4.1 Triangles and Angles

What you should learn

GOAL 1 Classify triangles by their sides and angles, as applied in **Example 2.**

GOAL 2 Find angle measures in triangles.

Why you should learn it

▼ To solve **real-life** problems, such as finding the measures of angles in a wing deflector in **Exs. 45 and 46.**

A wing deflector is used to change the velocity of the water in a stream.

GOAL 1 CLASSIFYING TRIANGLES

A **triangle** is a figure formed by three segments joining three noncollinear points. A triangle can be classified by its sides and by its angles, as shown in the definitions below.

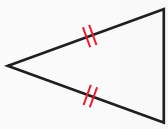
NAMES OF TRIANGLES

Classification by Sides

EQUILATERAL TRIANGLE

3 congruent sides

ISOSCELES TRIANGLE



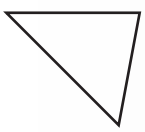
At least 2 congruent sides

SCALENE TRIANGLE

No congruent sides

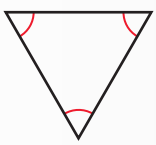
Classification by Angles

ACUTE TRIANGLE



3 acute angles

EQUIANGULAR TRIANGLE

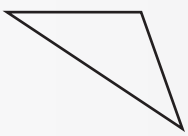


3 congruent angles

RIGHT TRIANGLE

1 right angle

OBTUSE TRIANGLE



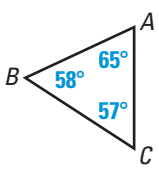
1 obtuse angle

Note: An equiangular triangle is also acute.

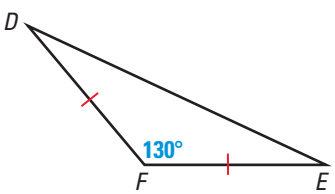
EXAMPLE 1 *Classifying Triangles*

When you classify a triangle, you need to be as specific as possible.

a. $\triangle ABC$ has three acute angles and no congruent sides. It is an acute scalene triangle. ($\triangle ABC$ is read as "triangle ABC.")

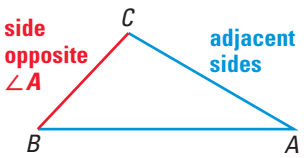


b. $\triangle DEF$ has one obtuse angle and two congruent sides. It is an obtuse isosceles triangle.



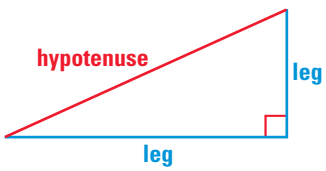
Each of the three points joining the sides of a triangle is a **vertex**. (The plural of vertex is *vertices*.) For example, in $\triangle ABC$, points A, B, and C are vertices.

In a triangle, two sides sharing a common vertex are **adjacent sides**. In $\triangle ABC$, $\overline{CA}$ and $\overline{BA}$ are adjacent sides. The third side, $\overline{BC}$, is the side *opposite* $\angle A$.

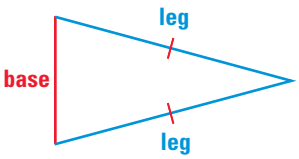


RIGHT AND ISOSCELES TRIANGLES The sides of right triangles and isosceles triangles have special names. In a right triangle, the sides that form the right angle are the **legs** of the right triangle. The side opposite the right angle is the **hypotenuse** of the triangle.

An isosceles triangle can have three congruent sides, in which case it is equilateral. When an isosceles triangle has only two congruent sides, then these two sides are the **legs** of the isosceles triangle. The third side is the **base** of the isosceles triangle.



Right triangle



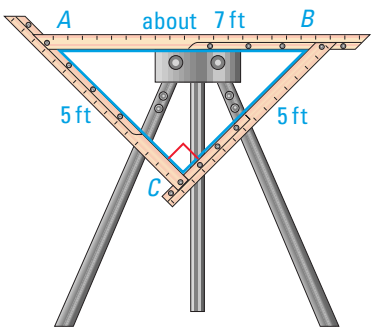
Isosceles triangle

EXAMPLE 2 *Identifying Parts of an Isosceles Right Triangle*

FOCUS ON APPLICATIONS

WEAVING Most looms are used to weave rectangular cloth. The loom shown in the photo is used to weave triangular pieces of cloth. A piece of cloth woven on the loom can use about 550 yards of yarn.

The diagram shows a triangular loom.



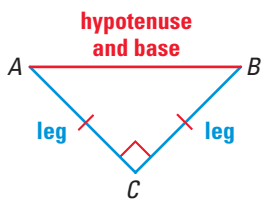
a. Explain why $\triangle ABC$ is an isosceles right triangle.

b. Identify the legs and the hypotenuse of $\triangle ABC$. Which side is the base of the triangle?

SOLUTION

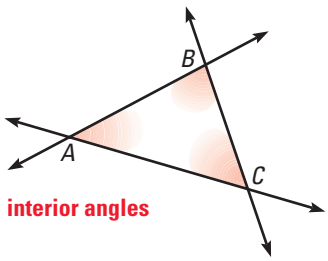
a. In the diagram, you are given that $\angle C$ is a right angle. By definition, $\triangle ABC$ is a right triangle. Because $AC = 5$ ft and $BC = 5$ ft, $\overline{AC} \cong \overline{BC}$. By definition, $\triangle ABC$ is also an isosceles triangle.

b. Sides $\overline{AC}$ and $\overline{BC}$ are adjacent to the right angle, so they are the legs. Side $\overline{AB}$ is opposite the right angle, so it is the hypotenuse. Because $\overline{AC} \cong \overline{BC}$, side $\overline{AB}$ is also the base.

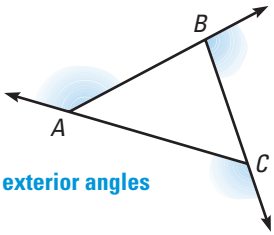


GOAL 2 USING ANGLE MEASURES OF TRIANGLES

When the sides of a triangle are extended, other angles are formed. The three original angles are the **interior angles**. The angles that are adjacent to the interior angles are the **exterior angles**. Each vertex has a *pair* of congruent exterior angles. It is common to show only *one* exterior angle at each vertex.



interior angles



exterior angles

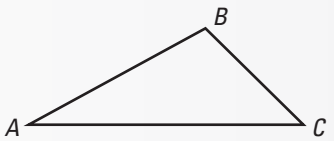
In Activity 4.1 on page 193, you may have discovered the *Triangle Sum Theorem*, shown below, and the *Exterior Angle Theorem*, shown on page 197.

THEOREM

THEOREM 4.1 ***Triangle Sum Theorem***

The sum of the measures of the interior angles of a triangle is 180°.

$$m\angle A + m\angle B + m\angle C = 180°$$

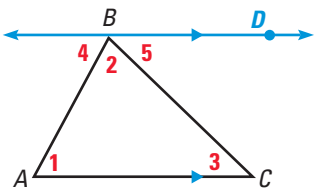


To prove some theorems, you may need to add a line, a segment, or a ray to the given diagram. Such an *auxiliary line* is used to prove the Triangle Sum Theorem.

Proof

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Plan for Proof By the Parallel Postulate, you can draw an auxiliary line through point B and parallel to $\overline{AC}$. Because $\angle 4$, $\angle 2$, and $\angle 5$ form a straight angle, the sum of their measures is 180°. You also know that $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 5$ by the Alternate Interior Angles Theorem.

Statements	Reasons
1. Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	**1.** Parallel Postulate
2. $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	**2.** Angle Addition Postulate and definition of straight angle
3. $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 5$	**3.** Alternate Interior Angles Theorem
4. $m\angle 1 = m\angle 4$ and $m\angle 3 = m\angle 5$	**4.** Definition of congruent angles
5. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	**5.** Substitution property of equality

STUDENT HELP

Study Tip

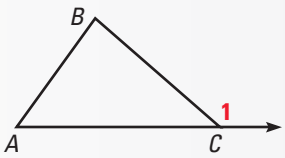
An auxiliary line, segment, or ray used in a proof must be justified with a reason.

THEOREM

THEOREM 4.2 ***Exterior Angle Theorem***

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

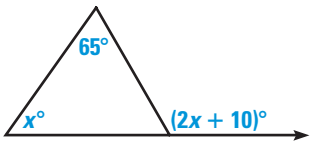
$m\angle 1 = m\angle A + m\angle B$



Using Algebra

EXAMPLE 3 *Finding an Angle Measure*

You can apply the Exterior Angle Theorem to find the measure of the exterior angle shown. First write and solve an equation to find the value of x:



$x° + 65° = (2x + 10)°$ **Apply the Exterior Angles Theorem.**

$55 = x$ **Solve for x.**

▶ So, the measure of the exterior angle is $(2 \cdot 55 + 10)°$, or $120°$.

STUDENT HELP

Skills Review
For help with solving equations, see p. 790.

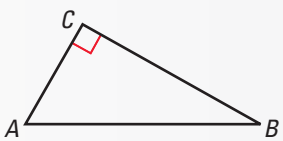
A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

COROLLARY

COROLLARY TO THE TRIANGLE SUM THEOREM

The acute angles of a right triangle are complementary.

$m\angle A + m\angle B = 90°$

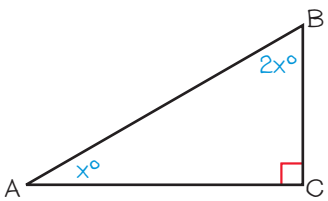


EXAMPLE 4 *Finding Angle Measures*

The measure of one acute angle of a right triangle is two times the measure of the other acute angle. Find the measure of each acute angle.

SOLUTION

Make a sketch. Let $x° = m\angle A$. Then $m\angle B = 2x°$.



$x° + 2x° = 90°$ **The acute angles of a right triangle are complementary.**

$x = 30$ **Solve for x.**

▶ So, $m\angle A = 30°$ and $m\angle B = 2(30°) = 60°$.

GUIDED PRACTICE

Vocabulary Check ✓

1. Sketch an obtuse scalene triangle. Label its interior angles 1, 2, and 3. Then draw its exterior angles. Shade the exterior angles.

Concept Check ✓

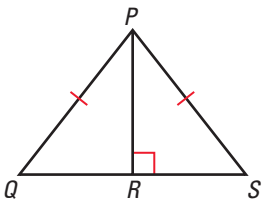
In the figure, $\overline{PQ} \cong \overline{PS}$ and $\overline{PR} \perp \overline{QS}$. Complete the sentence.

2. $\overline{PQ}$ is the ___?___ of the right triangle $\triangle PQR$.

3. In $\triangle PQR$, $\overline{PQ}$ is the side opposite angle ___?___.

4. $\overline{QS}$ is the ___?___ of the isosceles triangle $\triangle PQS$.

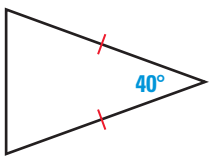
5. The legs of $\triangle PRS$ are ___?___ and ___?___.



Skill Check ✓

In Exercises 6–8, classify the triangle by its angles and by its sides.

6.



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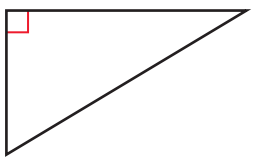
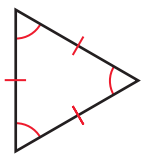


8.



9. The measure of one interior angle of a triangle is 25°. The other interior angles are congruent. Find the measures of the other interior angles.

PRACTICE AND APPLICATIONS

STUDENT HELP

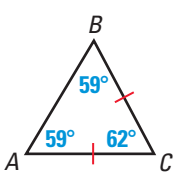
Extra Practice to help you master skills is on p. 809.

MATCHING TRIANGLES **In Exercises 10–15, match the triangle description with the most specific name.**

10. Side lengths: 2 cm, 3 cm, 4 cm	**A.** Equilateral
11. Side lengths: 3 cm, 2 cm, 3 cm	**B.** Scalene
12. Side lengths: 4 cm, 4 cm, 4 cm	**C.** Obtuse
13. Angle measures: 60°, 60°, 60°	**D.** Equiangular
14. Angle measures: 30°, 60°, 90°	**E.** Isosceles
15. Angle measures: 20°, 145°, 15°	**F.** Right

CLASSIFYING TRIANGLES **Classify the triangle by its angles and by its sides.**

16.



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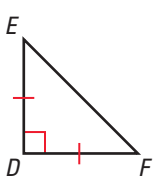

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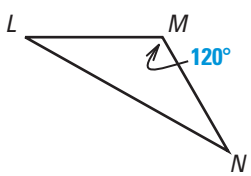
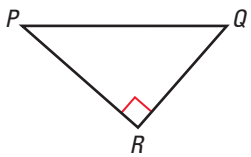


STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 10–26, 34–36
Example 2: Exs. 27, 28, 45
Example 3: Exs. 31–39
Example 4: Exs. 41–44

19.



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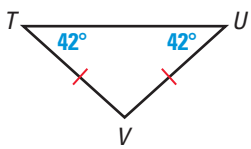

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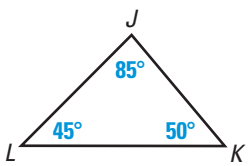


LOGICAL REASONING **Complete the statement using *always, sometimes,* or *never.***

22. An isosceles triangle is __?__ an equilateral triangle.

23. An obtuse triangle is __?__ an isosceles triangle.

24. An interior angle of a triangle and one of its adjacent exterior angles are __?__ supplementary.

25. The acute angles of a right triangle are __?__ complementary.

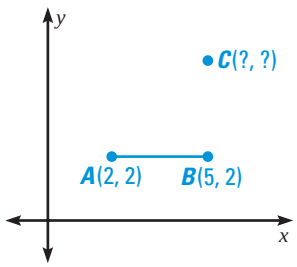
26. A triangle __?__ has a right angle and an obtuse angle.

IDENTIFYING PARTS OF TRIANGLES **Refer to the triangles in Exercises 16–21.**

27. Identify the legs and the hypotenuse of any right triangles.

28. Identify the legs and the base of any isosceles triangles. Which isosceles triangle has a base that is also the hypotenuse of a right triangle?

USING ALGEBRA **Use the graph. The segment $\overline{AB}$ is a leg of an isosceles right triangle.**



29. Find the coordinates of point C. Copy the graph and sketch $\triangle ABC$.

30. Find the coordinates of a point D that forms a different isosceles right triangle with leg $\overline{AB}$. Include a sketch with your answer.

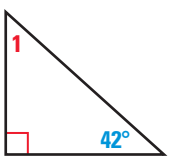
STUDENT HELP

INTERNET HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for help with Exs. 31–33.

FINDING ANGLE MEASURES **Find the measure of the numbered angles.**

31.



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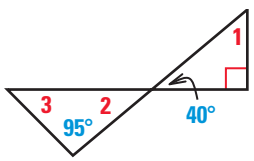

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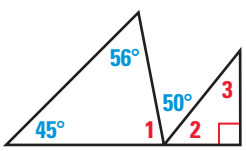


USING ALGEBRA **The variable expressions represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles.**

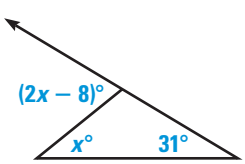
34. $m\angle A = x°$
$m\angle B = 2x°$
$m\angle C = (2x + 15)°$

35. $m\angle R = x°$
$m\angle S = 7x°$
$m\angle T = x°$

36. $m\angle W = (x - 15)°$
$m\angle Y = (2x - 165)°$
$m\angle Z = 90°$

EXTERIOR ANGLES **Find the measure of the exterior angle shown.**

37.



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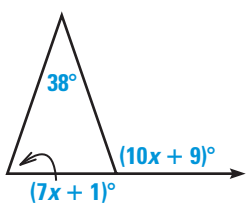

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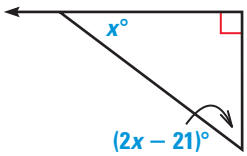

40. **TECHNOLOGY** Use geometry software to demonstrate the Triangle Sum Theorem or the Exterior Angle Theorem. Describe your procedure.

41. **USING ALGEBRA** In $\triangle PQR$, the measure of $\angle P$ is 36°. The measure of $\angle Q$ is five times the measure of $\angle R$. Find $m\angle Q$ and $m\angle R$.

42. **USING ALGEBRA** The measure of an exterior angle of a triangle is 120°. The interior angles that are not adjacent to this exterior angle are congruent. Find the measures of the interior angles of the triangle.

43. **BILLIARD RACK** You want to make a wooden billiard rack. The rack will be an equilateral triangle whose side length is 33.5 centimeters. You have a strip of wood that is 100 centimeters long. Do you need more wood? Explain.

44. **COAT HANGER** You are bending a wire to make a coat hanger. The length of the wire is 88 centimeters, and 20 centimeters are needed to make the hook portion of the hanger. The triangular portion of the hanger is an isosceles triangle. The length of one leg of this triangle is $\frac{3}{5}$ the length of the base. Sketch the hanger. Give the dimensions of the triangular portion.

FOCUS ON CAREERS

HYDROLOGY A hydrologist studies how water circulates in the atmosphere, on the ground, and under the ground. A hydrologist might use a wing deflector to minimize the effects of erosion on the bank of a stream.

CAREER LINK
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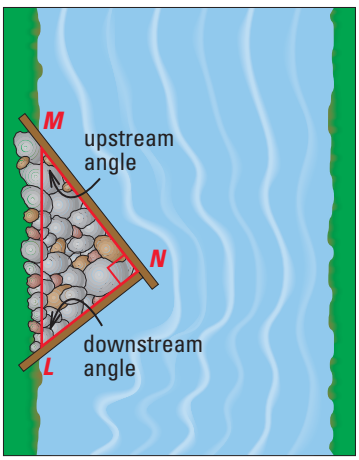
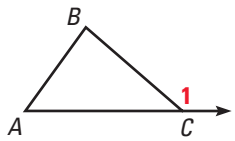
WING DEFLECTORS **In Exercises 45 and 46, use the information about wing deflectors.** A wing deflector is a structure built with rocks to redirect the flow of water in a stream and increase the rate of the water's flow. Its shape is a right triangle.



45. Identify the legs and the hypotenuse of the right triangle formed by the wing deflector.

46. It is generally recommended that the upstream angle should range from 30° to 45°. Give a range of angle measures for the downstream angle.

47. **DEVELOPING PROOF** Fill in the missing steps in the two-column proof of the Exterior Angle Theorem.



GIVEN ▶ $\angle 1$ is an exterior angle of $\triangle ABC$.

PROVE ▶ $m\angle 1 = m\angle A + m\angle B$

Statements	Reasons
1. $\angle 1$ is an exterior angle of $\triangle ABC$.	**1.** Given
2. $\angle ACB$ and $\angle 1$ are a linear pair.	**2.** Definition of exterior angle
3. $m\angle ACB + m\angle 1 = 180°$	**3.** ?
4. ?	**4.** Triangle Sum Theorem
5. $m\angle ACB + m\angle 1 = m\angle A + m\angle B + m\angle ACB$	**5.** ?
6. $m\angle 1 = m\angle A + m\angle B$	**6.** ?

48. **TWO-COLUMN PROOF** Write a two-column proof of the Corollary to the Triangle Sum Theorem on page 197.

Test Preparation

49. MULTIPLE CHOICE The lengths of the two legs of an isosceles triangle are represented by the expressions $(2x - 5)$ and $(x + 7)$. The perimeter of the triangle is 50 cm. Find the length of the base of the triangle.

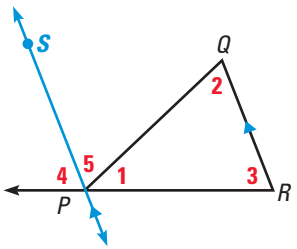
Ⓐ 11 cm Ⓑ 19 cm Ⓒ 12 cm Ⓓ 26 cm Ⓔ 32 cm

50. MULTIPLE CHOICE Which of the terms below can be used to describe a triangle with two 45° interior angles?

Ⓐ Acute Ⓑ Right Ⓒ Scalene Ⓓ Obtuse Ⓔ Equilateral

Challenge

51. ALTERNATIVE PROOFS There is often more than one way to prove a theorem. In the diagram, $\overline{SP}$ is constructed parallel to $\overline{QR}$. This construction is the first step of a proof of the Triangle Sum Theorem. Use the diagram to prove the Triangle Sum Theorem.



GIVEN ▶ $\triangle PQR$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

EXTRA CHALLENGE
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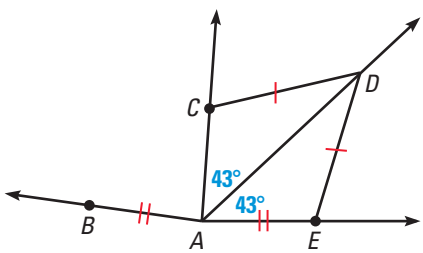
MIXED REVIEW

EVALUATING STATEMENTS **Use the figure to determine whether the statement is *true* or *false*.** (Review 1.5 for 4.2)



52. $\overline{AE} \cong \overline{BA}$

53. $\angle CAD \cong \angle EAD$

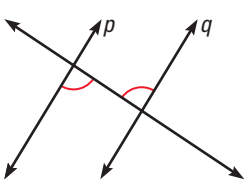
54. $m\angle CAD + m\angle EAB = 86°$

55. $\overline{CD} \cong \overline{AC}$

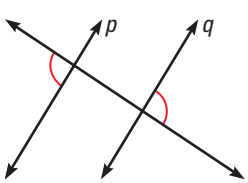
56. $\overrightarrow{AD}$ bisects $\angle CAE$.

DEVELOPING PROOF **Is it possible to prove that lines *p* and *q* are parallel? If so, state the postulate or theorem you would use.** (Review 3.4)

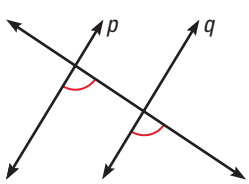
57.



58.



59.



WRITING EQUATIONS **Write an equation of the line that passes through the given point *P* and has the given slope.** (Review 3.6)

60. $P(0, -2), m = 0$

61. $P(4, 7), m = 1$

62. $P(-3, -5), m = -1$

63. $P(9, -1), m = \frac{2}{3}$

64. $P(-1, -1), m = \frac{3}{4}$

65. $P(-2, -3), m = -\frac{7}{2}$

66. $P(5, 2), m = 0$

67. $P(8, 3), m = -\frac{3}{2}$

68. $P(-6, -4), m = -\frac{1}{3}$