TRIANGLE RELATIONSHIPS

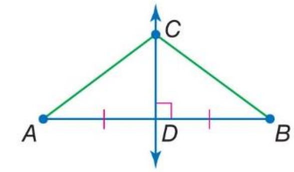
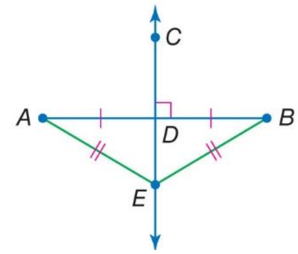
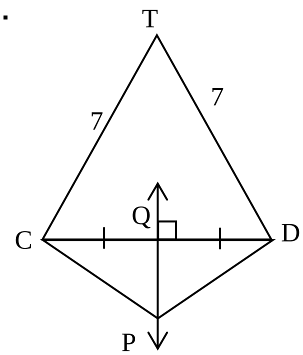
Chapter 5 – Unit 7

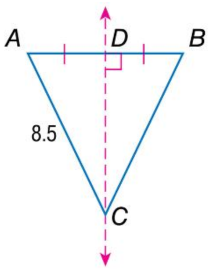
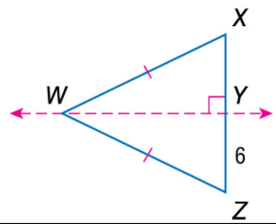
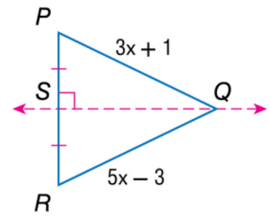
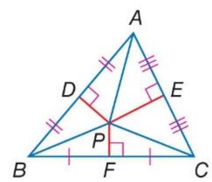
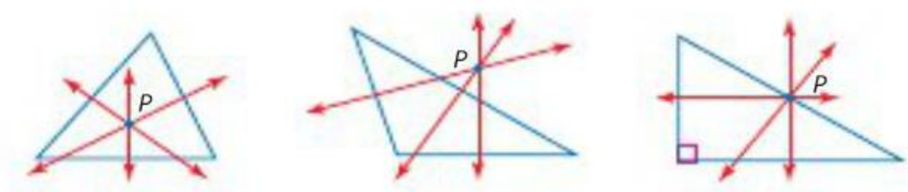
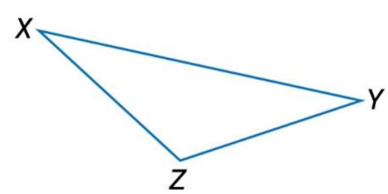
Geometry- Rushing

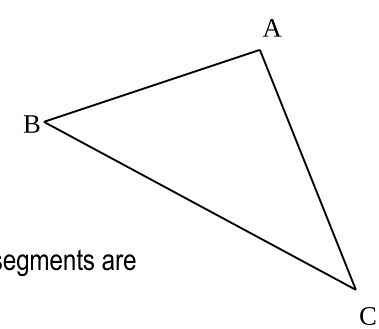
Name ______________________________

Hour ____________________

5.1 Bisectors of Triangles

I can…	1. Identify and use perpendicular bisectors in triangles. 2. Identify and use angle bisectors in triangles.
	Theorems Perpendicular Bisectors **5.1 Perpendicular Bisector Theorem** If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. **Example:** If $\overline{CD}$ is a $\perp$ bisector of $\overline{AB}$, then $AC = BC$.  **5.2 Converse of the Perpendicular Bisector Theorem** If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. **Example:** If $AE = BE$, then E lies on $\overline{CD}$, the $\perp$ bisector of $\overline{AB}$. 
Definitions	**Perpendicular Bisector –** a line, segment, or ray that passes through the midpoint of a side and is perpendicular to that side. **Equidistant from two points –** The distance between two lines measured along a perpendicular line is always the same. **Distance from a point to a line (p. 215) –** the length of the segment perpendicular to the line from the point. **Equidistant from two lines –** the distance between two lines measured along a perpendicular line is always the same.
	In the diagram shown, $\overleftrightarrow{PQ}$ is the perpendicular bisector $\overline{CD}$.  a. What segment lengths in the diagram are equal? b. Explain why T is on $\overleftrightarrow{PQ}$.

<table>
<tr><td>Perpendicular Bisector Theorems</td><td>A. Find BC.
</td><td>B. Find XY.
</td><td>C. Find PQ.
</td></tr>
<tr><td></td><td colspan="3">__________ point of intersection forms the __________ of the triangle.</td></tr>
<tr><td></td><td colspan="3">Theorem 5.3 Circumcenter Theorem
Words The perpendicular bisectors of a triangle intersect at a point called the circumcenter that is equidistant from the vertices of the triangle.
Example If P is the circumcenter of $\triangle ABC$, then $PB = PA = PC$.
</td></tr>
<tr><td>Use the Circumcenter Theorem</td><td colspan="3">The circumcenter can be on the interior, exterior, or side of a triangle.
acute triangle   obtuse triangle   right triangle</td></tr>
<tr><td></td><td colspan="3">A triangular-shaped garden is shown. Can a fountain be placed at the circumcenter and still be inside the garden?
</td></tr>
<tr><td>Using Circumcenter</td><td colspan="3">Three people need to decide on a location to hold a monthly meeting. They will all be coming from different places in the city and they want to make the meeting location the same distance for each person.

a. Explain why using circumcenter as the location for the meeting would be fairest for all.

b. Locate the circumcenter of the triangle and tell what segments are congruent.</td></tr>
</table>

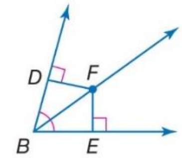
<table>
<tr><td></td><td colspan="3">

Theorems Angle Bisectors

5.4 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

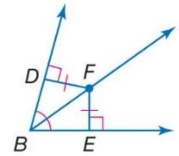
Example: If $\overrightarrow{BF}$ bisects $\angle DBE$, $\overline{FD} \perp \overrightarrow{BD}$, and $\overline{FE} \perp \overrightarrow{BE}$, then $DF = FE$.



5.5 Converse of the Angle Bisector Theorem

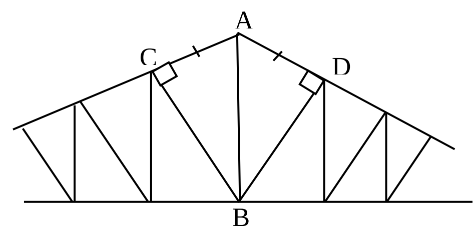
If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.

Example: If $\overline{FD} \perp \overrightarrow{BD}$, $\overline{FE} \perp \overrightarrow{BE}$, and $DF = FE$, then $\overrightarrow{BF}$ bisects $\angle DBE$.



</td></tr>
<tr><td></td><td colspan="3">

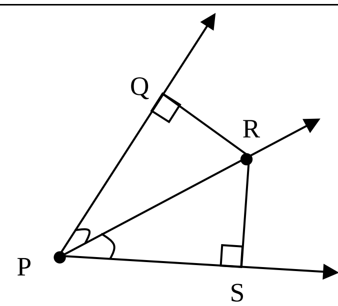
Suppose that in constructing a wooden roof truss, BC and BD are installed so that ∠ ACB and ∠ ADB are right angles and that AC = AD. What can you say about ∠ CBA and ∠ ABD?



</td></tr>
<tr><td></td><td colspan="3">

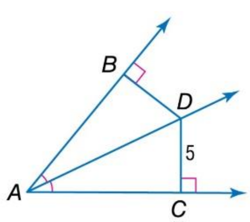
Given: R is on the bisector of ∠ QPS.
RQ ⊥ PQ, RS ⊥ PS

Prove: $\overline{RQ} \cong \overline{RS}$



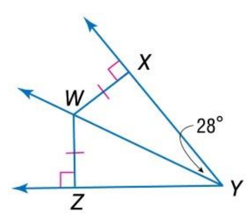
</td></tr>
<tr><td>Use the Angle Bisector Theorems</td><td>

A. Find *DB*.



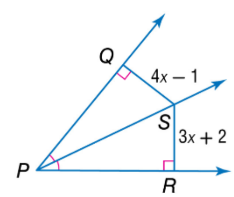
</td><td>

B. Find $m\angle WYZ$.



</td><td>

C. Find *QS*.



</td></tr>
<tr><td>Definitions</td><td colspan="3">

Perpendicular Bisector of a Triangle - a line perpendicular to the side and passing through its midpoint.

Concurrent Lines (or rays or segments) – 3 or more lines that intersect

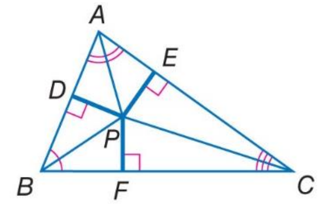
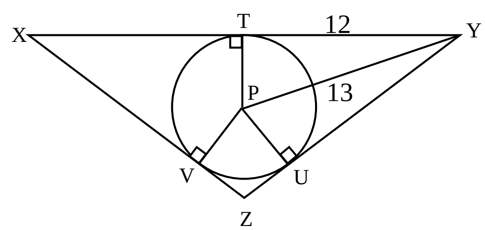
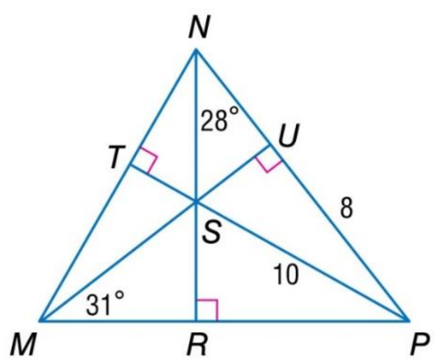
Point of Concurrency – the point of intersection of concurrent lines

Circumcenter – the point of concurrency of the perpendicular bisectors of a triangle.

Angle Bisector of a Triangle - if a ray or segment bisects an angle of a triangle then it divides the two segments on either side proportionally.

Incenter of a Triangle – a point of concurrency of the angle bisectors of a triangle.

</td></tr>
</table>

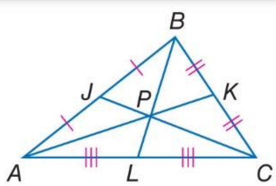
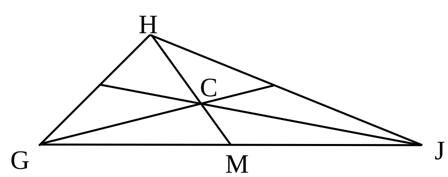
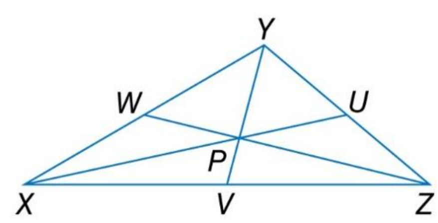
	Theorem 5.6 Incenter Theorem **Words** The angle bisectors of a triangle intersect at a point called the *incenter* that is equidistant from the sides of the triangle. **Example** If *P* is the incenter of $\triangle ABC$, then $PD = PE = PF$. 
	_____ __________ point of intersection forms the __________ of the triangle. This point is ___________ from the sides of the triangle.
Using Angle Bisectors	The angle bisector of $\triangle$ XYZ meet at point P. a. What segments are congruent?  b. Find PT. PV.
Use the Incenter Theorem	A. Find *SU* if *S* is the incenter of Δ*MNP*.  B. Find $m\angle SPU$ if *S* is the incenter of Δ*MNP*.

5.2 Means and Altitudes of Triangles

<table>
<tr><td>I can…</td><td colspan="2">1. Identify and use medians in triangles.
2. Identify and use altitudes in triangles.</td></tr>
<tr><td>Definitions</td><td colspan="2">Median of a Triangle – A line segment with endpoints that are a vertex of a triangle and the midpoint of the side opposite the vertex.

Centroid of a Triangle – The point of concurrency of the medians.

Altitude of a Triangle – a segment from a vertex of the triangle to the line containing the opposite side and perpendicular to that side.

Orthocenter of the Triangle – the point of concurrency of the altitudes of a triangle.</td></tr>
<tr><td></td><td colspan="2">__________ point of intersection forms the __________ of the triangle. This point is ___________ from the vertex to the midpoint of the opposite side of the triangle.</td></tr>
<tr><td></td><td colspan="2">Theorem 5.7 Centroid Theorem
The medians of a triangle intersect at a point called the centroid that is two thirds of the distance from each vertex to the midpoint of the opposite side.
Example If P is the centroid of $\triangle ABC$, then $AP = \frac{2}{3}AK$, $BP = \frac{2}{3}BL$, and $CP = \frac{2}{3}CJ$.

</td></tr>
<tr><td rowspan="2">Use the Centroid Theorem</td><td>a. C is the centroid of $\triangle GHJ$ and CM = 8. Find HM and CH.

</td><td>b. In ΔXYZ, P is the centroid and YV = 12. Find YP and PV.

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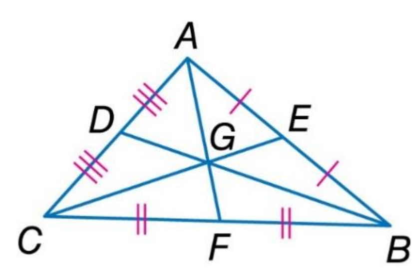
<tr><td colspan="2">c. In ΔABC, CG = 4. Find GE.

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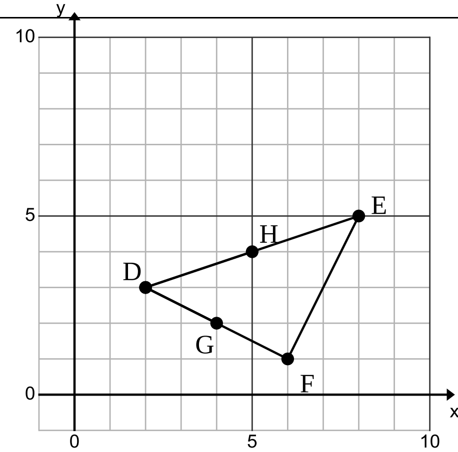
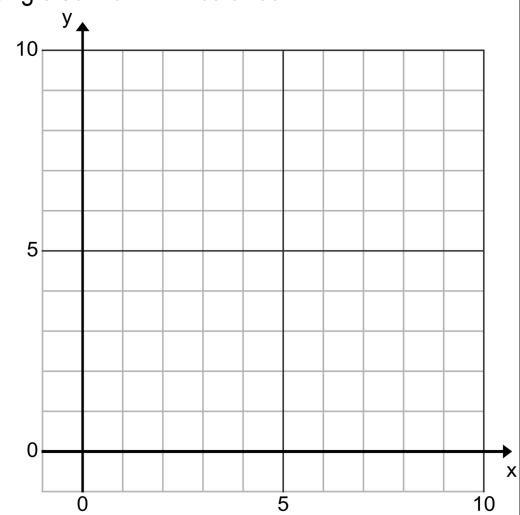
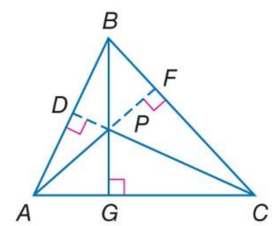
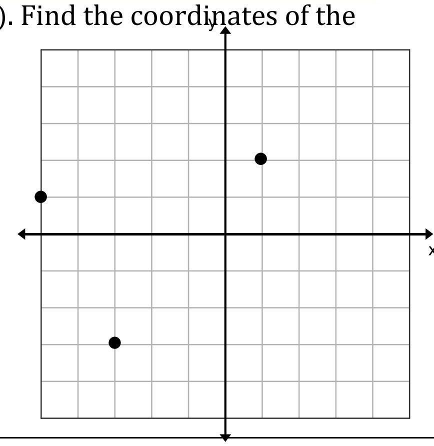
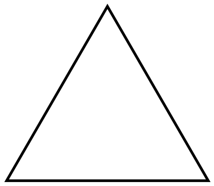
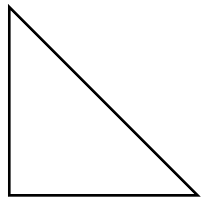
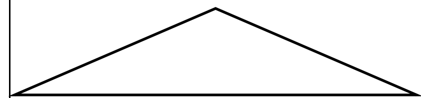
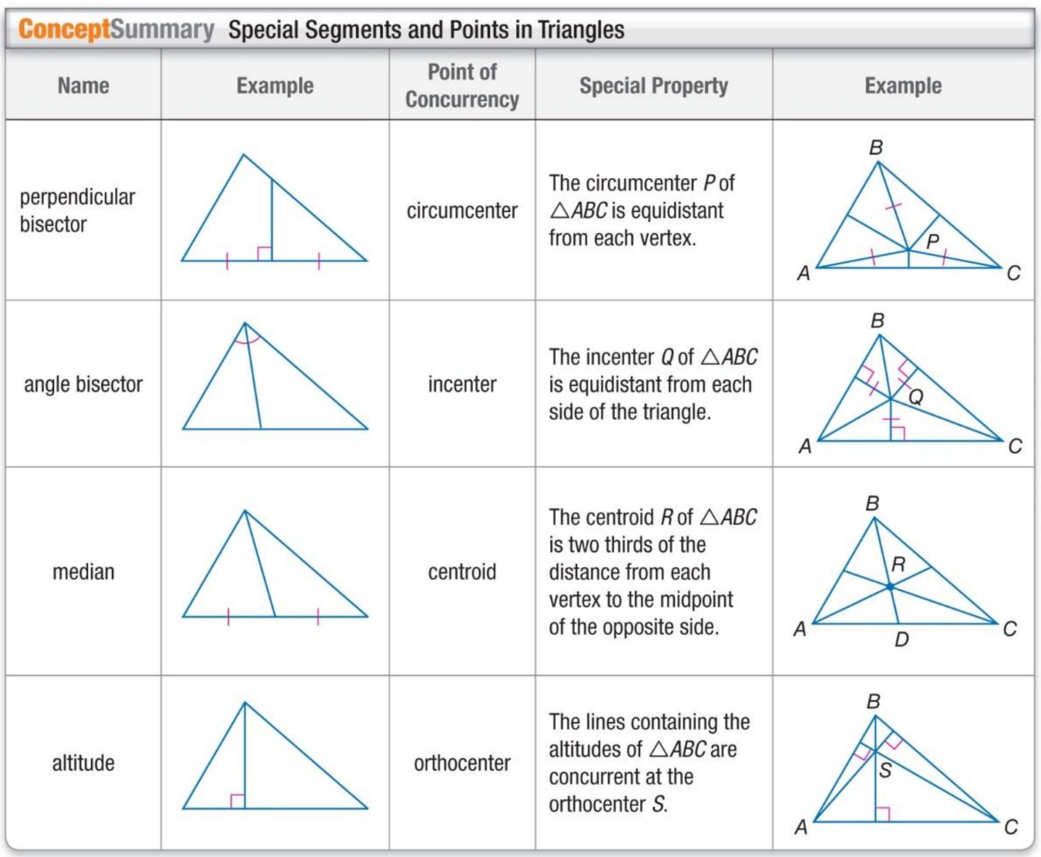
</table>

Find the Centroid on a Coordinate Plane	Find the coordinate of the centroid P of $\triangle DEF$. D (2, 3) E (8, 5) F (6, 1) G (4, 2) H (5, 4) Segments EG and HF point of intersection is the centroid. 
Find the Centroid on a Coordinate Plane	An artist is designing a sculpture that balances a triangle on top of a pole. In the artist's design on the coordinate plane, the vertices are located at $(7, 4)$, $(3, 0)$, and $(3, 8)$. What are the coordinates of the point where the artist should place the pole under the triangle so that it will balance? 
__________ point of intersection forms the __________ of the triangle.	**KeyConcept Orthocenter** The lines containing the altitudes of a triangle are concurrent, intersecting at a point called the **orthocenter**. **Example** The lines containing altitudes $\overline{AF}$, $\overline{CD}$, and $\overline{BG}$ intersect at *P*, the orthocenter of $\triangle ABC$. 
Find the Orthocenter on a Coordinate Plane	The vertices of ΔHIJ are $H(1, 2)$, $I(-3, -3)$, and $J(-5, 1)$. Find the coordinates of the orthocenter of ΔHIJ. 

Orthocenter	Where is the orthocenter located in $\Delta\, ABC$? Is it ***inside, outside*** or ***on*** the triangle?		
	a. if m∠A = m∠B = m∠C? 	b. m∠A = m∠B = 45°? 	c. If m∠A = 110°? 



ConceptSummary Special Segments and Points in Triangles

Name	Example	Point of Concurrency	Special Property	Example
perpendicular bisector		circumcenter	The circumcenter *P* of △*ABC* is equidistant from each vertex.	B, A, C, P
angle bisector		incenter	The incenter *Q* of △*ABC* is equidistant from each side of the triangle.	B, A, C, Q
median		centroid	The centroid *R* of △*ABC* is two thirds of the distance from each vertex to the midpoint of the opposite side.	B, A, C, R, D
altitude		orthocenter	The lines containing the altitudes of △*ABC* are concurrent at the orthocenter *S*.	B, A, C, S

5.3 Inequalities in Triangles

I can…

3. Recognize and apply properties of inequalities to the measures of the angles of a triangle.
4. Recognize and apply properties of inequalities to the relationships between the angles and sides of a triangle.

KeyConcept Properties of Inequality for Real Numbers

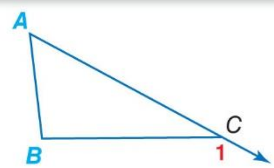
The following properties are true for any real numbers a, b, and c.

Comparison Property of Inequality	$a < b$, $a = b$, or $a > b$
Transitive Property of Inequality	**1.** If $a < b$ and $b < c$, then $a < c$. **2.** If $a > b$ and $b > c$, then $a > c$.
Addition Property of Inequality	**1.** If $a > b$, then $a + c > b + c$. **2.** If $a < b$, then $a + c < b + c$.
Subtraction Property of Inequality	**1.** If $a > b$, then $a - c > b - c$. **2.** If $a < b$, then $a - c < b - c$.

Theorem 5.8 Exterior Angle Inequality

The measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles.

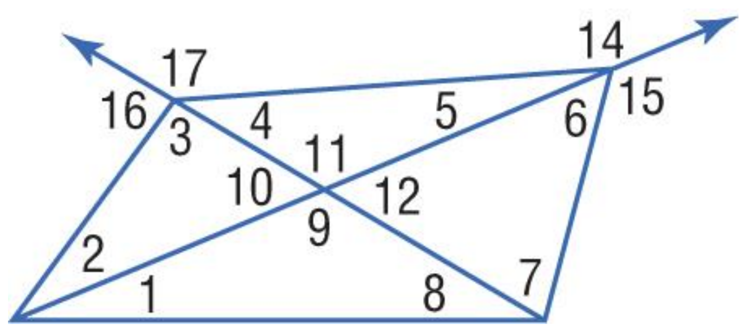
Example: $m\angle 1 > m\angle A$
$m\angle 1 > m\angle B$



Use the Exterior Angle Inequality Theorem

A. List all angles whose measures are less than $m\angle 14$.

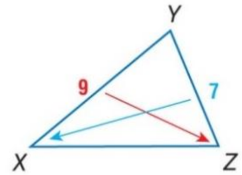
B. List all angles whose measures are greater than $m\angle 5$.



Theorems Angle-Side Relationships in Triangles

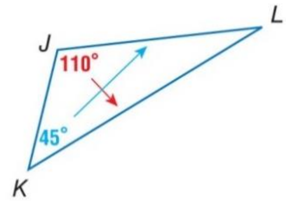
5.9 If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

Example: $XY > YZ$, so $m\angle Z > m\angle X$.



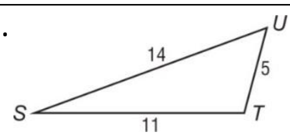
5.10 If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

Example: $m\angle J > m\angle K$, so $KL > JL$.



Order Triangle Angle/Side Measures

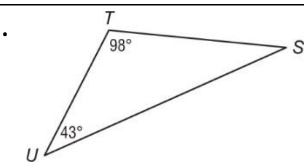
1.



Write the angles in order from smallest to largest.

_______ <_______<_______

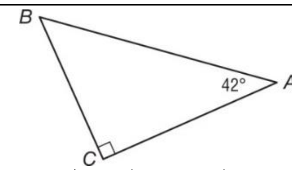
2.



Write the sides in order from shortest to longest.

_______ <_______<_______

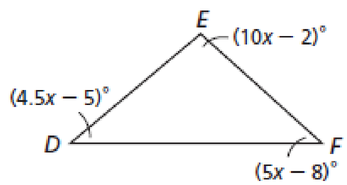
3.



Write the sides in order from longest to shortest.

_______ <_______<_______

4. List the sides of the triangle shortest to longest.



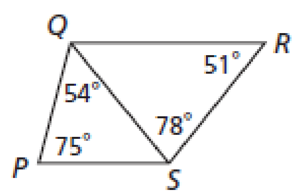
_______ <_______<_______

Compare. Write <, >, or =.

5. *PQ* *QS*

6. *QS* *RS*

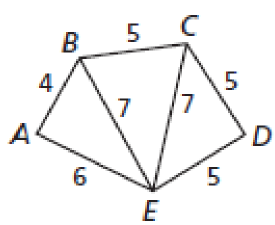
7. *QR* *RS*



8. $m\angle CBE$ $m\angle CEB$

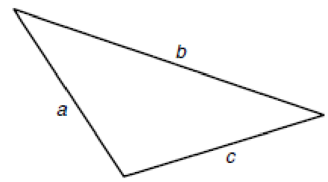
9. $m\angle DCE$ $m\angle CDE$

10. $m\angle EBC$ $m\angle ECB$

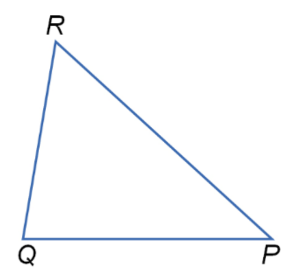


5.5 The Triangle Inequality

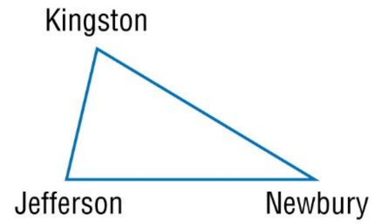
<table>
<tr><td>I can…</td><td>1. Use the Triangle Inequality Theorem to identify possible triangles.
2. Prove triangle relationships using the Triangle Inequality Theorem.</td></tr>
<tr><td></td><td>
<table>
<tr><th>Theorem</th><th>Example</th></tr>
<tr><td>Triangle Inequality Theorem
The sum of any two side lengths of a triangle is greater than the third side length.</td><td>



$a + b > c$
$b + c > a$
$c + a > b$</td></tr>
</table>
</td></tr>
<tr><td>Constructing a Triangle</td><td>Tell whether a trianlge can have sides witht the given lenths. Explain.
1. 3 in, 3 in, 8 in   2. 6 in, 6in, 12 in   3. 9 in, 5 in, 11 in</td></tr>
<tr><td>Possible triangles given side lenghts</td><td>The lengths of two sides of a triangle are given. Find the rangel of possible lenghts for the third side.
4. 4 and 19   5. 3.07 and 1.89   6. $3\frac{5}{6}$ and $6\frac{1}{2}$</td></tr>
<tr><td>Find Possible Side Lengths</td><td>In $\Delta\ PQR, PQ = 7.2$ and $QR = 5.2$. which measure cannot be PR?

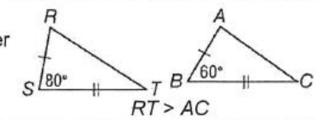
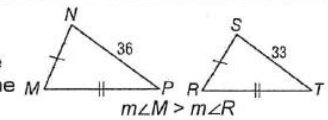


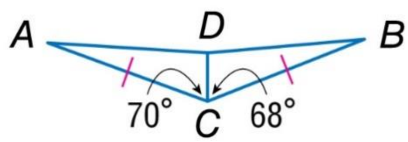
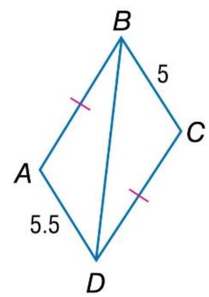
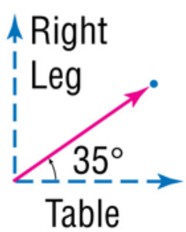
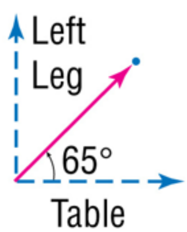
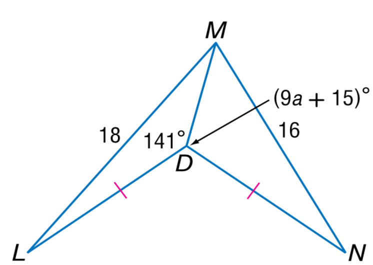
</td></tr>
</table>

Proof Using Triangle Inequality Theorem	**The towns of Jefferson, Kingston, and Newbury are shown in the map below. Prove that driving first from Jefferson to Kingston and then Kingston to Newbury is a greater distance than driving from Jefferson to Newbury.** 
Finding Possible Side Lengths and Angle Measures	A. In ΔABC and ΔDEF, $\overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}, AB = 11\ in, ED = 15\ in$, and $m\angle F = 58°$. Which of the following is a possible measure for $m\angle C$: 45°, 58°, 80°, or 90°?
	B. In ΔGHI and ΔJKL, $\overline{GH} \cong \overline{JK}, \overline{HI} \cong \overline{KL}, GI = 9cm, m\angle H = 45°$ and $m\angle K = 65°$. Which of the following is a possible length for $\overline{JL}$: 5cm, 7cm, 9cm or 11cm?

5.6 Inequalities in Two Triangles

I can...	1. Apply the Hinge Theorem or its converse to make comparisons in two triangles. 2. Prove triangle relationships using the Hinge Theorem or its converse.

Hinge Theorem	If two sides of a triangle are congruent to two sides of another triangle and the included angle of the first is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.	
Converse of the Hinge Theorem	If two sides of a triangle are congruent to two sides of another triangle, and the third side in the first is longer than the third side in the second, then the included angle in the first triangle is greater than the included angle in the second triangle.	

Example 5-6-1: **Use the Hinge Theorem and Its Converse**	**A.** Compare the measures AD and BD.  **B.** Compare the measures $m\angle ABD$ and $m\angle BDC$. 
Example 5-6-2: **Use the Hinge Theorem**	Doctors use a straight-leg-raising test to determine the amount of pain felt in a person's back. The patient lies flat on the examining table, and the doctor raises each leg until the patient experiences pain in the back area. Nitan can tolerate the doctor raising his right leg 35° and his left leg 65° from the table. Which leg can Nitan raise higher above the table?  Since Nitan's legs are the same length and his left leg and the table is the same length in both situations, the Hinge Theorem says ____________________
Example 5-6-3: **Apply Algebra to the Relationships in Triangles**	Find the range of possible values for *a* 

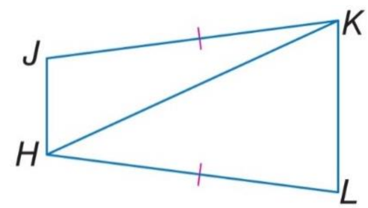
Example 5-6-4: **Prove Triangle Relationships Using Hinge Theorem**

Write a two-column proof.

Given: $JK = HL$; $JH \parallel KL$

$m\angle JKH + m\angle HKL < m\angle JHK + m\angle KHL$

Prove: $JH < KL$

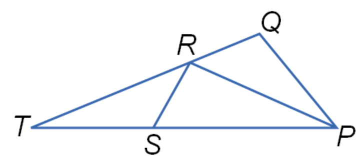


Statements	Reasons
1. $JK = HL$	1. Given
2. HK = HK	2.
3. $m\angle JKH + m\angle HKL < m\angle JHK + m\angle KHL$	3. Given
4. $m\angle HKL = m\angle JHK$	4.
5. $m\angle JKH + m\angle JHK < m\angle JHK + m\angle KHL$	5.
6. $m\angle JHK < m\angle KHL$	6.
7. $JH < KL$	7.

Example 5-6-5: **Prove Relationships Using Converse of Hinge Theorem**

Given: $ST = PQ$; $SR = QR$; $ST = \frac{2}{3}SP$

Prove: $m\angle SRP > m\angle PRQ$



Statements	Reasons
1. $SR = QR$	1. Given
2.	2.
3. $ST = PQ$	3. Given
4. $ST = \frac{2}{3}SP$, SP > ST	4. Given
5.	5.
6. $m\angle SRP > m\angle PRQ$	6.