

Trading the Line Strategy

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Outline

- 1 Overview & Subfields
- 2 Trading the Line Strategy
- 3 Analysis
- 4 Trinomial Model
- 5 Conclusions



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- **Public/International/Corporate Finance.** (We won't go there at all).

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In your insurance contract, there is something called a deductible, (say K). If you suffer a damage of amount S , then the insurance will cover $(S - K)^+$ amount. That is, you will exercise this option (go to the insurance company and demand that it pay $(S - K)^+$ when it is positive. Otherwise, you will keep quiet (will not exercise your option).

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Insurance people call it a stop-loss contract and looks like the usual call option. But, it differs from the call option in a fundamental way. Call option is based on an underlying which is openly traded, whereas your insurance contract is based on your own accident loss — not a traded entity.

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Stop-Loss strategy simply involves setting a limit (a barrier) to take action as soon as the underlying process hits the barrier.

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In contrast a day-to-day trader faces a different structure. Since his trades are often of very short duration, the impact of any interest on the investment is negligible. Therefore, his main goal is to make money by employing the age old cliché, **buy low and sell high**. That is, decide when to **get in**, and when to **get out**.

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In a fully functional market knowing the future price of any traded asset or option is any one's guess (assuming there is no insider information). In such a situation the mathematical problem is to define **stopping rules** that indicate the getting in time and the getting out times.

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Our aim here is not to recommend one trading strategy over another, but rather to give a platform over which one can compare the performance of these strategies to have a better feeling of pluses and minuses of any of these strategies.

Even with this simple goal in mind, the problem is surprisingly complicated as far as mathematical analysis is concerned.

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Having said this, the talk is primarily focused on the mathematical aspects, along with some unsolved issues.

Analysis of Trading

To just gain a foot hold into this extremely complex field, one can start off by using the binominal tree, or trinomial tree or the Black-Scholes model (or more sophisticated the so called jump-diffusion processes) for the price dynamics. Then study the salient features of the trading strategies.

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Each process has its own benefits and drawbacks. Hopefully, the results obtained will not differ much. However, should the results differ, then such a trading strategy would be very sensitive to the correct modeling of the price dynamics, and hence would need to be used with extreme caution.

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Principle 1: So, as a first step, don't be ashamed to study a trading strategy under **ANY** pseudo-realistic price process.

Trading Strategies

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Without commenting on the last three, our aim is to focus on the first one. There are some obvious ways mathematics should be able to help.

Two Examples

Before you can study a strategy,

Principle 2: Make sure that a trading strategy is fully defined.

Every one has his/her favorite “strategy”. The problem is that they are often not well defined. We can only focus on the “well defined ones”.

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- **Stop-loss strategy,**
- **Trailing stop strategy,** (also known as the trading the line strategy). In this strategy one “rides” the price curve as long as it goes in the trader's favor. This strategy also sets a barrier, but the barrier is moved as long as the price curve moves in the favor of the trader. Otherwise, the position is closed once the barrier is hit. (in quality control literature it is precisely the CUSUM procedure.)

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- **Religion (rumour) based strategy**, i.e., the stopping time N may be taken to be probabilistically independent of the price-information filtration.

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- The sensitivity of the above parameters to any “tuning constants” of the trading strategy.
- The sensitivity of the above parameters to the various price dynamics.
- Comparison of the strategies for these (and other) statistics.

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For instance, when the price process follows a log-normal model, a closed form expression for the expected stopping time for the trailing stops strategy in discrete time settings is still an **open problem**.

Fortunately, most of the above cited statistics do already have the necessary mathematics well-developed (although in a different language) for both binomial and trinomial random walk models and both stop-loss and trading the line strategies. Even some other models can be handled as well.

Trading the Line Strategy

In a rather informal language, let me ask you a question.

- if a blind person is not allowed to use his stick to feel what is in front of him, how can he walk and detect if the path ahead is going to go up or go down?

Trading the Line Strategy

In a rather informal language, the trading the line strategy says that

- the blind person should use his stick behind him and try to detect if the slope in the immediate past is upward or downwards and make a decision.

Trading the Line Strategy

Let us fix the notation first. Let Y_1, Y_2, \dots , be independent and identically distributed random variables representing the returns of the security that is being traded. The random variables Y_j could be discrete or continuous. Let $S_n = Y_1 + \dots + Y_n$, and let $P_n = P_0 e^{S_n}$, $n \geq 1$, be the price.

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Trading the line strategy can be defined in several ways. For instance, by using the log of the price process, S_n , for the get out position, we may define the stopping rule

$$L_h := \inf \left\{ n \geq 1 : \max_{k \leq n} S_k - S_n \geq h \right\},$$

where $h > 0$ is the specified amount by which the log-price has to drop to trigger sellout. The corresponding trailing stops strategy to get in is

$$N_h := \inf \left\{ n \geq 1 : S_n - \min_{k \leq n} S_k \geq k \right\},$$

where $k > 0$ is the specified amount by which the log-price has to rise to trigger getting in.

Trading the Line Strategy

One can use the two trailing stop strategies in tandem. That is, use N_h to trigger getting in. After getting in, start the getting out strategy. Once the get-out-strategy triggers, get out and start the getting-in strategy. So on, the circle continues. Hopefully, at each junction one makes enough money in the process to pay off the transaction costs and still makes some profit.

The SPRT

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The main tool in their analysis is another stopping rule, called the Wald's **sequential probability ratio test**, (SPRT). It is defined as

$$\tau_{a,h} := \inf \{n \geq 1 : S_n \notin (a, h)\},$$

where $S_0 = 0$ and $a < 0 < h$. (Its one-sided version gives the stop-loss strategy.)

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It says to stop as soon as the log price hits either of the upper or lower barriers. Of course, it does not say what course of action should one take. All it indicates is that there is perhaps some sort of shift taking place in the price dynamics, which the trader should avail.

Optimal Strategy in Continuous Time

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More precisely, if the trader is not allowed to keep his positions open for more than a day, one can consider a day as one unit of time and the trader must get out by the end of the day, regardless of his position. Under this scenario, it is known that the “optimal” strategy is the following

$$A_h := \inf \left\{ t > 0 : \max_{s < t} S(s) - S(t) \geq h\sqrt{1-t} \right\}.$$

Note its resemblance with the discrete time trailing stops strategy L_h .

Correlation

Religion/Rumor Strategy: For the sake of comparison, we start off by considering a trading strategy, ν , that is **independent** of the log price process, $\{S_n, n \geq 0\}$. The following result shows how the correlation of S_ν and ν is dependent on the basic statistics of the stopping rule.

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Let Y_1, Y_2, \dots be any sequence of independent and identically distributed random variables with $E(Y_1^2) < \infty$. If ν is any stopping rule independent of the log return process, Y_1, Y_2, \dots , having positive finite variance, and $S_n = Y_1 + \dots + Y_n$, then the correlation between ν and S_ν is,

$$\text{Corr}(\nu, S_\nu) = \frac{E(Y_1)}{\sqrt{E(Y_1^2) + \text{Var}(Y_1)E(\nu)/\text{Var}(\nu)}}.$$

Next consider the adapted case.

correlation

Let Y_1, Y_2, \dots be any sequence of independent and identically distributed random variables with $0 < E(Y_1^2) < \infty$, $E(Y_1) \neq 0$ and let

$S_n = Y_1 + \dots + Y_n$. If ν is any stopping rule **adapted** to the filtration $\sigma(Y_1, Y_2, \dots, Y_n)$, $n \geq 1$, having finite positive variance, then

$$\text{Corr}(\nu, S_\nu) = \frac{\text{Var}(S_\nu) + (E(Y_1))^2 \text{Var}(\nu) - \text{Var}(Y_1)E(\nu)}{2E(Y_1)\sqrt{\text{Var}(\nu)\text{Var}(S_\nu)}}.$$

SPRT

The above two propositions show that the key statistics are $E(\nu)$, $\text{Var}(\nu)$, $\text{Var}(S_\nu)$ and the Laplace transform $E(e^{-s\nu})$ for any trading strategy ν . In our case the trading strategy is N_h , which is adapted to the filtration $\mathcal{F}_n = \sigma(Y_1, Y_2, \dots, Y_n)$, $n \geq 1$.

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Once again, the key statistics that are needed are the average amount of time the position will remain open, $E(N_h)$, the variance of the time the position will remain open, $\text{Var}(N_h)$, $\text{Var}(S_{N_h})$, and the Laplace transform $E(e^{-sN_h})$, for appropriate values of s . In order to study the properties of the short position trailing stops, N_h , and the corresponding stopped log price, S_{N_h} , and the time discounted gain/loss $P_0 - e^{-rN_h}P_{N_h}$, the SPRT,

$$\tau_{a,h} := \inf \{n \geq 1 : S_n \notin (a, h)\},$$

plays an important role, where $a < 0 < h$.

Identity

The stopping rules N_h and $\tau_{0,h}$ are related through the following identity.

$$N_h = \tau_{0,h} + N_{h,\tau_{0,h}} I(S_{\tau_{0,h}} \leq 0),$$

where $N_{h,\tau_{0,h}}$ is an identical copy of N_h which is independent of $S_{\tau_{0,h}}$ when $\tau_{0,h}$ is given. Here and elsewhere $I(A)$ is the indicator random variable taking value one when A occurs and zero otherwise.

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With the help of this link we easily deduce the Laplace transform of N_h in terms of the Laplace transform of the SPRT. Therefore the statistics of N_h are, in turn, linked to the statistics of the SPRT.

Laplace Transform

For any $s > 0$, we have

$$E(e^{-sN_h}) = \frac{E(e^{-s\tau_{0,h}}) - E(e^{-s\tau_{0,h}} | (S_{\tau_{0,h}} \leq 0))}{1 - E(e^{-s\tau_{0,h}} | (S_{\tau_{0,h}} \leq 0))}.$$

The above Laplace transform of N_h and its relationship with $\tau_{0,h}$ has been known in the quality control literature for some time. The main stumbling block has been the calculations of the expressions on the right hand side.

Assumptions

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- (A1). The moment generating function of log returns, $\phi(\theta) = E(e^{\theta Y_i})$, exists.

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- (A1). The moment generating function of log returns, $\phi(\theta) = E(e^{\theta Y_i})$, exists.
- (A2). For each small $s > 0$, there exist two values of θ , say $\theta_1(s), \theta_2(s)$ so that $\phi(\theta) = e^s$ holds for $\theta = \theta_1(s), \theta_2(s)$.

Assumptions

Consider a random walk, whose iid components, Y_i , obey the following assumptions:

- (A1). The moment generating function of log returns, $\phi(\theta) = E(e^{\theta Y_i})$, exists.
- (A2). For each small $s > 0$, there exist two values of θ , say $\theta_1(s), \theta_2(s)$ so that $\phi(\theta) = e^s$ holds for $\theta = \theta_1(s), \theta_2(s)$.
- (A3). There exist functions $K(\theta), k(x), R(\theta), g(x)$, so that for $\theta = 0, \theta_1, \theta_2$,

$$H(x) := E(e^{\theta Y_1} I(Y_1 \leq x)) = K(\theta)k(x)e^{\theta x}, \quad x < 0,$$

$$G(x) := E(e^{\theta Y_1} I(Y_1 \geq x)) = R(\theta)g(x)e^{\theta x}, \quad x > 0.$$

Here, in the notation $H(x), G(x)$, we suppress the dependence on θ and s .

Assumptions

As examples, the binomial, trinomial, two-sided geometric and double exponential models, for the log-returns, obey the above sets of conditions.

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Under the above mentioned assumptions, the following two theorems are the main results that give the probabilistic properties of the trailing stops strategy for the short position.

Analogous results can be obtained, by replacing Y_i by $-Y_i$, for the trailing stops strategy in the long position, we therefore omit the straight forward details.

Results

(Nondegenerate case) Under the above mentioned assumptions, when $K(\theta)$, $R(\theta)$ are not constant functions (non-degenerate case), the short position trailing stops strategy, $N = N_h$, has the following Laplace transform, mean and variance.

$$E(e^{-sN}) = \frac{R(0)\{K(\theta_1) - K(\theta_2)\}}{\{K(\theta_1) - K(0)\}R(\theta_2)e^{\theta_2 h} - \{K(\theta_2) - K(0)\}R(\theta_1)e^{\theta_1 h}}, \quad s > 0,$$

$$E(N) = \frac{1}{E(Y)} \left\{ h + \frac{R'(0)}{R(0)} - \frac{K'(0)\{R(0) - R(\theta^*)e^{\theta^* h}\}}{R(0)(K(0) - K(\theta^*))} \right\}, \quad E(Y) \neq 0,$$

where $\theta_1(s) \rightarrow 0$ and $\theta_2(s) \rightarrow \theta^*$ as $s \rightarrow 0$.

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where $\theta_1(s) \rightarrow 0$ and $\theta_2(s) \rightarrow \theta^*$ as $s \rightarrow 0$. When $E(Y) = 0$, we get

$$E(N) = \frac{1}{\text{Var}(Y)} \left\{ h^2 + h \frac{2R'(0)K'(0) - R(0)K''(0)}{R(0)K'(0)} + \frac{K'(0)R''(0) - R'(0)K''(0)}{R(0)K'(0)} \right\}.$$

Furthermore, when $\theta'_1(s) + \theta'_2(s) = 0$ in a neighborhood of zero, and $E(Y) \neq 0$, the variance simplifies to

$$\text{Var}(N) = \frac{1}{(E(Y))^2} \left\{ \text{Var}(Y)E(N) + 2(E(Y)E(N) - h) \frac{K'(0) + K'(\theta^*)}{K(0) - K(\theta^*)} \right. \\ \left. + (E(Y)E(N) - h)^2 + \frac{K''(0)\{R(0) - R(\theta^*)e^{\theta^* h}\}}{R(0)(K(0) - K(\theta^*))} - \frac{R''(0)}{R(0)} \right. \\ \left. - 2 \frac{K'(\theta^*)R'(0) - K'(0)R'(\theta^*)e^{\theta^* h}}{R(0)(K(0) - K(\theta^*))} + 4h \frac{K'(0)R(\theta^*)e^{\theta^* h}}{R(0)(K(0) - K(\theta^*))} \right\}.$$

Results

(Degenerate case) When Y is an integer valued random variable and $K(\theta), R(\theta)$ are constants, the trailing stops strategy for the short position, $N = N_h$, has the following Laplace transform, mean and variance

$$E(e^{-sN}) = \frac{e^{\theta_2} - e^{\theta_1}}{(1 - e^{\theta_1})e^{\theta_2(h+1)} - (1 - e^{\theta_2})e^{\theta_1(h+1)}}, \quad s > 0,$$

$$E(N) = \frac{1}{E(Y)} \left\{ h + \frac{1 - e^{\theta^* h}}{1 - e^{-\theta^*}} \right\}, \quad E(Y) \neq 0,$$

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Furthermore, when $\theta_1'(s) + \theta_2'(s) = 0$ in a neighborhood of zero, where $\theta_1(s) \rightarrow 0$ and $\theta_2(s) \rightarrow \theta^*$ as $s \rightarrow 0$, and $E(Y) \neq 0$, the variance simplifies to

$$\text{Var}(N) = \frac{\text{Var}(Y)}{(E(Y))^3} \left\{ h + \frac{1 - e^{\theta^* h}}{1 - e^{-\theta^*}} \right\} + \frac{\{e^{\theta^*(h+1)} + 3\} \{e^{\theta^* h} - 1\}}{(E(Y))^2 (1 - e^{-\theta^*})(e^{\theta^*} - 1)} + \frac{4he^{\theta^*(h+1)}}{(E(Y))^2 (1 - e^{\theta^*})}.$$

Trinomial model

Recall the Cox, Ross, Rubinstein (CRR) model of option pricing. In this model, the log returns, Y_i , can take two possible values, say -1 , 1 for simplicity, with respective probabilities q , p . In the trinomial case, we allow 0 as another possibility with probability $1 - p - q$.

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Assume that the price process has some drift, $E(Y_1) = p - q \neq 0$. One can give analytic expressions for the expected open position, $E(N_h)$, the expected stopped log-price, $E(S_{N_h})$, and the variance of the stopped log-price, $Var(S_{N_h})$. With some effort, one can also compute the correlation between the duration of the position and the log-price at the stopping time, $Corr(N_h, S_{N_h})$.

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It turns out that the correlation between N_h and S_{N_h} is negative when $E(Y_1) = 0$, indicating that the longer N_h remains open, the lower the log price is going to be at the stopping time.

Simulation

The glimpse of the last results indicates some of the type of mathematics one can do. The problem is that sooner or later mathematics will become intractable. In such situations simulation (Monte Carlo) methods become the last resort, which we now explain. For instance, consider the following results from a trinomial model.

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When $h = 1$,

Trinomial Model	$E(S_{N_h})$	$Var(S_{N_h})$	$E(N_h)$	$Var(N_h)$	Sample Size
$p = 0.06, q = 0.56$	-8.33	96.44	16.67	261.11	
Simulation	-8.17	89.48	16.36	241.69	1222
Error %	-1.93	-7.22	-1.84	-7.44	

When $h = 2$

$p = 0.12, q = 0.42$	-13.75	263.81	45.83	1916	
Simulation	-13.52	266.01	44.90	1930.9	1336
Error %	-1.69	0.83	-2.03	0.78	

Simulation

The amount of expected gain/loss, when $P_0 = 1$, is presented in the following table.

$$p = 0.06, q = 0.10, h = 1$$

r	$1 - E(e^{-rN_h + S_{N_h}})$	Std.Err	Sample Size
0.05	0.0855	0.03	1168
0.04	0.0193	0.03	1188
0.03	-0.0358	0.03	1171
0.02	-0.1419	0.03	1219
0.01	-0.2549	0.03	1242

Simulation

Note that $E(Y_1) = p - q = -0.04$, which might suggest that a short position might be profitable. However, note that $E(e^{Y_1}) = 1.0399$ and $E(e^{Y_1}) > e^r$ if and only if $r < 0.0391$. Hence, it seems as if the short position trailing stops strategy requires that $E(e^{Y_1}) < e^r$ to remain profitable, and not just that $E(Y_1) < 0$. Needless to say that the corresponding conclusion goes for the long position of the trailing stops strategy, as well.

Binomial model

To compute the correlation, $\text{Corr}(N_h, S_{N_h})$, we use the results of the last section.

For a given $s > 0$, we may solve $\phi(\theta) = e^s$, and get two solutions for θ .

$$e^{\theta_1} = \frac{e^s - \sqrt{e^{2s} - 4p(1-p)}}{2p}, \quad e^{\theta_2} = \frac{e^s + \sqrt{e^{2s} - 4p(1-p)}}{2p}.$$

Note that $\theta'_1 + \theta'_2 = 0$. When $p < \frac{1}{2}$, we see that $\theta_1 \rightarrow 0$ and $\theta_2 \rightarrow \theta^* = \ln(q/p)$.

Binomial model

The Laplace transform

$$E(e^{-sN_h}) = \frac{e^{\theta_2} - e^{\theta_1}}{(1 - e^{\theta_1})e^{\theta_2(h+1)} - (1 - e^{\theta_2})e^{\theta_1(h+1)}}, \quad s > 0.$$

The Laplace transform completely characterizes the probability distribution of the duration N_h . In principle the probability distribution can be obtained by the standard inversion theory of Laplace transforms. The moments of the trading strategy can be obtained via differentiation. For instance,

$$E(N_h) = - \left. \frac{d}{ds} E(e^{-sN_h}) \right|_{s=0}.$$

Binomial model

Higher moments are obtained similarly by taking higher order derivatives of the Laplace transform and then inserting $s = 0$. After omitting the simple, but a substantial amount of algebra, the following moments of the trading strategy can be deduced from the above stated Laplace transform.

$$E(N_h) = \frac{h}{p - q} + \frac{q}{(p - q)^2} \{(q/p)^h - 1\}, \quad p \neq \frac{1}{2},$$

$$E(N_h) = h(h + 1), \quad p = \frac{1}{2},$$

$$\begin{aligned} \text{Var}(N_h) = & \frac{4p(1 - p)}{(2p - 1)^3} \left\{ h + \frac{1 - ((1 - p)/p)^h}{1 - (p/(1 - p))} \right\} + \frac{4ph((1 - p)/p)^{h+1}}{(2p - 1)^3} \\ & + \frac{p(1 - p)\{((1 - p)/p)^{h+1} + 3\}\{((1 - p)/p)^h - 1\}}{(2p - 1)^4}, \quad p < \frac{1}{2}, \end{aligned}$$

$$\text{Var}(N_h) = \frac{h(h + 1)}{3} (2h^2 + 2h - 1), \quad p \rightarrow \frac{1}{2}.$$

Binomial model

By Wald's equation, we see that

$$E(S_{N_h}) = E(Y_1)E(N_h) = h - \frac{1-p}{2p-1} \{1 - ((1-p)/p)^h\}, \quad p \neq \frac{1}{2},$$

$$E(S_{N_h}) = 0, \quad p = \frac{1}{2},$$

$$\text{Var}(S_{N_h}) = \frac{1-p}{2p-1} \left\{ 1 - ((1-p)/p)^h + (1-p) \frac{(((1-p)/p)^h - 1)^2}{2p-1} \right\}, \quad p \neq \frac{1}{2},$$

$$\text{Var}(S_{N_h}) = h(h+1) = E(N_h), \quad p = \frac{1}{2},$$

$$\text{Corr}(N_h, S_{N_h}) = \frac{\text{Var}(S_{N_h}) - E(N_h)\text{Var}(Y_1) + (E(Y_1))^2\text{Var}(N_h)}{2E(Y_1)\sqrt{\text{Var}(N_h)\text{Var}(S_{N_h})}},$$

$$\text{Corr}(N_h, S_{N_h}) = \frac{-(2h+1)}{\sqrt{3}\sqrt{2h(h+1)-1}}, \quad p \rightarrow \frac{1}{2}.$$

The last expression shows that the correlation between N_h and S_{N_h} is negative when $E(Y_1) = 0$, indicating that the longer the short position remains open, the lower the log price is going to be at the stopping time. In a short position the lower S_{N_h} is the less the loss will be.

Corr Sign ($\mu < 0$)

- The sign of the correlation between S_ν and ν is always controlled by the factor

$$\frac{\text{Var}(S_\nu) + (E(Y_1))^2 \text{Var}(\nu) - \text{Var}(Y_1)E(\nu)}{E(Y_1)}$$

for any trading strategy that is adapted to the log returns filtration, $\sigma(Y_1, \dots, Y_n)$, $n \geq 1$. As shown by the binomial and trinomial models, when $E(Y_1) < 0$, this amount is negative and rather close to negative one for the trading the line strategy, $\nu = N_h$. The same conclusions seem to hold for all the other models.

Monotonicity ($\mu < 0$)

- $E(S_{N_h})$ is a decreasing function of h , when $E(Y_1) < 0$. Indeed, consider $\nu_2 = N_{h_2}$, and $\nu_1 = N_{h_1}$, when $h_2 > h_1$. Now, $S_n - nE(Y_1)$, being a martingale,

$$\begin{aligned} E(S_{\nu_2} - S_{\nu_1} | \mathcal{F}_{\nu_1}) &= E(S_{\nu_2} - S_{\nu_1} - (\nu_2 - \nu_1)E(Y_1) | \mathcal{F}_{\nu_1}) \\ &\quad + E(\nu_2 - \nu_1 | \mathcal{F}_{\nu_1}) E(Y_1) \\ &\leq 0, \end{aligned} \tag{1}$$

where \mathcal{F}_{ν_1} is the information up to time N_{h_1} . The last inequality follows since $\nu_2 = N_{h_2} \geq N_{h_1} = \nu_1$ and $E(Y_1) < 0$. The inequality becomes strict if $N_{h_2} < N_{h_1}$ with positive probability. Hence, $E(S_{N_h})$ is a nonincreasing function of the triggering constant h , whenever $E(Y_1) < 0$. However, as the last item (below) explains, this does not necessarily mean that we will gain more with h_2 versus h_1 , when $E(Y_1) < 0$.

Corr Sign ($\mu \geq 0$)

- If $E(Y_1) = 0$, i.e., no drift in the log returns, then for any strategy, ν , that is independent of the filtration of the log returns, $Corr(\nu, S_\nu) = 0$. However, this is not so when $\nu = N_h$. For the models we presented, $Corr(N_h, S_{N_h}) < 0$, even when $E(Y_1) = 0$. This means that the longer we wait with the trailing stop strategy, N_h , the more we expect to gain, when $E(Y_1) \leq 0$.

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- Even when $E(Y_1) > 0$ and small, the correlation of N_h, S_{N_h} did not change sign in the examples for which we were able to find a closed form expression for $\text{Corr}(N_h, S_{N_h})$ and as well as via simulation for the other examples. Therefore, we may conclude that $\text{Corr}(N_h, S_{N_h}) < 0$ for all values of $E(Y_1)$ in a neighborhood of zero (and not just at zero), since the correlation is a continuous function of $E(Y_1)$. When $E(Y_1) > 0$, by a similar argument as above, we should remark that $E(S_{N_h})$ becomes a nondecreasing function of h .

Parabolic link

- The variance of the duration that the position remains open, $Var(N_h)$, seems to have a parabolic relationship with the expected duration, $E(N_h)$. The coefficients of the parabola are functions of the trigger size h . Furthermore, the larger we pick the triggering constant, h , the larger is the expected duration and, in turn, the more unstable the strategy becomes.

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- The variance of S_{N_h} also seems to have a parabolic relation with $E(S_{N_h})$. For instance, for both the binomial and trinomial models,

$$Var(S_{N_h}) = h - E(S_{N_h}) + (E(S_{N_h}) - h)^2.$$

Since $E(S_{N_h}) = E(Y_1)E(N_h)$, we also have

$$Var(S_{N_h}) = h - E(Y_1)E(N_h) + (E(Y_1)E(N_h) - h)^2.$$

This indicates that, when $E(Y_1) < 0$, one cannot hope to find an optimal h that simultaneously minimizes the expected stopped amount, $E(S_{N_h})$, and also minimizes the variance, $Var(S_{N_h})$.

Gains ($\mu < 0$)

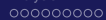
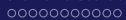
- It seems reasonable to propose that the distribution of a constant multiple of N_h , when $E(Y) < 0$, is approximately a geometric random variable, at least when the assumptions (A1), (A2), and (A3) hold. This should be compared to the known approximation of N_h , which states that $N_h e^{-\theta^* h}$ is approximately an exponential random variable, as h gets large.

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- If the price dynamics are assumed to follow geometric random walk, even when $E(Y_1) < 0$, any trading strategy in short position, that is independent of the filtration $\sigma(Y_1, \dots, Y_n)$, $n \geq 1$, will give time discounted gains to be negative provided $E(e^{Y_1}) > e^r$, where r is the continuously compounding discount rate. That is, for those price dynamics in which $E(Y_1) < 0$ and $E(e^{Y_1}) > e^r$, there cannot exist a trading strategy, that is independent of the log returns process, and gives positive time discounted gains. In fact, this result remains valid for a wide class of adaptive strategies. A similar observation holds for the long position trading strategies.

Short vs Long Position

- For the trailing stops strategy, N_h , adapted to the filtration, $\sigma(Y_1, \dots, Y_n)$, $n \geq 1$, we observe the phenomenon of the last item. In fact, for the geometric model this observation takes even a more acute form. This is due to the fact that, on the one hand, the geometric random variable can take arbitrarily large values and, on the other hand, the price can never go below zero, regardless of how large a magnitude of the negative values of the double sided geometric random variable is. Since, in a short position, by investing \$1, one cannot hope to gain more than one dollar, however, there is no limit to how much one may lose. Hence, the short position is inherently more **risky** than the long position. Furthermore, an indicator of a possible gain is the criterion, $E(e^{Y_1}) < e^r$, where r is the risk free rate, and not just that $E(Y_1) < 0$.



Thank You!