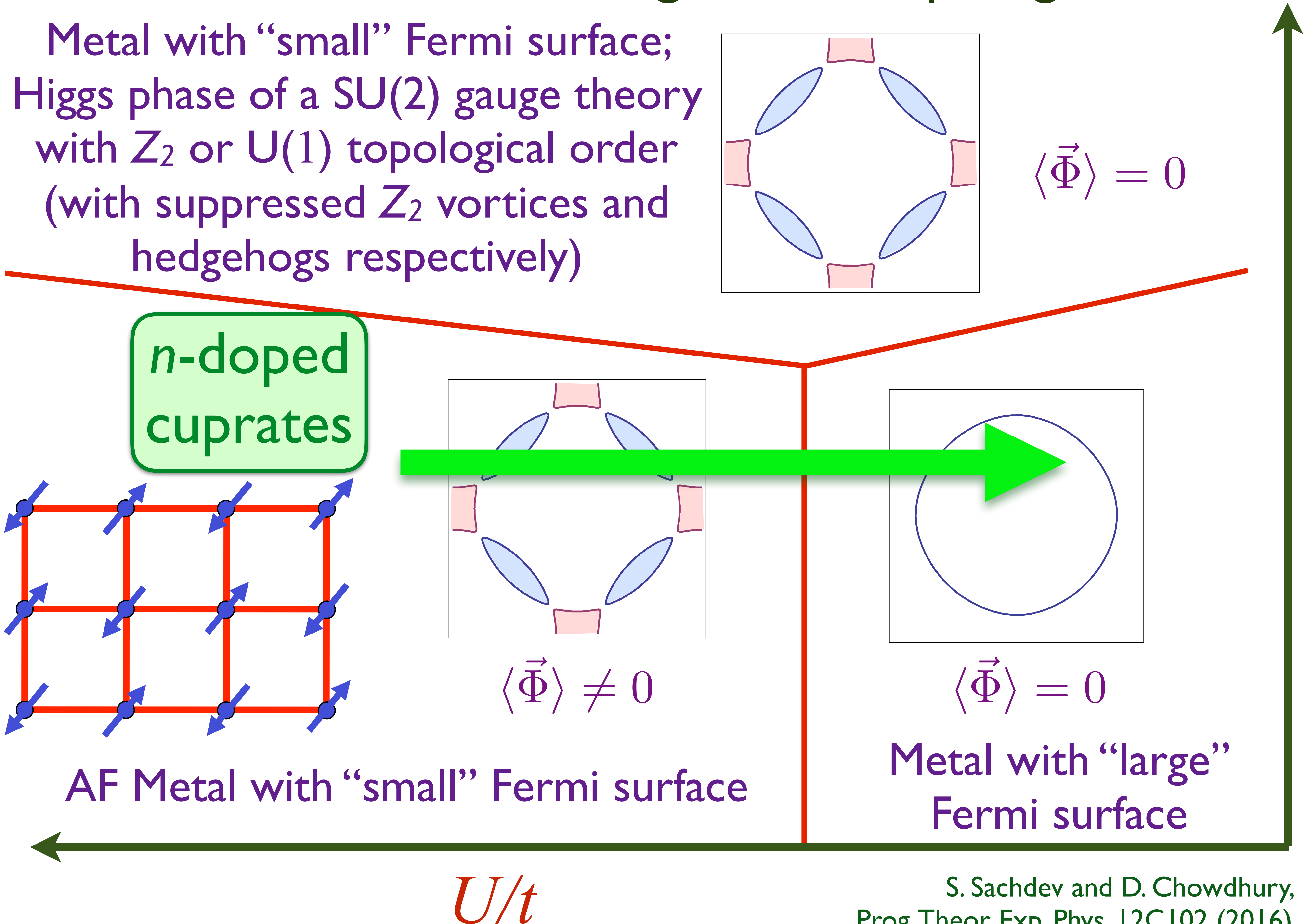
Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

If present at $T=0$, a metal with a size p Fermi surface (and translational symmetry preserved) *must* have *topological order*



Fermi surface+antiferromagnetism+topological order
Metal with "small" Fermi surface; Higgs phase of a SU(2) gauge theory with Z_2 or U(1) topological order (with suppressed Z_2 vortices and hedgehogs respectively)
$\langle \vec{\Phi} \rangle = 0$
$\langle \vec{\Phi} \rangle \neq 0$
$\langle \vec{\Phi} \rangle = 0$
AF Metal with "small" Fermi surface
Metal with "large" Fermi surface
U/t
S. Sachdev and D. Chowdhury, Prog. Theor. Exp. Phys. 12C102 (2016)

Fermi surface+antiferromagnetism+topological order



S. Sachdev and D. Chowdhury,
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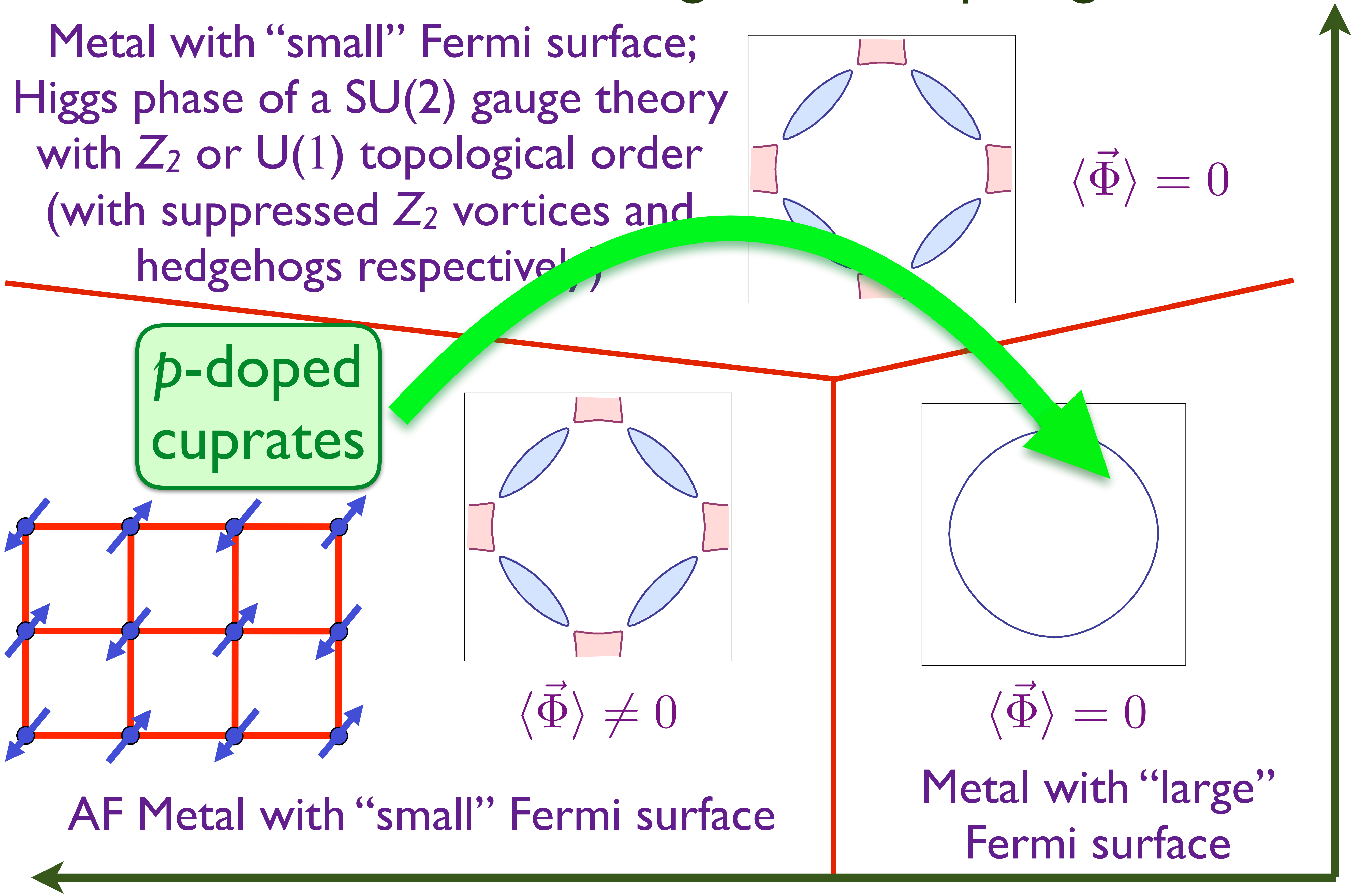
Fermi surface+antiferromagnetism+topological order



U/t

S. Sachdev and D. Chowdhury,
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- New classes of quantum states with topological order

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- Can be understood as:
  (a) defect suppression in states with fluctuating order associated with broken symmetries
  (b) Higgs phases of emergent gauge fields

- New classes of quantum states with topological order

- Can be understood as:
  (a) defect suppression in states with fluctuating order associated with broken symmetries
  (b) Higgs phases of emergent gauge fields

- A metal with bulk topological order (*i.e.* long-range quantum entanglement) can explain existing experiments in cuprates, and agrees well with cluster-DMFT