Transformations and Congruence

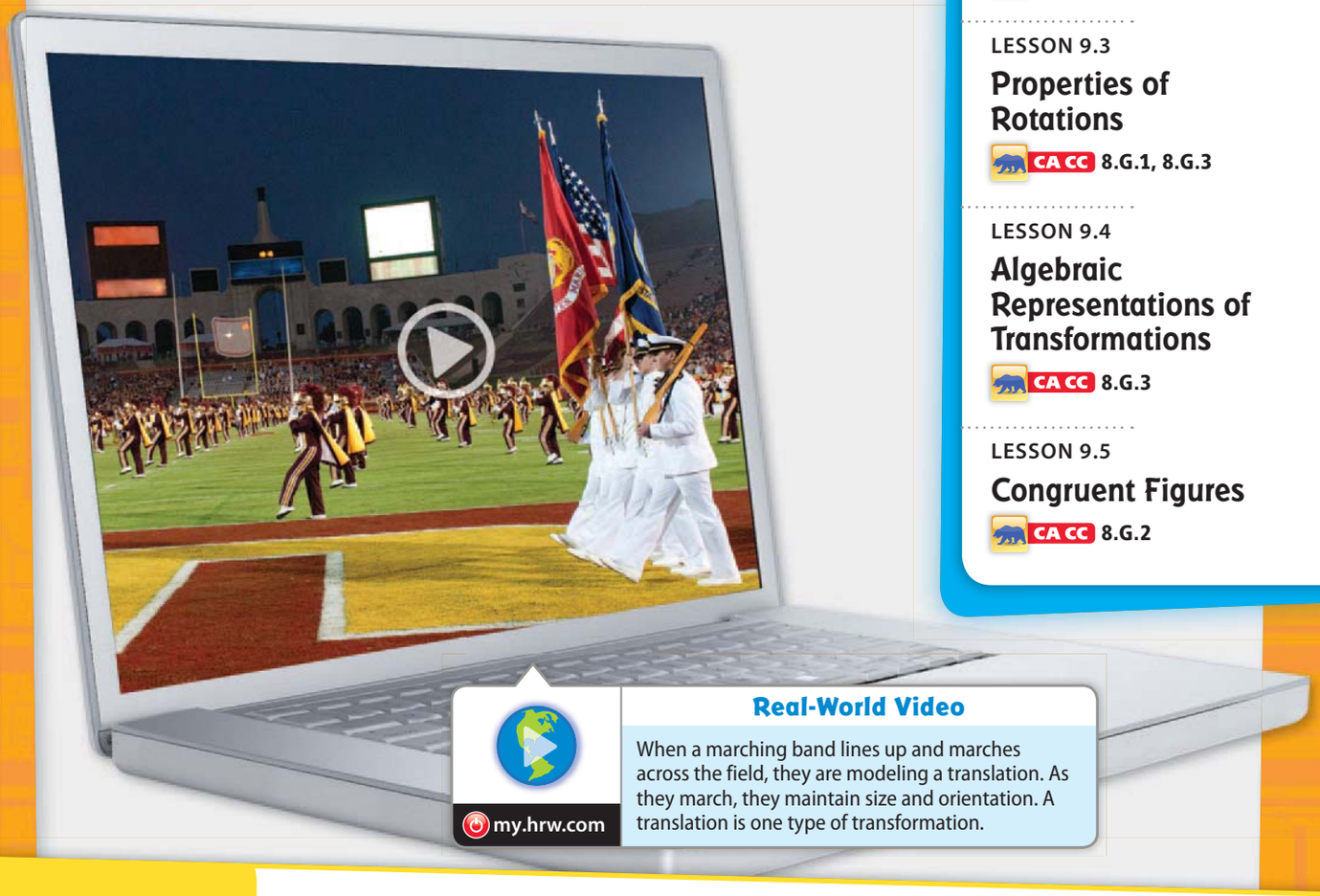
MODULE 9

CALIFORNIA

ESSENTIAL QUESTION

How can you use transformations and congruence to solve real-world problems?

You can analyze how real-world objects are affected when they undergo reflections, translations, rotations, and dilations.



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Real-World Video

When a marching band lines up and marches across the field, they are modeling a translation. As they march, they maintain size and orientation. A translation is one type of transformation.

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Are You Ready?

Assess Readiness

Use the assessment on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.

RtI Response to Intervention

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Intervention	Enrichment

Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Skills Intervention worksheets
- Skill 47 Integer Operations
- Skill 89 Measure Angles

Differentiated Instruction
- Challenge worksheets

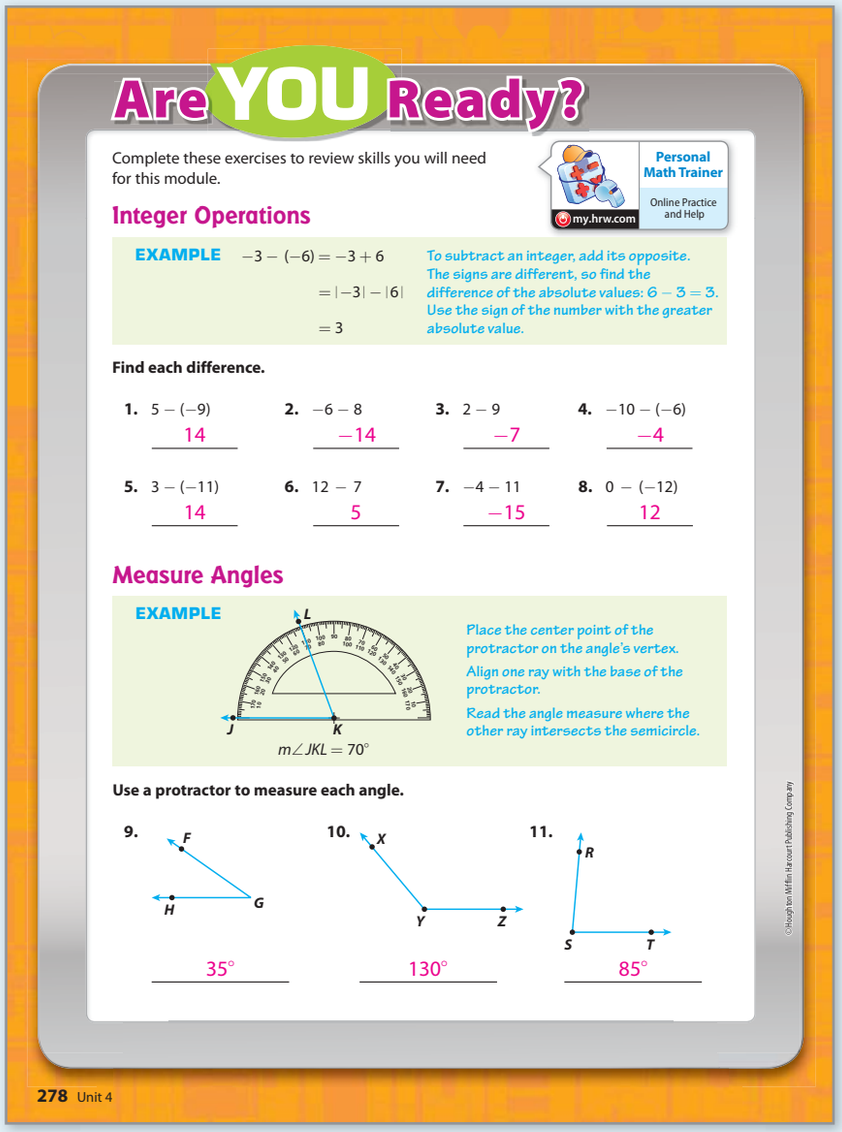
PRE-AP

Extend the Math PRE-AP
Lesson Activities in TE

Real-World Video Viewing Guide

After students have watched the video, discuss the following:

- What are some ways mentioned in the video that transformations are used in the real world?
- How do you move the band formation by using a transformation?
  Move each person the same number of steps up or down and left or right.



Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Integer Operations

EXAMPLE $-3-(-6)=-3+6$
$=|-3|-|6|$
$=3$

To subtract an integer, add its opposite. The signs are different, so find the difference of the absolute values: $6-3=3$. Use the sign of the number with the greater absolute value.

Find each difference.

1. $5-(-9)$ 14
2. $-6-8$ -14
3. $2-9$ -7
4. $-10-(-6)$ -4
5. $3-(-11)$ 14
6. $12-7$ 5
7. $-4-11$ -15
8. $0-(-12)$ 12

Measure Angles

EXAMPLE

$m\angle JKL = 70°$

Place the center point of the protractor on the angle's vertex.
Align one ray with the base of the protractor.
Read the angle measure where the other ray intersects the semicircle.

Use a protractor to measure each angle.

9.

35°

10.

130°

11.

85°

278 Unit 4

PROFESSIONAL DEVELOPMENT VIDEO

Author Juli Dixon models successful teaching practices as she explores the concept of real numbers in an actual eighth-grade classroom.

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Interactive Answers and Solutions
Customize answer keys to print or display in the classroom. Choose to include answers only or full solutions to all lesson exercises.

Interactive Whiteboards
Engage students with interactive whiteboard-ready lessons and activities.

Personal Math Trainer: Online Assessment and Intervention
Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated practice tests aligned with Common Core.

Reading Start-Up EL

Have students complete the activities on this page by working alone or with others.

Strategies for English Learners

Each lesson in the TE contains specific strategies to help English Learners of all levels succeed.

Emerging: Students at this level typically progress very quickly, learning to use English for immediate needs as well as beginning to understand and use academic vocabulary and other features of academic language.

Expanding: Students at this level are challenged to increase their English skills in more contexts, and learn a greater variety of vocabulary and linguistic structures, applying their growing language skills in more sophisticated ways appropriate to their age and grade level.

Bridging: Students at this level continue to learn and apply a range of high-level English language skills in a wide variety of contexts, including comprehension and production of highly technical texts.

Active Reading

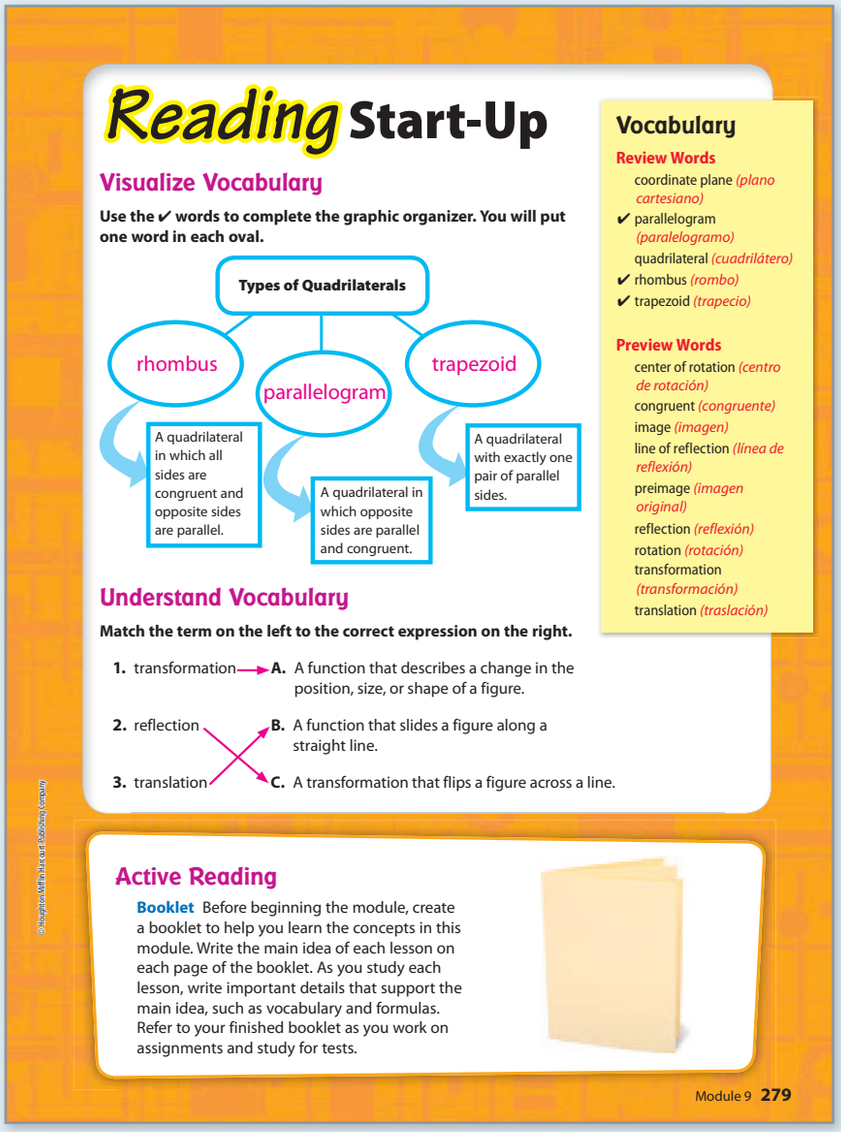
Integrating Language Arts

Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

Additional Resources

Differentiated Instruction

- Reading Strategies EL



Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic organizer. You will put one word in each oval.

Types of Quadrilaterals

rhombus — A quadrilateral in which all sides are congruent and opposite sides are parallel.

parallelogram — A quadrilateral in which opposite sides are parallel and congruent.

trapezoid — A quadrilateral with exactly one pair of parallel sides.

Understand Vocabulary

Match the term on the left to the correct expression on the right.

1. transformation → A. A function that describes a change in the position, size, or shape of a figure.
2. reflection → C. A transformation that flips a figure across a line.
3. translation → B. A function that slides a figure along a straight line.

Vocabulary

Review Words
- coordinate plane (plano cartesiano)
- ✔ parallelogram (paralelogramo)
- quadrilateral (cuadrilátero)
- ✔ rhombus (rombo)
- ✔ trapezoid (trapecio)

Preview Words
- center of rotation (centro de rotación)
- congruent (congruente)
- image (imagen)
- line of reflection (línea de reflexión)
- preimage (imagen original)
- reflection (reflexión)
- rotation (rotación)
- transformation (transformación)
- translation (traslación)

Active Reading

Booklet Before beginning the module, create a booklet to help you learn the concepts in this module. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.

Module 9 279

CA CC Focus | Coherence | Rigor

Tracking Your Learning Progression

Before	In this module	After
Students understand: • how to classify and draw plane figures • how to graph plane figures on the coordinate plane • congruence	Students use transformational geometry to represent: • properties of orientation and congruence of translations in a coordinate plane • properties of orientation and congruence of reflections in a coordinate plane • properties of orientation and congruence of rotations in a coordinate plane • the effect of translations, reflections, and rotations in a coordinate plane using an algebraic representation	Students will connect: • transformations and congruence • reflections over an axis and symmetry • algebra and coordinate geometry

GETTING READY FOR

Transformations and Congruence

Use the examples on the page to help students know exactly what they are expected to learn in this module.

CA Common Core Standards

Content Areas

CA CC Geometry—8.G

Cluster Understand congruence and similarity using physical models, transparencies, or geometry software.

Go online to see a complete unpacking of the CA Common Core Standards.

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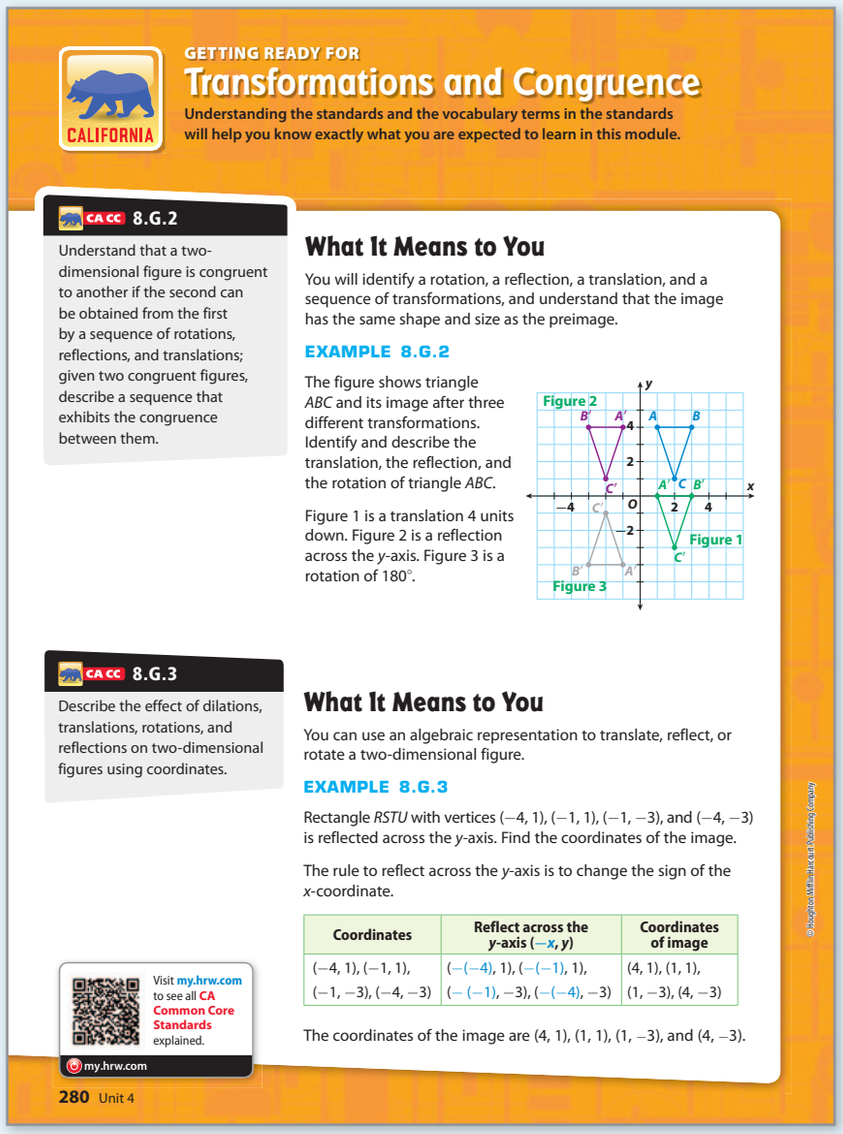


California Common Core Standards	Lesson 9.1	Lesson 9.2	Lesson 9.3	Lesson 9.4	Lesson 9.5
CA CC 8.G.1 Verify experimentally the properties of rotations, reflections, and translations: **a** Lines are taken to lines, and line segments to line segments of the same length. **b** Angles are taken to angles of the same measure. **c** Parallel lines are taken to parallel lines.	●	●	●		
CA CC 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.					●
CA CC 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	●	●	●	●	

Lesson Support

Content Objective Students will learn how to describe the properties of translations and their effect on the congruence and orientation of figures.

Language Objective Students will describe the properties of translation and their effect on the congruence and orientation of figures.

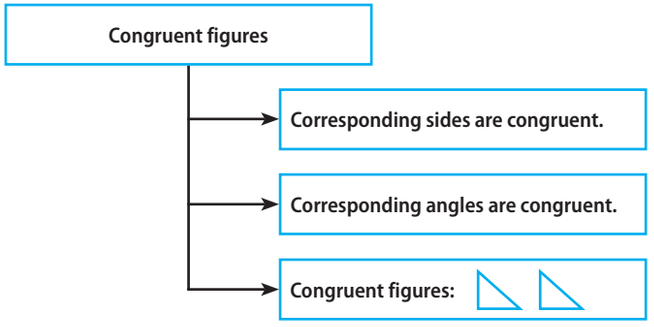
California Common Core Standards

CA CC **8.G.1**	Verify experimentally the properties of rotations, reflections, and translations. a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
CA CC **8.G.3**	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
CA CC **MP.6**	Attend to precision.

CA CC **Focus | Coherence | Rigor**

Building Background

Eliciting Prior Knowledge Review what it means for two polygons to have corresponding sides and corresponding angles. Then ask students to create a definition and example chart for congruent figures. Transition into the lesson by asking whether two congruent figures remain congruent if one of the figures is moved up or down or to the right or left.



Learning Progressions

In this lesson, students verify experimentally the properties of translations by translating polygons in the coordinate plane. Important understandings for students include the following:

- Investigate the properties of translations.
- Graph translations.

Students are familiar with translations from earlier grades. This lesson provides a review of the properties of translations, which students have likely encountered before. They verify the properties by translating figures on a coordinate plane. They begin to connect the geometry of the translation to the algebraic concept of a translation as a function with an input (the preimage) and the output (the image).

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Understand congruence and similarity using physical models, transparencies, or geometry software.*** Tell students that line segment AB has endpoints $A(2, 1)$ and $B(5, 3)$. Ask them to describe how each translation changes the coordinates of the endpoints of the preimage to create the coordinates of the endpoints of the image:

(1) translate $\overline{AB}$ 4 units to the right;
(2) translate $\overline{AB}$ 4 units to the left;
(3) translate $\overline{AB}$ 4 units up;
(4) translate $\overline{AB}$ 4 units down

Sample answer: (1) add 4 to the x-coordinate; y-coordinate stays same; (2) subtract 4 from the x-coordinate; y-coordinate stays same; (3) x-coordinate stays same; add 4 to the y-coordinate; (4) x-coordinate stays same; subtract 4 from the y-coordinate.

Language Support

California ELD Standards

CA ELD **Emerging 2.I.5. Listening actively** – Demonstrate active listening in oral presentation activities by asking and answering basic questions with prompting and substantial support.

CA ELD **Expanding 2.I.5. Listening actively** – Demonstrate active listening in oral presentation activities by asking and answering detailed questions with occasional prompting and moderate support.

CA ELD **Bridging 2.I.5. Listening actively** – Demonstrate active listening in oral presentation activities by asking and answering detailed questions with minimal prompting and support.

Linguistic Support

Academic/Content Vocabulary

This lesson on properties of translations relies on students' understanding of the meaning of several words as they are used in mathematics: *transformation*, *preimage*, *image*, and *translation*. While English learners at the expanding and bridging levels of English proficiency might have encountered these words in other classes, they may be new to them in mathematics. Even with the definition in the lesson, students may benefit from adding an illustration along with the word to their word journals.

Building Background

pre- – The term *preimage* is introduced in this lesson. *Pre-* is a prefix meaning *before*, *earlier*, or *in front of*. Other useful words beginning with the prefix *pre-* are *precook*, *predict*, and *precaution*.

-tion – The terms *transformation* and *translation* are also introduced in this lesson. The suffix *-tion* means *action*. Other words ending in *-tion* are *competition*, *exploration*, and *organization*.

Leveled Strategies for English Learners

Emerging Have students at this level of English proficiency work in pairs to review and copy on graph paper the translations in Explore Activities 1 & 2. If possible, have them discuss in their primary language the steps to take to accomplish the translations.

Expanding Have students at this level of English proficiency work in pairs to redraw the translations in Explore Activities 1 & 2. Then have them list the steps they took to accomplish the translations.

Bridging Have students at this level of English proficiency work in pairs to redraw the translations in Explore Activities 1 & 2. Then have them describe for each other the steps they took to accomplish these translations.

Math Talk

Write out and model for students a sentence frame to begin their answer.

Yes, the figures are congruent because_____.

LESSON

9.1 Properties of Translations

CA Common Core Standards

The student is expected to:

CA CC Geometry—8.G.1

Verify experimentally the properties of rotations, reflections, and translations.

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure

c. Parallel lines are taken to parallel lines.

CA CC Geometry—8.G.3

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices

CA CC MP.6 Precision

Engage

ESSENTIAL QUESTION

How do you describe the properties of translation and their effect on the congruence and orientation of figures? Sample answer: Translations preserve size, shape, and orientation.

Motivate the Lesson

Ask: What changes when you slide an object, such as a book, from one corner of your desk to different corners of your desk? Does the size or shape of the object change? Begin the Explore Activity to find out how to describe this action mathematically.

Explore

EXPLORE ACTIVITY 1

Focus on Modeling Mathematical Practices

Ask students to move the triangle from the image position back to the preimage position and describe the movement. 7 units left and 5 units up How does the description of the movement change? How does the description stay the same? Students should see that the magnitude of the movement stays the same, but the direction changes.

Explain

EXPLORE ACTIVITY 2

Connect Vocabulary EL

Emphasize that a *transformation* is a function that describes a change in the position, size, or shape of a figure, and a *translation* is a *type* of transformation in which a shape changes position but not size or orientation. Students often mix up these two terms.

Questioning Strategies Mathematical Practices

- How many different ways could trapezoid *TRAP* be translated? Justify your answer. It can be translated an infinite number of ways. The rule for the translation would be different for each way.
- What characteristics do you look for in an image to know that it has been translated from a preimage? The corresponding sides have the same lengths, and the corresponding angles have the same measures. The shape and size of the image is the same as that of the preimage. The orientation of the image and preimage are the same.

Engage with the Whiteboard

You may wish to have students measure the actual side lengths of the projected image and preimage on the whiteboard, or you can have students count the lengths of the sides in grid units (using the Distance Formula for *PT* and *P'T'*). Point out to students that although the lengths of the sides in centimeters on the projected image will not match the lengths in their books, the angle measurements will be the same.

LESSON 9.1

Properties of Translations

CA CC 8.G.1a

Verify experimentally the properties of rotations, reflections, and translations: Lines are taken to lines, and line segments to line segments of the same length. *Also 8.G.1b, 8.G.1c, 8.G.3*

ESSENTIAL QUESTION

How do you describe the properties of translation and their effect on the congruence and orientation of figures?

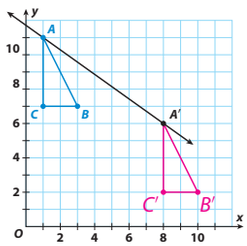
EXPLORE ACTIVITY 1 CA CC 8.G.1a

Exploring Translations

You learned that a function is a rule that assigns exactly one output to each input. A **transformation** is a function that describes a change in the position, size, or shape of a figure. The input of a transformation is the **preimage**, and the output of a transformation is the **image**.

A **translation** is a transformation that slides a figure along a straight line.

The triangle shown on the grid is the preimage (input). The arrow shows the motion of a translation and how point *A* is translated to point *A′*.



A Trace triangle *ABC* and line *AA′* onto a piece of paper.

B Slide your triangle along the line to model the translation that maps point *A* to point *A′*.

C The image of the translation is the triangle produced by the translation. Sketch the image of the translation.

D The vertices of the image are labeled using prime notation. For example, the image of *A* is *A′*. Label the images of points *B* and *C*.

E Describe the motion modeled by the translation.

Move 7 units right and 5 units down.

F Check that the motion you described in part **E** is the same motion that maps point *A* onto *A′*, point *B* onto *B′*, and point *C* onto *C′*.

Reflect

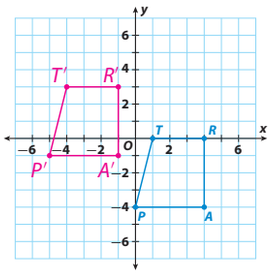
1. How is the orientation of the triangle affected by the translation?

It is not affected. The orientation stays the same.

EXPLORE ACTIVITY 2 CA CC 8.G.1a

Properties of Translations

Use trapezoid *TRAP* to investigate the properties of translations.



A Trace the trapezoid onto a piece of paper. Cut out your traced trapezoid.

B Place your trapezoid on top of the trapezoid in the figure. Then translate your trapezoid 5 units to the left and 3 units up. Sketch the image of the translation by tracing your trapezoid in this new location. Label the vertices of the image *T′*, *R′*, *A′*, and *P′*.

C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

$TR =$ 1.3 cm $RA =$ 1.7 cm $AP =$ 1.7 cm $TP =$ 1.75 cm

D Use a ruler to measure the sides of trapezoid *T′R′A′P′* in centimeters.

$T'R' =$ 1.3 cm $R'A' =$ 1.7 cm $A'P' =$ 1.7 cm $T'P' =$ 1.75 cm

E What do you notice about the lengths of corresponding sides of the two figures?

The lengths of corresponding sides are the same.

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T =$ 104° $m\angle R =$ 90° $m\angle A =$ 90° $m\angle P =$ 76°

G Use a protractor to measure the angles of trapezoid *T′R′A′P′*.

$m\angle T' =$ 104° $m\angle R' =$ 90° $m\angle A' =$ 90° $m\angle P' =$ 76°

H What do you notice about the measures of corresponding angles of the two figures?

The measures of corresponding angles are the same.

I Which sides of trapezoid *TRAP* are parallel? How do you know?

$\overline{TR}$ and $\overline{AP}$; They both lie along horizontal grid lines.

Which sides of trapezoid *T′R′A′P′* are parallel? $\overline{T'R'}$ and $\overline{A'P'}$

What do you notice? The sides that were parallel in the preimage remain parallel in the image.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.6

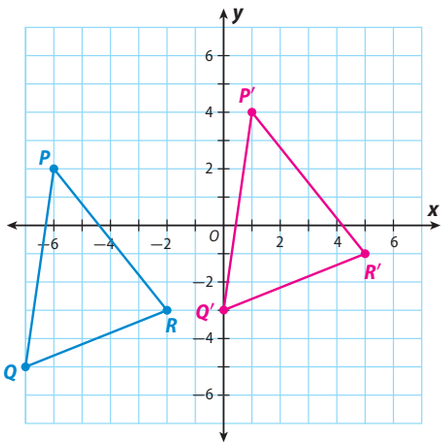
This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to communicate precisely. Students translate a figure on a coordinate grid following a given translation rule. Then, students measure the lengths of the sides and the degrees of the angles to show that the corresponding sides and angles are congruent. Finally, students make a conjecture about the preservation of the size and shape of a figure.

Math Background

In future geometry courses, students will learn special conditions that guarantee that two triangles are congruent. It is not necessary to verify that all three pairs of sides are congruent and all three pairs of angles are congruent. There are numerous "shortcuts," such as the Side-Side-Side (SSS) Congruence Postulate, which states that two triangles are congruent if the corresponding sides of one triangle are congruent to the corresponding sides of the other triangle. In other words, if the corresponding sides are congruent, the angles must also be congruent. However, having all three corresponding angles congruent does not guarantee that the corresponding sides are necessarily congruent.

ADDITIONAL EXAMPLE 1

The figure shows triangle *PQR*. Graph the image of the triangle after a translation of 7 units to the right and 2 units up.



Interactive Whiteboard
Interactive example available online

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EXAMPLE 1

Questioning Strategies Mathematical Practices

- What do you notice about the purple lines in the first grid that show the translation of each vertex? They are parallel to each other. They have the same length and slope.
- What do you notice about the corresponding sides of the preimage and the image in Step 4? The corresponding sides are parallel and have the same length and slope.

Engage with the Whiteboard

On the second grid, have a volunteer plot points and draw triangle *X″Y″Z″*, which is the image of *X′Y′Z′* after a translation exactly half the units of the one given in the example.

YOUR TURN

Avoid Common Errors

Students can check that they have not miscounted units in the translation of any one point by checking that the image and preimage have the same size, shape, and orientation.

Talk About It

Check for Understanding

Ask: What translation rule would have moved side *DC* to coincide with the *x*-axis? a translation of 1 unit up and any distance left or right

Elaborate

Talk About It

Summarize the Lesson

Ask: How do you know when a transformation is a translation? The image will have the same size, shape, and orientation as the preimage.

GUIDED PRACTICE

Engage with the Whiteboard

To help students visualize Exercises 3–4, have volunteers sketch the images and preimages on a coordinate grid. In Exercise 3, have the students assign letters to the vertices as well.

Avoid Common Errors

Exercises 3–5 Remind students that a translation is a type of transformation. A translation only causes a change in the position of the figure; everything else remains the same.

Exercise 5 Remind students that the image will have its vertices labeled with the same letters as the corresponding preimage vertices, plus the symbol ′.

Reflect

2. **Make a Conjecture** Use your results from parts E, H, and I to make a conjecture about translations.

Sample answer: Translations preserve the size and shape of a figure, as well as its orientation.

3. Two figures that have the same size and shape are called *congruent*. What can you say about translations and congruence?

A translation produces a figure that is congruent to the original figure.

Graphing Translations

To translate a figure in the coordinate plane, translate each of its vertices. Then connect the vertices to form the image.

Math On the Spot
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EXAMPLE 1

CA CC 8.G.3

The figure shows triangle *XYZ*. Graph the image of the triangle after a translation of 4 units to the right and 1 unit up.

STEP 1 Translate point *X*.

Count right 4 units and up 1 unit and plot point *X′*.

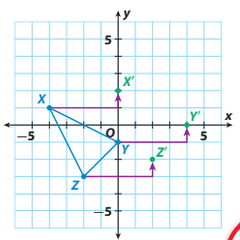
STEP 2 Translate point *Y*.

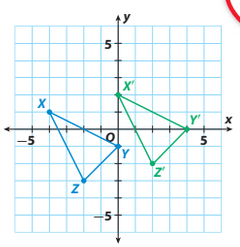
Count right 4 units and up 1 unit and plot point *Y′*.

STEP 3 Translate point *Z*.

Count right 4 units and up 1 unit and plot point *Z′*.

STEP 4 Connect *X′*, *Y′*, and *Z′* to form triangle *X′Y′Z′*.





Each vertex is moved 4 units right and 1 unit up.

Math Talk
Mathematical Practices
Is the image congruent to the preimage? How do you know?

Yes, the figures are congruent. Translations preserve size and shape.

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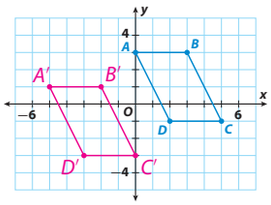
YOUR TURN

4. The figure shows parallelogram *ABCD*. Graph the image of the parallelogram after a translation of 5 units to the left and 2 units down.



Guided Practice

1. **Vocabulary** A transformation is a change in the position, size, or shape of a figure.

2. **Vocabulary** When you perform a transformation of a figure on the coordinate plane, the input of the transformation is called the preimage, and the output of the transformation is called the image.

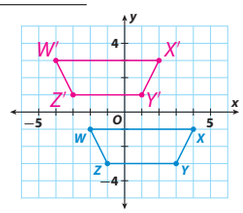
3. Joni translates a right triangle 2 units down and 4 units to the right. How does the orientation of the image of the triangle compare with the orientation of the preimage? (Explore Activity 1)

The orientation will be the same.

4. Rashid drew rectangle *PQRS* on a coordinate plane. He then translated the rectangle 3 units up and 3 units to the left and labeled the image *P′Q′R′S′*. How do rectangle *PQRS* and rectangle *P′Q′R′S′* compare? (Explore Activity 2)

They are congruent.

5. The figure shows trapezoid *WXYZ*. Graph the image of the trapezoid after a translation of 4 units up and 2 units to the left. (Example 1)



ESSENTIAL QUESTION CHECK-IN

6. What are the properties of translations?

Sample answer: Translations preserve the size, shape, and orientation of a figure.

DIFFERENTIATE INSTRUCTION

Curriculum Integration

Have groups of students make up a pattern of dance steps formed by translating shapes, which represent dancers' feet, on a grid. Then have students show the class their "dance" using the tiles on the floor as an enlarged grid.

World History

A zoetrope is a device that consists of a cylinder with slits cut vertically around its sides. On the inside of the cylinder is drawn a series of pictures of the same object translated to different positions. When the zoetrope is spun, a person looking through the slits sees what appears to be the object in motion. The earliest known zoetrope was created in China around 180 CE.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

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9.1 LESSON QUIZ

CA CC **8.G.1, 8.G.3**

Graph triangle *ABC* with vertices *A*(−3, 4), *B*(0, 2), and *C*(−2, 1) on a coordinate grid.

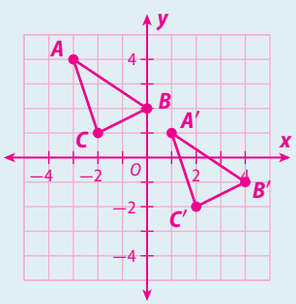
1. Graph the image of triangle *ABC* after a translation of 4 units right and 3 units down.
2. Which side of the image is congruent to side $\overline{AB}$?
3. Which angle in the image is congruent to angle *B*?
4. Angle *G* in quadrilateral *FGHJ* measures 135°. Brent translates the quadrilateral 3 units right and 1 unit up. What is the measure of the image of angle *G*?

Lesson Quiz available online

my.hrw.com

Answers

1.



2. $\overline{A'B'}$
3. angle *B*′
4. 135°

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.G.1, 8.G.1a, 8.G.1b, 8.G.1c, 8.G.3**

Concepts & Skills	Practice
Explore Activity 1 Exploring Translations	Exercises 1–3, 11–12
Explore Activity 2 Properties of Translations	Exercises 4, 7–8
Example 1 Graphing Translations	Exercises 5, 7–10

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
7	**2** Skills/Concepts	**MP.6** Precision
8	**2** Skills/Concepts	**MP.7** Using Structure
9–10	**2** Skills/Concepts	**MP.8** Patterns
11	**2** Skills/Concepts	**MP.4** Modeling
12	**2** Skills/Concepts	**MP.3** Logic
13	**3** Strategic Thinking H.O.T.	**MP.6** Precision
14	**3** Strategic Thinking H.O.T.	**MP.8** Patterns
15	**3** Strategic Thinking H.O.T.	**MP.3** Logic

Additional Resources

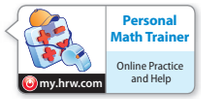
Differentiated Instruction includes:

- Leveled Practice worksheets

Name _____ Class _____ Date _____

9.1 Independent Practice

CA CC 8.G.1a, 8.G.1b, 8.G.1c, 8.G.3



7. The figure shows triangle *DEF*.

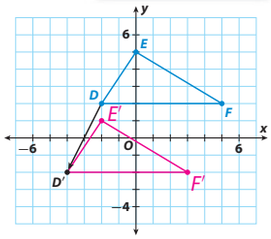
a. Graph the image of the triangle after the translation that maps point *D* to point *D′*.

b. How would you describe the translation?

The translation moved the triangle 2 units to the left and 4 units down.

c. How does the image of triangle *DEF* compare with the preimage?

They are congruent.



8. **a.** Graph quadrilateral *KLMN* with vertices *K*(−3, 2), *L*(2, 2), *M*(0, −3), and *N*(−4, 0) on the coordinate grid.

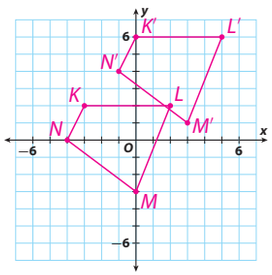
b. On the same coordinate grid, graph the image of quadrilateral *KLMN* after a translation of 3 units to the right and 4 units up.

c. Which side of the image is congruent to side $\overline{LM}$?

$\overline{L'M'}$

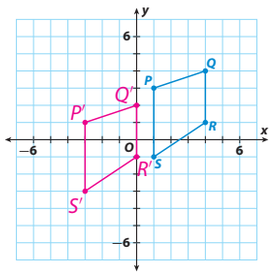
Name three other pairs of congruent sides.

$\overline{KL}$ and $\overline{K'L'}$, $\overline{MN}$ and $\overline{M'N'}$, $\overline{KN}$ and $\overline{K'N'}$

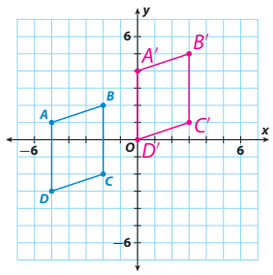


Draw the image of the figure after each translation.

9. 4 units left and 2 units down

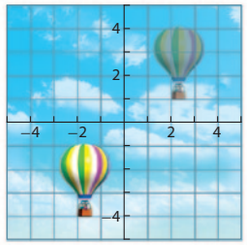


10. 5 units right and 3 units up



11. The figure shows the ascent of a hot air balloon. How would you describe the translation?

The hot air balloon was translated 4 units to the right and 5 units up.



12. **Critical Thinking** Is it possible that the orientation of a figure could change after it is translated? Explain.

No; when a figure is translated, it is slid to a new location. Since it is not turned or flipped, the orientation will remain the same.

H.O.T. FOCUS ON HIGHER ORDER THINKING

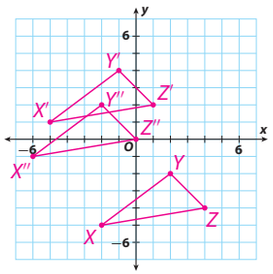
13. **a.** **Multistep** Graph triangle *XYZ* with vertices *X*(−2, −5), *Y*(2, −2), and *Z*(4, −4) on the coordinate grid.

b. On the same coordinate grid, graph and label triangle *X′Y′Z′*, the image of triangle *XYZ* after a translation of 3 units to the left and 6 units up.

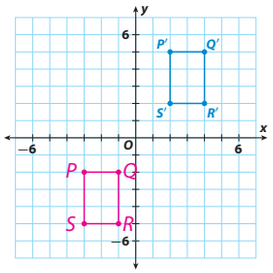
c. Now graph and label triangle *X″Y″Z″*, the image of triangle *X′Y′Z′* after a translation of 1 unit to the left and 2 units down.

d. **Analyze Relationships** How would you describe the translation that maps triangle *XYZ* onto triangle *X″Y″Z″*?

Sample answer: The original triangle was translated 4 units up and 4 units to the left.



14. **Critical Thinking** The figure shows rectangle *P′Q′R′S′*, the image of rectangle *PQRS* after a translation of 5 units to the right and 7 units up. Graph and label the preimage *PQRS*.



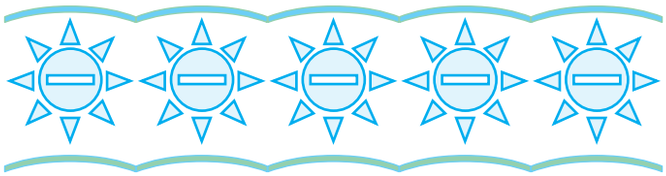
15. **Communicate Mathematical Ideas** Explain why the image of a figure after a translation is congruent to its preimage.

Sample answer: Since every point of the original figure is translated the same number of units up/down and left/right, the image is exactly the same size and shape as the preimage. Only the location is different.

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity A strip pattern is a design that repeats itself along a straight line. Some strip patterns, like the one shown, are examples of translations. On a strip of paper, create a design. Then repeat the design by translating it along a straight line to create your own strip pattern.



Lesson Support

Content Objective Students will learn how to describe the properties of reflections and their effect on the congruence and orientation of figures.

Language Objective Students will explain how to describe the properties of reflection and their effect on the congruence and orientation of figures.

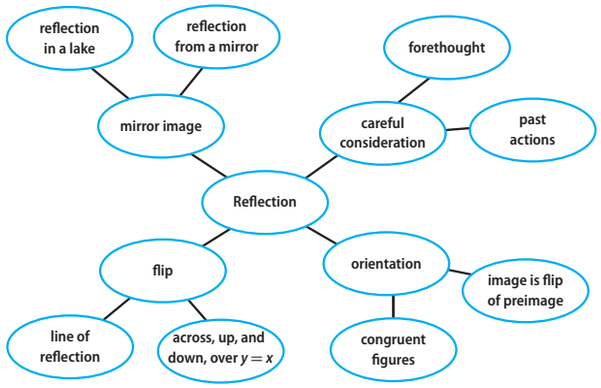
California Common Core Standards

CA CC **8.G.1**	Verify experimentally the properties of rotations, reflections, and translations. a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
CA CC **8.G.3**	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
CA CC **MP.5**	Use appropriate tools strategically.

CA CC Focus | Coherence | Rigor

Building Background

Eliciting Prior Knowledge Ask students to work with a partner or in small groups to create a concept map based on reflections. The map should include both real-life connections and whatever geometrical concepts students can recall about reflections.



Learning Progressions

In this lesson, students verify experimentally the properties of reflections by reflecting polygons in the coordinate plane across the axes. Important understandings for students include the following:

- Investigate the properties of reflections.
- Graph reflections.

Students are familiar with reflections, or flips, from earlier grades. This lesson provides an examination of the properties of reflections by having students measure and compare lengths and angles of corresponding parts of preimages and images. Students apply their understanding by graphing reflections of polygons in the coordinate plane.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Understand congruence and similarity using physical models, transparencies, or geometry software.*** Tell students that triangle *ABC* has vertices *A*(1, 1), *B*(3, 1), and *C*(2, 4). Ask them to describe how each reflection changes the coordinates of the vertices of the preimage to create the coordinates of the vertices of the image:

(1) across the *x*-axis

(2) across the *y*-axis

Sample answer: (1) the *x*-coordinates stay the same and the *y*-coordinates have opposite signs from the original; (2) the *x*-coordinates have opposite signs from the original and the *y*-coordinates stay the same.

Language Support

California ELD Standards

CA ELD Emerging 2.I.1. Exchanging information/ideas – Contribute to conversations and express ideas by asking and answering *yes-no* and *wh-* questions and responding using short phrases.

CA ELD Expanding 2.I.1. Exchanging information/ideas – Contribute to class, group, and partner discussions, including sustained dialogue, by following turn-taking rules, asking relevant questions, affirming others, and adding relevant information.

CA ELD Bridging 2.I.1. Exchanging information/ideas – Contribute to class, group, and partner discussions, including sustained dialogue, by following turn-taking rules, adding relevant information, building on responses, and providing useful feedback.

Linguistic Support

Academic/Content Vocabulary

This lesson relies on students' understanding of the meaning of *reflection* and *line of reflection*. Point out that these terms are defined in the context of the sentence in which they are introduced. Also, have students turn to the glossary to see the new word defined. Often there is a visual diagram or other support, including a Spanish language explanation, in the glossary. For a glossary resource in 13 world languages, be sure to take students to the Online Multilingual Glossary.

Building Background

Help English learners figure out the meaning of unknown words in the lesson by pointing out patterns in words. In Explore Activity 1 of this lesson, students are instructed to *fold* and *unfold* a piece of paper. The prefix *un-* means *not*. So, *unfold* means *not fold*. Other examples of words that begin with this prefix are *able/unable, do/undo*, *fair/unfair*, and *decided/undecided*. Suggest they add this information to their math journals.

Leveled Strategies for English Learners

Emerging Have students at this level of English proficiency work in pairs to illustrate and label on graph paper a translation and reflection of the same size and shape. Ask them to show or point to how they are similar and how they are different.

Expanding Have students at this level of English proficiency work in pairs to define and illustrate on graph paper a translation and reflection of the same size and shape. Have students write down how the two are different.

Bridging Have students at this level of English proficiency work in pairs to discuss, illustrate, and label the differences between translation and reflection of the same size and shape.

Math Talk

Write out and model for students a sentence frame to begin their answer.

A reflection produces a figure that is congruent _____.

LESSON

9.2 Properties of Reflections

CA Common Core Standards

The student is expected to:

CA CC Geometry—8.G.1

Verify experimentally the properties of rotations, reflections, and translations.

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure

c. Parallel lines are taken to parallel lines.

CA CC Geometry—8.G.3

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices

CA CC MP.5 Using Tools

Engage

ESSENTIAL QUESTION

How do you describe the properties of reflection and their effect on the congruence and orientation of figures? Sample answer: Reflections preserve size and shape, but not orientation.

Motivate the Lesson

Ask: What changes when you flip an object, such as a book, in any direction? Does the size or shape of the object change? Begin the Explore Activity to find out how to describe this action mathematically.

Explore

EXPLORE ACTIVITY 1

Focus on Modeling **Mathematical Practices**

After students have folded their paper across the axes and drawn both reflections, have them fold their papers over both axes at the same time, folding the paper into quarters. If they have drawn the reflections correctly, all three figures should match up exactly.

Explain

EXPLORE ACTIVITY 2

Connect Vocabulary EL

Point out that translations and *reflections* are both types of transformations. While a translation does not change the orientation of a figure, a reflection does. Emphasize that a *line of reflection* is often one of the axes, but it can be any line, including lines that are not horizontal or vertical.

Questioning Strategies

- How many different ways could trapezoid *TRAP* be reflected? Justify your answer. It can be reflected an infinite number of ways. Any line can be a line of reflection, and there is an infinite number of lines.
- What characteristics do you look for in an image to know that it has been reflected from a preimage? The corresponding sides have the same lengths, and the corresponding angles have the same measures. The shape and size of the image is the same as that of the preimage. The only difference is the image is a mirror image of the preimage.

Engage with the Whiteboard

Have students draw the reflection of trapezoid *TRAP* across the *x*-axis. Name the new image $T''R''A''P''$. Students can also draw the reflection of $T'R'A'P'$ across the *x*-axis and name the new image $T'''R'''A'''P'''$.

LESSON 9.2

Properties of Reflections

CA CC 8.G.1b
Verify experimentally the properties of rotations, reflections, and translations: Angles are taken to angles of the same measure. *Also 8.G.1a, 8.G.1c, 8.G.3*

ESSENTIAL QUESTION How do you describe the properties of reflection and their effect on the congruence and orientation of figures?

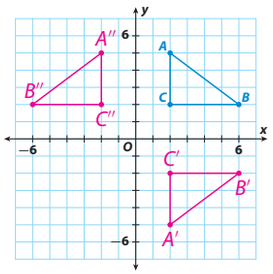
EXPLORE ACTIVITY 1

CA CC 8.G.1b

Exploring Reflections

A **reflection** is a transformation that flips a figure across a line. The line is called the **line of reflection**. Each point and its image are the same distance from the line of reflection.

The triangle shown on the grid is the preimage. You will explore reflections across the *x*- and *y*-axes.



A Trace triangle *ABC* and the *x*- and *y*-axes onto a piece of paper.

B Fold your paper along the *x*-axis and trace the image of the triangle on the opposite side of the *x*-axis. Unfold your paper and label the vertices of the image *A′*, *B′*, and *C′*.

C What is the line of reflection for this transformation?

the *x*-axis

D Find the perpendicular distance from each point to the line of reflection.

Point *A* 5 units Point *B* 2 units Point *C* 2 units

E Find the perpendicular distance from each point to the line of reflection.

Point *A′* 5 units Point *B′* 2 units Point *C′* 2 units

F What do you notice about the distances you found in **D** and **E**?

They are the same for a point and its reflection.

Reflect

1. Fold your paper from **A** along the *y*-axis and trace the image of triangle *ABC* on the opposite side. Label the vertices of the image *A″*, *B″*, and *C″*. What is the line of reflection for this transformation?

the *y*-axis

2. How does each image in your drawings compare with its preimage?

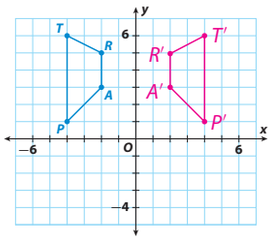
Sample answer: $\triangle A'B'C'$ is $\triangle ABC$ flipped across the *x*-axis. $\triangle A''B''C''$ is $\triangle ABC$ flipped across the *y*-axis.

EXPLORE ACTIVITY 2

CA CC 8.G.1b

Properties of Reflections

Use trapezoid *TRAP* to investigate the properties of reflections.



A Trace the trapezoid onto a piece of paper. Cut out your traced trapezoid.

B Place your trapezoid on top of the trapezoid in the figure. Then reflect your trapezoid across the *y*-axis. Sketch the image of the reflection by tracing your trapezoid in this new location. Label the vertices of the image *T′*, *R′*, *A′*, and *P′*.

C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

TR = 0.9 cm RA = 0.8 cm AP = 1.2 cm TP = 2.1 cm

D Use a ruler to measure the sides of trapezoid *T′R′A′P′* in centimeters.

$T'R'$ = 0.9 cm $R'A'$ = 0.8 cm $A'P'$ = 1.2 cm $T'P'$ = 2.1 cm

E What do you notice about the lengths of corresponding sides of the two figures?

The lengths of corresponding sides are the same.

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T$ = 63° $m\angle R$ = 117° $m\angle A$ = 135° $m\angle P$ = 45°

G Use a protractor to measure the angles of trapezoid *T′R′A′P′*.

$m\angle T'$ = 63° $m\angle R'$ = 117° $m\angle A'$ = 135° $m\angle P'$ = 45°

H What do you notice about the measures of corresponding angles of the two figures?

The measures of corresponding angles are the same.

I Which sides of trapezoid *TRAP* are parallel? $\overline{TP}$ and $\overline{RA}$

Which sides of trapezoid *T′R′A′P′* are parallel? $\overline{T'P'}$ and $\overline{R'A'}$

What do you notice?

The sides that were parallel in the preimage remain parallel in the image.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.5

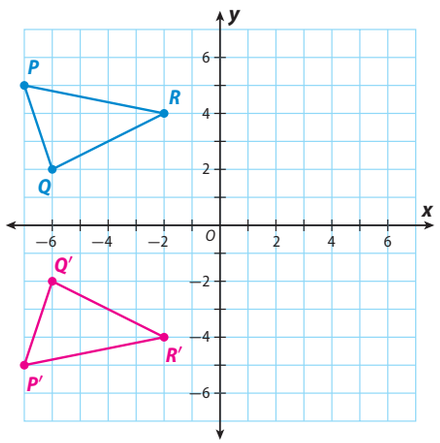
This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to use tools such as models, rulers, and pencil and paper to analyze relationships. Students use the results of the Explore Activities to make a conjecture that reflections preserve the size and shape of a figure. They find the measures of the angles and side lengths of the image and its preimage and use them to justify their conjecture.

Math Background

Translations, reflections, and rotations are examples of rigid motions. Reflections are sometimes called improper rigid motions as they can cause shapes to flip into a new orientation. However, applying the same reflection twice results in the original orientation. Translations and rotations are sometimes called proper rigid motions as they never cause the shape to flip.

ADDITIONAL EXAMPLE 1

The figure shows triangle *PQR*. Graph the image of the triangle after a reflection across the *x*-axis.



Interactive Whiteboard
Interactive example available online

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EXAMPLE 1

Questioning Strategies Mathematical Practices

- What do you notice about the purple lines that show the reflection of each vertex? They are parallel to each other. The line of reflection divides each line in half, but all three lines are not the same length.
- Why is triangle *X′Y′Z′* not a translation of triangle *XYZ*? The figures do not have the same orientation.

Engage with the Whiteboard

Have a volunteer plot a new triangle, *X″Y″Z″*, that is a reflection of *X′Y′Z′* across the *y*-axis. Have students compare the orientations of the three triangles.

YOUR TURN

Avoid Common Errors

Students may plot the vertices of the image correctly but label them incorrectly. Suggest that they label each vertex of the image as they plot the point and confirm that each letter matches the letter of the corresponding vertex in the preimage.

Talk About It

Check for Understanding

Ask: How do the coordinates of the image differ from the coordinates of the preimage when a figure is reflected across the *y*-axis? The *x*-values are the opposite of the preimage's *x*-values, but the *y*-values remain the same.

Elaborate

Talk About It

Summarize the Lesson

Ask: How do you know when a transformation is a reflection? The image will have the same size and shape as the preimage, but the orientation will not be the same. There will be a line of reflection such that each image point will be the same distance from that line as its corresponding preimage point.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 2 have students graph the reflections of trapezoid *ABCD* across both the *x*-axis and the *y*-axis on the coordinate grid. Label the images *A′B′C′D′* and *A″B″C″D″*.

Avoid Common Errors

Exercise 2a Remind students that the image will have its vertices labeled with the same letters as the corresponding preimage vertices, plus the symbol ′.

Reflect

3. **Make a Conjecture** Use your results from E, H, and I to make a conjecture about reflections.

Sample answer: Reflections preserve the size and shape of a figure, but the orientation changes to a mirror image of the original.

Math Talk
Mathematical Practices
What can you say about reflections and congruence?

Math Talk: A reflection produces a figure that is congruent to the original figure.

Graphing Reflections

To reflect a figure across a line of reflection, reflect each of its vertices. Then connect the vertices to form the image. Remember that each point and its image are the same distance from the line of reflection.

Math On the Spot
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EXAMPLE 1

CA CC 8.G.3

The figure shows triangle *XYZ*. Graph the image of the triangle after a reflection across the *x*-axis.

STEP 1 Reflect point *X*.

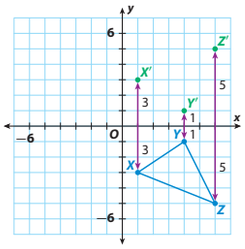
Point *X* is 3 units below the *x*-axis. Count 3 units above the *x*-axis and plot point *X*′.

STEP 2 Reflect point *Y*.

Point *Y* is 1 unit below the *x*-axis. Count 1 unit above the *x*-axis and plot point *Y*′.

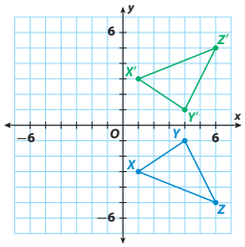
STEP 3 Reflect point *Z*.

Point *Z* is 5 units below the *x*-axis. Count 5 units above the *x*-axis and plot point *Z*′.



STEP 4 Connect *X*′, *Y*′, and *Z*′ to form triangle *X*′*Y*′*Z*′.

Each vertex of the image is the same distance from the *x*-axis as the corresponding vertex in the original figure.



My Notes

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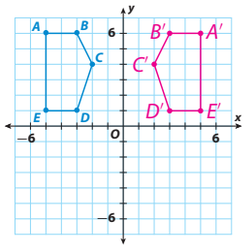
YOUR TURN

4. The figure shows pentagon *ABCDE*. Graph the image of the pentagon after a reflection across the *y*-axis.



Guided Practice

1. **Vocabulary** A reflection is a transformation that flips a figure across a line called the line of reflection.

2. The figure shows trapezoid *ABCD*. (Explore Activities 1 and 2 and Example 1)

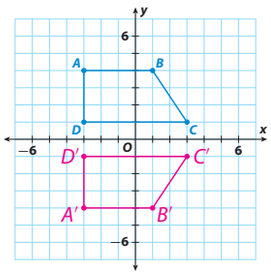
   a. Graph the image of the trapezoid after a reflection across the *x*-axis. Label the vertices of the image.

   b. How do trapezoid *ABCD* and trapezoid *A′B′C′D′* compare?

   They are congruent.

   c. **What If?** Suppose you reflected trapezoid *ABCD* across the *y*-axis. How would the orientation of the image of the trapezoid compare with the orientation of the preimage?

   The orientation would be reversed horizontally. That is, the figure from left to right in the preimage would match the figure from right to left in the image.



ESSENTIAL QUESTION CHECK-IN

3. What are the properties of reflections?

Sample answer: Reflections preserve size and shape but not orientation.

DIFFERENTIATE INSTRUCTION

Cooperative Learning

Provide each student with a full sheet of grid paper. Have each student fold their paper into quarters along grid lines. They should mark the fold lines as the *x*- and *y*-axes. Each student should draw half of a face or design to the left of the *y*-axis. The right edge of the face or design should touch the *y*-axis. Students then trade with another student to complete the face or design by drawing a reflection of the drawing across the *y*-axis.

Critical Thinking

Pose this question to your students: If a cat is sitting 8 inches away from the front of a mirror, how far away will the cat's reflection appear to be for the cat? The reflection of the cat will appear to be twice the distance the cat is from the mirror, or 16 inches.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

9.2 LESSON QUIZ

CA CC **8.G.1, 8.G.3**

Graph triangle *ABC* with vertices *A*(−4, 1), *B*(−2, 1), and *C*(−1, −2) on a coordinate grid.

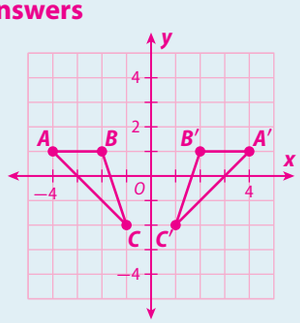
1. Graph the image of triangle *ABC* after a reflection across the *y*-axis.
2. Which side of the image is congruent to side $\overline{AB}$?
3. Which angle in the image is congruent to angle *B*?
4. If a point *M*, 5 units from the *x*-axis, is reflected across the *x*-axis, how far is the image of the point, *M*′, from the *x*-axis?
5. Angle *G* in trapezoid *FGHJ* measures 135°. Jasmine reflects the trapezoid over the *x*-axis. What is the measure of the image of angle *G*?

Lesson Quiz available online

my.hrw.com

Answers

1.



2. $\overline{A'B'}$
3. angle *B*′
4. 5 units
5. 135°

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.G.1, 8.G.1a, 8.G.1b, 8.G.1c, 8.G.3**

Concepts & Skills	Practice
Explore Activity 1 Exploring Reflections	Exercises 2, 4–5, 9
Explore Activity 2 Properties of Reflections	Exercises 2, 4–5, 7–8
Example 1 Graphing Reflections	Exercises 2, 8

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
4–5	**1** Recall of Information	**MP.2** Reasoning
6	**2** Skills/Concepts	**MP.6** Precision
7	**1** Recall of Information	**MP.3** Logic
8	**2** Skills/Concepts	**MP.6** Precision
9	**3** Strategic Thinking H.O.T.	**MP.3** Logic
10–11	**3** Strategic Thinking H.O.T.	**MP.6** Precision

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

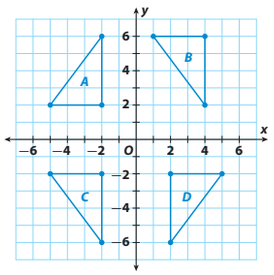
Name _____ Class _____ Date _____

9.2 Independent Practice

CA CC 8.G.1a, 8.G.1b, 8.G.1c, 8.G.3

Personal Math Trainer — Online Practice and Help — my.hrw.com

The graph shows four right triangles. Use the graph for Exercises 4–7.



4. Which two triangles are reflections of each other across the *x*-axis?

A and *C*

5. For which two triangles is the line of reflection the *y*-axis?

C and *D*

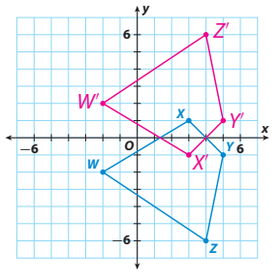
6. Which triangle is a translation of triangle *C*? How would you describe the translation?

triangle *B*; a translation of 8 units up and 6 units right

7. Which triangles are congruent? How do you know?

Sample answer: Since each triangle is either a reflection or translation of triangle *C*, they are all congruent.

8. a. Graph quadrilateral *WXYZ* with vertices *W*(−2, −2), *X*(3, 1), *Y*(5, −1), and *Z*(4, −6) on the coordinate grid.



b. On the same coordinate grid, graph quadrilateral *W′X′Y′Z′*, the image of quadrilateral *WXYZ* after a reflection across the *x*-axis.

c. Which side of the image is congruent to side $\overline{YZ}$?

$\overline{Y'Z'}$

Name three other pairs of congruent sides.

$\overline{WX}$ and $\overline{W'X'}$, $\overline{XY}$ and $\overline{X'Y'}$, $\overline{WZ}$ and $\overline{W'Z'}$

d. Which angle of the image is congruent to $\angle X$?

$\angle X'$

Name three other pairs of congruent angles.

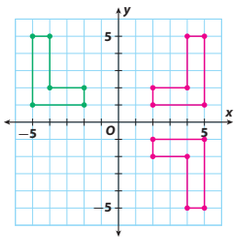
$\angle W$ and $\angle W'$, $\angle Y$ and $\angle Y'$, $\angle Z$ and $\angle Z'$

9. Critical Thinking Is it possible that the image of a point after a reflection could be the same point as the preimage? Explain.

Yes; if the point lies on the line of reflection, then the image and the preimage will be the same point.

H.O.T. FOCUS ON HIGHER ORDER THINKING

10. a. Graph the image of the figure shown after a reflection across the *y*-axis.



b. On the same coordinate grid, graph the image of the figure you drew in part **a** after a reflection across the *x*-axis.

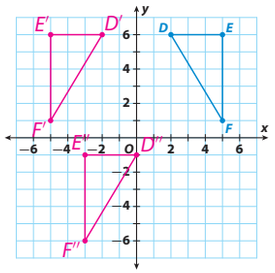
c. Make a Conjecture What other sequence of transformations would produce the same final image from the original preimage? Check your answer by performing the transformations. Then make a conjecture that generalizes your findings.

The same image can be obtained by reflecting first across the *x*-axis and then across the *y*-axis. In general, reflecting a figure first across the *y*-axis and then across the *x*-axis produces the same result as reflecting first across the *x*-axis and then across the *y*-axis.

11. a. Graph triangle *DEF* with vertices *D*(2, 6), *E*(5, 6), and *F*(5, 1) on the coordinate grid.

b. Next graph triangle *D′E′F′*, the image of triangle *DEF* after a reflection across the *y*-axis.

c. On the same coordinate grid, graph triangle *D″E″F″*, the image of triangle *D′E′F′* after a translation of 7 units down and 2 units to the right.



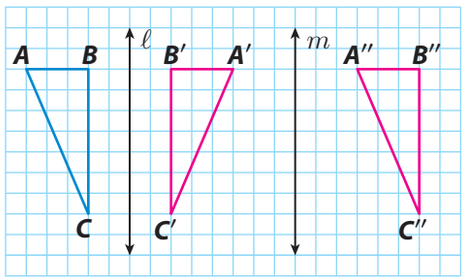
d. Analyze Relationships Find a different sequence of transformations that will transform triangle *DEF* to triangle *D″E″F″*.

Sample answer: Translate triangle *DEF* 7 units down and 2 units to the left. Then reflect the image across the *y*-axis.

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Reflect triangle *ABC* across line ℓ. Label the image triangle *A′B′C′*. Then reflect triangle *A′B′C′* across line m. Label the image triangle *A″B″C″*. What other transformation could you have performed on triangle *ABC* to get triangle *A″B″C″*? Do you think this would be true of any shape that goes through the same process?



Sample answer: A translation of triangle *ABC* could have produced *A″B″C″*. Yes, any shape would be reversed after one reflection, then reversed again to the original figure after the second reflection only if the lines of reflection are parallel.

Lesson Support

Content Objective Students will learn how to describe the properties of rotations and their effects on the congruence and orientation of figures.

Language Objective Students will describe the properties of rotations and their effects on the congruence and orientation of figures.

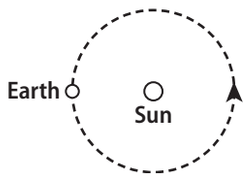
California Common Core Standards

CA CC	**8.G.1**	Verify experimentally the properties of rotations, reflections, and translations. a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
CA CC	**8.G.3**	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
CA CC	**MP.2**	Reason abstractly and quantitatively.

CA CC Focus | Coherence | Rigor

Building Background

Connecting to Everyday Life Ask students to explain what a rotation means in their own words. Then have them describe examples of rotations with which they are familiar. Possible examples include an analog clock, gears, wind and water mills, Ferris wheels, tires, planetary orbits, and Earth's rotation. Elicit the observation that all of the examples rotate about a center point, the center of rotation.



Learning Progressions

In this lesson, students verify experimentally the properties of reflections by rotating polygons in the coordinate plane around a given center of rotation. Important understandings for students include the following:

- Investigate the properties of rotations.
- Graph rotations.

Like the other rigid transformations, students learn that a rotation preserves congruence and like reflections can change orientation. They rotate figures 90°, 180°, and 270°, clockwise and counterclockwise. Students wind up their review and practice with rigid transformations in this lesson in preparation for analyzing these transformations as functions algebraically in the next lesson.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Understand congruence and similarity using physical models, transparencies, or geometry software.*** The center of rotation of a polygon can be inside the polygon. Have students graph an octogon with vertices $A(1, 3)$, $B(1, 1)$, $C(2, 0)$, $D(2, -2)$, $E(-2, -2)$, $F(-2, 0)$, $G(-1, 1)$, and $H(-1, 3)$. Ask students to rotate the octagon about the origin clockwise 90°, 180°, and 270°. Have them record the coordinates of preimage vertex A and the corresponding rotation images of A', A'', and A'''.

$A(2, 3)$, $A'(3, -2)$, $A''(-2, -3)$, $A'''(-3, 2)$

Language Support

California ELD Standards

CA ELD Emerging 2.I.12b. Selecting language resources – Use knowledge of morphology to appropriately select affixes in basic ways.

CA ELD Expanding 2.I.12b. Selecting language resources – Use knowledge of morphology to appropriately select affixes in a growing number of ways to manipulate language.

CA ELD Bridging 2.I.12b. Selecting language resources – Use knowledge of morphology to appropriately select affixes in a variety of ways to manipulate language.

Linguistic Support

Academic/Content Vocabulary

This lesson relies on students' understanding of the meanings of *rotation* and *center of rotation*. These terms are defined in the context of the sentence in which they are introduced. Have students also turn to the glossary to see the new word defined as well as a visual diagram. For a glossary resource for 13 world languages, including Spanish, be sure to take students to the Online Multilingual Glossary.

Building Background

The words *rotate, rotation*, *reflect,* and *reflections* are found throughout this lesson. Many English verbs can be turned into nouns by adding the suffix *-tion* to them. Words that end in *-tion* in English are often cognates in Spanish (*rotación*, *reflexión*).

The Prefix *counter-*

The prefix *counter-* is also found in this lesson in the word *counterclockwise*. Provide a demonstration of *clockwise* first. Then point out that *counter* is like *contra*, it means *against*.

Leveled Strategies for English Learners

Emerging When proficiency in English is limited, having students use their primary language in peer-to-peer discussion encourages higher-level thinking.

Expanding Have students at this level of English proficiency work in small groups of mixed language proficiency to list the properties of rotations.

Bridging To make sure that the nuances of language have not prevented students from understanding the concepts, have them separate the parts of the essential question to help them answer it.

Math Talk

Write out and model for students a sentence frame to begin their answer.

The orientation of the triangle is affected by _____.

LESSON

9.3 Properties of Rotations

CA Common Core Standards

The student is expected to:

CA CC Geometry—8.G.1

Verify experimentally the properties of rotations, reflections, and translations.

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure

c. Parallel lines are taken to parallel lines.

CA CC Geometry—8.G.3

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices

CA CC MP.2 Reasoning

Engage

ESSENTIAL QUESTION

How do you describe the properties of rotation and their effect on the congruence and orientation of figures? Sample answer: Rotations preserve size and shape, but change orientation.

Motivate the Lesson

Ask: What changes when you turn an object, such as a book, around a point? Does the size or shape of the object change? Begin the Explore Activity to find out how to describe this action mathematically.

Explore

EXPLORE ACTIVITY 1

Focus on Modeling Mathematical Practices

Make sure students understand that point A is the same as point A' because A lies at the center of rotation. The next Explore Activity shows a rotation where none of the vertices lie at the center of rotation, and therefore all of the vertices change position.

Explain

EXPLORE ACTIVITY 2

Connect Vocabulary EL

Emphasize that a *transformation* is a function that describes a change in the position, size, or shape of a figure, and a *rotation* is a *type* of transformation. The measures of the figure's sides and angles do not ever change in a rotation. In most rotations, the figure's position and orientation change.

Questioning Strategies Mathematical Practices

- If you were to draw segments from T to the origin and from T' to the origin, what angle would the two segments form? They would form a 180° or straight angle.
- How does the distance from the origin to T compare to the distance from the origin to T'? The distance is the same.
- Would you give the same answers to the previous two questions for each of the other vertices and their images? Yes, each pair of vertices (preimage and image) would form a 180° angle with the origin, and their distance from the origin would be the same.

Engage with the Whiteboard

Have students draw semicircular arrows, with the center of the semicircle at the origin, connecting T with T', A with A', R with R' and P with P'. Point out that in this case, counterclockwise and clockwise arrows are equally valid.

Focus on Critical Thinking

Point out to students that a clockwise rotation of 270° results in the same image as a counterclockwise rotation of 90°. Ask students to examine this claim, discuss why it is true, and justify it with a logical argument. Sample answer: Since $270° + 90° = 360°$, and a full rotation is 360°, then rotating 270° in one direction is the same as rotating 90° in the opposite direction.

LESSON 9.3

Properties of Rotations

CA CC 8.G.1c

Verify experimentally the properties of rotations, reflections, and translations: Parallel lines are taken to parallel lines. *Also 8.G.1a, 8.G.1b, 8.G.3*

ESSENTIAL QUESTION

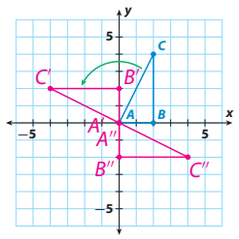
How do you describe the properties of rotation and their effect on the congruence and orientation of figures?

EXPLORE ACTIVITY 1 CA CC 8.G.1c

Exploring Rotations

A **rotation** is a transformation that turns a figure around a given point called the **center of rotation**. The image has the same size and shape as the preimage.

The triangle shown on the grid is the preimage. You will use the origin as the center of rotation.



A Trace triangle *ABC* onto a piece of paper. Cut out your traced triangle.

B Rotate your triangle 90° counterclockwise about the origin. The side of the triangle that lies along the *x*-axis should now lie along the *y*-axis.

C Sketch the image of the rotation. Label the images of points *A*, *B*, and *C* as *A′*, *B′*, and *C′*.

D Describe the motion modeled by the rotation.

Rotate 90 degrees counterclockwise about the origin.

E Check that the motion you described in **D** is the same motion that maps point *A* onto *A′*, point *B* onto *B′*, and point *C* onto *C′*.

Reflect

1. **Communicate Mathematical Ideas** How are the size and the orientation of the triangle affected by the rotation?

The size stays the same, but the orientation changes in that the triangle is turned or tilted left – what was "up" is now "left".

2. Rotate triangle *ABC* 90° clockwise about the origin. Sketch the result on the coordinate grid above. Label the image vertices *A″*, *B″*, and *C″*.

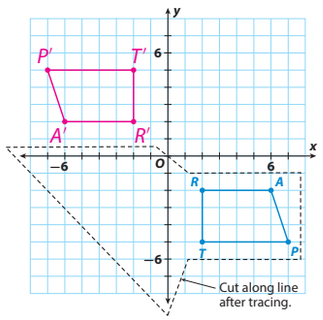
EXPLORE ACTIVITY 2 CA CC 8.G.1c

Properties of Rotations

Use trapezoid *TRAP* to investigate the properties of rotations.

A Trace the trapezoid onto a piece of paper. Include the portion of the *x*- and *y*-axes bordering the third quadrant. Cut out your tracing.

B Place your trapezoid and axes on top of those in the figure. Then use the axes to help rotate your trapezoid 180° counterclockwise about the origin. Sketch the image of the rotation of your trapezoid in this new location. Label the vertices of the image *T′*, *R′*, *A′*, and *P′*.



C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

TR = 1.3 cm RA = 1.7 cm

AP = 1.5 cm TP = 2.2 cm

D Use a ruler to measure the sides of trapezoid *T′R′A′P′* in centimeters.

$T'R'$ = 1.3 cm $R'A'$ = 1.7 cm

$A'P'$ = 1.5 cm $T'P'$ = 2.2 cm

E What do you notice about the lengths of corresponding sides of the two figures?

The lengths of corresponding sides are the same.

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T$ = 90° $m\angle R$ = 90° $m\angle A$ = 108° $m\angle P$ = 72°

G Use a protractor to measure the angles of trapezoid *T′R′A′P′*.

$m\angle T'$ = 90° $m\angle R'$ = 90° $m\angle A'$ = 108° $m\angle P'$ = 72°

H What do you notice about the measures of corresponding angles of the two figures?

The measures of corresponding angles are the same.

I Which sides of trapezoid *TRAP* are parallel? $\overline{TP}$ and $\overline{RA}$

Which sides of trapezoid *T′R′A′P′* are parallel? $\overline{T'P'}$ and $\overline{R'A'}$

What do you notice? The sides that were parallel in the preimage remain parallel in the image.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.2

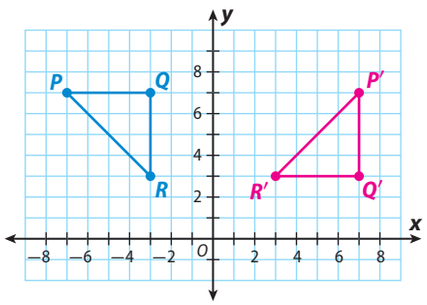
This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to make sense of relationships in a problem. Students use coordinate grids to visualize a relationship between a preimage and a rotation that results in an image. Then students use words to describe the relationship between the preimage and the image following a rotation.

Math Background

A rotation is a mathematical model of the motion of turning. It is a transformation. To rotate a figure you must be given or know three things: the center of rotation, the magnitude (number of degrees), and direction (clockwise or counterclockwise) of the rotation.

ADDITIONAL EXAMPLE 1

The figure shows triangle *PQR*. Graph the image of the triangle after a clockwise rotation of 90° about the origin.



Interactive Whiteboard
Interactive example available online

my.hrw.com

Animated Math
Explore Transformations

Using an interactive model, students explore the effect of translations, reflections, and rotations.

my.hrw.com

EXAMPLE 1

Questioning Strategies

Mathematical Practices

- About what point do you rotate triangle *ABC*? point *A*.
- Triangle *ABC* is in the first and second quadrants. In which quadrants will the image lie? Quadrants I and IV.

Engage with the Whiteboard

Have a student draw triangle *A″B″C″*, which is *A′B′C′* after a 90° clockwise rotation. Have another student draw *A‴B‴C‴* after another 90° clockwise rotation. Have students predict what another 90° rotation would produce.

YOUR TURN

Avoid Common Errors

Students often confuse clockwise and counterclockwise when performing a rotation. Draw or show the diagram below to show the meanings of the words.

Talk About It

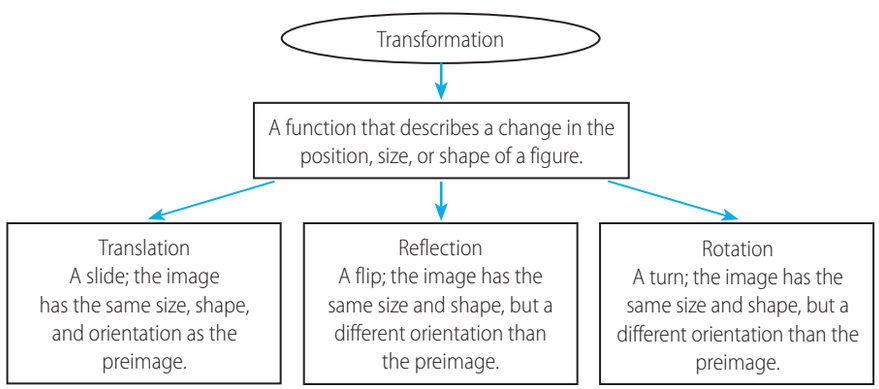
Check for Understanding

Ask: Looking at your answer to Exercise 6, what indicates that quadrilateral *ABCD* was not translated to get quadrilateral *A′B′C′D′*? The size and shape of the figures are the same, but the orientation is different.

Elaborate

Talk About It

Summarize the Lesson



GUIDED PRACTICE

Engage with the Whiteboard

For Exercises 2–5, have volunteers circle the degree, direction, and center of rotation of the figures. In Exercises 4 and 5, draw curved arrows in the direction of rotation.

Avoid Common Errors

Exercise 5 Make sure students label the image correctly. Some students may swap *B′* and *D′* out of carelessness or thinking the order of the labels doesn't matter.

Reflect

3. **Make a Conjecture** Use your results from E, H, and I to make a conjecture about rotations.

Sample answer: Rotations preserve size and shape, or congruence, but change a figure's orientation by turning it.

4. Place your tracing back in its original position. Then perform a 180° *clockwise* rotation about the origin. Compare the result with the result in B.

A 180° clockwise rotation gives the same image as a 180° counterclockwise rotation.

Graphing Rotations

To rotate a figure in the coordinate plane, rotate each of its vertices. Then connect the vertices to form the image.

Math On the Spot
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Animated Math
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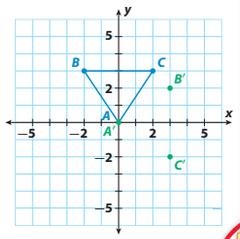
EXAMPLE 1

CA CC 8.G.3

The figure shows triangle *ABC*. Graph the image of triangle *ABC* after a rotation of 90° clockwise.

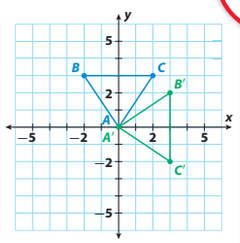


STEP 1 Rotate the figure clockwise from the *y*-axis to the *x*-axis. Point *A* will still be at (0, 0).

Point *B* is 2 units to the left of the *y*-axis, so point *B*′ is 2 units above the *x*-axis.

Point *C* is 2 units to the right of the *y*-axis, so point *C*′ is 2 units below the *x*-axis.

STEP 2 Connect *A*′, *B*′, and *C*′ to form the image triangle *A*′*B*′*C*′.



Reflect

5. Is the image congruent to the preimage? How do you know?

Sample answer: Yes, you can see from the grid squares that the side lengths and angle measures are the same.

Math Talk
Mathematical Practices
How is the orientation of the triangle affected by the rotation?

Sample answer: The triangle is turned to the right about the origin by the angle of rotation.

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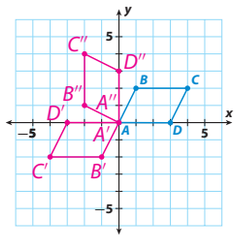
YOUR TURN

Graph the image of quadrilateral *ABCD* after each rotation.

6. 180°
7. 270° clockwise
8. Find the coordinates of Point *C* after a 90° counterclockwise rotation followed by a 180° rotation.

(2, −4)



Guided Practice

1. **Vocabulary** A rotation is a transformation that turns a figure around a given point called the center of rotation.

Siobhan rotates a right triangle 90° counterclockwise about the origin.

2. How does the orientation of the image of the triangle compare with the orientation of the preimage? (Explore Activity 1)

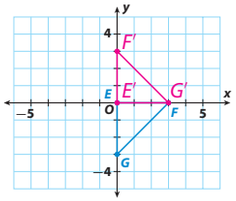
Each leg in the preimage is perpendicular to its corresponding leg in the image.

3. Is the image of the triangle congruent to the preimage? (Explore Activity 2)

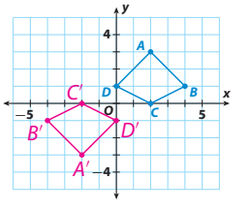
Yes, the figures are congruent.

Draw the image of the figure after the given rotation about the origin. (Example 1)

4. 90° counterclockwise



5. 180°



ESSENTIAL QUESTION CHECK-IN

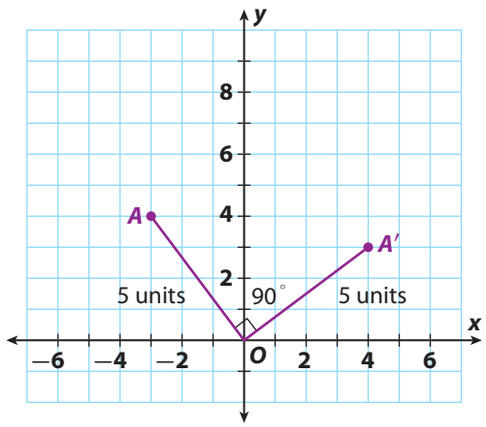
6. What are the properties of rotations?

Sample answer: Rotations preserve size and shape but change orientation.

DIFFERENTIATE INSTRUCTION

Modeling

Students who have trouble finding the location of an image after a rotation may benefit from finding the image of just one point after a rotation. Provide students with examples like the one shown here. The purple lines (which should be drawn by the students) show the angle of rotation and that the image and preimage are the same distance from the point of rotation (the origin). Point out how a 3–4–5 triangle can be used to find the distance from the origin to *A* and *A*′.



Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

9.3 LESSON QUIZ

CA CC **8.G.1, 8.G.3**

Graph triangle *ABC* with vertices *A*(−4, 1), *B*(−2, 1), and *C*(−1, −2) on a coordinate grid.

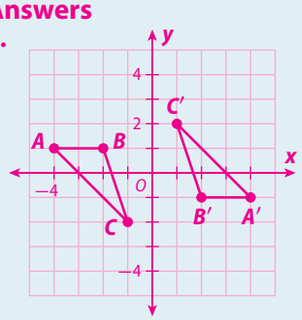
1. Graph the image of triangle *ABC* after a 180° clockwise rotation about the origin.
2. Which side of the image is congruent to side $\overline{AB}$?
3. Which angle in the image is congruent to angle *B*?
4. If a point *M*, located at (3, −2), is rotated clockwise 90° about the origin, what are the coordinates of its image, *M*′?
5. Angle *G* in trapezoid *FGHJ* measures 135°. If Lee rotates the trapezoid 270° counterclockwise about the origin, what will be the measure of angle *G*′ in the image?

Lesson Quiz available online

my.hrw.com

Answers

1.



2. $\overline{A'B'}$
3. angle *B*′
4. (−2, −3)
5. 135°

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.G.1, 8.G.1a, 8.G.1b, 8.G.1c, 8.G.3**

Concepts & Skills	Practice
Explore Activity 1 Exploring Rotations	Exercises 2, 9, 16
Explore Activity 2 Properties of Rotations	Exercises 3, 7–9
Example 1 Graphing Rotations	Exercises 4–5, 10–15

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
7–16	**2** Skills/Concepts	**MP.2** Reasoning
17	**3** Strategic Thinking H.O.T.	**MP.4** Modeling
18	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning
19	**3** Strategic Thinking H.O.T.	**MP.3** Logic

Additional Resources

Differentiated Instruction includes:

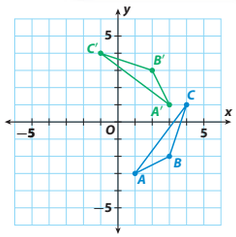
- Leveled Practice worksheets

Name _____ Class _____ Date _____

9.3 Independent Practice

CA CC 8.G.1a, 8.G.1b, 8.G.1c, 8.G.3

7. The figure shows triangle *ABC* and a rotation of the triangle about the origin.



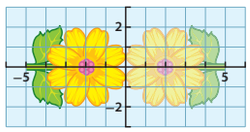
a. How would you describe the rotation?

ABC was rotated 90° counterclockwise about the origin.

b. What are the coordinates of the image?

A′ (3, 1), *B′* (2, 3), *C′* (−1, 4)

8. The graph shows a figure and its image after a transformation.



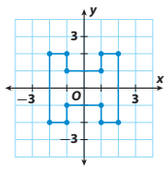
a. How would you describe this as a rotation?

The figure was rotated 180° about the origin.

b. Can you describe this as a transformation other than a rotation? Explain.

Yes, you can also describe it as a reflection across the *y*-axis.

9. What type of rotation will preserve the orientation of the H-shaped figure in the grid?



180° rotation about the origin

10. A point with coordinates (−2, −3) is rotated 90° clockwise about the origin. What are the coordinates of its image?

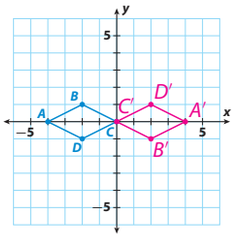
(−3, 2)

Complete the table with rotations of 180° or 90°. Include the direction of rotation for rotations of 90°.

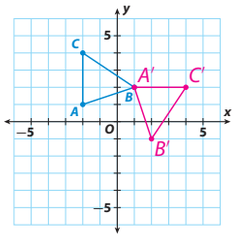
	Shape in quadrant	Image in quadrant	Rotation
11.	I	IV	90° clockwise
12.	III	I	180°
13.	IV	III	90° clockwise

Draw the image of the figure after the given rotation about the origin.

14. 180°



15. 270° counterclockwise



16. Is there a rotation for which the orientation of the image is always the same as that of the preimage? If so, what?

Yes, a 360° rotation

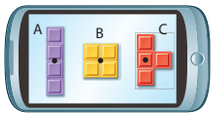
H.O.T. FOCUS ON HIGHER ORDER THINKING

17. Problem Solving Lucas is playing a game where he has to rotate a figure for it to fit in an open space. Every time he clicks a button, the figure rotates 90 degrees clockwise. How many times does he need to click the button so that each figure returns to its original orientation?

Figure A 2 times

Figure B 1 time

Figure C 4 times



18. Make a Conjecture Triangle *ABC* is reflected across the *y*-axis to form the image *A′B′C′*. Triangle *A′B′C′* is then reflected across the *x*-axis to form the image *A″B″C″*. What type of rotation can be used to describe the relationship between triangle *A″B″C″* and triangle *ABC*?

Triangle *A″B″C″* is a 180° rotation of triangle *ABC*.

19. Communicate Mathematical Ideas Point *A* is on the *y*-axis. Describe all possible locations of image *A′* for rotations of 90°, 180°, and 270°. Include the origin as a possible location for *A*.

Sample answer: If *A* is at the origin, *A′* for any rotation about the origin is at the origin. Otherwise, *A′* is on the *x*-axis for 90° and 270° rotations and on the *y*-axis for a 180° rotation.

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity The transformed image of point *A* located at (3, 3) is point *A′* located at (−3, −3). Explain how this image could be produced by a translation, a rotation, and by one or more reflections.

By a translation: move 6 units left and 6 units down.

By a rotation: rotate 180° about the origin.

By a reflection or reflections: the point is reflected across the line $y = -x$, or is reflected across the *x*-axis and the *y*-axis sequentially in either order.

Lesson Support

Content Objective Students will learn how to describe the effect of a translation, rotation, or reflection on coordinates using an algebraic representation.

Language Objective Students will demonstrate how to describe the effect of a translation, rotation, or reflection on coordinates using an algebraic representation.

California Common Core Standards

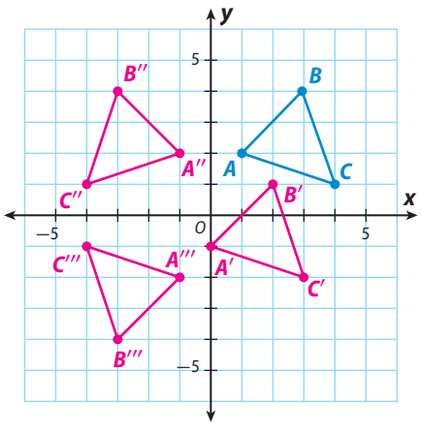
CA CC **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

CA CC **MP.3** Construct viable arguments and critique the reasoning of others.

CA CC Focus | Coherence | Rigor

Building Background

Visualizing Math Have students work in pairs. Ask them to use triangle *ABC* and perform the following transformations: translate the triangle 1 unit left and 3 units down; reflect the triangle across the *y*-axis; rotate the triangle 180°. As they perform the transformations, ask them to look for patterns in how the coordinates of each vertex in the preimage change to create the coordinates of the corresponding vertices in the image. Discuss the patterns students observe.



Learning Progressions

In this lesson, students are introduced to the rules that describe how coordinates change when a figure is transformed by a rigid transformation. Important understandings for students include the following:

- Describe the effect of a translation on the coordinates of the vertices of a geometric figure.
- Describe the effect of a reflection on the coordinates of the vertices of a geometric figure.
- Describe the effect of a rotation on the coordinates of the vertices of a geometric figure.

In earlier lessons, students may have noticed patterns in the way the coordinates of vertices change in rigid transformations. In this lesson they learn the formal rules for the transformations and apply the rules to perform given transformations. They continue to graph the preimage and image to both check and visualize the transformations.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Understand congruence and similarity using physical models, transparencies, or geometry software.*** Triangle *QRS* has vertices *Q*(2, 2), *R*(−2, 3), and *S*(−2, −2). Ask students to perform the following series of transformations, in order, on triangle *QRS*.

Rule 1: $(x, y) \rightarrow (x, -y)$
Rule 2: $(x, y) \rightarrow (-x, y)$
Rule 3: $(x, y) \rightarrow (x, -y)$
Rule 4: $(x, y) \rightarrow (-x, y)$

What is the final result of the transformations? Encourage students to verify their result by graphing the transformations. The final result is the original triangle *QRS*.

Language Support

California ELD Standards

CA ELD Emerging 2.II.3. Using verbs and verb phrases – Use a variety of verbs in different tenses and aspects appropriate for the text type and discipline on familiar topics.

CA ELD Expanding 2.II.3. Using verbs and verb phrases – Use a variety of verbs in different tenses and aspects appropriate for the task, text type, and discipline on an increasing variety of topics.

CA ELD Bridging 2.II.3. Using verbs and verb phrases – Use a variety of verbs in different tenses, aspects, voices, and moods appropriate for the task, text type, and discipline on a variety of topics.

Linguistic Support

Academic/Content Vocabulary

Following Example 2 in the lesson, there is a table that points to the coordinates of the vertices of an image when points are rotated about the origin. English learners may find this especially helpful to tab in their book or to copy into their math journals.

Building Background

Words that are spelled the same yet have different pronunciations and meanings are called *homographs*. While the word *coordinate* in this lesson refers to one of the values in an ordered pair (*x*-coordinate or *y*-coordinate), it also has another meaning and pronunciation as a verb. Discuss these different meanings, and have students add them to their word journals.

Leveled Strategies for English Learners

Emerging When proficiency in English is limited, having students use their primary language in peer-to-peer discussion encourages higher-level thinking. Have students illustrate rotating a triangle and labeling the vertices.

Expanding Working in small groups is an excellent way for English learners to deepen concept knowledge and practice the academic language and vocabulary. Have students work together to illustrate rotating a triangle and label the new *x*- and *y*-coordinates.

Bridging Have students discuss and illustrate rotating a triangle and labeling the new *x*- and *y*-coordinates and then take turns explaining how they did it.

Math Talk

Write out and model for students a sentence frame to begin their answer.

When you translate a figure to the left, you add to or subtract from the _____.

LESSON

9.4 Algebraic Representations of Transformations

CA Common Core Standards

The student is expected to:

CA CC Geometry—8.G.3

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Mathematical Practices

CA CC MP.3 Logic

Engage

ESSENTIAL QUESTION

How can you describe the effect of a translation, rotation, or reflection on coordinates using an algebraic representation? Sample answer: For a given transformation, the change in the coordinates can be described algebraically following specific rules for that transformation.

Motivate the Lesson

Ask: How can you find the coordinates of the vertices of an image after a translation, rotation, or reflection without graphing?

Explore

What are the signs of the coordinates of a point in each quadrant of the coordinate plane? How do the coordinates change as you move left and right? up and down?

Explain

EXAMPLE 1

Questioning Strategies Mathematical Practices

- Which coordinate changes and how does it change as you translate a vertex right? *x*; increase Left? *x*; decrease Up? *y*; increase Down? *y*; decrease
- How does the distance between vertices of the image change as compared to the distance between vertices in the preimage? The distances stay the same.

Focus on Modeling Mathematical Practices

Explain to students that if you are at a 2 on a horizontal number line and move 3 units to the right, you are now at 2 + 3 or 5. A move left of 3 units moves you to 2 − 3 or −1. On a vertical number line, you add when moving up and subtract when moving down.

YOUR TURN

Avoid Common Errors

Students may make changes to the wrong coordinate. Help them understand that a change to the left or right affects the *x*-coordinate, and a change up or down affects the *y*-coordinate.

EXAMPLE 2

Questioning Strategies

- In a reflection over the *x*- or *y*-axis, what are the only ways in which the coordinates will change? One of the coordinates will be multiplied by −1.
- Would a translation to the right by 5 units produce the same transformation? No; the labels on the vertices would be different if the rectangle was translated.

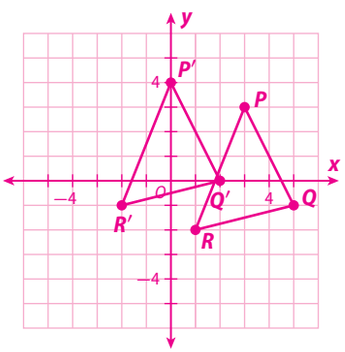
Engage with the Whiteboard

Extend the table by two columns, and use the coordinate plane to step through and graph a reflection of *RSTU* across the *x*-axis.

ADDITIONAL EXAMPLE 1

Triangle *PQR* has vertices *P* (3, 3), *Q* (5, −1), and *R* (1, −2). Find the vertices of triangle *P'Q'R'* after a translation of 3 units to the left and 1 unit up. Then graph the triangle and its image.

P'(0, 4), *Q'*(2, 0), and *R'*(−2, −1)



Interactive Whiteboard
Interactive example available online

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LESSON 9.4

Algebraic Representations of Transformations

CA CC 8.G.3
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

ESSENTIAL QUESTION

How can you describe the effect of a translation, rotation, or reflection on coordinates using an algebraic representation?

Algebraic Representations of Translations

The rules shown in the table describe how coordinates change when a figure is translated up, down, right, and left on the coordinate plane.

Math On the Spot
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Translations

Right a units	Add a to the x-coordinate: $(x, y) \rightarrow (x + a, y)$
Left a units	Subtract a from the x-coordinate: $(x, y) \rightarrow (x - a, y)$
Up b units	Add b to the y-coordinate: $(x, y) \rightarrow (x, y + b)$
Down b units	Subtract b from the y-coordinate: $(x, y) \rightarrow (x, y - b)$

EXAMPLE 1

CA CC 8.G.3

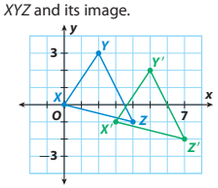
Triangle XYZ has vertices $X(0, 0)$, $Y(2, 3)$, and $Z(4, -1)$. Find the vertices of triangle $X'Y'Z'$ after a translation of 3 units to the right and 1 unit down. Then graph the triangle and its image.

STEP 1 Apply the rule to find the vertices of the image.

Add 3 to the x-coordinate of each vertex and subtract 1 from the y-coordinate of each vertex.

Vertices of $\triangle XYZ$	Rule: $(x + 3, y - 1)$	Vertices of $\triangle X'Y'Z'$
$X(0, 0)$	$(0 + 3, 0 - 1)$	$X'(3, -1)$
$Y(2, 3)$	$(2 + 3, 3 - 1)$	$Y'(5, 2)$
$Z(4, -1)$	$(4 + 3, -1 - 1)$	$Z'(7, -2)$

STEP 2 Graph triangle XYZ and its image.



You add to or subtract from the x-coordinate.

Math Talk
Mathematical Practices
When you translate a figure to the left or right, which coordinate do you change?

Personal Math Trainer
Online Practice and Help
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YOUR TURN

1. A rectangle has vertices at $(0, -2)$, $(0, 3)$, $(3, -2)$, and $(3, 3)$. What are the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x - 6, y - 3)$? Describe the translation.

$(-6, -5)$, $(-6, 0)$, $(-3, -5)$, and $(-3, 0)$; the rectangle is translated 6 units to the left and 3 units down.

Math On the Spot
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Algebraic Representations of Reflections

The signs of the coordinates of a figure change when the figure is reflected across the x-axis and y-axis. The table shows the rules for changing the signs of the coordinates after a reflection.

Reflections

Across the x-axis	Multiply each y-coordinate by -1: $(x, y) \rightarrow (x, -y)$
Across the y-axis	Multiply each x-coordinate by -1: $(x, y) \rightarrow (-x, y)$

EXAMPLE 2

CA CC 8.G.3

Rectangle $RSTU$ has vertices $R(-4, -1)$, $S(-1, -1)$, $T(-1, -3)$, and $U(-4, -3)$. Find the vertices of rectangle $R'S'T'U'$ after a reflection across the y-axis. Then graph the rectangle and its image.

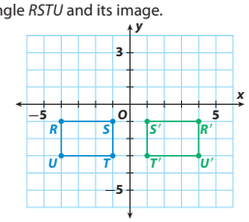
My Notes

STEP 1 Apply the rule to find the vertices of the image.

Multiply the x-coordinate of each vertex by -1.

Vertices of $RSTU$	Rule: $(-1 \cdot x, y)$	Vertices of $R'S'T'U'$
$R(-4, -1)$	$(-1 \cdot (-4), -1)$	$R'(4, -1)$
$S(-1, -1)$	$(-1 \cdot (-1), -1)$	$S'(1, -1)$
$T(-1, -3)$	$(-1 \cdot (-1), -3)$	$T'(1, -3)$
$U(-4, -3)$	$(-1 \cdot (-4), -3)$	$U'(4, -3)$

STEP 2 Graph rectangle $RSTU$ and its image.



PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.3

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to use logic to analyze situations. Students use the rules for translations, reflections, and rotations to find the vertices of the image using an algebraic representation instead of graphs. Also, students use an algebraic rule to create a graph of an image, then use the graph to describe the transformation.

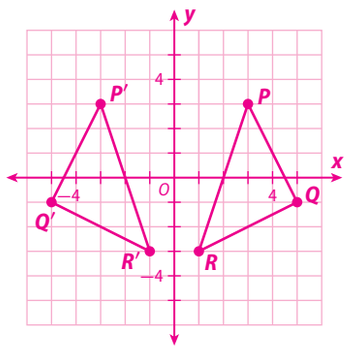
Math Background

Having a good understanding of transformations in both their graphical and algebraic representations will be beneficial for students in more advanced levels of algebra. For example, students will be using a parent parabola that has a vertex at the origin, and then translating it while maintaining its shape. Students will use the equation for the original parabola and the translation rule to write the equation of the translated parabola.

ADDITIONAL EXAMPLE 2

Triangle PQR has vertices P (3, 3), Q (5, −1), and R (1, −3). Find the vertices of triangle $P'Q'R'$ after a reflection across the y-axis. Then graph the triangle and its image.

$P'(-3, 3)$, $Q'(-5, -1)$, and $R'(-1, -3)$



Interactive Whiteboard
Interactive example available online

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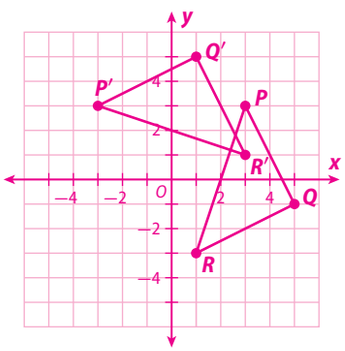
ADDITIONAL EXAMPLE 3

Triangle PQR has vertices P (3, 3), Q (5, −1), and R (1, −3). Find the vertices of triangle $P'Q'R'$ after a 90° counterclockwise rotation about the origin. Then graph the triangle and its image.

$P'(-3, 3)$, $Q'(1, 5)$, and $R'(3, 1)$



Interactive Whiteboard
Interactive example available online

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YOUR TURN

Avoid Common Errors

Make sure that students understand they do not just make the y-value a negative number. The y-value of the image must have a sign opposite that of the preimage's y-value.

EXAMPLE 3

Questioning Strategies Mathematical Practices

- In a rotation of 180°, what is the relationship between the coordinates of the preimage and the coordinates of the image? The numbers are the same, but the signs of the coordinates of the image are the opposite of the signs of the coordinates of the preimage.
- If the point (−2, 5) is rotated 90° clockwise, what are the new coordinates? (5, 2) What if (−2, 5) is rotated 180°? (2, −5)

Focus on Modeling Mathematical Practices

In Step 2, be sure students understand why D and D' are plotted at the same point.

Integrating Language Arts EL

Encourage a broad class discussion on the Reflect. English learners will benefit from hearing and participating in classroom discussions.

YOUR TURN

Avoid Common Errors

It does not matter if students multiply the y-values by −1 before or after the coordinates are switched in Exercise 4, but multiplying by −1 first may prevent errors.

Elaborate

Talk About It

Summarize the Lesson

Ask: Is it possible to determine if a translation, reflection, or rotation occurred by examining the coordinates of the image and preimage? The rules for translations, reflections, and rotations affect coordinates of the ordered pair in distinct ways, so it is often possible to determine which transformation occurred.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 2, graph a point such as (2, 2) and reflect it across the x-axis on the grid for Exercise 1.

Avoid Common Errors

Exercise 3 Remind students that they must state the number of degrees, point of rotation, and the direction of the rotation.

YOUR TURN

2. Triangle ABC has vertices $A(-2, 6)$, $B(0, 5)$, and $C(3, -1)$. Find the vertices of triangle $A'B'C'$ after a reflection across the x-axis.

$A'(-2, -6)$, $B'(0, -5)$, and $C'(3, 1)$

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Algebraic Representations of Rotations

When points are rotated about the origin, the coordinates of the image can be found using the rules shown in the table.

Rotations

90° clockwise	Multiply each x-coordinate by -1; then switch the x- and y-coordinates: $(x, y) \rightarrow (y, -x)$
90° counterclockwise	Multiply each y-coordinate by -1; then switch the x- and y-coordinates: $(x, y) \rightarrow (-y, x)$
180°	Multiply both coordinates by -1: $(x, y) \rightarrow (-x, -y)$

EXAMPLE 3

CA CC 8.G.3

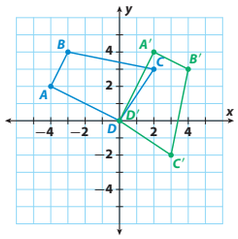
Quadrilateral $ABCD$ has vertices $A(-4, 2)$, $B(-3, 4)$, $C(2, 3)$, and $D(0, 0)$. Find the vertices of quadrilateral $A'B'C'D'$ after a 90° clockwise rotation. Then graph the quadrilateral and its image.

STEP 1 Apply the rule to find the vertices of the image.

Multiply the x-coordinate of each vertex by -1, and then switch the x- and y-coordinates.

Vertices of $ABCD$	Rule: $(y, -x)$	Vertices of $A'B'C'D'$
$A(-4, 2)$	$(2, -1 \cdot (-4))$	$A'(2, 4)$
$B(-3, 4)$	$(4, -1 \cdot (-3))$	$B'(4, 3)$
$C(2, 3)$	$(3, -1 \cdot 2)$	$C'(3, -2)$
$D(0, 0)$	$(0, -1 \cdot 0)$	$D'(0, 0)$

STEP 2 Graph the quadrilateral and its image.



Reflect

3. **Communicate Mathematical Ideas** How would you find the vertices of an image if a figure were rotated 270° clockwise? Explain.

A 270° clockwise rotation is the same as a 90° counterclockwise rotation. Change the sign of the y-coordinate and switch the coordinates.

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Online Practice and Help
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YOUR TURN

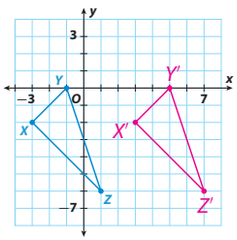
4. A triangle has vertices at $J(-2, -4)$, $K(1, 5)$, and $L(2, 2)$. What are the coordinates of the vertices of the image after the triangle is rotated 90° counterclockwise?

$J'(4, -2)$, $K'(-5, 1)$, and $L'(-2, 2)$

Guided Practice

1. Triangle XYZ has vertices $X(-3, -2)$, $Y(-1, 0)$, and $Z(1, -6)$. Find the vertices of triangle $X'Y'Z'$ after a translation of 6 units to the right. Then graph the triangle and its image. (Example 1)

$X'(3, -2)$, $Y'(5, 0)$, and $Z'(7, -6)$

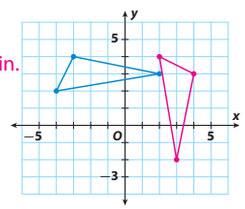


2. Describe what happens to the x- and y-coordinates after a point is reflected across the x-axis. (Example 2)

The x-coordinate remains the same, while the y-coordinate changes sign.

3. Use the rule $(x, y) \rightarrow (y, -x)$ to graph the image of the triangle at right. Then describe the transformation. (Example 3)

The triangle is rotated 90° clockwise about the origin.



ESSENTIAL QUESTION CHECK-IN

4. How do the x- and y-coordinates change when a figure is translated right a units and down b units?

The x-coordinates increase by a, and the y-coordinates decrease by b.

DIFFERENTIATE INSTRUCTION

Communicating Math

Have students work in pairs. One student provides a transformation. The other describes how the ordered pairs change using an algebraic representation.

Student 1: Translate right 3 units and up 2 units.

Student 2: Add 3 to the x-value, and add 2 to the y-value: $(x, y) \rightarrow (x + 3, y + 2)$.

Visual Cues

On three separate index cards, have students write the rules for how to change the coordinates for a figure when it is translated, rotated, and reflected. Have students use colored pencils or markers to emphasize the changes made to the x- and y-values.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

9.4 LESSON QUIZ

CA CC **8.G.3**

Triangle *ABC* has vertices *A*(−4, 1), *B*(−2, 1), and *C*(−1, −2).

1. Find the coordinates of the vertices of triangle *A′B′C′* after a 90° clockwise rotation about the origin.
2. Find the coordinates of the vertices of triangle *A′B′C′* after triangle *ABC* is reflected across the *y*-axis.
3. Find the coordinates of the vertices of triangle *A′B′C′* after triangle *ABC* is translated using the rule $(x, y) \rightarrow (x + 5, y - 3)$. Then describe the translation.
4. Point *M* has coordinates (3, −2). The coordinates of point *M′* after a single transformation are (−3, 2). Name a transformation that could have done this.

Lesson Quiz available online

my.hrw.com

Answers

1. *A′*(1, 4), *B′*(1, 2), and *C′*(−2, 1)
2. *A′*(4, 1), *B′*(2, 1), and *C′*(1, −2)
3. *A′*(1, −2), *B′*(3, −2), and *C′*(4, −5); The triangle is translated 5 units to the right and 3 units down.
4. Sample answers: translation 6 units left and 4 units up; rotation of 180°; reflection across the line $y = x$

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.G.3**

Concepts & Skills	Practice
Example 1 Algebraic Representations of Translations	Exercises 1, 5, 7, 9–11
Example 2 Algebraic Representations of Reflections	Exercises 2, 8
Example 3 Algebraic Representations of Rotations	Exercises 3, 6, 12

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
5–9	**2** Skills/Concepts	**MP.6** Precision
10	**2** Skills/Concepts	**MP.7** Using Structure
11–12	**2** Skills/Concepts	**MP.6** Precision
13	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning
14	**3** Strategic Thinking H.O.T.	**MP.3** Logic
15	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

CLUSTER CONNECTION **Exercise 13** combines concepts from the California Common Core cluster "Understand congruence and similarity using physical models, transparencies, or geometry software."

Name _____ Class _____ Date _____

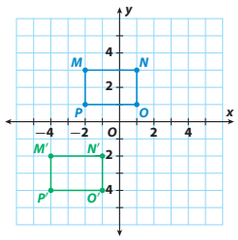
9.4 Independent Practice

CA CCS 8.G.3

Personal Math Trainer — Online Practice and Help — my.hrw.com

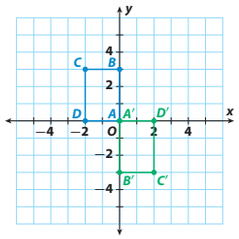
Write an algebraic rule to describe each transformation. Then describe the transformation.

5.



$(x, y) \rightarrow (x - 2, y - 5)$; translation of 2 units to the left and 5 units down

6.



$(x, y) \rightarrow (-x, -y)$; rotation of 180°

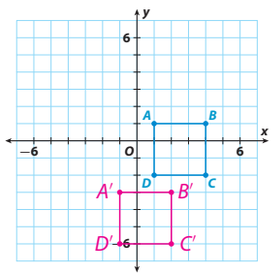
7. Triangle XYZ has vertices $X(6, -2.3)$, $Y(7.5, 5)$, and $Z(8, 4)$. When translated, X' has coordinates $(2.8, -1.3)$. Write a rule to describe this transformation. Then find the coordinates of Y' and Z'.

$(x, y) \rightarrow (x - 3.2, y + 1)$; $Y'(4.3, 6)$, $Z'(4.8, 5)$

8. Point L has coordinates $(3, -5)$. The coordinates of point L' after a reflection are $(-3, -5)$. Without graphing, tell which axis point L was reflected across. Explain your answer.

y-axis; when you reflect a point across the y-axis, the sign of the x-coordinate changes and the sign of the y-coordinate remains the same.

9. Use the rule $(x, y) \rightarrow (x - 2, y - 4)$ to graph the image of the rectangle. Then describe the transformation.

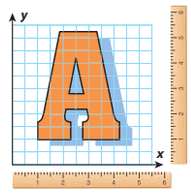


The rectangle is translated 2 units to the left and 4 units down.

10. Parallelogram $ABCD$ has vertices $A(-2, -5\frac{1}{2})$, $B(-4, -5\frac{1}{2})$, $C(-3, -2)$, and $D(-1, -2)$. Find the vertices of parallelogram $A'B'C'D'$ after a translation of $2\frac{1}{2}$ units down.

$A'(-2, -8)$, $B'(-4, -8)$, $C'\left(-3, -4\frac{1}{2}\right)$, and $D'\left(-1, -4\frac{1}{2}\right)$

11. Alexandra drew the logo shown on half-inch graph paper. Write a rule that describes the translation Alexandra used to create the shadow on the letter A.



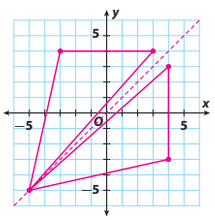
$(x, y) \rightarrow (x + 0.5, y - 0.25)$

12. Kite $KLMN$ has vertices at $K(1, 3)$, $L(2, 4)$, $M(3, 3)$, and $N(2, 0)$. After the kite is rotated, K' has coordinates $(-3, 1)$. Describe the rotation, and include a rule in your description. Then find the coordinates of L', M', and N'.

90° counterclockwise; $(x, y) \rightarrow (-y, x)$; $L'(-4, 2)$, $M'(-3, 3)$, and $N'(0, 2)$

H.O.T. FOCUS ON HIGHER ORDER THINKING

13. Make a Conjecture Graph the triangle with vertices $(-3, 4)$, $(3, 4)$, and $(-5, -5)$. Use the transformation (y, x) to graph its image.



a. Which vertex of the image has the same coordinates as a vertex of the original figure? Explain why this is true.

$(-5, -5)$; x and y are equal, so switching x and y has no effect on the coordinates.

b. What is the equation of a line through the origin and this point?

$y = x$

c. Describe the transformation of the triangle.

The triangle is reflected across the line $y = x$.

14. Critical Thinking Mitchell says the point $(0, 0)$ does not change when reflected across the x- or y-axis or when rotated about the origin. Do you agree with Mitchell? Explain why or why not.

Yes; reflecting across the x- or y-axis changes the sign of the y- or x-coordinate; 0 cannot change signs. Rotating about the origin doesn't change the origin, (0, 0).

15. Analyze Relationships Triangle ABC with vertices $A(-2, -2)$, $B(-3, 1)$, and $C(1, 1)$ is translated by $(x, y) \rightarrow (x - 1, y + 3)$. Then the image, triangle $A'B'C'$, is translated by $(x, y) \rightarrow (x + 4, y - 1)$, resulting in $A''B''C''$.

a. Find the coordinates for the vertices of triangle $A''B''C''$.

$A''(1, 0)$, $B''(0, 3)$, and $C''(4, 3)$

b. Write a rule for one translation that maps triangle ABC to triangle $A''B''C''$.

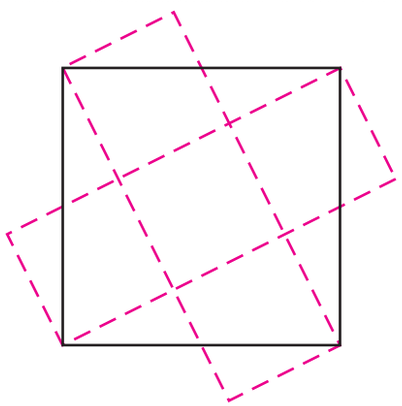
$(x, y) \rightarrow (x + 3, y + 2)$

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Transform the square shown into five smaller, but equal squares, with only four cuts. The sum of the areas of the five smaller squares must total the area of the larger square. Pieces of the larger square, produced when the four cuts are made, can be rotated, reflected, or translated and combined to form the five smaller squares.



Lesson Support

Content Objective Students will learn how transformations can be used to verify that two figures have the same shape and size.

Language Objective Students will show how transformations can be used to verify that two figures have the same shape and size.

California Common Core Standards

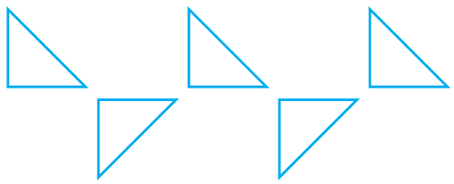
CA CC 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

CA CC MP.6 Attend to precision.

CA CC Focus | Coherence | Rigor

Building Background

Visualizing Math Have students create a pattern by using a series of rigid transformations. Ask them to draw a simple geometric figure, such as a triangle, quadrilateral, or pentagon. Then ask them to translate, reflect, or rotate the figure several times to create a pattern. It may be helpful to have students use graph paper. Encourage them to combine transformations. For example, they can create a slide image by reflecting and translating the figure successively. Discuss whether the figures in the pattern are congruent and why or why not.



Learning Progressions

In this lesson, students are introduced to a formal definition of congruence in the coordinate plane. Important understandings for students include the following:

- Combine transformations to create a congruent figure.
- Identify a sequence of transformations that create a congruent figure.

Students conclude the work with rigid transformations that they have been studying throughout this module with this lesson. They understand that two figures in the plane are congruent if one can be transformed into the other using a series of rigid transformations. They also apply their experience with the transformations to describe a sequence that will transform one figure into the other in the coordinate plane.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Understand congruence and similarity using physical models, transparencies, or geometry software.*** Have students graph triangle ABC with vertices $A(-1, 1)$, $B(-3, 2)$, and $C(-3, 1)$. Then ask them to perform the following series of transformations, in order, on triangle ABC.

(1) Rotate the triangle 90° clockwise.
(2) Reflect the rotated triangle over the x-axis.
(3) Translate the reflected triangle 3 units left and 1 unit up.

What are the coordinates of the vertices of the final image?

$(-2, 0), (-1, -2), (-2, -3)$

Language Support

California ELD Standards

CA ELD Emerging 2.II.3. Using verbs and verb phrases – Use a variety of verbs in different tenses and aspects appropriate for the text type and discipline on familiar topics.

CA ELD Expanding 2.II.3. Using verbs and verb phrases – Use a variety of verbs in different tenses and aspects appropriate for the task, text type, and discipline on an increasing variety of topics.

CA ELD Bridging 2.II.3. Using verbs and verb phrases – Use a variety of verbs in different tenses, aspects, voices, and moods appropriate for the task, text type, and discipline on a variety of topics.

Linguistic Support

Academic/Content Vocabulary

This lesson opens with an Explore Activity in which students are provided instructions for combining transformations. Understanding detailed instructions like these requires a high level of English proficiency. To assure that English learners catch the details described, you may want to form groups of students of mixed English proficiency levels to solve problems. Provide them with sentence frames to support their responses. Encourage students to use visuals and check each step as information is shared.

Rules and Patterns

When English learners read the instructions on tests and in textbooks, they encounter the imperative form or command form. For example, if the instructions tell the student *complete,* it means they are to finish out the exercise. Among the verbs in the imperative form in this lesson are *apply, label, compare, identify, graph*, and *describe*. Have English learners add these to their word journals for future reference.

Leveled Strategies for English Learners

Emerging When proficiency in English is limited, having students use their primary language in peer-to-peer discussion encourages higher-level thinking. Have students check the glossary, review the definition of *congruent figures*, and then illustrate and label an example on graph paper.

Expanding Have students at this level of English proficiency work in pairs to review the definition of *congruent figures* and then illustrate examples of a congruent figures with different transformations on graph paper.

Bridging Have students at this level of English proficiency work in pairs to review and rephrase the definition of *congruent figures* and then illustrate examples of congruent figures with different transformations on graph paper.

Math Talk

Write out and model for students a sentence frame to begin their answer.

The sequence of transformations must include a rotation when _____.

LESSON

9.5 Congruent Figures

CA Common Core Standards

The student is expected to:

CA CC Geometry—8.G.2

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Mathematical Practices

CA CC MP.6 Precision

Engage

ESSENTIAL QUESTION

How can transformations be used to verify that two figures have the same shape and size?
Sample answer: If two figures have the same shape and size, then there exists a sequence of translations, reflections, and/or rotations that transforms one into the other.

Motivate the Lesson

Ask: What effect will a combination of rotations, translations, and/or reflections have on a triangle's size and shape? Take a guess. Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY

Engage with the Whiteboard

Color coding the corresponding sides or labeling the vertices of the triangle and its images will help students better visualize the movement of the triangle.

Explain

EXAMPLE 1

Questioning Strategies Mathematical Practices

- How do you know when a rotation is part of the transformation? The image will be turned when compared to the original figure.
- In Parts B and C, why do you think the rotation is performed before the translation? Sample answer: It can be difficult to see exactly where the figure will end up after a rotation. By rotating first, you can simply translate the figure into the correct place, avoiding this issue.

Engage with the Whiteboard

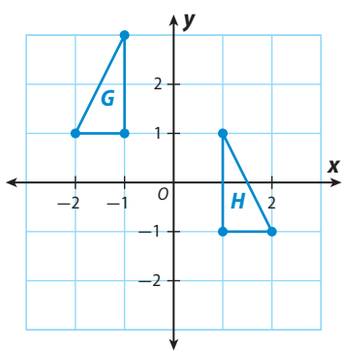
For each part, have a volunteer graph the intermediate step in producing the final image. For example, in Part A, the volunteer would graph the reflection of figure *A* over the *y*-axis before it is translated 1 unit left.

Focus on Communication Mathematical Practices

In Part C, encourage students to suggest other approaches that might map figure *D* on to figure *E*. For example, figure *D* could have been reflected over the *x*-axis, rotated 90° clockwise about the origin, and translated 1 unit down. The algebraic sequence of transformations is $(x, y) \rightarrow (-x, y)$, $(x, y) \rightarrow (y, -x)$, $(x, y) \rightarrow (x, y - 1)$.

ADDITIONAL EXAMPLE 1

A Identify a sequence of transformations that will transform figure *G* into figure *H*.



$(x, y) \rightarrow (-x, y)$, $(x, y) \rightarrow (x, y - 2)$

(Continued on page 307)

Interactive Whiteboard
Interactive example available online

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LESSON

9.5 Congruent Figures

CA CC 8.G.2

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

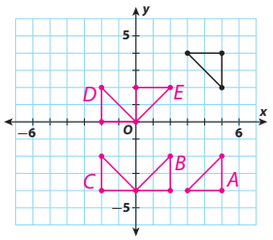
ESSENTIAL QUESTION

How can transformations be used to verify that two figures have the same shape and size?

EXPLORE ACTIVITY CA CC 8.G.2

Combining Transformations

Apply the indicated series of transformations to the triangle. Each transformation is applied to the image of the previous transformation, not the original figure. Label each image with the letter of the transformation applied.



A. Reflection across the *x*-axis

B. $(x, y) \rightarrow (x - 3, y)$

C. Reflection across the *y*-axis

D. $(x, y) \rightarrow (x, y + 4)$

E. Rotation 90° clockwise around the origin

F. Compare the size and shape of the final image to that of the original figure.

They have the same size and shape, just a different orientation.

Reflect

1. Which transformation(s) change the orientation of figures? Which do not?

Reflections and rotations; translations

2. **Make a Conjecture** After a series of transformations, two figures have the same size and shape, but different orientations. What does this indicate about the transformations?

None of the transformations had properties that effected the congruency of the figures, but at least one of the transformations was a reflection or a rotation.

Math On the Spot
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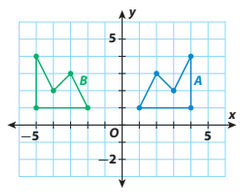
Congruent Figures

Recall that segments and their images have the same length and angles and their images have the same measure under a translation, reflection, or rotation. Two figures are said to be **congruent** if one can be obtained from the other by a sequence of translations, reflections, and rotations. Congruent figures have the same size and shape.

When you are told that two figures are congruent, there must be a sequence of translations, reflections, and/or rotations that transforms one into the other.

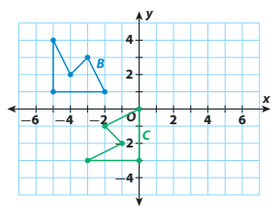
EXAMPLE 1 CA CC 8.G.2

A Identify a sequence of transformations that will transform figure *A* into figure *B*.



To transform figure *A* into figure *B*, you need to reflect it over the *y*-axis and translate one unit to the left. A sequence of transformations that will accomplish this is $(x, y) \rightarrow (-x, y)$ and $(x, y) \rightarrow (x - 1, y)$.

B Identify a sequence of transformations that will transform figure *B* into figure *C*.



Any sequence of transformations that changes figure *B* into figure *C* will need to include a rotation. A 90° counterclockwise rotation around the origin would properly orient figure *B*, but not locate it in the same position as figure *C*. The rotated figure would be 2 units below and 1 unit to the left of where figure *C* is. You would need to translate the rotated figure up 2 units and right 1 unit.

A sequence of transformations is a 90° counterclockwise rotation about the origin, $(x, y) \rightarrow (-y, x)$, followed by $(x, y) \rightarrow (x + 1, y + 2)$.

In both cases, the image is turned sideways from the original figure. Only a rotation will do this.

Math Talk Mathematical Practices

How do you know that the sequence of transformations in Parts B and C must include a rotation?

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.6

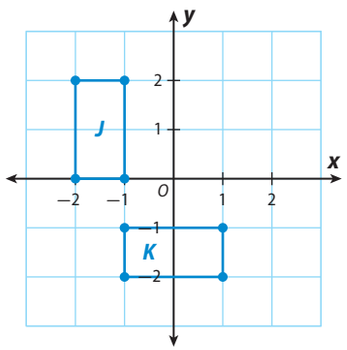
This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to attend to precision. Students pay close attention to the coordinates of the vertices of a figure in order to apply a given sequence of transformations and graph the resulting image. Each transformation must be carefully and precisely applied to obtain the desired outcome. Students also must pay close attention to the coordinates of the vertices of a figure and its images when determining the sequence of transformations that result in a figure being transformed into a particular image.

Math Background

If there exists a sequence of translations, reflections, and/or rotations that will transform one figure into the other, the two figures are congruent. Note that dilations are not included in this list of transformations. The image after a dilation is either an enlargement or reduction of the original figure, making the two figures similar but not congruent. While dilations preserve the shape of a figure, they do not preserve the size. A dilation is often called a similarity transformation.

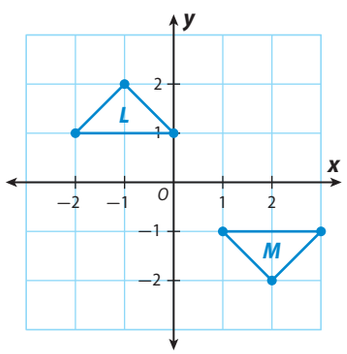
ADDITIONAL EXAMPLE 1

B Identify a sequence of transformations that will transform figure *J* into figure *K*.



$(x, y) \rightarrow (-y, x)$, $(x, y) \rightarrow (x + 1, y)$

C Identify a sequence of transformations that will transform figure *L* into figure *M*.



$(x, y) \rightarrow (-x, y)$, $(x, y) \rightarrow (x, -y)$, $(x, y) \rightarrow (x + 1, y)$

Interactive Whiteboard
Interactive example available online

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YOUR TURN

Avoid Common Errors

If students cannot visualize the sequence of transformations, suggest they use a cut-out paper triangle the same size and shape as figure *A* that they can rotate and translate on the coordinate grid.

Talk About It

Check for Understanding

Ask: How do you know if a two-dimensional figure is congruent to another? Two figures are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and translations. The two figures will have the same size and shape.

Elaborate

Talk About It

Summarize the Lesson

Ask: When two figures have different orientations, what clues help you decide which transformations were performed in the sequence of transformations? In order to get a turned image, a rotation must have occurred. In order to get a mirror image, a reflection must have occurred.

GUIDED PRACTICE

Engage with the Whiteboard

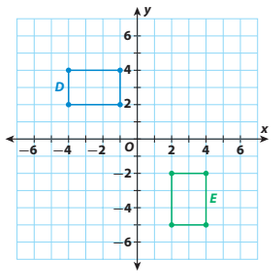
For Exercise 1, have five volunteers take turns drawing the indicated series of transformations on the coordinate grid provided.

Avoid Common Errors

Exercise 2 Students might think a rotation was used to transform figure *A* into figure *B*. Suggest students use a paper cut-out of triangle *A* and actually rotate it about point (0, 2) to see that the orientation of figure *B* is not right for the transformation to have been a rotation.

Exercise 4 Before trying to write the algebraic sequence of transformations used, suggest that students label figures *A*, *B*, and *C* with the ordered pairs for each of the vertices. Then, analyze the pairs to understand the changes to the *x* and *y* values.

C Identify a sequence of transformations that will transform figure *D* into figure *E*.



A sequence of transformations that changes figure *D* to figure *E* will need to include a rotation. A 90° clockwise rotation around the origin would result in the figure being oriented as figure *E*.

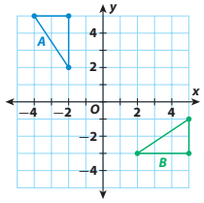
However, the rotated figure would be 6 units above where figure *E* is. You would need to translate the rotated figure down 6 units.

A sequence of transformations is a 90° clockwise rotation about the origin, $(x, y) \rightarrow (y, -x)$, followed by $(x, y) \rightarrow (x, y - 6)$.

My Notes

YOUR TURN

3. Identify a sequence of transformations that will transform figure *A* into figure *B*.



Rotation 90° clockwise about origin, translation 5 units down; $(x, y) \rightarrow (y, -x)$, $(x, y) \rightarrow (x, y - 5)$

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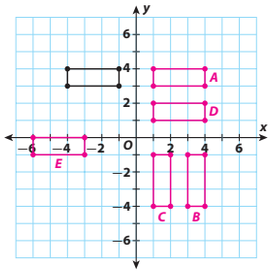
Guided Practice

1. Apply the indicated series of transformations to the rectangle. Each transformation is applied to the image of the previous transformation, not the original figure. Label each image with the letter of the transformation applied. (Explore Activity)



A. Reflection across the *y*-axis

B. Rotation 90° clockwise around the origin

C. $(x, y) \rightarrow (x - 2, y)$

D. Rotation 90° counterclockwise around the origin

E. $(x, y) \rightarrow (x - 7, y - 2)$

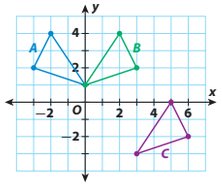
Identify a sequence of transformations that will transform figure *A* into figure *C*. (Example 1)

2. What transformation is used to transform figure *A* into figure *B*?

reflection across the *y*-axis

3. What transformation is used to transform figure *B* into figure *C*?

translation 3 units right and 4 units down



4. What sequence of transformations is used to transform figure *A* into figure *C*? Express the transformations algebraically.

$(x, y) \rightarrow (-x, y)$, $(x, y) \rightarrow (x + 3, y - 4)$

5. **Vocabulary** What does it mean for two figures to be congruent?

The figures have the same size and the same shape.

ESSENTIAL QUESTION CHECK-IN

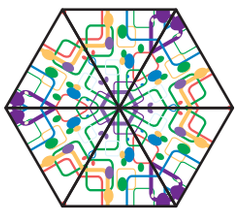
6. After a sequence of translations, reflections, and rotations, what is true about the first figure and the final figure?

They have the same size and the same shape. (They are congruent.)

DIFFERENTIATE INSTRUCTION

Multiple Representations

Provide students with 6 congruent equilateral triangles. Have students arrange the triangles to form a hexagon. Students should then draw a simple but colorful design on one of the triangles and then reflect that design around the hexagon 5 times onto the other triangles. Explain that their final design will be a kaleidoscope image, as shown here.



Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

9.5 LESSON QUIZ

CA CC **8.G.2**

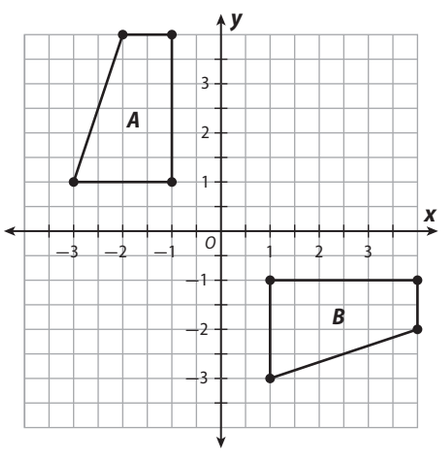
1. On a coordinate grid, graph a triangle with its vertices at (−2, 1), (−4, 1), and (−1, 4). Then apply the indicated series of transformations to the triangle. Each transformation is applied to the image of the previous transformation. Label each image with the letter of the transformation applied.

A Rotation 90° clockwise around the origin

B $(x, y) \rightarrow (x, y - 3)$

C $(x, y) \rightarrow (x - 3, y - 2)$

2. Identify a sequence of transformations that will transform figure *A* into figure *B*.



Lesson Quiz example available online

my.hrw.com

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.G.2**

Concepts & Skills	Practice
Explore Activity Combining Transformations	Exercises 1, 7–10, 12
Example 1 Congruent Figures	Exercises 2–5, 11–12

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
7–10	**1** Recall of Information	**MP.4** Modeling
11	**2** Skills/Concepts	**MP.4** Modeling
12	**2** Skills/Concepts	**MP.3** Logic
13	**3** Strategic Thinking H.O.T.	**MP.3** Logic
14	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning

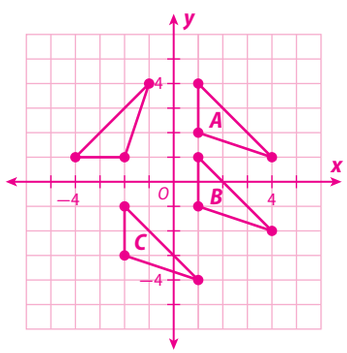
Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

Answers

1.



2. Reflection across the *y*-axis, rotation 90° clockwise about the origin; $(x, y) \rightarrow (-x, y)$, $(x, y) \rightarrow (y, -x)$

Name _____ Class _____ Date _____

9.5 Independent Practice

CA CC 8.G.2

Personal Math Trainer — Online Practice and Help — my.hrw.com

For each given figure *A*, graph figures *B* and *C* using the given sequence of transformations. State whether figures *A* and *C* have the same or different orientation.

7.

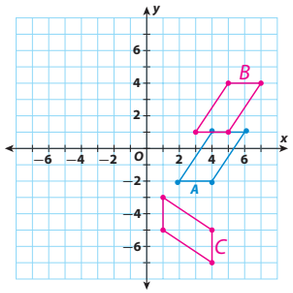


Figure *B*: a translation of 1 unit to the right and 3 units up

Figure *C*: a 90° clockwise rotation around the origin

Different orientation

8.

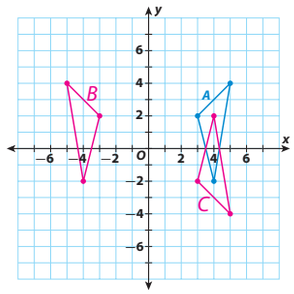


Figure *B*: a reflection across the *y*-axis

Figure *C*: a 180° rotation around the origin

Different orientation

9.

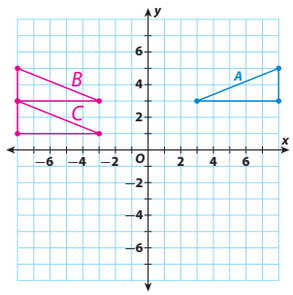


Figure *B*: a reflection across the *y*-axis

Figure *C*: a translation 2 units down

Different orientation

10.

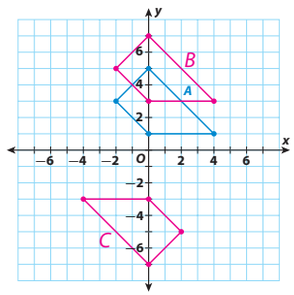
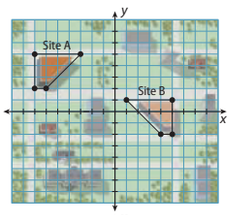


Figure *B*: a translation 2 units up

Figure *C*: a rotation of 180° around the origin

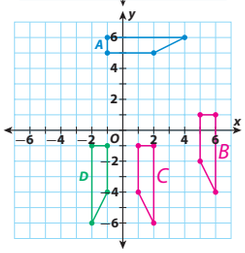
Different orientation

11. Represent Real-World Problems A city planner wanted to place the new town library at site *A*. The mayor thought that it would be better at site *B*. What transformations were applied to the building at site *A* to relocate the building to site *B*? Did the mayor change the size or orientation of the library?



Sample answer: translation 2 units right and 4 units down, reflection across *y*-axis; size: no; orientation: yes

12. Persevere in Problem Solving Find a sequence of three transformations that can be used to obtain figure *D* from figure *A*. Graph the figures *B* and *C* that are created by the transformations.



Sample answer: Figure *B*: rotation 90° clockwise around origin; figure *C*: translation 4 units left and 2 units down; figure *D*: reflection across *y*-axis

H.O.T. FOCUS ON HIGHER ORDER THINKING

13. Counterexamples The Commutative Properties for Addition and Multiplication state that the order of two numbers being added or multiplied does not change the sum or product. Are translations and rotations commutative? If not, give a counterexample.

No; the point (1, 2), translated 2 units to the right, becomes (3, 2), then rotated 90° around the origin it becomes (2, −3). The point (1, 2) rotated 90° around the origin becomes (2, −1), then translated 2 units to the right it becomes (4, −1), which is not the same.

14. Multiple Representations For each representation, describe a possible sequence of transformations.

a. $(x, y) \rightarrow (-x - 2, y + 1)$

Sample answer: translation 2 units right and 1 unit up, reflection across *y*-axis

b. $(x, y) \rightarrow (y, -x - 3)$

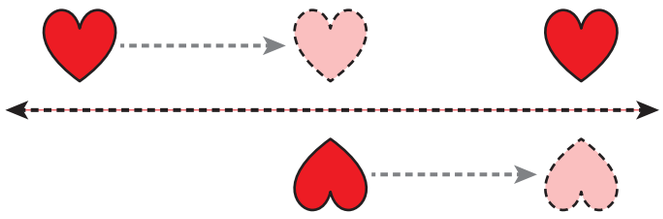
Sample answer: rotation 90° clockwise around the origin, translation 3 units down

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity A translation followed by a reflection about a line that is parallel to the line of translation is called a glide reflection. The heart shown below has been glided and reflected twice.



Draw a horizontal line on a piece of paper, choose a shape, and make a glide reflection pattern.

Ready to Go On?

Assess Mastery

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

RtI Response to Intervention

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Intervention	Enrichment

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

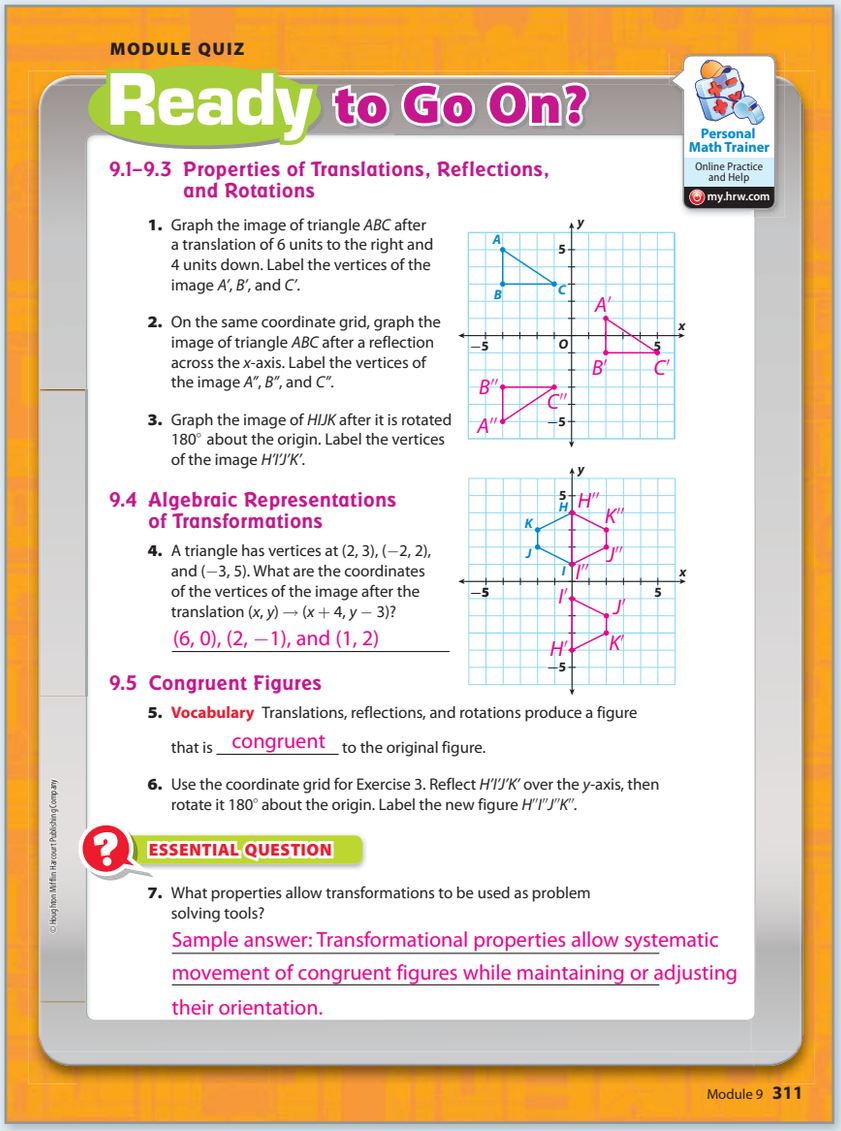
Differentiated Instruction
- Reteach worksheets
- Reading Strategies EL
- Success for English Learners EL

Differentiated Instruction
- Challenge worksheets PRE-AP

Extend the Math PRE-AP Lesson Activities in TE

Additional Resources

Assessment Resources includes:
- Leveled Module Quizzes



MODULE QUIZ

Ready to Go On?

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9.1–9.3 Properties of Translations, Reflections, and Rotations

1. Graph the image of triangle *ABC* after a translation of 6 units to the right and 4 units down. Label the vertices of the image *A′*, *B′*, and *C′*.

2. On the same coordinate grid, graph the image of triangle *ABC* after a reflection across the *x*-axis. Label the vertices of the image *A″*, *B″*, and *C″*.

3. Graph the image of *HIJK* after it is rotated 180° about the origin. Label the vertices of the image *H′I′J′K′*.

9.4 Algebraic Representations of Transformations

4. A triangle has vertices at (2, 3), (−2, 2), and (−3, 5). What are the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x + 4, y - 3)$?
(6, 0), (2, −1), and (1, 2)

9.5 Congruent Figures

5. **Vocabulary** Translations, reflections, and rotations produce a figure that is congruent to the original figure.

6. Use the coordinate grid for Exercise 3. Reflect *H′I′J′K′* over the *y*-axis, then rotate it 180° about the origin. Label the new figure *H″I″J″K″*.

ESSENTIAL QUESTION

7. What properties allow transformations to be used as problem solving tools?
Sample answer: Transformational properties allow systematic movement of congruent figures while maintaining or adjusting their orientation.

Module 9 311

California Common Core Standards

Lesson	Exercises	Common Core Standards
9.1	1	**8.G.1, 8.G.3**
9.2	2	**8.G.1, 8.G.3**
9.3	3	**8.G.1, 8.G.3**
9.4	4	**8.G.3**
9.5	5–6	**8.G.2**

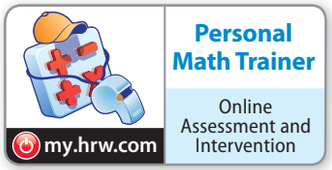
Assessment Readiness

Scoring Guide

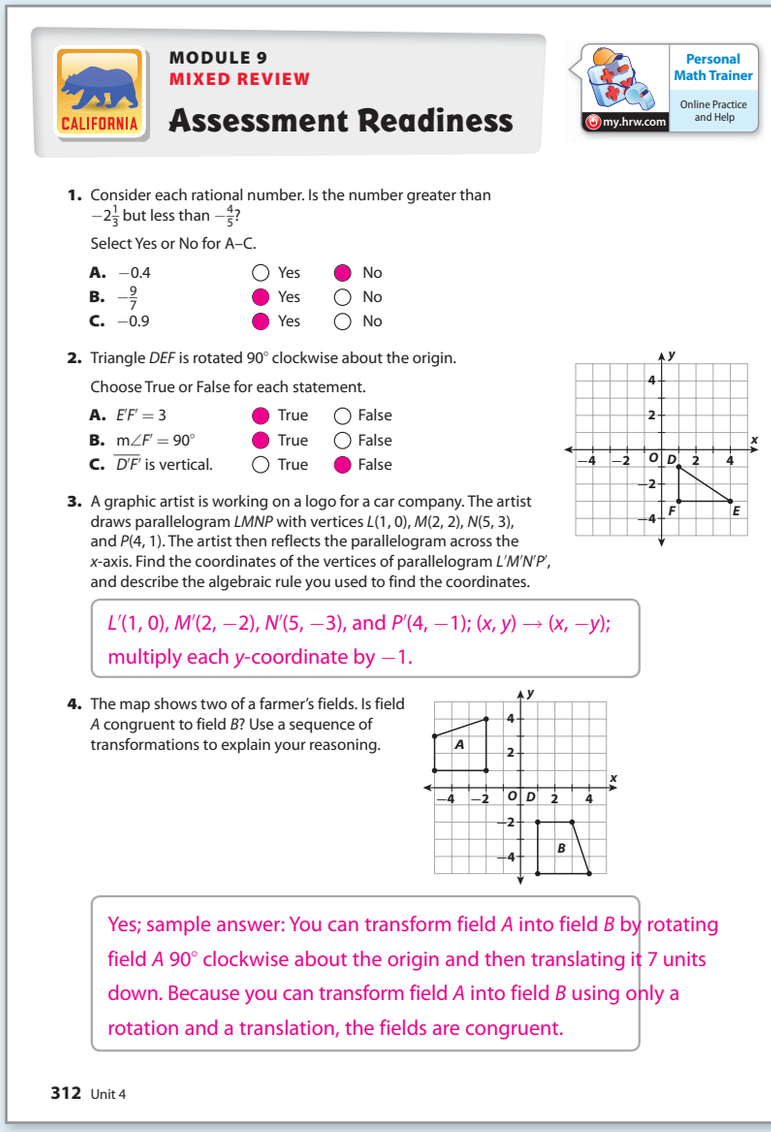
Item 3 Award the student 1 point for finding the coordinates of the vertices of parallelogram *L′M′N′P′* and 1 point for describing the algebraic rule used to find the coordinates.

Item 4 Award the student 1 point for stating that the fields are congruent and 1 point for explaining how a sequence of transformations shows that the fields are congruent.

Additional Resources



To assign this assessment online, login to your Assignment Manager at **my.hrw.com**.



MODULE 9
MIXED REVIEW

Assessment Readiness

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1. Consider each rational number. Is the number greater than $-2\frac{1}{3}$ but less than $-\frac{4}{5}$?

Select Yes or No for A–C.

A. -0.4	○ Yes	● No
B. $-\frac{9}{7}$	● Yes	○ No
C. -0.9	● Yes	○ No

2. Triangle *DEF* is rotated 90° clockwise about the origin.

Choose True or False for each statement.

A. $E'F' = 3$	● True	○ False
B. $m\angle F' = 90°$	● True	○ False
C. $\overline{D'F'}$ is vertical.	○ True	● False

3. A graphic artist is working on a logo for a car company. The artist draws parallelogram *LMNP* with vertices *L*(1, 0), *M*(2, 2), *N*(5, 3), and *P*(4, 1). The artist then reflects the parallelogram across the *x*-axis. Find the coordinates of the vertices of parallelogram *L′M′N′P′*, and describe the algebraic rule you used to find the coordinates.

L′(1, 0), *M′*(2, −2), *N′*(5, −3), and *P′*(4, −1); (*x*, *y*) → (*x*, −*y*); multiply each *y*-coordinate by −1.

4. The map shows two of a farmer's fields. Is field *A* congruent to field *B*? Use a sequence of transformations to explain your reasoning.

Yes; sample answer: You can transform field *A* into field *B* by rotating field *A* 90° clockwise about the origin and then translating it 7 units down. Because you can transform field *A* into field *B* using only a rotation and a translation, the fields are congruent.

312 Unit 4

California Common Core Standards

Items	Grade 8 Standards	Mathematical Practices
1*	8.NS.2	MP.2
2	8.G.1, 8.G.3	MP.5, MP.6
3	8.G.3	MP.6
4	8.G.2, 8.G.3	MP.3, MP.4

* Item integrates mixed review concepts from previous modules or a previous course.

CLUSTER CONNECTION

Item 4 combines concepts from the California Common Core cluster "Understand congruence and similarity using physical models, transparencies, or geometry software."