TRANSFORMERS

A. PREPARATION

1. *Introduction*
2. *Physics of Coupled Networks*
3. *Magnetic Materials*
4. *A Network Approach*
5. *Core Loss and Jordan-type Loss Coefficients*
6. *The Equivalent Circuit of a Real Transformer at Low Frequency*
7. *Simple Open-circuit and Short-circuit Tests*
8. *Three Phase Transformers*
9. *Autotransformers*
10. *Practical Considerations*
11. *Standard (ANSI C57.12.90-1973) Electrical Tests*
12. *References*

B. EXPERIMENT

1. *Equipment List*
2. *The Single Phase Transformer*
    a. Winding resistances
    b. Open circuit test
    c. Short circuit test
3. *Maximum Power Transfer*
    a. Maximum Power Transfer Circuit
    b. Load Resistance
4. *Report*

II. TRANSFORMERS

A. *PREPARATION*

1. *Introduction*

The device which more than any other has made possible the long distance, large-scale transmission of electric power and the modern power grid which supplies thousands of square kilometers with power at a wide variety of different voltages[1] is the transformer by means of which the current and voltage levels characterizing a given power flow in a circuit are readily traded off against each other. Thus, a given amount of energy can be shipped at high voltage and low current (low ohmic loss along the line) and used at low voltage and high current (high consumer convenience).

It is just this flexibility provided by the transformer which gave alternating current its big initial edge over direct current, for only an alternating voltage will work in a transformer. It was the engineering realities of transformer action

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[1]Union Electric here in St. Louis offers, for example, the following standard hookups. Local: 120/240 1ϕ, 240 delta, 120/208 wye, 480 delta, 277/480 wye. Primary service: 4160 delta (mostly inside Lindbergh), 12470 delta (mostly outside Lindbergh). Subtransmission: 34.5 kV delta. Transmission: 138 kV delta.

which were the principal determinants of the world's choice of power line frequencies (50 Hz or 60 Hz). And it is the existence and utility of the transformer principle which have made induction motors the end users of over half the electric power generated in the United States. In short, if you don't understand transformers you don't understand electric power.

Unfortunately, within the understanding of transformers, there are more layers than in an onion's skin; and if you peel them all away in an effort to get at the heart of the matter you end up with lots of individual isolated bits of information and little grasp of the whole. We shall therefore proceed as simply as possible to discuss the physics of magnets (leaving details to a course on electromagnetic theory), the special properties of iron (leaving details to a course on magnetic materials), the basic equivalent circuits for the transformer (leaving details to a course in network analysis), and the fundamentals of transformer losses (leaving details to a course in solid state physics) so that we can describe some few practical properties of and tests upon real power transformers. Naturally, we can not begin to offer even a comprehensive introduction to the arcana of real power transformers. But we can perhaps (Indeed, it is our goal.) provide enough background to enable the student to meet simple challenges on his own and to delve successfully into the more advanced literature including that bible of the transformer world the J&P Transformer Book (Stigant and Franklin, 1973).

2. *Physics of Coupled Networks*

Consider the physical realization of a single mesh circuit of arbitrary geometric form. And in particular let it be assumed that all conducting regions (wires, plates, etc.) are uninsulated (i.e., not embedded in a dielectric). Then dip the mesh into a solution of suitable properties (e.g. soapy) so that, when it is withdrawn, a visible film forms defining a surface S bounded by the conducting mesh $\vec{s}$; let $\hat{n}$ be the right-handed normal to S . When this film covered mesh is placed in a time varying magnetic field $\vec{B}$ $\left[W/m^2\right]$, it is possible to define a quantity Φ [Wb] called the "flux" such that

$$\Phi = \int_S \hat{n} \cdot \vec{B} \, dS \, , \qquad (2.1)$$

where dS $\left[m^2\right]$ is an element of area in S . Faraday's Law then states that the time rate of change of the flux induces a voltage v_{ind} [V] in the mesh such that (*cf*. King, 1963), at sufficiently low frequencies,

$$v_{ind} = - \frac{\partial \Phi}{\partial t} \, , \qquad (2.2)$$

where v_{ind} is measured by circling the mesh in a counterclockwise direction.

However, since the electric scalar potential of a network is a single-valued function of position, a counterclockwise circling of the mesh can yield no *net* voltage change and v_{ind} must therefore be balanced by a compensating voltage drop due, for example, to resistance. That is, v_{ind} will cause a current to flow in the mesh.

Consider, on the other hand, a closed contour s of arbitrary geometric form winding about in a region of current density $\vec{J}$. If the $\vec{H}$-vector[2] in the region (related to $\vec{B}$ by way of magnetization $\vec{M}$ [A/m]) is given by

$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M} , \qquad (2.3)$$

where μ_0 [$\equiv 4\pi \times 10^{-7}$ H/m] is the permeability of free space, then the low frequency form of Ampere's Law becomes approximately (*cf.* King, 1963)

$$\oint_s \vec{H} \cdot d\vec{s} = \int_S \hat{n} \cdot \vec{J} \, d S , \qquad (2.4)$$

where $d\vec{s}$ is a counterclockwise-directed infinitesimal along s . What this in effect says is that current flow can produce magnetic field.

To examine these concepts more deeply, consider a planar loop of some simple shape carrying a uniform counterclockwise current i(t) [A] and assume that the loop is embedded in free

[2]The magnetic B-vector is sometimes called the *flux density*, and the magnetic H-vector is sometimes called the *field strength*.

space. If one then applies to an element of this loop the consideration of Eq. (2.4) in the form of the right hand rule, it follows that within the plane of the loop

$$\vec{B}_{loop} = \hat{z}\mu_0 i(t)\beta \tag{2.5}$$

where the positive quantity $\beta \left[m^{-1}\right]$ is a function of (i) loop geometry and (ii) position within the loop. Thus, by Eqs. (2.1) and (2.5),

$$\Phi_{loop} = L_{loop} i(t) \tag{2.6}$$

where L_{loop} [H] is a positive constant called the *inductance* which depends solely upon loop geometry. Thus, the flow of the current $i(t)$ will set up an induced voltage given by Eq. (2.2) so that Kirchhoff's current law yields

$$0 = v_{gen}(t) - L_{loop}\frac{di}{dt} - iR_{loop} \tag{2.7}$$

where v_{gen} [V] is the voltage of the generator driving i and R_{loop} [Ω] is the loop resistance. Clearly, the net effect is for the magnetic field generated by the current i to oppose changes in i.

Further insight can be gained by considering two arbitrary coils, each of which is carrying a current. The net induced voltage in coil-1 will be

$$v_{ind,\,1} = -\frac{\partial\Phi_1}{\partial t} = -\left[\frac{\partial\Phi_{11}}{\partial t} + \frac{\partial\Phi_{12}}{\partial t}\right]$$

$$= -\left[L_{11}\frac{\partial i_1}{\partial t} + L_{12}\frac{\partial i_2}{\partial t}\right] \qquad (2.8a)$$

and that in coil-2 will be

$$v_{ind,\,2} = -\left[L_{21}\frac{\partial i_1}{\partial t} + L_{22}\frac{\partial i_2}{\partial t}\right]. \qquad (2.8b)$$

That is, spatially adjacent circuits can interact by virtue of the magnetic fields which their currents establish.

3. *Magnetic Materials*

In the derivation leading to Eqs. (2.8) a most important difficulty was evaded: the Ampere relation of Eq. (2.4) explained the fashion in which the manetizing current density gave rise to the magnetic H-vector, whereas the Faraday relation of Eq. (2.2) explained the induced voltage in terms of the magnetic B-vector; and, as Eq. (2.3) reveals, the two are related by an as yet undefined quantity called the magnetization.

Magnetization can be explained as follows. What sets up a magnetic field is the circulation of current. Clearly, each atom has, by virtue of its structure, a circulating current associated with it. This current may be due either to orbital electron motion or to electron spin with electron spin dominating strongly in the ferrous materials used in electric power applications. Each such miniscule current contributes little to the magnetic H-vector. However, it is possible for groups of adjacent atoms to align to form extensive regions (called *domains*) where the contributions to $\vec{M}$ add and the domain becomes in effect a small magnet.

Consider now the case of an unmagnetized piece of material which is placed in the region of influence of a current carrying wire. An $\vec{H}$ will be set up in the material. This field will in turn align to a degree the magnetic movements of that material's domains so that not only $\vec{H}$ but also $\vec{M}$ will be monotone functions of the current. That is, from Eq. (2.5)

$$\vec{B} = \mu_0 [\vec{H} + \vec{M}] \tag{2.9}$$

and $\vec{M} = \chi \vec{H}$ where χ [dimensionless] is the susceptibility. Now $\vec{M}$ is due to alignment of intrinsic magnetic moments in the material; and, as there are only so many of these, there is a maximum magnetization such that

$$\lim_{|\vec{H}| \to \infty} \vec{M} = M_{sat} \frac{\vec{H}}{|\vec{H}|} \tag{2.10}$$

Hence, as shown in Fig.2.1, $\vec{B}$, at first, rises linearly in $\vec{H}$ and eventually rolls off into the asymptotic regime $\partial\vec{B}/\partial\vec{H} = \mu_0$; a quantity called the *permeability* of the medium is commonly defined as

$$\mu = [\partial \vec{B} / \partial \vec{H}\,]_{\vec{H}} \ , \tag{2.11}$$

and clearly varies with field strength. Suppose next that the H-vector is reduced. It is generally found, as illustrated in Fig. 2.1, that the 'normal" or "virgin" curve (i.e., initial path) is not retraced but that instead the domains resist reorientation and a so-called *hysteresis loop*[3] results (cf. Olsen, 1966; Watson, 1980); *it must be emphasized that the hysteresis loop is not a unique entity but that as* $|\vec{H}|$ *is -swept slowly from* 0 *to some* H_+ *and then back to some* H_- *and is then cycled between* H_- *and* H_+ *, a different loop is generated for each ordered pair* (H_-, H_+). Moreover, the qualitative shape of the hysteresis loop can and does vary greatly from one material to another, as the curves of Fig. 2.2 illustrate. Some few descriptive terms may be useful:

(α) A loop shaped like that of Fig. 2.1 is called "normal". If the loop is of small relative area (say $B_rH_c << (\mu_0 M_{sat})H_\sigma$), the material is said to be "soft";

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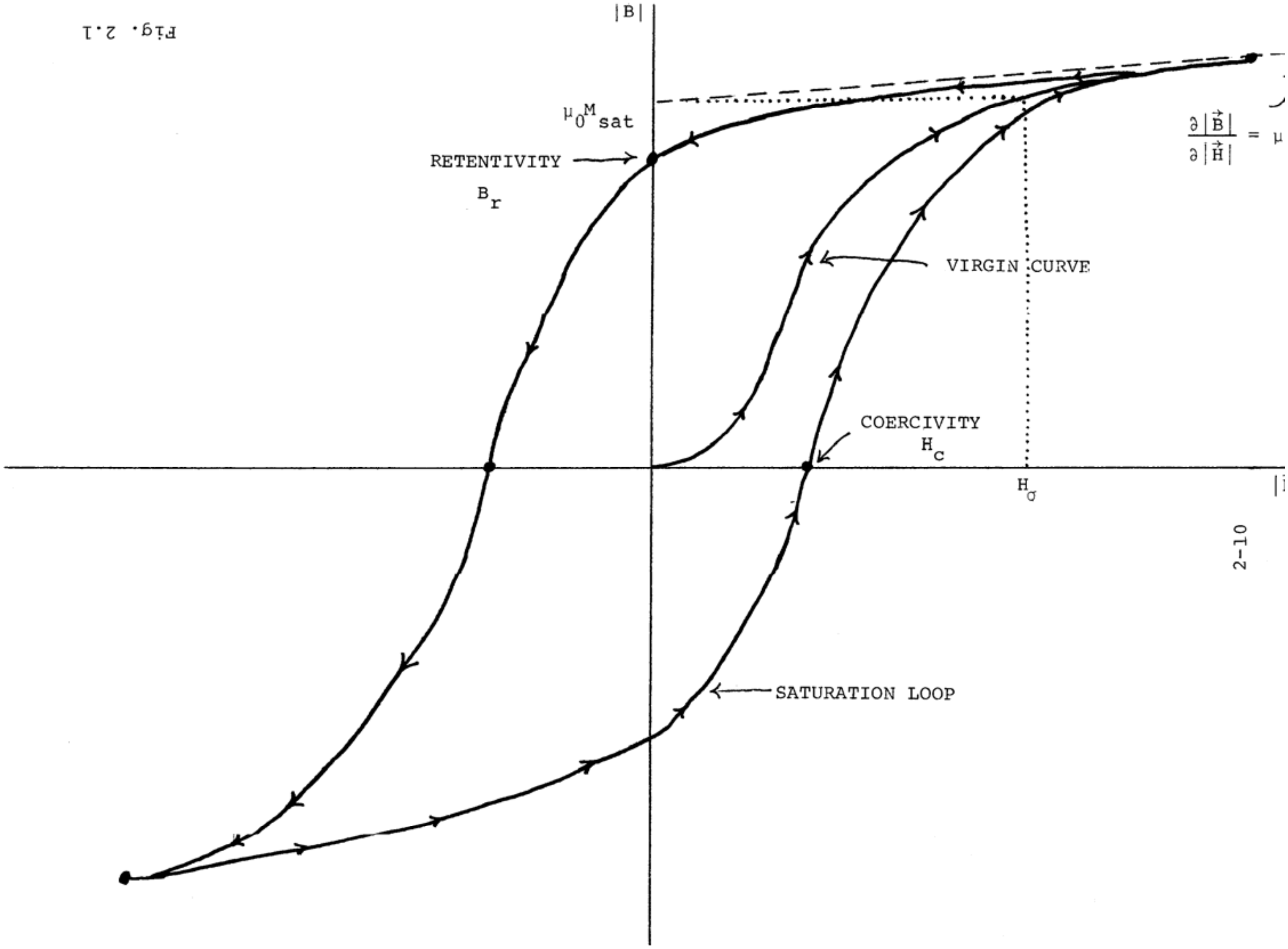
[3] The term *hysteresis* is derived from the Greek υατερησισ (a coming late or a delay) and can be traced back to the work of J. A. Ewing in the 1880's (*cf*. Heck, 1974).

Fig. 2.1

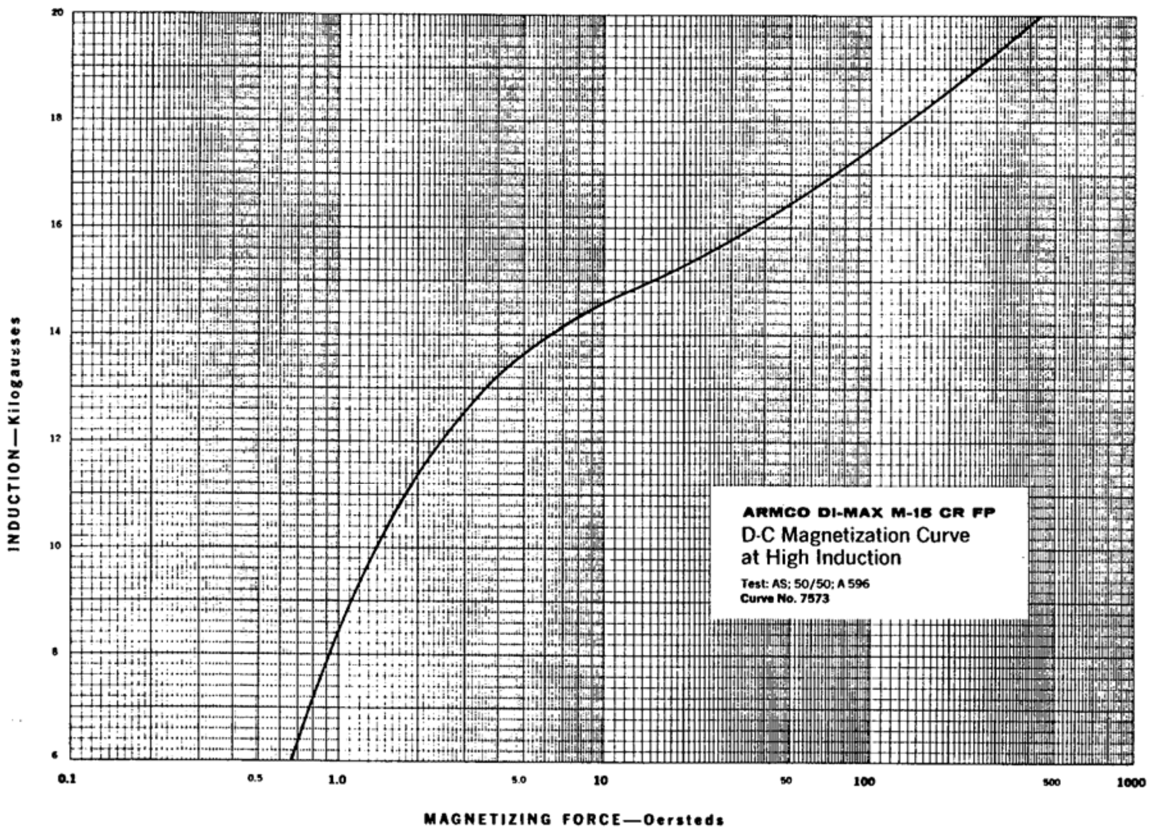

|B|
$\mu_0 M_{sat}$
RETENTIVITY
B_r
$\frac{\partial|\vec{B}|}{\partial|\vec{H}|} = \mu$
VIRGIN CURVE
COERCIVITY
H_c
H_σ
SATURATION LOOP
2-10

whereas, as $B_r H_c \to (\mu_0 M_{sat}) H_\sigma$, it is said to be "hard".

(β) If $H_c \doteq H_\sigma$ and $B_r \doteq \mu_0 M_{sat}$, the material is said to have a square loop.

(γ) If $\partial|\vec{B}| / \partial|\vec{H}| \doteq$ constant for $|\vec{H}| \lesssim H_\sigma$ and if $B_r \ll \mu_0 M_{sat}$, then the sides of the loop are straight and parallel and the loop is said to be "isoperm". It will be seen that the soft isoperm loop is highly desirable in power transformers.

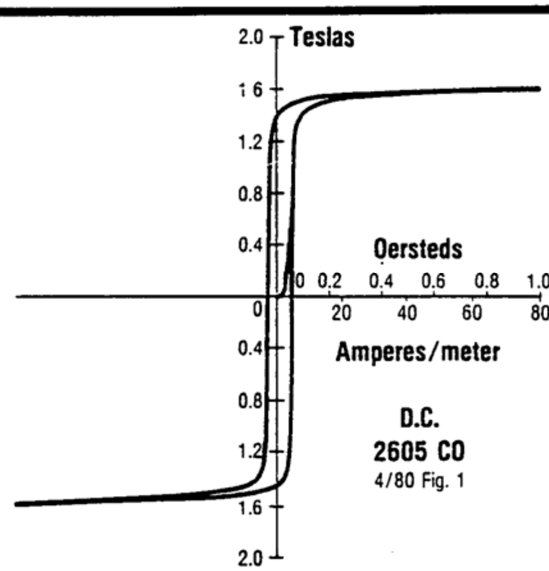
A. Silicon Steel



1 gauss = 10^{-4} tesla = 10^{-4} Wb/m^2 1 oersted = 79.57... A/m

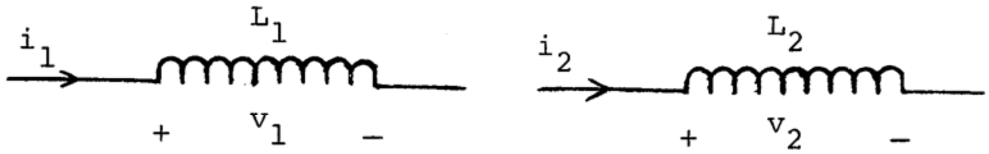
B. Metglas

METGLAS® magnetic properties



4. *A Networks Approach*

Consider now, two inductors embedded near one another in some soft, isoperm, but not necessarily homogeneous material.



It then should follow that the field of one inductor influences current flow in the other so that an experience akin to Eq. (2.8) obtains and

$$v_1 = L_1 \frac{di_1}{dt} \pm M_{12} \frac{di_2}{dt} \qquad (2.12a)$$

$$v_2 = \pm M_{21} \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \qquad (2.12b)$$

where $M_{12} = M_{21} = M > 0$ by "reciprocity considerations"[4]. The sign ambiguity can be resolved in one of two ways:

(i) Formal (The DOT convention)

Each inductor as placed in the circuit is provided with a single dot (•) located at one of its two terminals. The dots are placed so that, if one current enters at a dotted end and the other at an undotted end, the minus (−) sign is used; otherwise the plus (+)

[4] In fact, M_{12} may not equal M_{21} if saturable media are allowed, and M is positive primarily because one has choice over the + and − signs.

sign is used. To appreciate physically what this means, consider the situation of Fig. 2.3 in which the two inductors are wound in the same toroidal core. The total flux seen by Coil 1 will be (using Eqs. (2.1), (2.5), and (2.6))

$$\phi_1 = N_1AB_1 + N_2AB_2 = (N_1A)(N_1\mu_0 i_1\beta_1) + (N_2A)(\mu_0 N_2 i_2\beta_2)$$

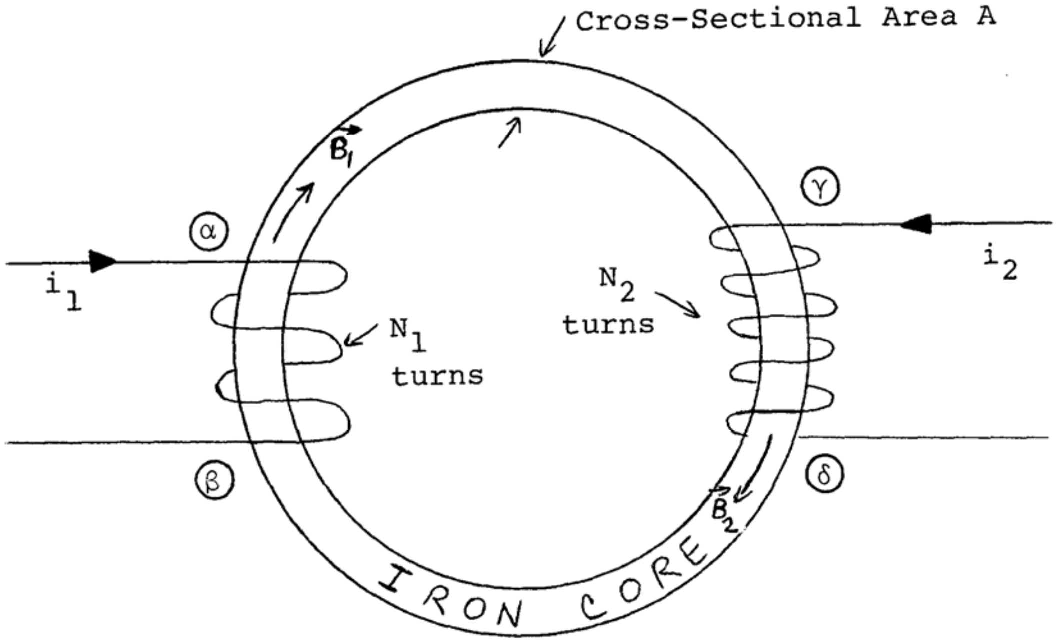
$$= L_{11}i_1 + L_{12}i_2 = L_1 i_1 + Mi_2 , \qquad (2.13)$$

where the plus (+) sign obtains since the fluxes produced by the two currents reinforce one another when $\text{sgn } i_1 = \text{sgn } i_2$ and where $L_1 \propto N_1^2$ and $M \propto N_1N_2$. Therefore, the dots must be placed at ⓐ and ⓨ or, equivalently, ⓑ and ⓓ.

(ii) Informal (Trial and error)

Choose the plus sign. Then, if the circuit doesn't work as anticipated, it will just by making the coupling inductance M negative. Alternatively, one can — in lieu of rectifying one's expectations for the circuit — rectify the circuit by unsoldering, reversing, and resoldering the leads on one of the inductors.

Before passing on, it should be noted that the time domain representation of Eqs. (2.12) becomes in the frequency domain just


Cross-Sectional Area A
B1
α
i1
N1
turns
β
γ
i2
N2
turns
δ
B2
IRON CORE

$$V_1 = I_1 \left[j\omega L_1 \right] + I_2 \left[\pm j\omega M \right] \tag{2.14a}$$

$$V_2 = I_1 \left[\pm j\omega M \right] + I_2 \left[j\omega L_2 \right] . \tag{2.14b}$$

Consider next $E(t)$ [J] , the total energy *delivered* to two coupled inductors. Clearly, by Eqs. (2.12) , it is

$$\begin{aligned} E(t) &= \int_{-\infty}^{t} \left[v_1(t') i_1(t') + v_2(t') i_2(t') \right] dt' \\ &= \frac{1}{2} L_1 i_1^{\,2} + \frac{1}{2} L_2 i_2^{\,2} \pm M \int_{-\infty}^{t} \left[i_1 \frac{di_2}{dt'} + i_2 \frac{di}{dt'} \right] dt' \\ &= \frac{1}{2} \left[L_1 i_1^{\,2} + L_2 i_2^{\,2} \pm 2M i_1 i_2 \right] \geq 0. \end{aligned} \tag{2.15}$$

Suppose that at an instant of time t , $i_2 = \zeta i_1$ where ζ is a dimensionless constant. At the instant considered

$$E(t) = \frac{1}{2} i_1^{\,2} \left[L_1 + L_2 \zeta^2 \pm 2M\zeta \right] \geq 0 \tag{2.15'}$$

This quadratic in ζ will have an extremum at

$$\pm \zeta_{ex} = - \frac{M}{L_2} \tag{2.16}$$

which corresponds to

$$E(t\,;\zeta_{ex}) = \frac{1}{2} i_1^{\,2} \left[L_1 - \frac{M^2}{L_2} \right] \geq 0 \tag{2.17}$$

And the inequality of Eq. (2.17) will be satisfied if and only if

$$L_1 L_2 \geq M^2 \tag{2.18}$$

or

$$M = k\sqrt{L_1 L_2} \tag{2.19a}$$

$$0 \leq k \leq 1 \tag{2.19b}$$

where k [dimensionless] is known as the *coefficient of coupling.*

To investigate a transformer we now consider the ratio

$$\frac{v_2}{v_1} = \frac{L_2 \frac{di_2}{dt} \pm k\sqrt{L_1 L_2}\frac{di_1}{dt}}{L_1 \frac{di_1}{dt} \pm k\sqrt{L_1 L_2}\frac{di_2}{dt}} = \pm\sqrt{\frac{L_2}{L_1}} \; \frac{L_2 \frac{di_2}{dt} \pm k\sqrt{L_1 L_2}\frac{di_1}{dt}}{\pm\sqrt{L_1 L_2}\frac{di_1}{dt} + k\, L_2 \frac{di_2}{dt}} \tag{2.20}$$

As $k \rightarrow 1$, Eq. (2.20) simplifies to the PERFECT TRANSFORMER limit

$$\left.\frac{v_2}{v_i}\right]_{k=1} = \pm\sqrt{\frac{L_2}{L_1}} \qquad \text{PERFECT TRANSFORMER} \tag{2.21}$$

But, by considerations akin to those which lead to Eq. (2.13),

$$L_1 \propto {N_1}^2 \tag{2.22a}$$

$$L_2 \propto {N_2}^2 \tag{2.22b}$$

so that, with a *turns ratio*

$$\eta = \frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}} \ , \tag{2.23}$$

we have

$$\left.\frac{v_2}{v_1}\right]_{k=1} = \pm \eta \tag{2.24a}$$

$$\left| \left.\frac{V_2}{V_1}\right]_{k=1} \right| = \eta \ . \tag{2.24b}$$

In the $k = 1$ limit, Eq. (2.12b) implies

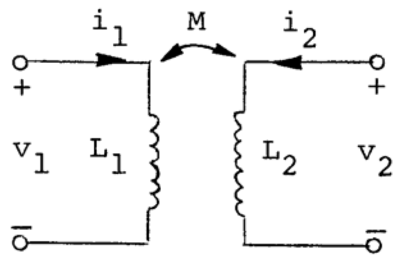
$$i_2(t) = \mp \frac{1}{\eta} \, i_1(t) + \frac{1}{L_2} \int_{-\infty}^{t} v_2(t')dt' \tag{2.25}$$

If now $L_2 \rightarrow \infty$ with $\eta = \sqrt{L_2/L_1}$ fixed, the IDEAL TRANSFORMER is achieved, and Eq. (2.25) reduces to

$$\left.\frac{i_2}{i_1}\right]_{\substack{k=1 \\ L_2=\infty}} = \mp \frac{1}{\eta} \qquad \text{IDEAL TRANSFORMER} \tag{2.26}$$

Both perfect and ideal transformers are lossless, as can be seen from Eq. (2.15) by considering the limit $i_1(t) = i_2(t) = 0$. Further, all ideal transformers are perfect. But only the $L_2 \rightarrow \infty$ subset of perfect transformers will be ideal. These considerations are summed up in the equivalent circuits of Fig. 2.4. The current i_{mag} in this diagram is the so-called *magnetizing*

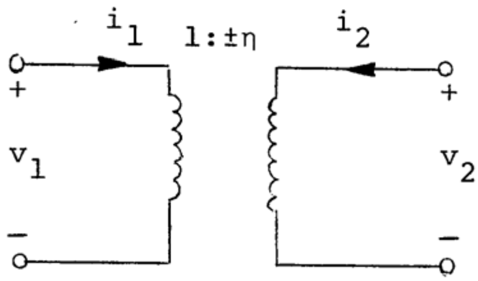
Lossless Transformer



$$v_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$v_2 = \pm M \frac{di_2}{dt} + L_2 \frac{di_2}{dt}$$

Ideal Transformer

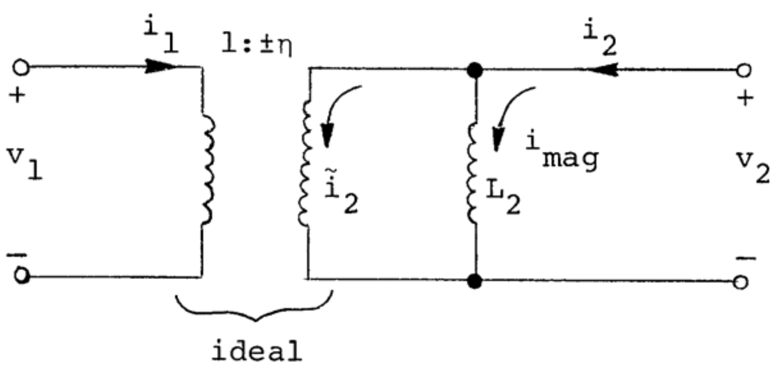


$$v_2 = \pm \eta \; v_1$$

$$i_2 = \mp \frac{1}{\eta} \, i_1$$

secondary primary

Perfect Transformer



$$v_2 = \pm \eta \; v_1$$

$$\tilde{i}_2 = \mp \frac{1}{\eta} \, i_1$$

$$i_{mag} = \frac{1}{L_2} \int_{-\infty}^{t} v_2(t')\,dt'$$

current and will figure prominently in later considerations.

5. *Core Loss and Jordan-type Loss Coefficients*

Of course, real transformers are not lossless, if for no other reasons than that (i) there will be ohmic losses in the wire of the inductor coils (*copper loss*) and (ii) there will be losses associated with the material upon which the coils of the primary and secondary are wound (*core loss*). Copper loss is seemingly straight-forward, although it may in fact contain some arcane points. Core loss is commonly split into three parts (Heck, 1974, p 35ff; Olsen, 1966, ch 48): (i) eddy-current; (ii) hysteresis loss; and (iii) residual loss.

The eddy-current loss is due to the alternating magnetic field which induces a voltage in the material of the core. This voltage then produces a current which in turn gives rise to an ohmic loss. For a core made up of thin high permeability sheets stacked closely together but not electrically in contact, the eddy current loss [W] can be shown to approximate

$$P_e = K_e\ f^2\ \hat{B}^2\ \delta^2\ \sigma\ V_c\ , \qquad (2.27)$$

where K_e [dimensionless] is a constant which depends upon the shape and material of the core, f [Hz] the applied frequency, $\hat{B}$ $[\mathrm{Wb/m^2}]$ the peak flux density, δ [m] the thickness of the sheet, σ [S/m] the conductivity of the sheet, and V_c $[\mathrm{m^3}]$ the volume of the core.

The hysteresis loss is due to collision-like effects as magnetic domains are reoriented. It is most frequently given by the empirical allometric relationship

$$P_h = K_h \, f \, \hat{B}^{\alpha} \, V_c \qquad (2.27)$$

where $K_h \left[\frac{J}{T^{\alpha} \cdot m^3}\right]$ is a constant which depends on the shape and material of the core and $1.6 < \alpha < 2.0$ is a constant dependent upon the material[5].

The residual loss, whose origin is obscure, is normally given by the empirical relationship

$$P_r = K_r \, f \, \hat{B}^2 \, , \qquad (2.28)$$

where $K_r \left[J/T^2\right]$ is a constant independent of both flux density and core geometry (Olsen, 1966, pg 59).

Suppose now (*cf*. Heck, 1974, p 35ff) that a set of measurements of total loss P_{loss} [W] is taken as a function of frequency for fixed $\hat{B}$ and the dc baseline found by extrapolating back to zero frequency. This baseline P_{dc} [W] is just the dc copper loss[6]. The total core loss is then by definition taken to be

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[5] Eq. (2.27) was presented by Steinmetz specifically to treat *high* induction levels in power transformers with steel cores. At lower $\vec{H}$ and with less saturated materials, the allometric relationship $P_h \propto H^3$ obtains.

[6] Observe that ac copper loss may, as a result of proximity and skin effects, be higher.

$$P_{core} = P_e + P_h + P_r = P_{loss} - P_{dc} \,. \tag{2.29}$$

The key idea of Jordan was that the three core terms behave differently in the $(f, \hat{B})$ plane and can thereby be told apart.

Transformers are seldom $\hat{B}$ driven, it being more common to employ instead the sinuisoidal voltage drive provided by the power company's "infinite busbar". Clearly, however, the laws of magnetic induction require that

$$\hat{B} \propto V_{rms} \tag{2.30}$$

so that Eqs. (2.27)-(2.28) can be used directly with an independent variable (V_{rms}) readily measured in the laboratory.

6. *The Equivalent Circuit of a Real Transformer at Low Frequencies.*

We come now to the problem of modeling a real transformer. We take as our basic variables the input line current $I_\ell(\omega)$ and input line voltage $V_\ell(\omega)$. This current unequivocally, physically passes through the primary winding. Hence, the actual voltage drop due to the transformer acting as a transformer will be

$$V_1(\omega) = V_\ell(\omega) - R_1 \, I_\ell(\omega) \tag{2.31}$$

where R_1 is the resistance due to copper in the primary.

We can abstract the I^2R loss of the windings as resistors in series with the primary and secondary windings.

We will ignore core loss for the moment. Considering the two coupled coils of page 2-13, we can show that a variety of circuit representations are possible. So as to avoid the use of negative circuit elements, it is advantageous to use an ideal transformer (ref. page 2-18) as part of the network. One such network is shown in fig. 2-5a. Many others are possible. Figure 2-5b shows fig. 2-5a with the effects of winding resistance included while fig. 2-5c has a shunt resistance added to include the effects of core loss. It should be noted that since core loss depends on applied voltage in a non-linear way, the resistor, R_c, can only be an approximation to the actual situation. Note that the turns ratio of our ideal transformer, which would also be the approximate primary/secondary voltage ratio of our actual transformer, depends on the primary/secondary inductance ratio and also on the coefficient of coulpling. This is certainly reasonable since if k were zero no secondary output could be had.

The inductor $L_1(1 - k^2)$ is non-zero because of imperfect coupling and so is called the 'leakage inductance', L_e, of the transformer. In the shunt arm the inductor k^2L_1 is the 'magnetizing inductance', L_m. Generally, for power transformers L_e is small, giving a voltage drop at rated current that is only a few percent of rated voltage; while at rated voltage, L_m is large, and will have a current through it of only a few percent of rated current.

Since L_m and R_c are parallel branches it is common to refer to them as a susceptance, B_m, and a conductance, G_c. In practice little error is incurred by abstracting B_m and G_c to the line terminals giving the equivalent network of fig. 2-5d.

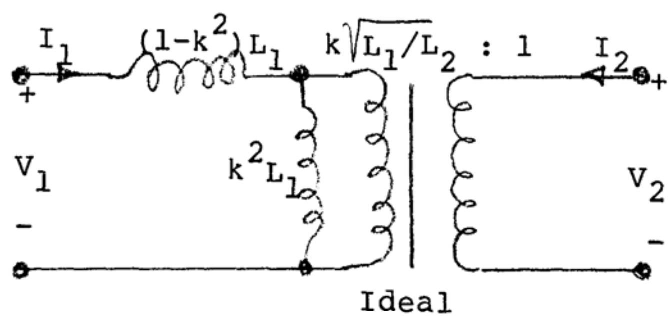


Figure 2-5a

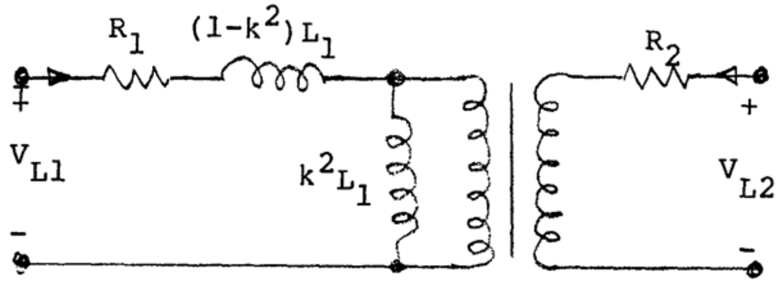


Figure 2-5b

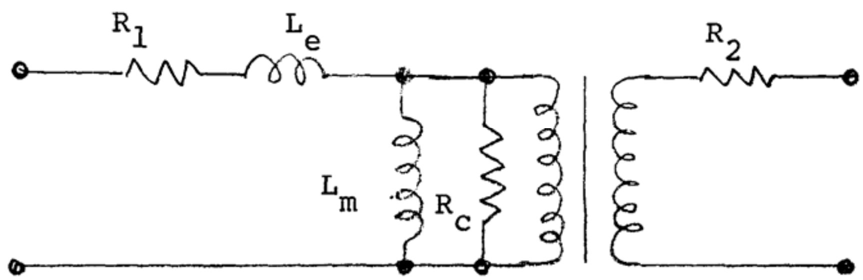


Figure 2-5c

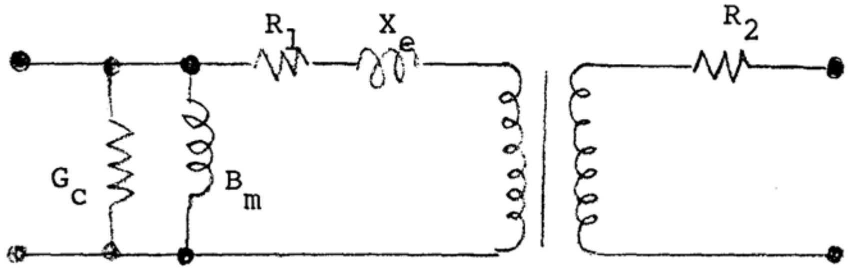


Figure 2-5d

7. EXPERIMENTAL TESTS

OPEN CIRCUIT

The equivalent network will be as shown in Fig. 2-6a.

Usually we can neglect R_1 and X_e for this case and have

$$P_{OC} = RE\{V_L I^*_{LOC}\} = G_C |V_L|^2 \qquad (2.32)$$

$$I_{LOC} = (G_c + jB_m)V_L \qquad (2.33)$$

So, measurement of power input, line current, and line voltage to the transformer with secondary open-circuited will yield the network elements G_c and B_m. Additionally, the transformation ratio, $a = k\sqrt{L_1/L_2}$ can be approximately obtained by measuring the voltage appearing across the secondary.

SHORT CIRCUIT

Figure 2-6b gives the equivalent network in this case. Sufficient accuracy can often be achieved by ignoring the shunt branch, $G_c + jB_m$ in which case we have

$$P_{SC} = |I_{LSC}|^2 R_1 + |I_2|^2 R_2 \ , \qquad (2.34)$$

or, using the transformation ratio, a , we have

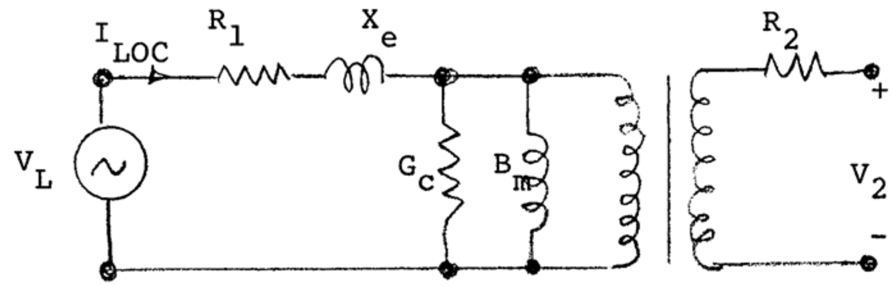
$$P_{SC} = |I_{LSC}|^2 \left[R_1 + a^2 R_2\right] \ . \qquad (2.35)$$

Additionally,

$$V_L = (R_1 + a^2 R_2 + jX_e)\, I_{LSC} \ , \qquad (2.36)$$

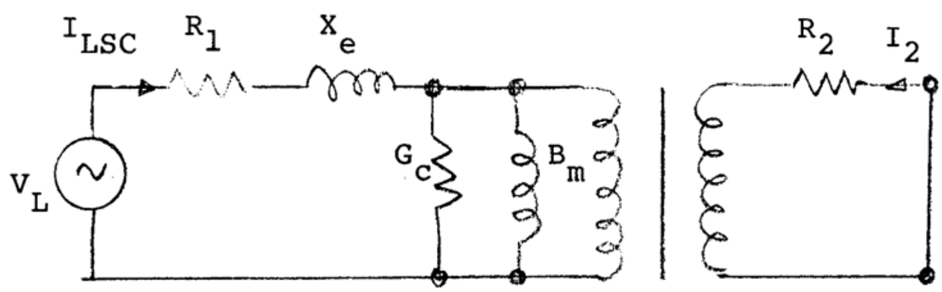
so the quantities $\left[R_1 + a^2 R_2\right]$ and X_e can be inferred from

measurements with the transformer secondary shorted. R_1 and R_2 can be separated by a DC ohmmeter measurement of R_1 .



Open-circuit test

Figure 2-6a



Short-circuit test

Figure 2-6bn

8. *Three Phase Transformers*

For the benefit of those students whose polyphase theory is somewhat rusty, a brief review is in order. This will be based primarily on the peerless treatise by A. T. Dover (1947).

Consider a system of three conductors 1, 2, 3 arranged clockwise in space and having with respect to some imagined reference point the potentials $T_1(j\omega)$, $T_2(j\omega)$, $T_3(j\omega)$. One then defines

$$V_A = T_2 - T_1 = V \qquad (2.44a)$$

$$V_B = T_3 - T_2 = Ve^{-j\frac{2\pi}{3}\delta} \qquad (2.44b)$$

$$V_C = T_1 - T_3 = Ve^{-j\frac{4\pi}{3}\delta} \qquad (2.44c)$$

where V [V] is an rms voltage and δ is a dimensionless constant being +1 for the positive phase (clockwise) sequence and -1 for the negative (counterclockwise) phase sequence. These three wires can be connected in either of two fashions as shown in Fig. 2.7. Fig. 2.7A shows the *delta* (or ring) configuration. Fig. 2.7B the *wye* (or star) configuration; if N (the *neutral* or star point) is taken to be at zero potential, then

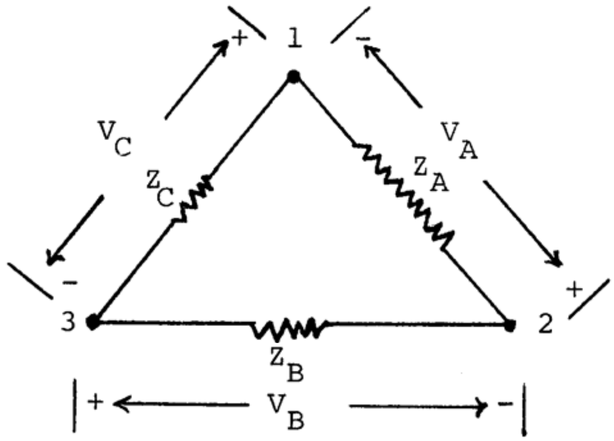
$$T_1 = V_{1N} = V_1 = \frac{V}{\sqrt{3}}\, e^{j\delta\frac{5\pi}{6}} \qquad (2.45a)$$

$$T_2 = V_{2N} = V_2 = \frac{V}{\sqrt{3}}\, e^{j\delta\frac{\pi}{6}} \qquad (2.45b)$$

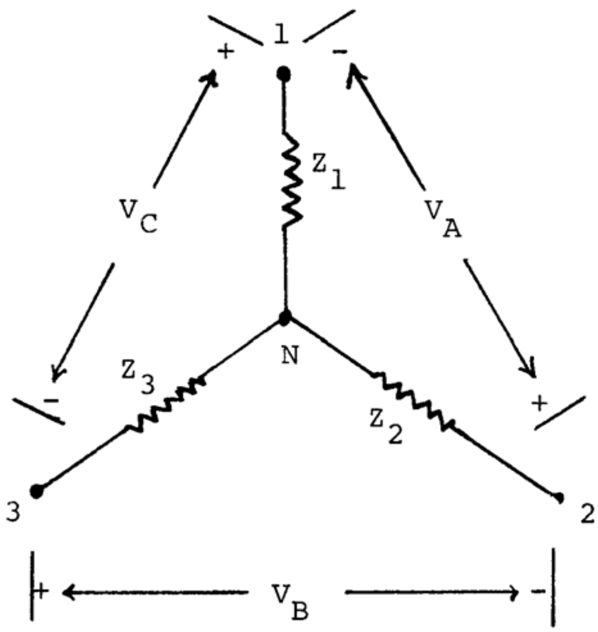
$$T_3 = V_{3N} = V_3 = \frac{V}{\sqrt{3}}\, e^{-j\delta\frac{\pi}{2}} \quad , \qquad (2.45c)$$

as can readily be shown (You should, as an exercise, do so.) if the sum of the T's is constrained to be zero. This brings us to the first great three phase result:

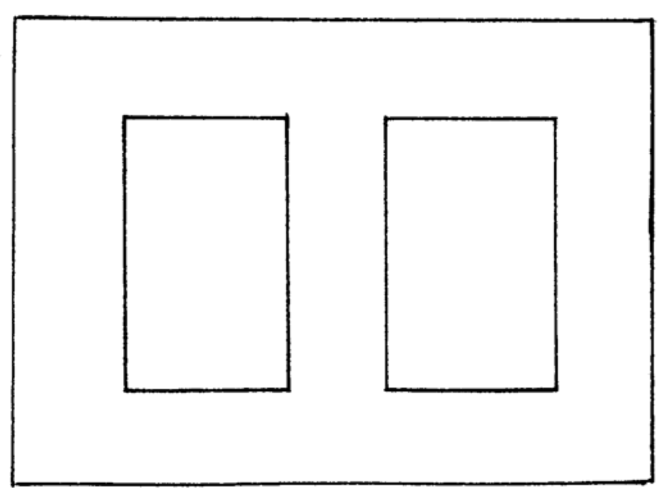
A.



B.



C.



rms line-neutral voltage $= \frac{1}{\sqrt{3}}$ *(rms line-line voltage).*

Since a three phase system can be thought of as just an ordinary network with three generators phase locked with respect to each other, ordinary mesh and nodal analysis will suffice to solve three phase systems. However, if it is assumed that the system is loaded so that either (delta) $Z_A = Z_B = Z_C = Z_\Delta$ or (wye) $Z_1 = Z_2 = Z_3 = Z_W$ the system is said to be *balanced*, and the analysis becomes simpler. For example, in a balanced delta, the line currents I_1, I_2 and I_3 are related to the load (phase) currents I_A, I_B, and I_C by the equations (see Fig. 2.7A)

$$I_1 = I_C - I_A = \frac{V}{Z_\Delta}^{-j\frac{4\pi}{3}\delta} - \frac{V}{Z_\Delta} = \frac{V}{Z_\Delta}\sqrt{3}e^{j\frac{5\pi}{6}\delta} \tag{2.46a}$$

$$I_2 = I_A - I_B = \frac{V}{Z_\Delta} - \frac{V}{Z_\Delta}e^{-j\frac{2\pi}{3}\delta} = \frac{V}{Z_\Delta}\sqrt{3}e^{+j\frac{\pi}{6}\delta} \tag{2.46b}$$

$$I_3 = I_B - I_C = \frac{V}{Z_\Delta}e^{-j\frac{2\pi}{3}\delta} - \frac{V}{Z_\Delta}e^{-j\frac{4\pi}{3}\delta} = \frac{V}{Z_\Delta}\sqrt{3}e^{-j\frac{\pi}{2}\delta} \quad . \tag{2.46c}$$

And this yields the second great three phase result:

For a balanced delta load, the rms line current is $\sqrt{3}$ *times the rms phase current.*

The power for a balanced load can be computed in terms of the line-line voltage V and the rms line current I. For a balanced star,

$$S = P + j\,Q = V_{1N} I_1^* + V_{2N} I_2^* + V_{3N} I_3^*$$

$$= \frac{V_{1N}\, V_{1N}^*}{Z_W^*} + \frac{V_{2N}\, V_{2N}^*}{Z_W^*} + \frac{V_{3N}\, V_{3N}^*}{Z_W^*}$$

$$= \sqrt{3}\ VI\ e^{j \arg Z_W}; \qquad (2.47a)$$

whereas for a balanced delta,

$$S = P + j\,Q = V_A I_A^* + V_B I_B^* + V_C I_C^*$$

$$= \frac{V_A\, V_A^*}{Z_\Delta^*} + \frac{V_B\, V_B^*}{Z_\Delta^*} + \frac{V_C\, V_C^*}{Z_\Delta^*}$$

$$= \sqrt{3}\ VI\ e^{j \arg Z_\Delta} \qquad (2.47b)$$

These formulas are essentially the same. In terms of the power factor

$$\cos\theta = \arctan\frac{X}{R}\ , \qquad (2.48)$$

the power is just

$$P = \sqrt{3}\ VI\ \mathrm{RE}\{e^{j\theta}\} = \sqrt{3}\ VI\ \cos\theta\ . \qquad (2.49)$$

Since each phase of a balanced three phase load draws the same power, the estimation of power is readily achieved by emplacing an ordinary wattmeter. If unbalance is the rule, this solution clearly will not work, However, while it may be conceptually pleasing to use three wattmeters, two frequently suffice, as can be seen by considering (for example) an unbalanced three wire delta as in Fig. 2.7A.

$$P = RE\{V_A I_A^* + V_B I_B^* + V_C I_C^*\}. \tag{2.50}$$

By Kirchhoff's current law

$$I_2 = I_A [1] + I_B [-1] + I_C [0] \tag{2.51a}$$

$$I_3 = I_A [0] + I_B [1] + I_C [-1] \tag{2.51b}$$

$$0 = I_A [1] + I_B [1] + I_C [1] \ ;$$

and thus

$$I_A = \frac{1}{3} \left[2I_2 + I_3\right] \tag{2.52a}$$

$$I_B = \frac{1}{3} \left[-I_2 + I_3\right] \tag{2.52b}$$

$$I_C = \frac{1}{3} \left[-I_2 - 2I_3\right] \tag{2.52c}$$

Moreover, by Kirchhoff's voltage law,

$$V_B = -(V_A + V_C) \ . \tag{2.53}$$

Hence

$$P = RE\{V_A I_2^* - V_C I_3^*\} \ . \tag{2.54}$$

Note that line 1 is the common reference in this instance. Observe also that it will be simple to mix up one's wires and hitch things backwards. Finally, as you may see in the lab, it is possible to put the two meter movements in one case and put on but a single pointer scale.

We now — almost as an afterthought — come to the question of the three phase transformer. One could, of course, use three single phase units and connect them into a suitable star or ring, but this is seldom done. Instead a laminated iron yoke of the form shown in Fig. 2.7C is used and one primary and one secondary wound on each yoke. The only caution is that the secondaries be correctly connected together. By way of illustrating this, consider a wye secondary with

$$V_{1N} = V \qquad (2.55a)$$

$$V_{2N} = Ve^{-j\delta\frac{2\pi}{3}} \qquad (2.55b)$$

$$V_{3N} = -Ve^{-j\delta\frac{4\pi}{3}} \qquad (2.55c)$$

The 3N phase is in backwards, and the result is

$$|V_A| = |V_2 - V_1| = \sqrt{3}V \qquad (2.56a)$$

$$|V_B| = |V_3 - V_2| = V \qquad (2.56b)$$

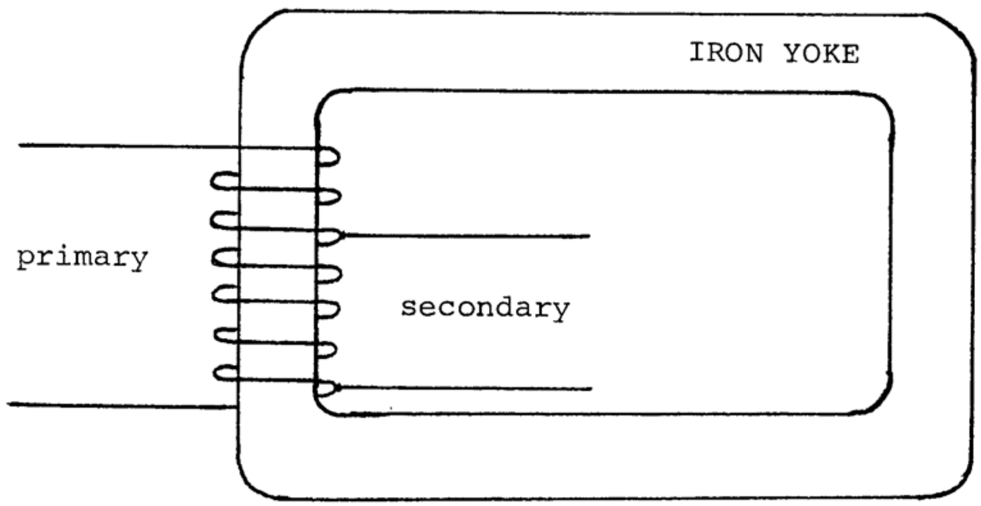
$$|V_C| = |V_1 - V_3| = V \qquad (2.56c)$$

9. *Autotransformers*

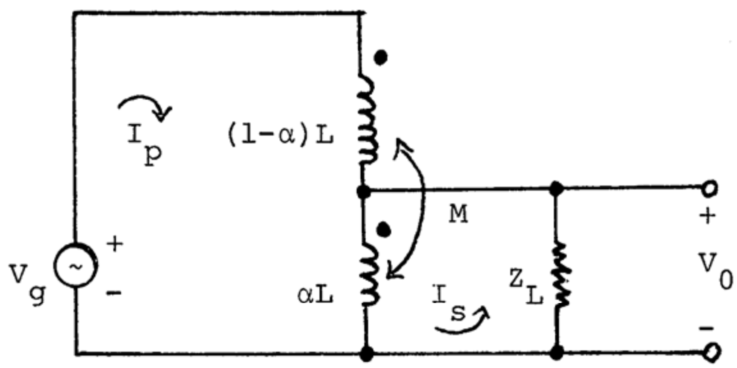
In the transformers connected to date, the primaries and secondaries were electrically isolated from each other. They need not be. In fact, one can define a circuit of the form shown in Fig. 2.8A Its lossless idealization as a coupled circuit is shown in Fig. 2.8B and the ideal transformer limit in Fig. 2.8C. Fig. 2.8C is readily solved to yield

$$V_0 = \frac{\eta}{1+\eta} V_g \ . \qquad (2.57)$$

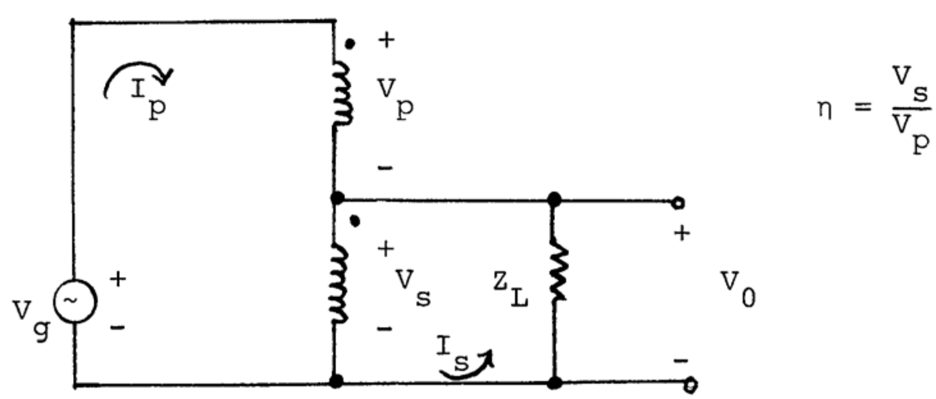
A.


IRON YOKE
primary
secondary

B.


I_p
(1-α)L
M
+
V_g
-
αL
I_s
Z_L
+
V_0
-

C.


+
I_p
V_p
-
+
+
V_s
Z_L
V_0
V_g
-
-
I_s
-
$\eta = \frac{V_s}{V_p}$

Now suppose that the primary has $N(1-\beta)$ turns and the secondary β turns. This implies $\eta = \beta/(1-\beta)$ or

$$V_0 = \beta V_g \ . \qquad (2.57'')$$

In practice such an effect is often achieved by connecting the high point of the secondary to a wiper arm and running it along the primary to give a variable autotransformer: such devices are commonly called Variacs after the model produced by Genrad (General Radio).

10. *Practical Considerations*

Even if you have now mastered faultlessly the above material, you are still not prepared to deal with specifying and installing a transformer because there is a plethora of practical information still missing in your background, and this short introduction can do no more than indicate where the more major deficiencies lie. Books such as that of Stigant and Franklin (1973) must be consulted for details about the niceties of grounding practice, cooling, regulation, performance under surge, insulation life, packaging, etc.

You should, however, be aware that transformers are generally specified by

(i) Primary voltage and secondary voltage.

(ii) Complex power put in (kVA).

In practice one would seek a family of transformers which had the required terminal voltages and then go up the family until a beefy enough one was reached. The reason that kVA rather than kW is specified is that line voltage is normally set by the power company's infinite busbar whereas load is set by the user. Hence, core loss (proportional roughly to the square of the terminal voltage) is fixed while the copper loss (proportional roughly to the square of the line voltage) can vary greatly. Thus, a highly reactive load could pull many kVAR (and kVA) without drawing all that many kW; and, therefore, a kW specification could lead to excessive copper loss and transformer burn-out.

11. *Standard (ANSI C57.12.90-1973) for Electrical Tests*

There are three ways of testing a transformer: the right way, the wrong way, and the official way. The official way is described by the above standard which is also known as IEEE Std 262-1973.

This standard describes (i) resistance measurements, (ii) dielectric tests, (iii) efficiency, loss, and impedance tests, (iv) ratio and regulation tests, (v) temperature rise tests, (vi) insulation power factor tests, and (vii) polarity and phase relation tests. The aim seems to be practical utility in industrial settings rather than deep understanding in the university laboratory. Nevertheless, you should be aware of this standard and refer to it once you enter the real world.

12. *References*

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B. *EXPERIMENT*

1. *Equipment List*

a. The standard equipment normally found at each station.

b. Various current shunts and probes.

c. One single-phase wattmeter (YEW model 2041 or Extech model 382860).

d. One variable autotransformer (Staco model 3PN1010).

e. One single-phase Transformer Test Rig (Stancor model 8666).

f . One rheostat module with two 50-Ω rheostats rated at 4.5 A each

g. One 200 Ohm, 20 Watt, resistor.

h. Various power cords.

2. *The Single Phase-Transformer*

a. Using a suitable method, accurately measure the DC resistances of the primary and the secondary[#]. (NOTE: Consult the connection diagram for the Transformer Test Rig.*)* **Constraint:** The current delivered to the winding under test must not exceed 1 Amp.

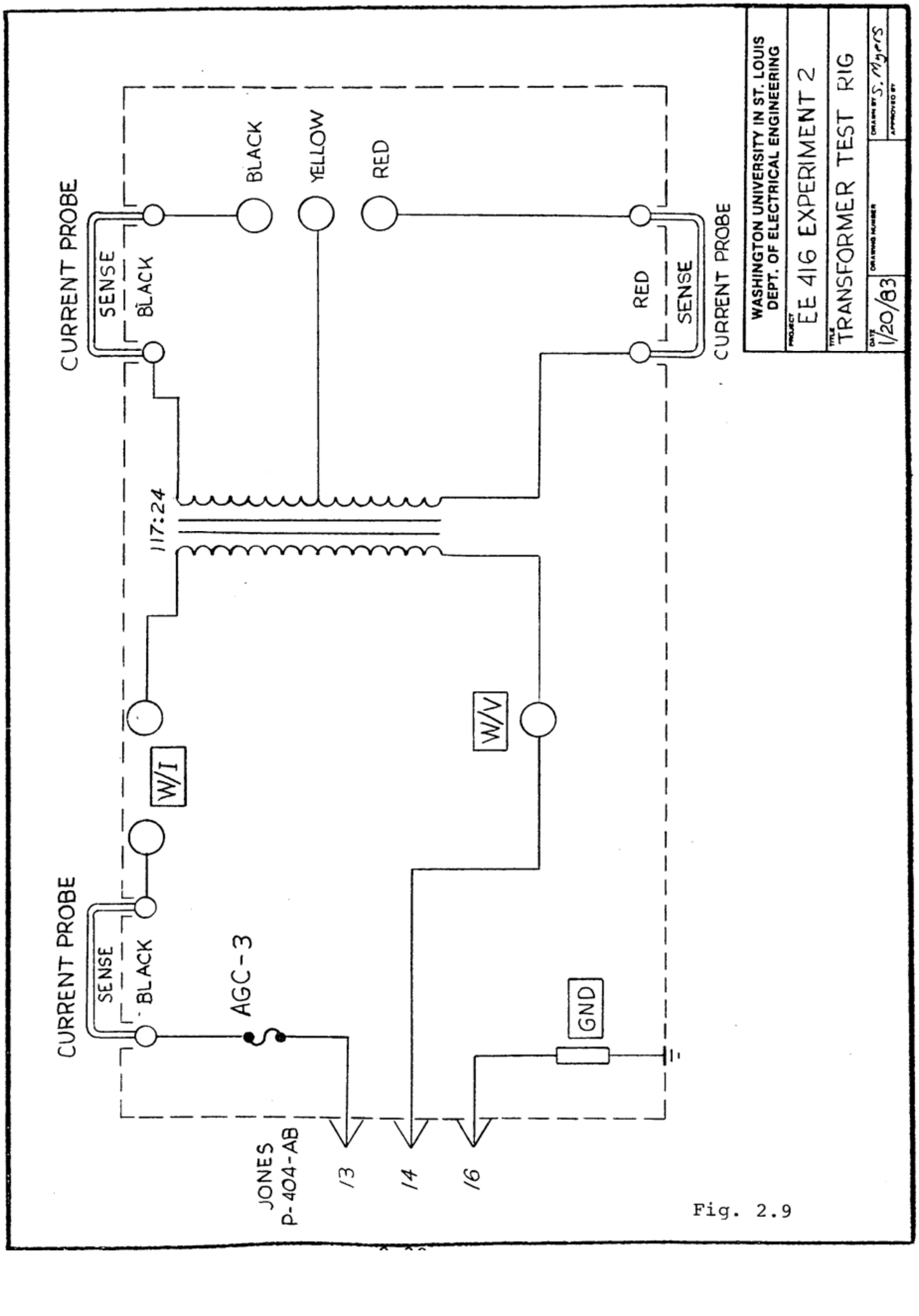
b. Connect the primary of the transformer to the autotransformer. Open circuit the secondary. Use a Wattmeter to determine P_{OC}, the open circuit input power. If "$_P$" and "$_S$" denote (respectively) primary and secondary rms quantities, measure V_P, I_P, and V_S over the V_P interval [10,140][*]. Be absolutely certain to use a *sensible* voltage grid[£]. And take care to use suitable meter ranges throughout. Turn off the autotransformer when finished.

c. Connect a shunt across the secondary and float the center tap. **Caution:** Start with very low autotransformer voltage and increase voltage slowly.

[#] Those of you who are unfamiliar with the up-sides and down-sides of the two-wire and four-wire techniques may wish to study and meditate upon them before coming to lab. Note also that the instructors have perversely neglected to tell you what frequency to use in these measurements. Be assured that you will need to discuss these points in the write-up.

[*] This is a POWER course, so we presumably mean volts. Sometimes one has to interpret instructions based upon their context.

[£] I've always liked to take a minimalist approach to data dredging. But of course it would never do to have the plotted points too far apart, especially since this might make difficult (impossible?) an accurate determination of the transformer's parameters.


CURRENT PROBE
SENSE
BLACK
YELLOW
RED
CURRENT PROBE
SENSE
RED
117:24
W/I
W/V
CURRENT PROBE
SENSE
BLACK
AGC-3
GND
JONES
P-404-AB
13
14
16
WASHINGTON UNIVERSITY IN ST. LOUIS
DEPT. OF ELECTRICAL ENGINEERING
EE 416 EXPERIMENT 2
TRANSFORMER TEST RIG
1/20/83
S. Myers
Fig. 2.9

Use the Wattmeter to measure P_{SC}, the short circuit input power. Measure also I_P, V_P, and I_S over the I_S interval [1,12]. Be certain to space your current readings *sensibly* and to employ *sensible* meter ranges.

d. Parts b. and c. above provide sufficient information to determine an equivalent circuit for the single-phase transformer. To obtain comparison data, connect the rheostats as a secondary load with 9 Amp capability and set the input voltage to rated voltage of 120 V_{rms}. Holding the input voltage at its rated value, adjust the rheostats and make measurements of P_P, V_P, I_P, V_S, and I_S for load currents I_S of 1, 3, 5, 7, and 9 A.

3. *Maximum Power Transfer*

a. Construct a power source consisting of the autotransformer and a series 200 Ohm, 20 Watt, resistor. Connect the primary of the single-phase transformer in the Transformer Test Rig to this power source. Construct a variable load using a single rheostat connected to the secondary of the single-phase transformer and set the load to maximum resistance. Instrument the set-up so that you can measure the voltage and power delivered by the autotransformer and the voltage and current in the variable load.

b. Set and maintain the autotransformer voltage to 100 V_{rms}. Adjust the load resistance until maximum power is being dissipated by the load and record all measurements. Be careful not to exceed the 4.5 Amp rating of the rheostat.

4. *REPORT*

a. In Part 2.a, what were measured DC $R_{primary}$ and $R_{secondary}$? Include the raw data as well as the derived resistances. Also, explain the resistance measuring technique you employed; and *justify your choice*.

b. All data measured in parts 2.b &2.c should be normalized relative to values measured at rated primary voltage or rated secondary current. Be sure to include the raw data and the normalized data in tabular form. Plot the normalized P_{OC}, I_P, & V_s data from your open circuit test vs. primary voltage V_P and plot normalized P_{SC} & I_P from your short circuit test vs. secondary current I_S. Carefully explaining (step by step) your procedure, construct an equivalent circuit for the transformer that was tested. Show all calculations. Your equivalent circuit should contain and have numerical values for all the elements of the last diagram on Figure 2-5d on page 24.

c. Using the equivalent circuit constructed in Part b. above, calculate and generate plots displaying % efficiency versus load R and % regulation versus the load R, where the values of R are those calculated from V_S and I_s measured in part 2.d. These quantities are defined as:

$$\% \text{ efficiency} = 100 \frac{P_{load}(R)}{P_{input}(R)}$$

$$\% \text{ regulation} = 100 \frac{V_S(\infty) - V_S(R)}{V_S(\infty)} .$$

Also, calculate the efficiency and regulation for the data taken in part 2.d. Plot these data points on the corresponding plots calculated from the equivalent circuit that you determined for the transformer.

d. What were the powers delivered by the autotransformer and dissipated by the load when maximum power transfer was obtained? What was the overall efficiency using this resistance matching approach? What was the resistance of the load when maximum power transfer occurred? How does this value of resistance compare to the theoretical value?