Transportation, Transshipment, and Assignment Problems

Chapter 6

Chapter Topics

- The Transportation Model
- Computer Solution of a Transportation Problem
- The Transshipment Model
- The Assignment Model
- Computer Solution of an Assignment Problem

Overview

- Part of a class of LP problems known as ***network flow models***.
- Special mathematical features that permit very efficient, unique solution methods (variations of traditional simplex procedure).
- Detailed description of methods is contained on the companion website
- Text focuses on model formulation and solution with Excel and QM for windows.
- Web site Module B addresses transportation and assignment solution methods

The Transportation Model: Characteristics

- A product is transported from a number of sources to a number of destinations at the ***minimum possible cost***.
- Each ***source is able to supply a fixed number*** of units of the product, and ***each destination has a fixed demand*** for the product.
- The linear programming model has ***constraints for supply*** at each source ***and demand*** at each destination.
- All constraints are equalities in a balanced transportation model where supply equals demand.
- Constraints contain inequalities in unbalanced models where supply does not equal demand.

Transportation Model Example Problem Definition and Data

How many tons of wheat to transport from each grain elevator to each mill on a monthly basis in order to minimize the total cost of transportation?

Grain Elevator	Supply	Mill	Demand
1. Kansas City	150	A. Chicago	220
2. Omaha	175	B. St. Louis	100
3. Des Moines	275	C. Cincinnati	300
Total	600 tons	Total	600 tons

Transport Cost from Grain Elevator to Mill ($/ton)

Grain Elevator	A. Chicago	B. St. Louis	C. Cincinnati
1. Kansas City	$ 6	$ 8	$ 10
2. Omaha	7	11	11
3. Des Moines	4	5	12

Transportation Model Example Transportation Network Routes

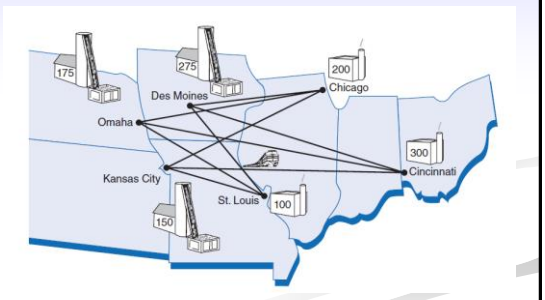


Figure 6.1 Network of transportation routes for wheat shipments

Transportation Model Example
Model Formulation

Minimize Z = 6x_{1A}$ + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} +
4x_{3A} + 5x_{3B} + 12x_{3C}

subject to:

$x_{1A} + x_{1B} + x_{1C} = 150$
$x_{2A} + x_{2B} + x_{2C} = 175$
$x_{3A} + x_{3B} + x_{3C} = 275$
$x_{1A} + x_{2A} + x_{3A} = 200$
$x_{1B} + x_{2B} + x_{3B} = 100$
$x_{1C} + x_{2C} + x_{3C} = 300$
$x_{ij} \geq 0$

x_{ij} = tons of wheat from each grain elevator, i, i = 1, 2, 3,
to each mill j, j = A,B,C

Transportation Model Example
Computer Solution with Excel (1 of 4)

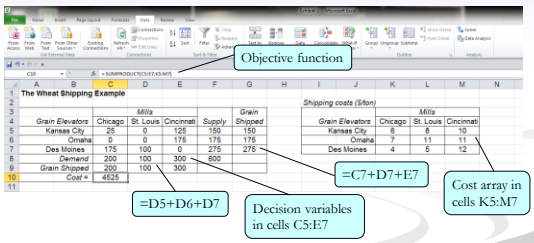


Exhibit 6.1

Transportation Model Example
Computer Solution with Excel (2 of 4)

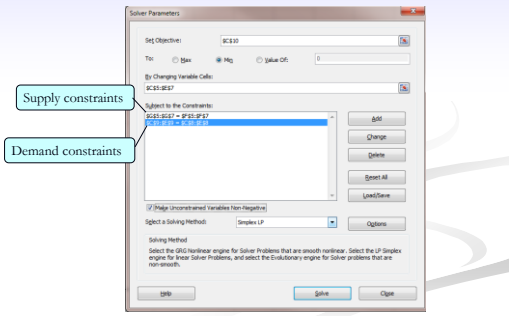
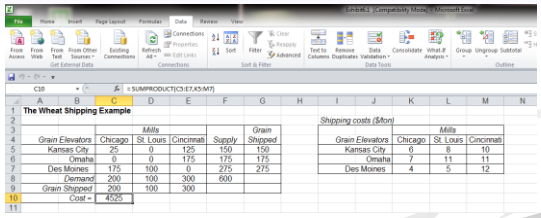


Exhibit 6.2

Transportation Model Example
Computer Solution with Excel (3 of 4)



The Wheat Shipping Example

Grain Elevators	Chicago	St. Louis	Cincinnati	Supply	Grain Shipped
Kansas City	25	0	125	150	150
Omaha	0	0	175	175	175
Des Moines	175	100	0	275	275
Demand	200	100	300	600	
Grain Shipped	200	100	300		
Cost =	4525				

Shipping costs ($/ton)

Grain Elevators	Mills: Chicago	Mills: St. Louis	Mills: Cincinnati
Kansas City	6	8	10
Omaha	7	11	11
Des Moines	4	5	12

Exhibit 6.3

Transportation Model Example
Computer Solution with Excel (4 of 4)

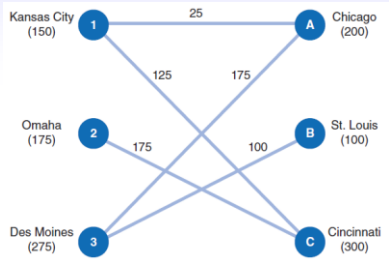


Figure 6.2 Transportation network solution for wheat-shipping example

Transportation Model Example
Computer Solution with Excel QM (1 of 3)

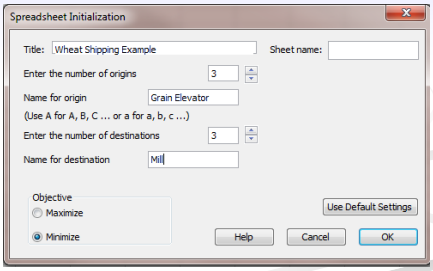
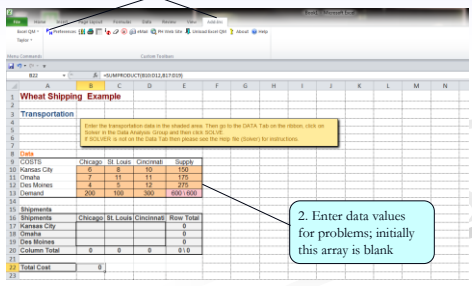


Exhibit 6.4

Transportation Model Example
Computer Solution with Excel QM (2 of 3)

1. Click on "Excel QM," then "B chapter" to access the macro menu



2. Enter data values for problems; initially this array is blank

Exhibit 6.5

Transportation Model Example
Computer Solution with Excel QM (3 of 3)

Click on "Data" tab and then "Solver"

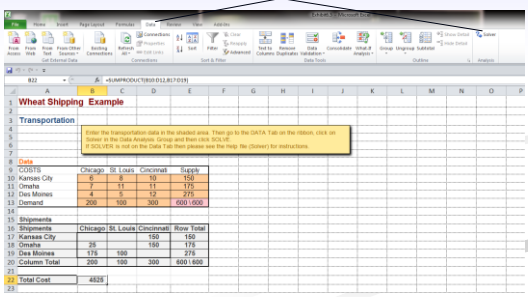


Exhibit 6.6

Transportation Model Example
Computer Solution with QM for Windows (1 of 4)

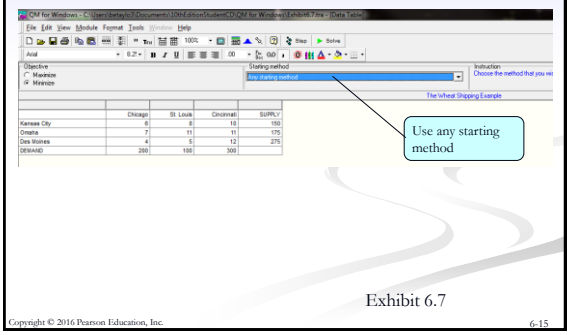


Exhibit 6.7

Transportation Model Example
Computer Solution with QM for Windows (2 of 4)

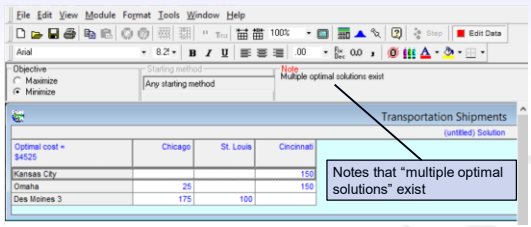
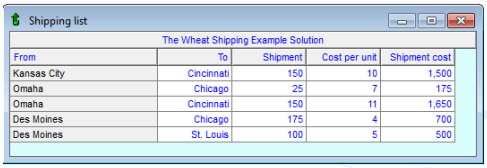


Exhibit 6.8

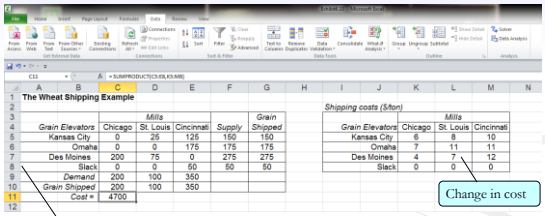
Transportation Model Example
Computer Solution with QM for Windows (3 of 4)



From	To	Shipment	Cost per unit	Shipment cost
Kansas City	Cincinnati	150	10	1,500
Omaha	Chicago	25	7	175
Omaha	Cincinnati	150	11	1,650
Des Moines	Chicago	175	4	700
Des Moines	St. Louis	100	5	500

Exhibit 6.9

Transportation Model Example
Computer Solution with QM for Windows (4 of 4)



The Wheat Shipping Example

Grain Elevators	Chicago	St. Louis	Cincinnati	Supply	Grain Shipped
Kansas City	0	25	125	150	150
Omaha	0	0	175	175	175
Des Moines	200	75	0	275	275
Slack	0	0	50	50	50
Demand	200	100	350		
Grain Shipped	200	100	350		
Cost =	4700				

Shipping costs ($/ton)

Grain Elevators	Chicago	St. Louis	Cincinnati
Kansas City	6	8	10
Omaha	7	11	11
Des Moines	4	7	12
Slack	0	0	0

Change in cost

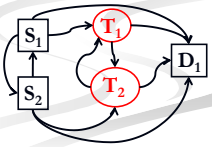
Added new row to reflect demand ≥ supply

Sensitivity analysis of transportation scenario

Exhibit 6.10

The Transshipment Model Characteristics

- Extension of the transportation model.
- Intermediate transshipment points are added between the sources and destinations.
- Items may be transported from:
  - Sources through transshipment points to destinations
  - One source to another
  - One transshipment point to another
  - One destination to another
  - Directly from sources to destinations
  - Some combination of these



Transshipment Model Example Problem Definition and Data

Extension of the transportation model in which intermediate transshipment points are added between sources and destinations.

Shipping Costs

	Grain Elevator		
Farm	3. Kansas City	4. Omaha	5. Des Moines
1. Nebraska	$16	10	12
2. Colorado	15	14	17

Transshipment Model Example Transshipment Network Routes

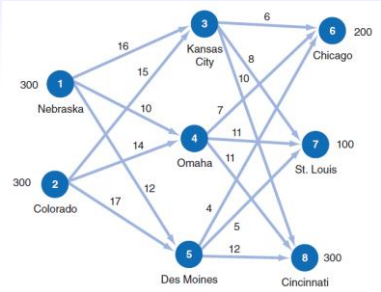


Figure 6.3 Network of transshipment routes

Transshipment Model Example
Model Formulation

Minimize Z = 16x_{13}$ + 10x_{14} + 12x_{15} + 15x_{23} + 14x_{24}
+ 17x_{25} + 6x_{36} + 8x_{37} + 10x_{38} + 7x_{46} + 11x_{47}
+ 11x_{48} + 4x_{56} + 5x_{57} + 12x_{58}

subject to:

$x_{13} + x_{14} + x_{15} = 300$
$x_{23} + x_{24} + x_{25} = 300$
$x_{36} + x_{46} + x_{56} = 200$
$x_{37} + x_{47} + x_{57} = 100$
$x_{38} + x_{48} + x_{58} = 300$
$x_{13} + x_{23} - x_{36} - x_{37} - x_{38} = 0$
$x_{14} + x_{24} - x_{46} - x_{47} - x_{48} = 0$
$x_{15} + x_{25} - x_{56} - x_{57} - x_{58} = 0$
$x_{ij} \geq 0$

Supply constraints for farms in Nebraska and Colorado

Demand constraints at the Chicago, St. Louis and Cincinnati mills

Transshipment Model Example
Computer Solution with Excel (1 of 3)

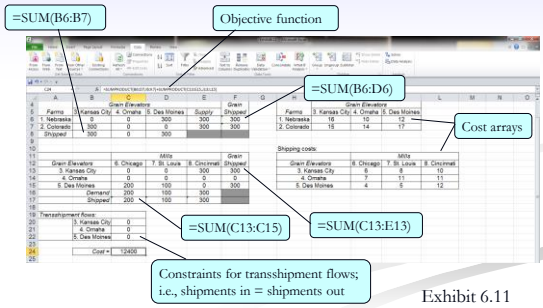


Exhibit 6.11

Transshipment Model Example
Computer Solution with Excel (2 of 3)

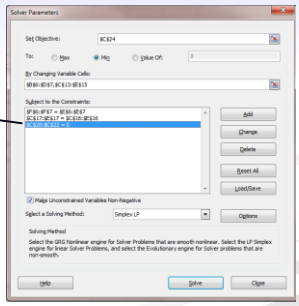


Exhibit 6.12

Transshipment Model Example
Network Solution for Wheat Shipping (3 of 3)

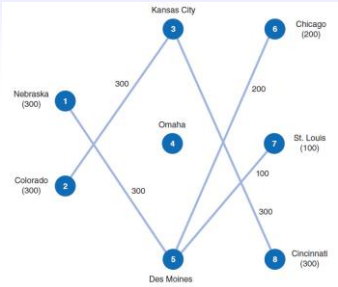


Figure 6.4 Transshipment network solution for wheat-shipping example

The Assignment Model Characteristics

- Special form of linear programming model similar to the transportation model.
- ***Supply*** at each source ***and demand*** at each destination ***limited to one unit***.
- In a balanced model supply equals demand.
- In an unbalanced model supply does not equal demand.

Assignment Model Example
Problem Definition and Data

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

	Game Sites			
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Assignment Model Example
Model Formulation

Minimize Z = $210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA}$
$+ 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD}$
$+ 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC}$

subject to:

$x_{AR} + x_{AA} + x_{AD} + x_{AC} = 1$
$x_{BR} + x_{BA} + x_{BD} + x_{BC} = 1$
$x_{CR} + x_{CA} + x_{CD} + x_{CC} = 1$
$x_{DR} + x_{DA} + x_{DD} + x_{DC} = 1$
$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$
$x_{AA} + x_{BA} + x_{CA} + x_{DA} = 1$
$x_{AD} + x_{BD} + x_{CD} + x_{DD} = 1$
$x_{AC} + x_{BC} + x_{CC} + x_{DC} = 1$

$x_{ij} \geq 0$

Assignment Model Example
Computer Solution with Excel (1 of 3)

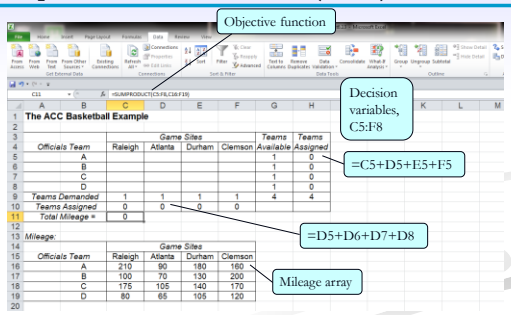


Exhibit 6.13

Assignment Model Example
Computer Solution with Excel (2 of 3)

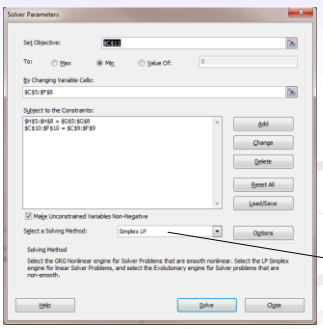
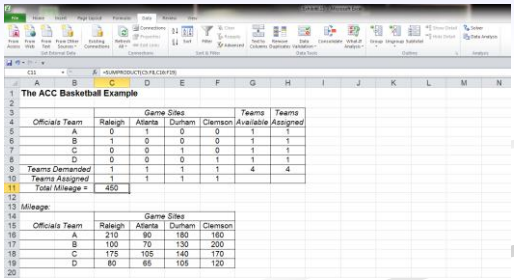


Exhibit 6.14

Assignment Model Example
Computer Solution with Excel (3 of 3)



The ACC Basketball Example

Officials Team	Game Sites: Raleigh	Atlanta	Durham	Clemson	Teams Available	Teams Assigned
A	0	1	0	0	1	1
B	1	0	0	0	1	1
C	0	0	1	0	1	1
D	0	0	0	1	1	1
Teams Demanded	1	1	1	1	4	4
Teams Assigned	1	1	1	1		
Total Mileage =	450					

Mileage:

Officials Team	Game Sites: Raleigh	Atlanta	Durham	Clemson
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Exhibit 6.15

Assignment Model Example
Assignment Network Solution

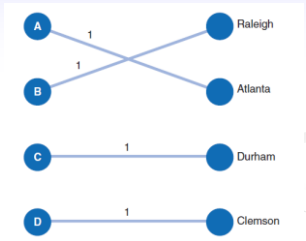
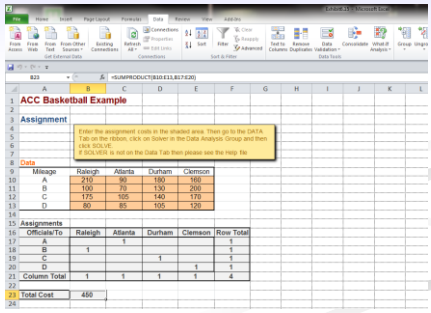


Figure 6.5 Assignment network solution for ACC officials

Assignment Model Example
Computer Solution with Excel QM



ACC Basketball Example

Assignment

Enter the assignment costs in the shaded area. Then go to the DATA Tab on the ribbon, click on Solver in the Data Analysis Group and then click SOLVE.
If SOLVER is not on the Data Tab then please see the Help file.

Data

Mileage	Raleigh	Atlanta	Durham	Clemson
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Assignments

Officials/To	Raleigh	Atlanta	Durham	Clemson	Row Total
A		1			1
B	1				1
C			1		1
D				1	1
Column Total	1	1	1	1	4

Total Cost 450

Exhibit 6.16

Assignment Model Example
Computer Solution with QM for Windows (1 of 2)

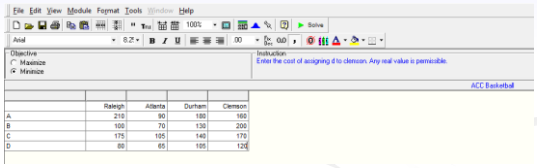


Exhibit 6.17

Assignment Model Example
Computer Solution with QM for Windows (2 of 2)

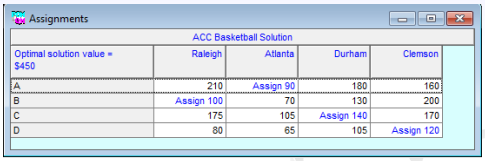


Exhibit 6.18

Example Problem Solution
Transportation Problem Statement

A concrete company transports concrete from three plants to three construction sites. The supply capacities of the three plants, the demand requirements at the three sites, and the transportation costs per ton are as follows:

	Construction site			
Plant	**A**	**B**	**C**	**Supply (tons)**
1	**$ 8**	**$ 5**	**$ 6**	**120**
2	**15**	**10**	**12**	**80**
3	**3**	**9**	**10**	**80**
Demand (tons)	**150**	**70**	**100**	

Determine the linear programming model formulation and solve using Excel.

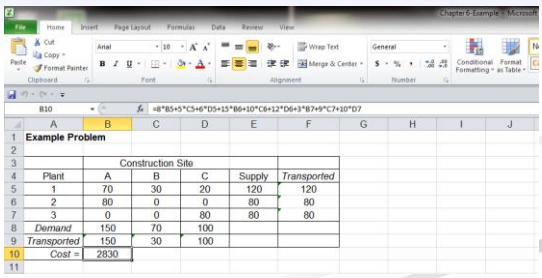
Example Problem Solution
Model Formulation

Minimize Z = $\$8x_{1A} + 5x_{1B} + 6x_{1C} + 15x_{2A} + 10x_{2B} + 12x_{2C} + 3x_{3A} + 9x_{3B} + 10x_{3C}$

subject to:

$$x_{1A} + x_{1B} + x_{1C} = 120$$
$$x_{2A} + x_{2B} + x_{2C} = 80$$
$$x_{3A} + x_{3B} + x_{3C} = 80$$
$$x_{1A} + x_{2A} + x_{3A} \leq 150$$
$$x_{1B} + x_{2B} + x_{3B} \leq 70$$
$$x_{1C} + x_{2C} + x_{3C} \leq 100$$
$$x_{ij} \geq 0$$

Example Problem Solution
Computer Solution with Excel



	A	B	C	D	E	F
1	Example Problem					
2						
3		Construction Site				
4	Plant	A	B	C	Supply	*Transported*
5	1	70	30	20	120	120
6	2	80	0	0	80	80
7	3	0	0	80	80	80
8	*Demand*	150	70	100		
9	*Transported*	150	30	100		
10	*Cost =*	2830				