Trichromatic Theory of Color Vision

Jonathan Pillow

Mathematical Tools for Neuroscience (NEU 314)
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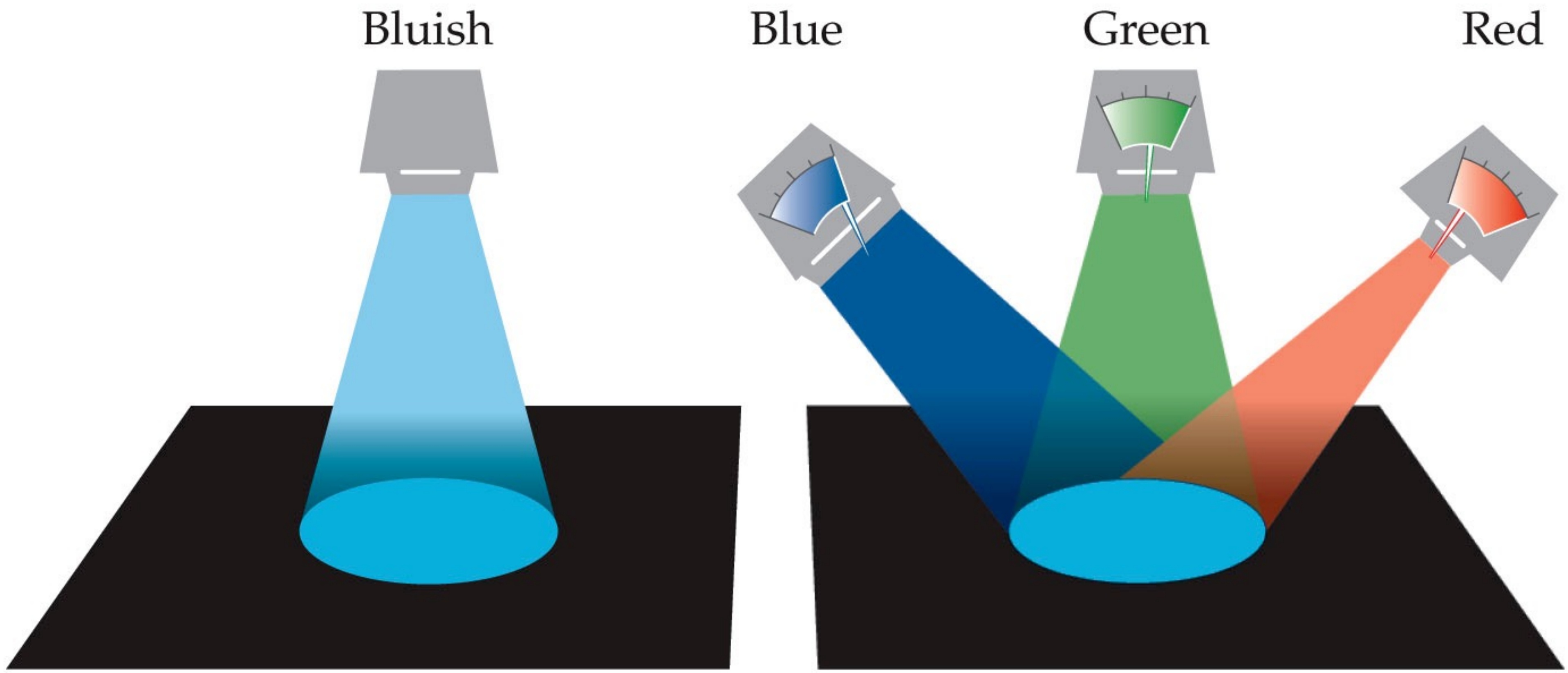
lecture 4

Motivation: James Maxwell's color-matching experiment

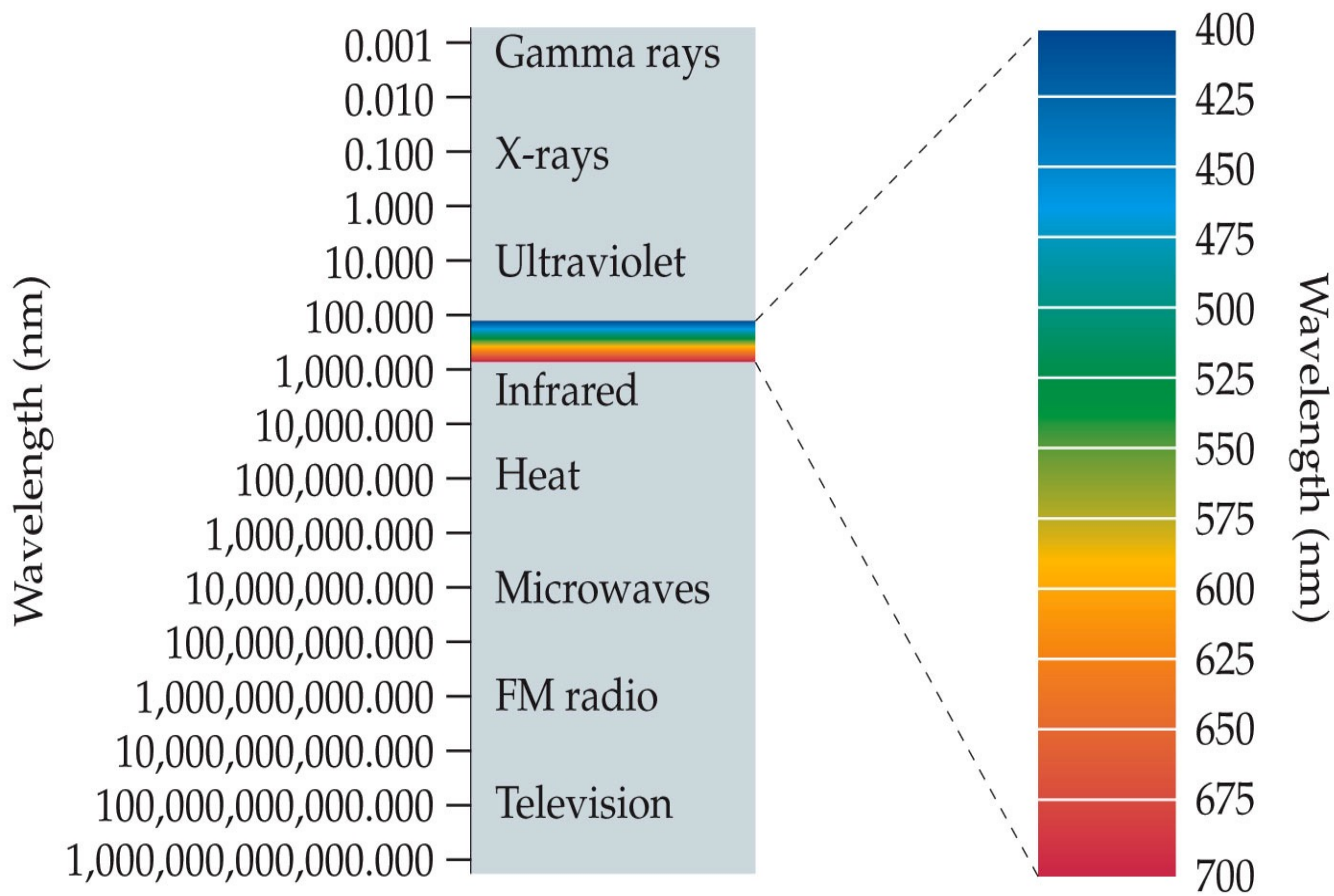


Given any "test" light, you can match it by adjusting the intensities of any three other lights

(2 is not enough; 4 is more than enough & produces non-unique matches)

A little physics background: *light*

- Really: just a particular range of the electromagnetic spectrum
- (We see only the part between 400 and 700 nm)

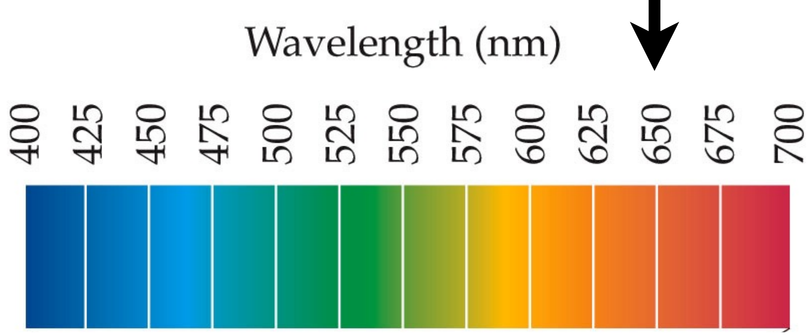


Q: How many numbers would you need to write down to specify the color of a light source?

Just one?

("the wavelength"?)

eg. "650"?

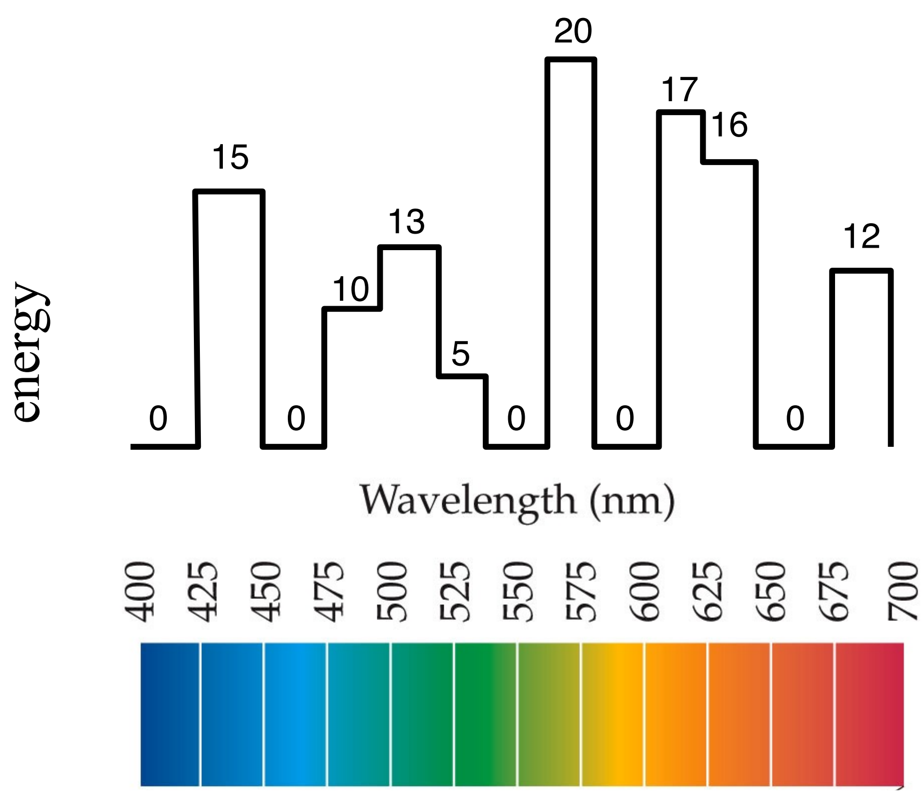


Q: How many numbers would you need to write down to specify the ~~color~~ of a light source?

spectral properties

A: It depends on how you "bin" up the spectrum

- One number for each spectral "bin":

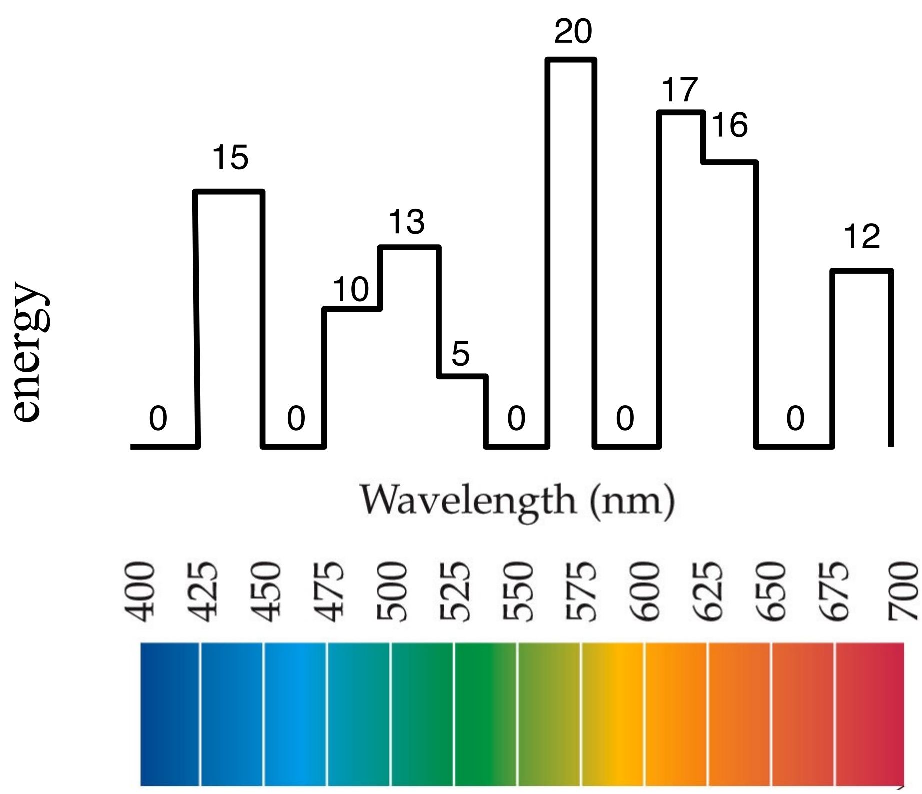


example: 13 bins

(a vector!)

Device: **hyper-spectral camera**

- measures amount of energy in each range of wavelengths
- can use thousands of bins (or “frequency bands”), instead of just the 13 shown here



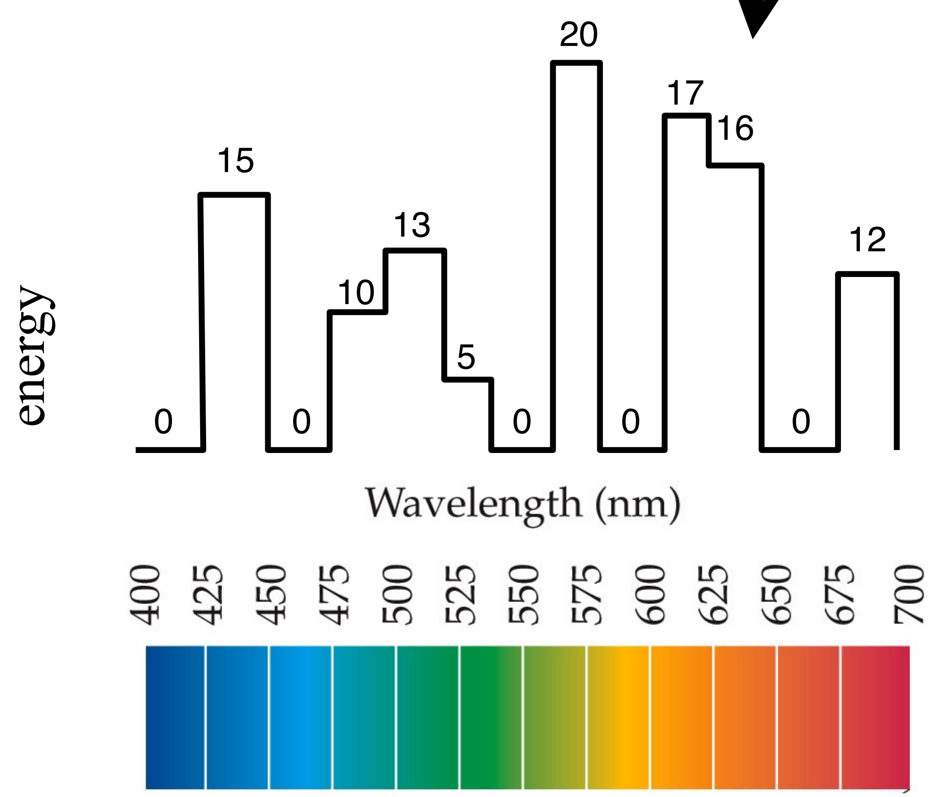
Some terminology for “colored” light:

spectral - referring to the wavelength of light

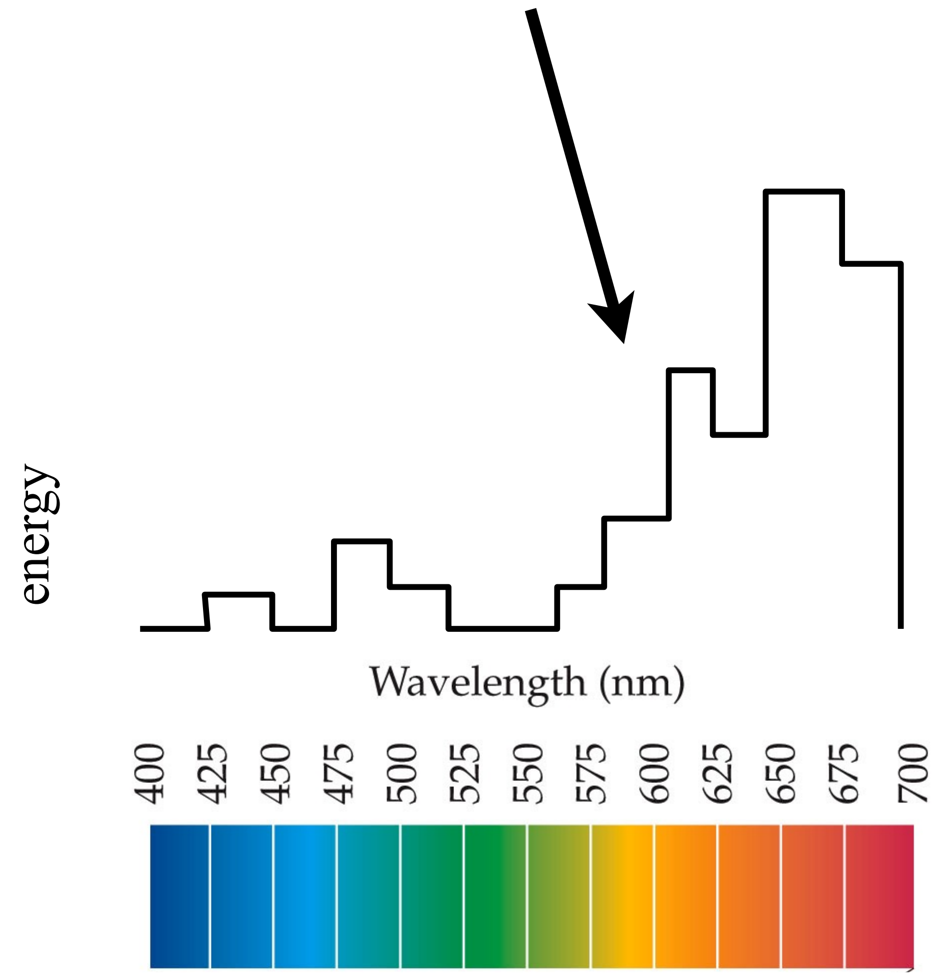
the **illuminant** - light source

illuminant power spectrum - this curve.

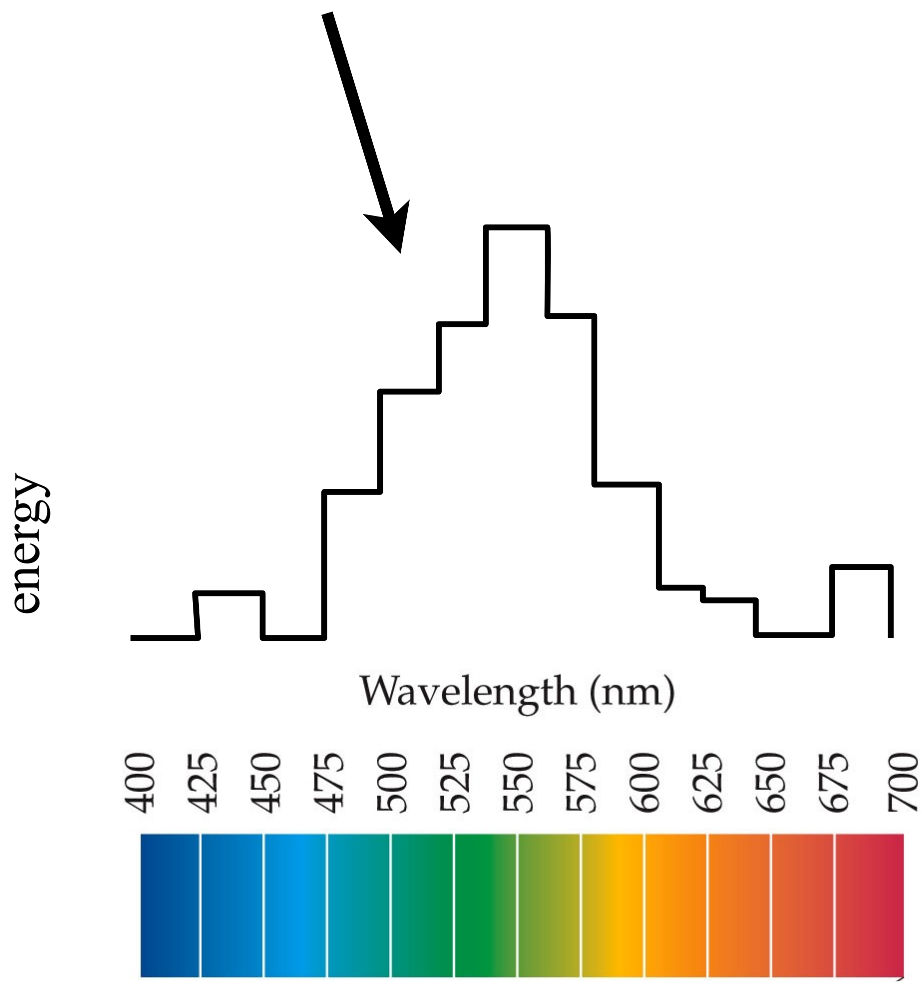
amount of energy (or power) at each frequency



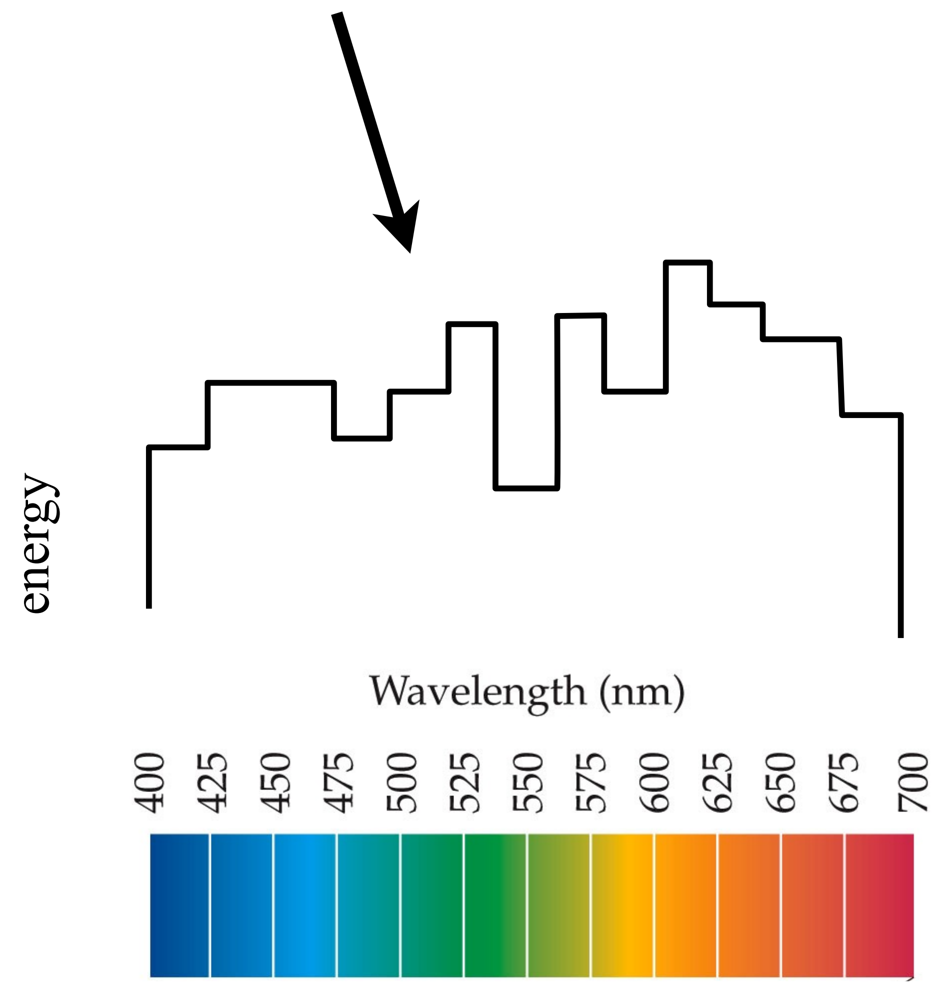
an illuminant with most power at long wavelengths (i.e., a *reddish* light source)



an illuminant with most power at medium wavelengths (i.e., a *greenish* light source)



an illuminant with power at all visible wavelengths (a *neutral* light source, or “white light”)



Q: How many measurements of this same spectrum does the human eye take (in bright conditions?)

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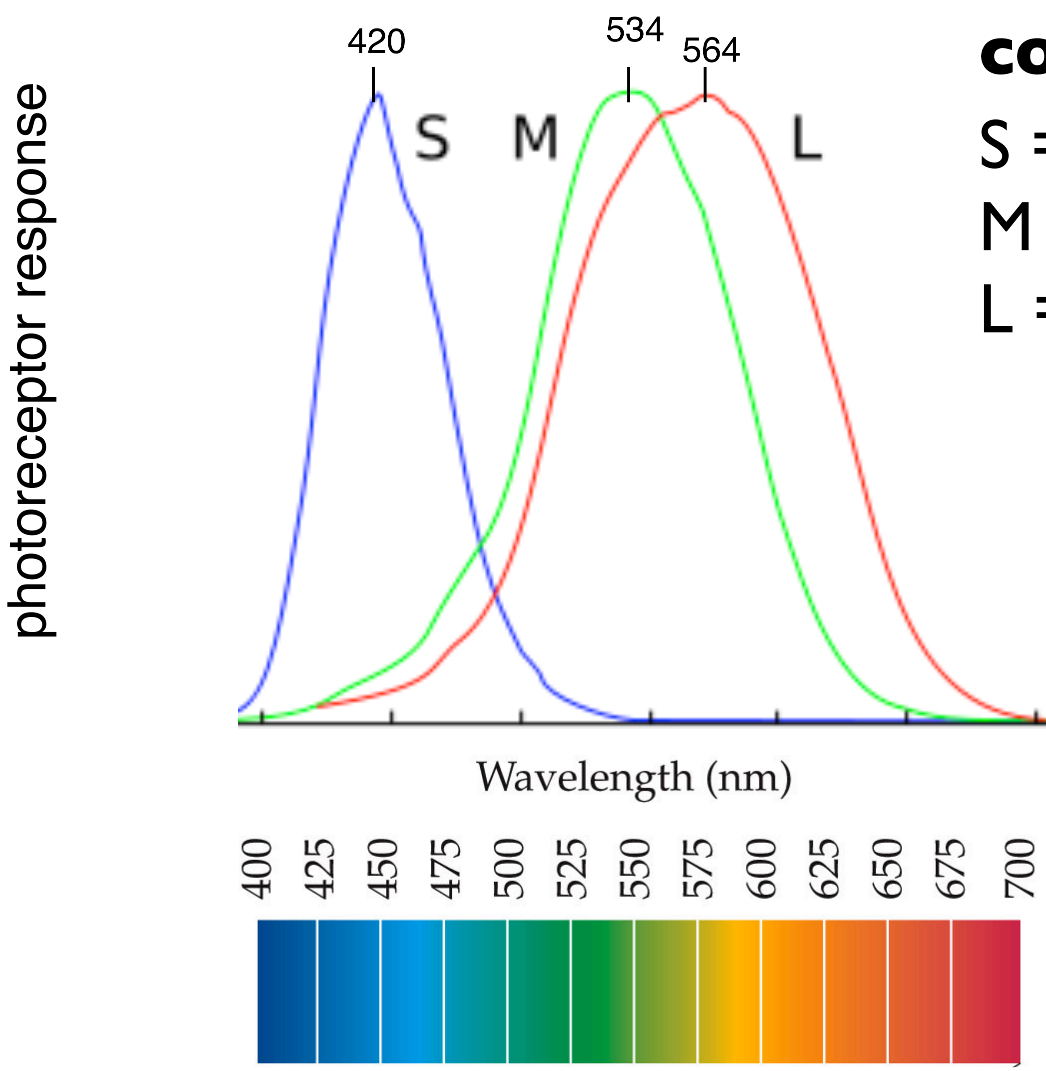
A: Only 3! One from each cone class



cone types

S = short (blue)

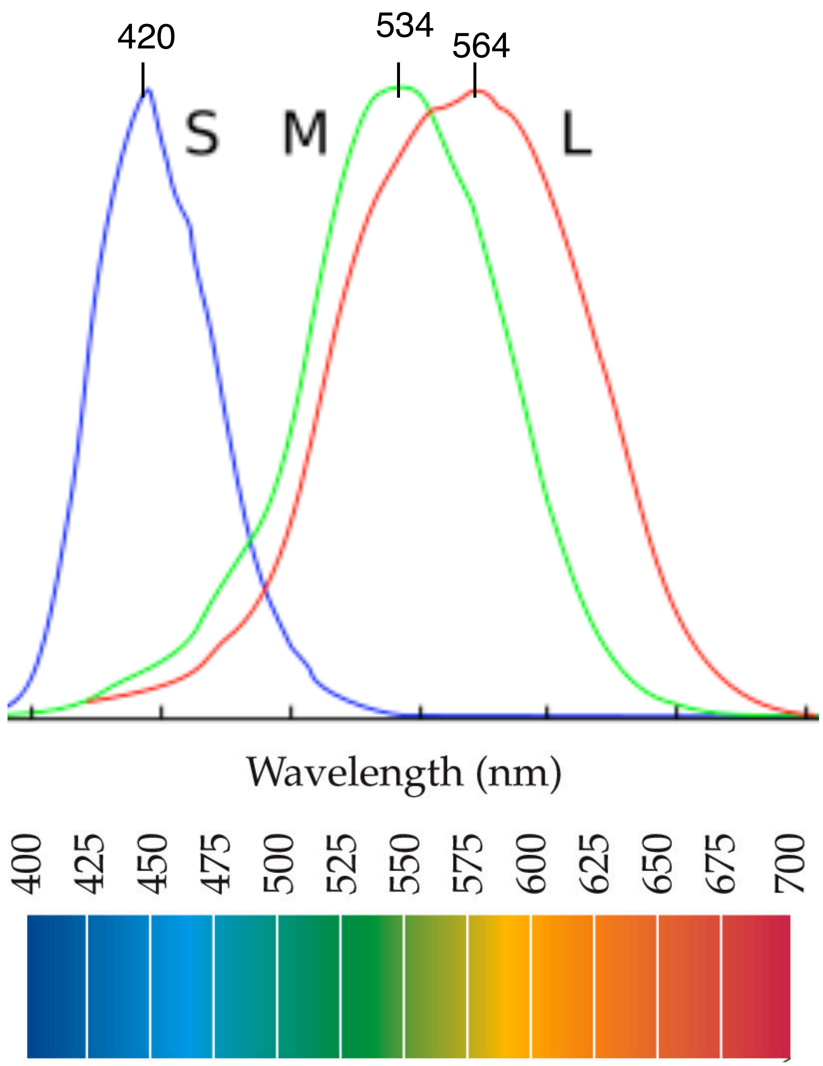
M = medium (green)

L = long (red)

Color vision: Relies entirely on comparison of responses from three cone types!

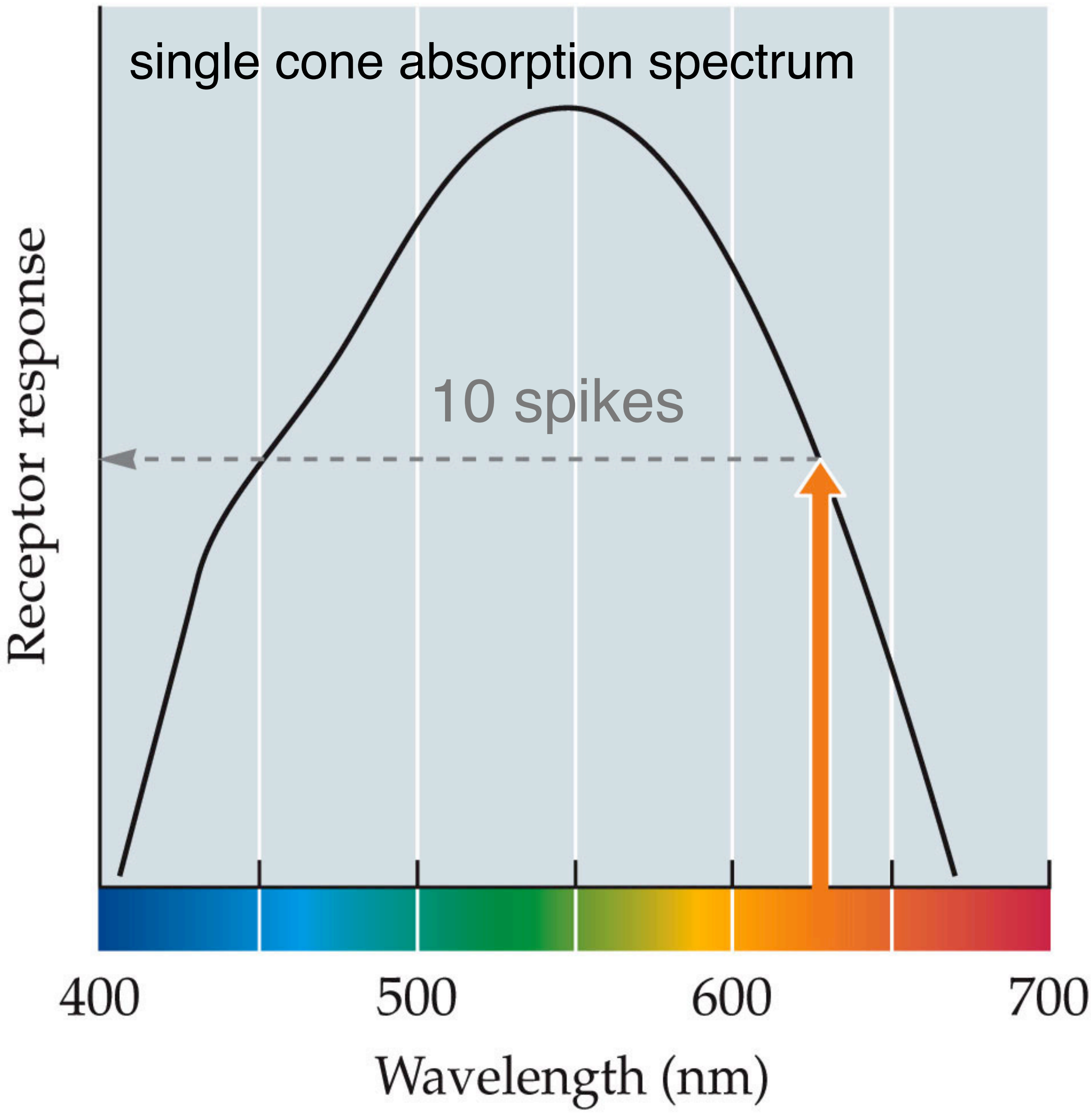
absorption spectrum - describes response (or “light absorption”) of a photoreceptor as a function of wavelength

photoreceptor response

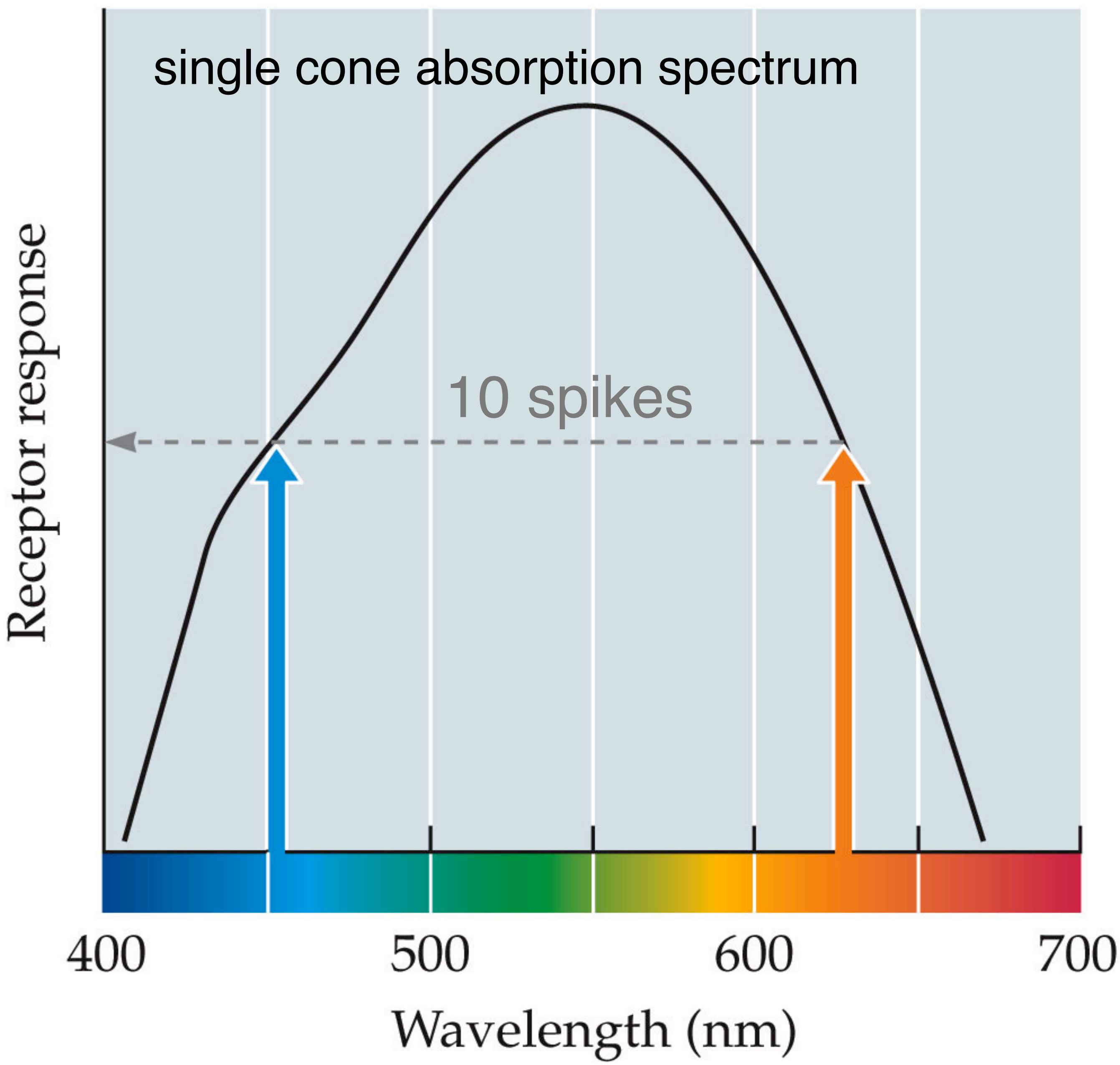


could also call this “sensitivity”

Problem: response from a single cone is ambiguous

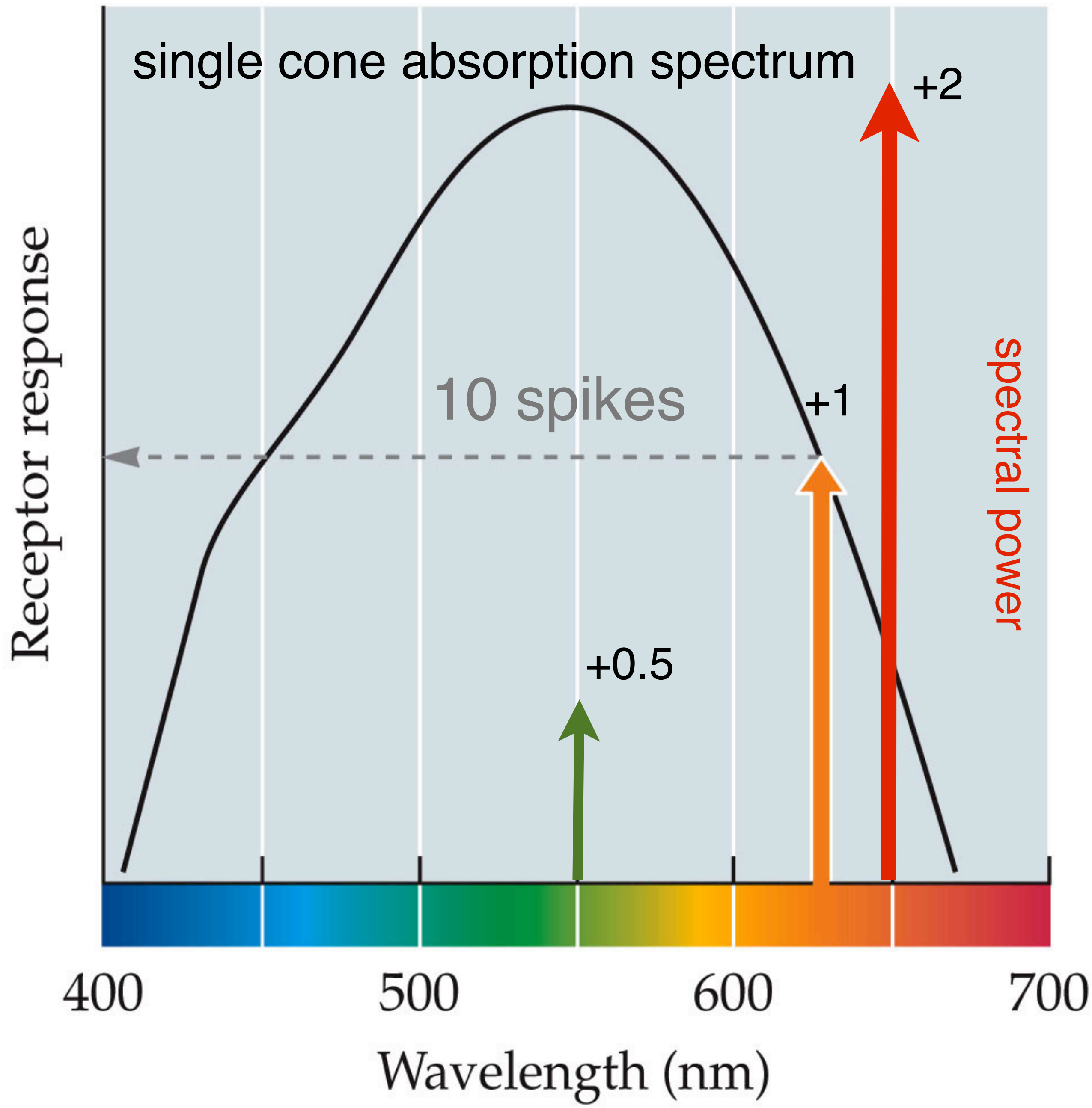


Problem: response from a single cone is ambiguous



- All the photoreceptor gives you is a “response”
- Can’t tell which light frequency gave rise to this response (blue or orange)

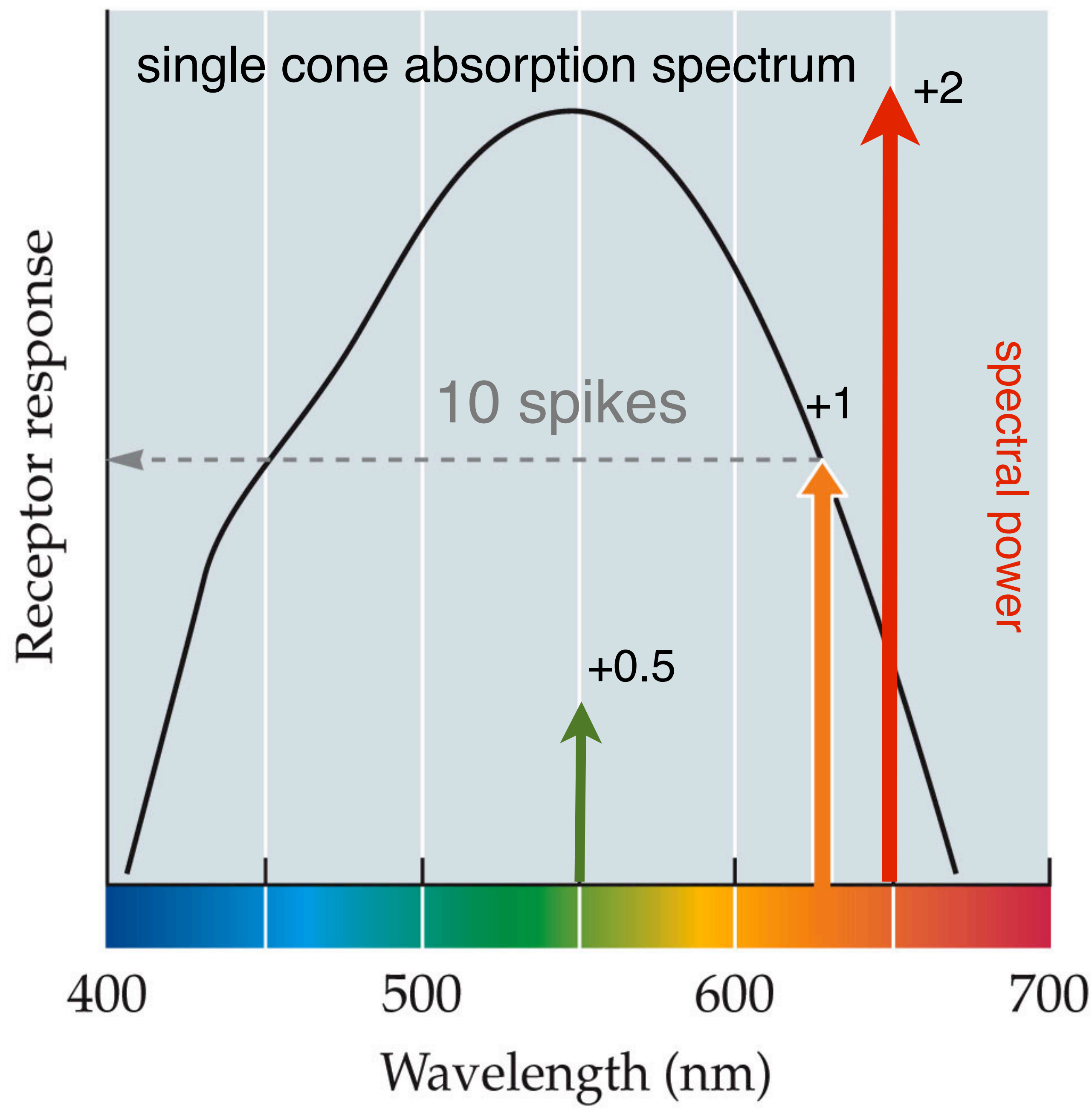
<u>Problem is actually much worse</u>: can't tell a weak signal at the peak sensitivity from a strong signal at an off-peak intensity



- All three of these lights give the same response from this cone

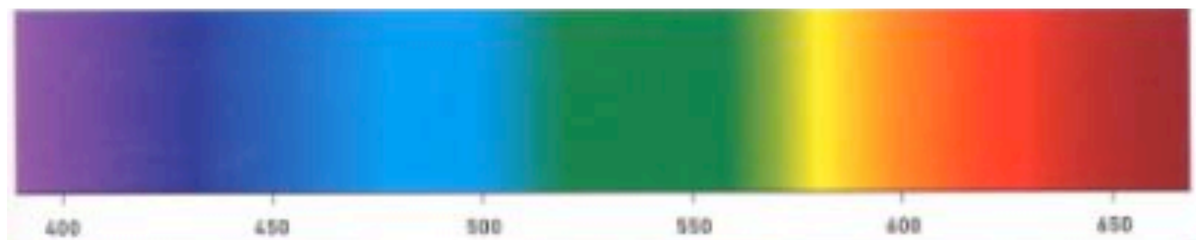
cone respone = *aborption spectrum* x *light intensity*

Problem of **univariance**: infinite set of wavelength+intensity combinations can elicit exactly the same response



So a single cone can’t tell you anything about the color of light!

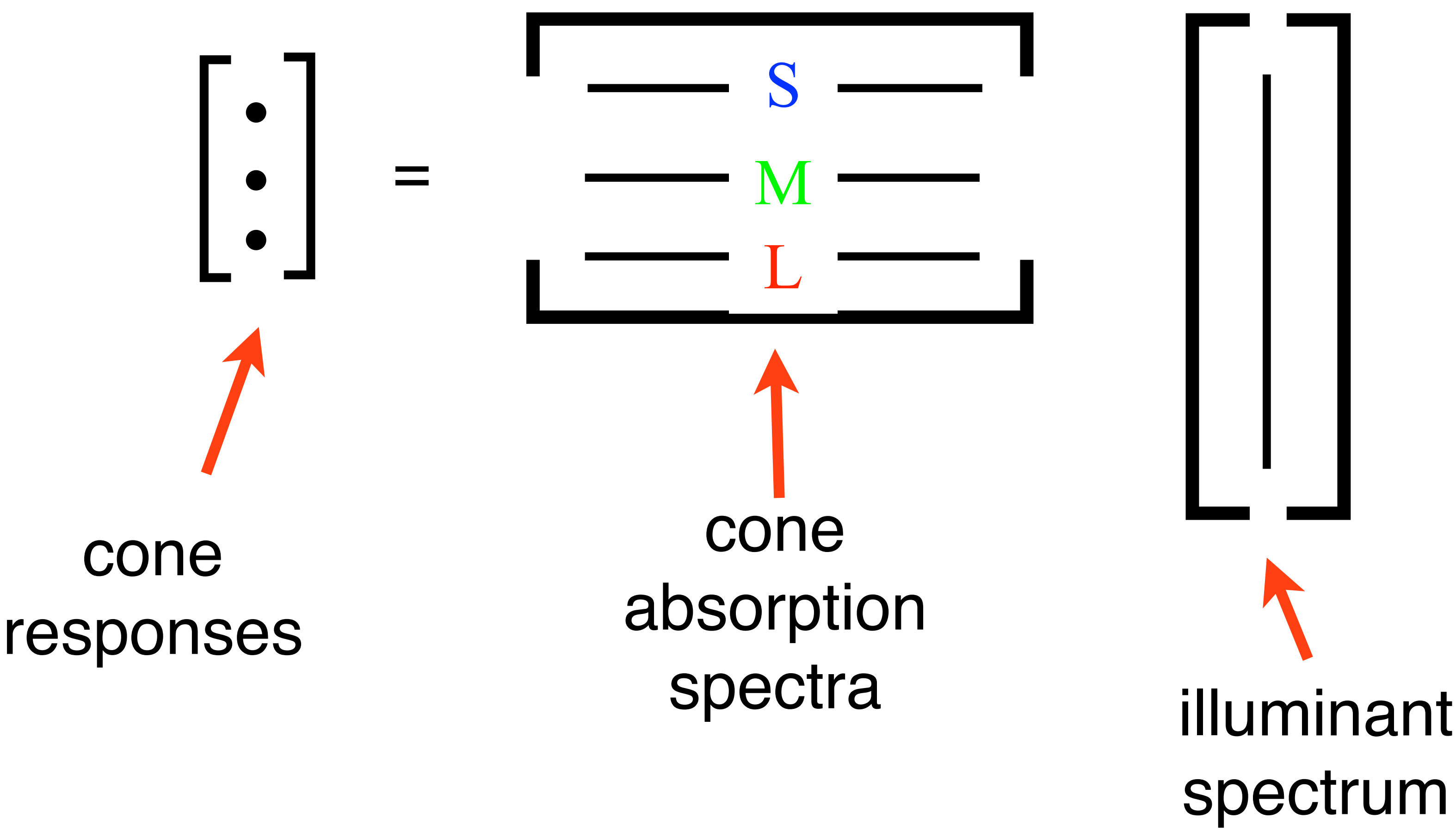
Colored stimulus

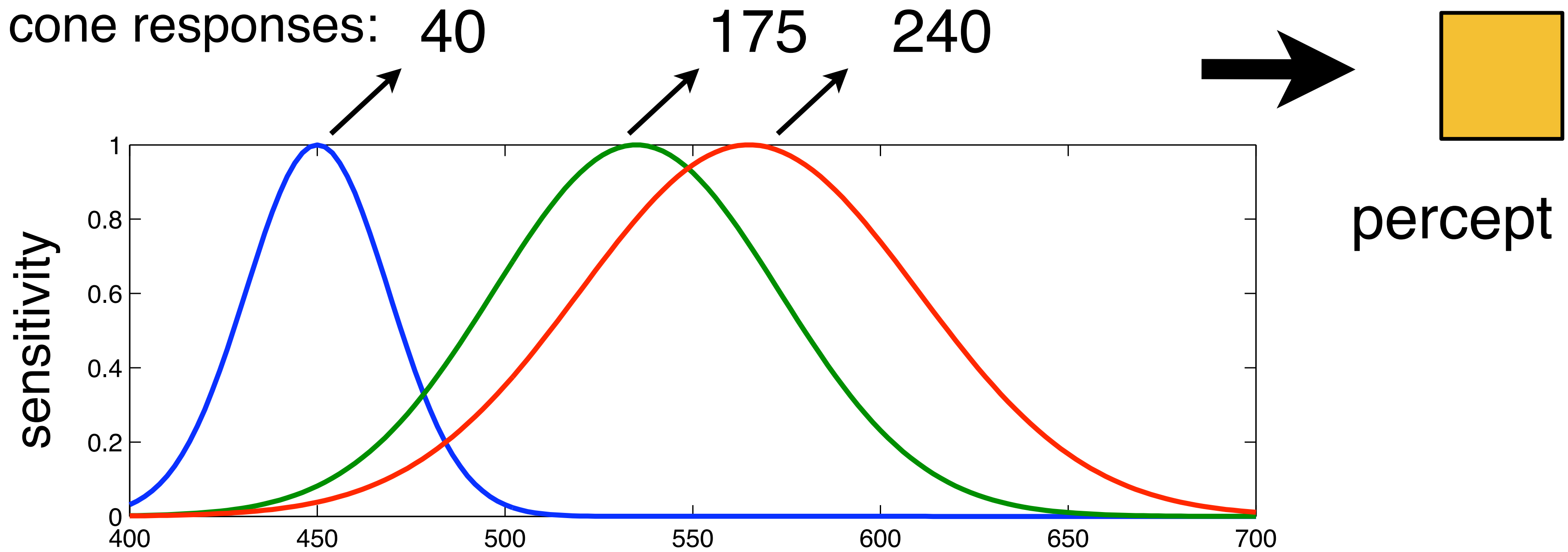


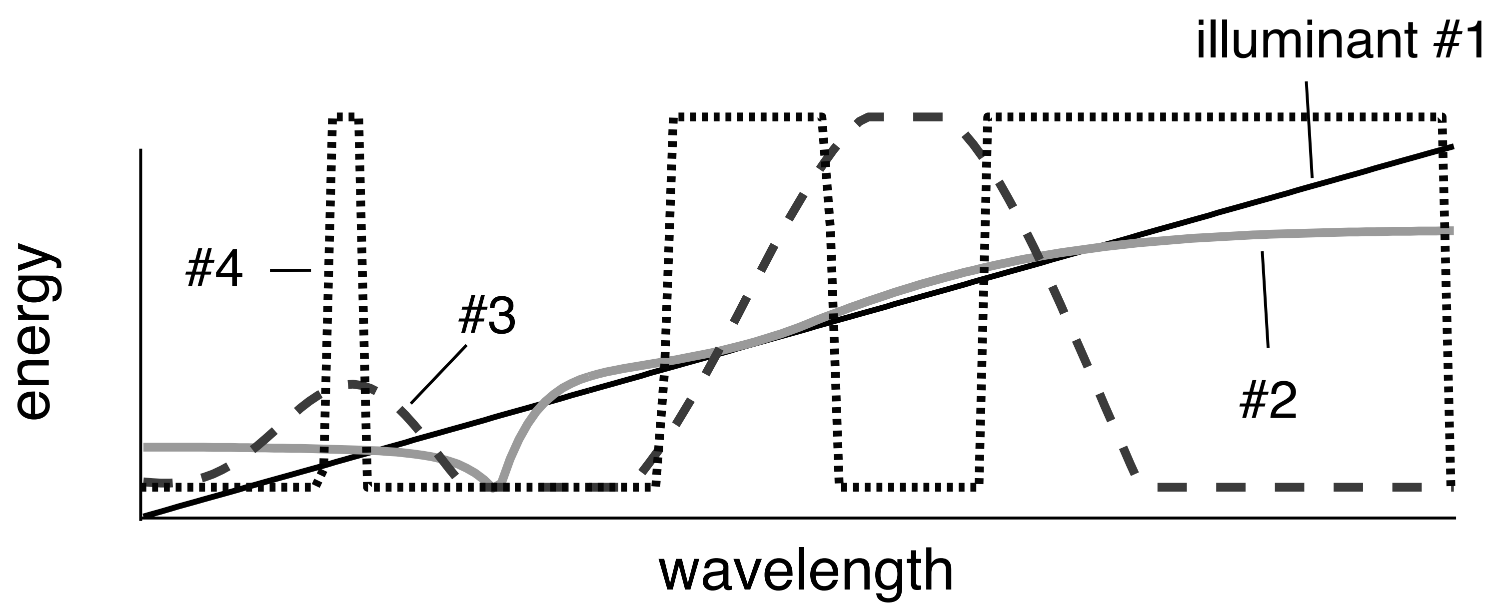
Response of your
“S” cones

Written in a linear algebra setting

$$\vec{y} = M\vec{x}$$



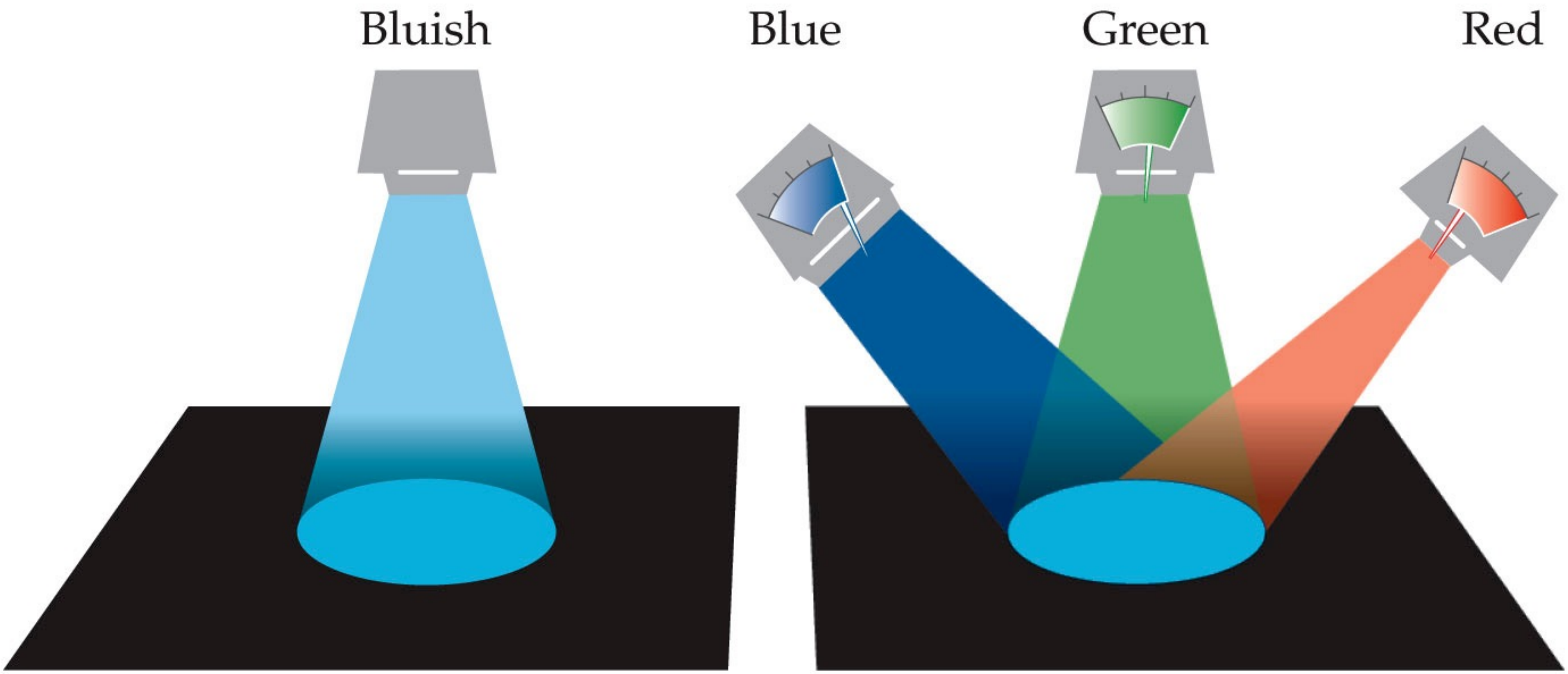




Metamers

- Illuminants that are physically distinct but perceptually indistinguishable

James Maxwell (1831–1879): color-matching experiment



- Any “test” light (“vector”), can be matched by adjusting the intensities of any three other lights (“basis vectors”)
- 2 is not enough; 4 is more than enough

Two lights x_1 and x_2 “match” iff

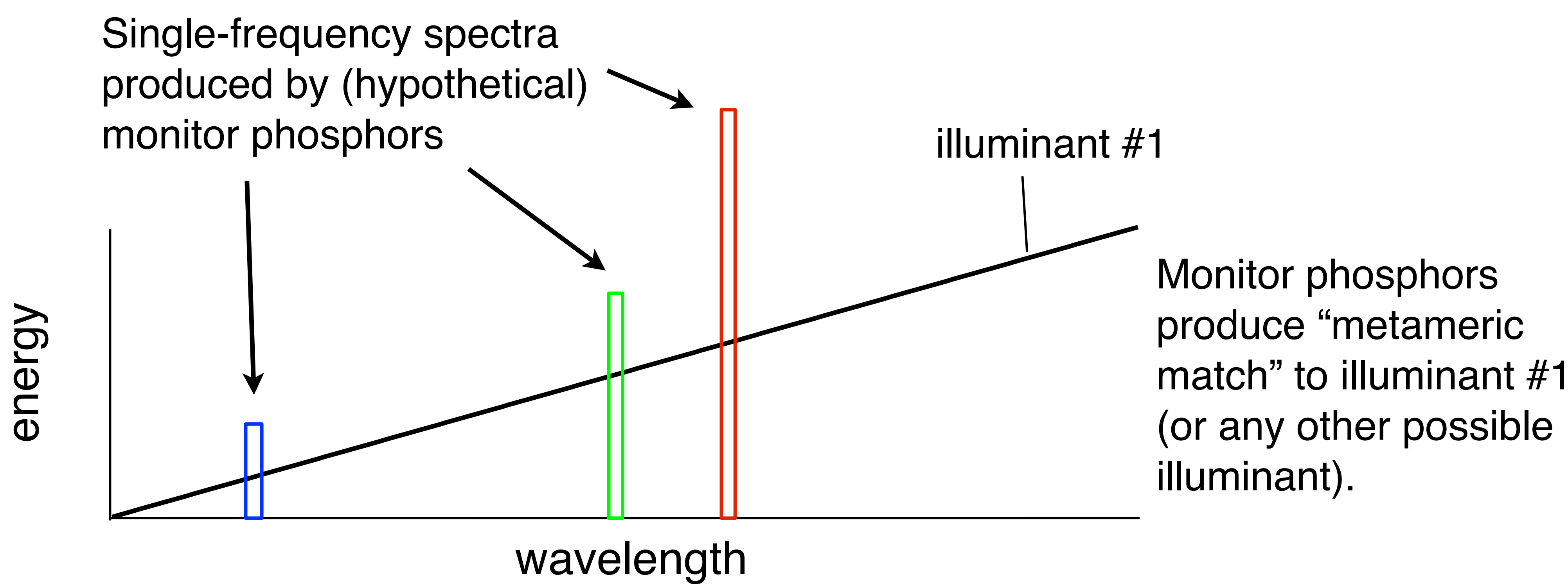
$$M\vec{x}_1 = M\vec{x}_2$$

(i.e., they evoke the same cone responses)

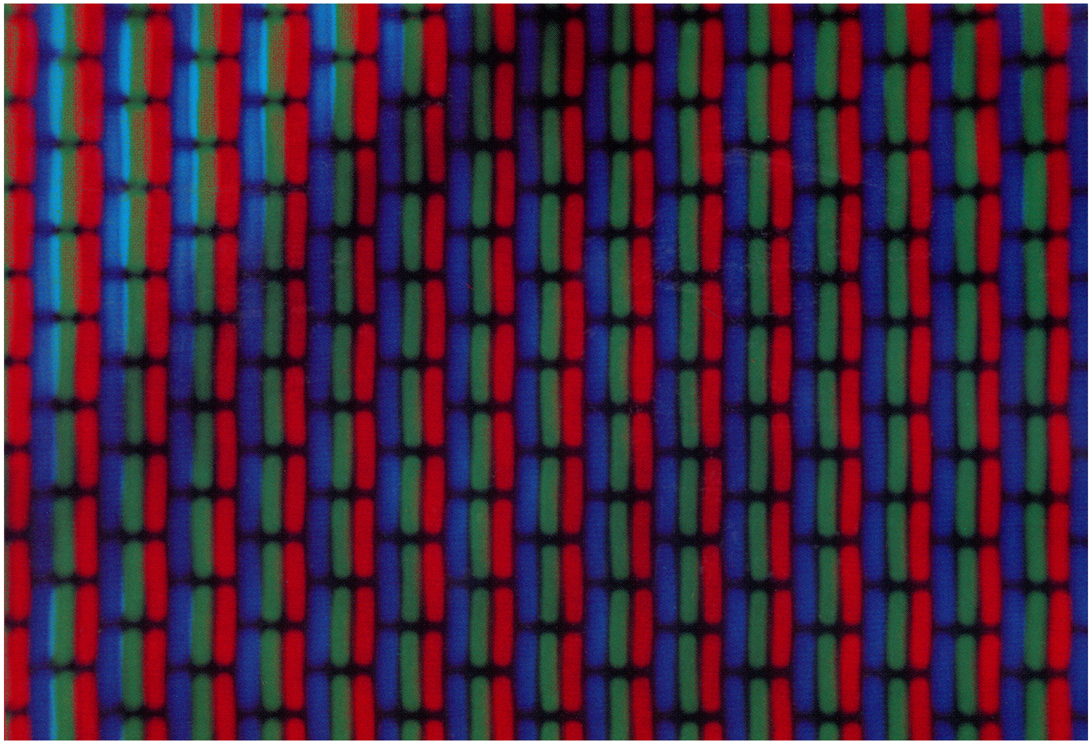
So in linear algebra terms: metamers refer to an entire subspace of lights that have the same linear projection onto the 3 cone absorption spectra basis vectors.

Implication: tons of things in the natural world have different spectral properties, but look the same to us.

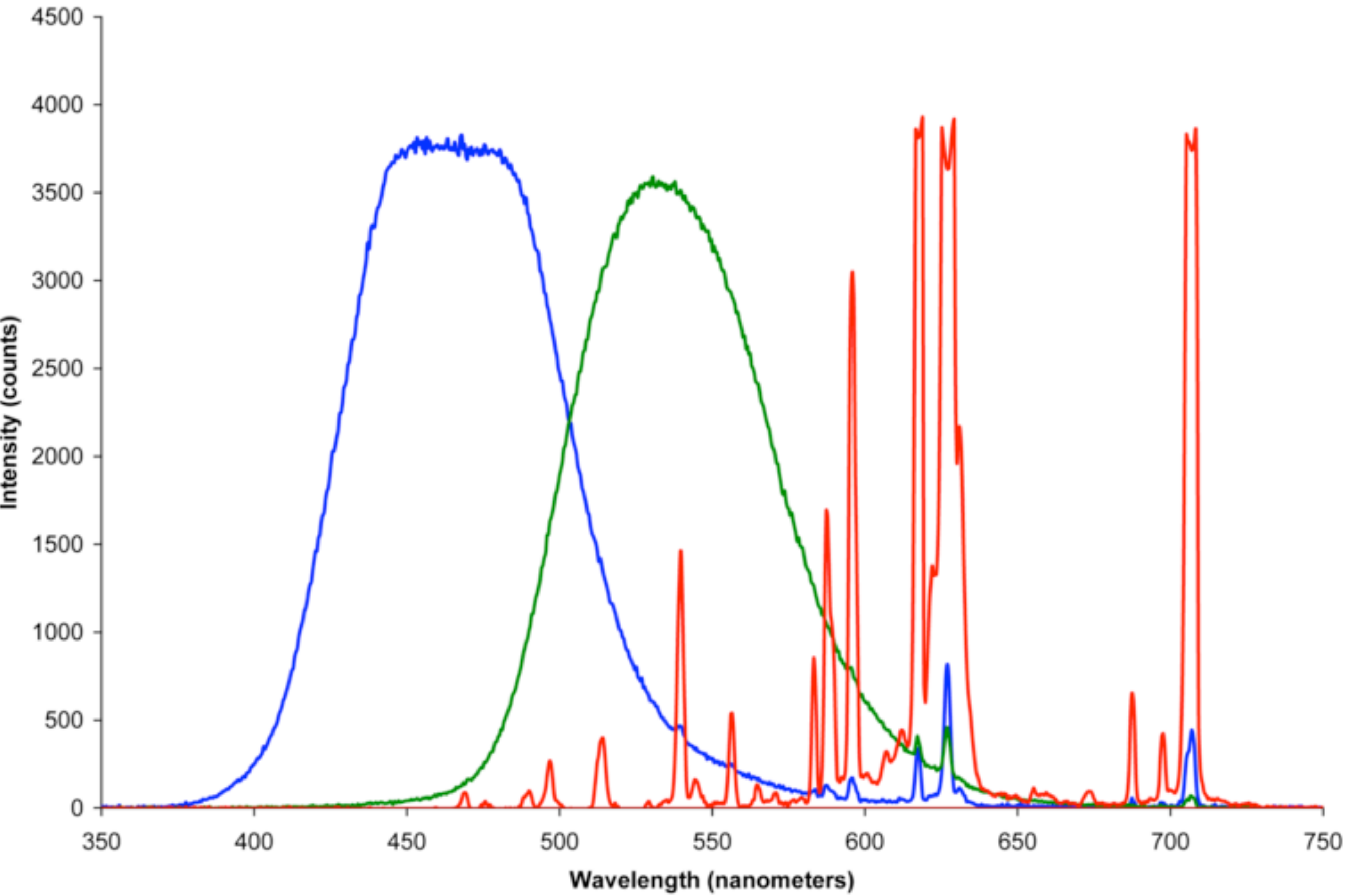
But, great news for the makers of TVs and Monitors: any three lights can be combined to approximate any color.



Close-up of computer monitor, showing three phosphors, (which can approximate any light color)

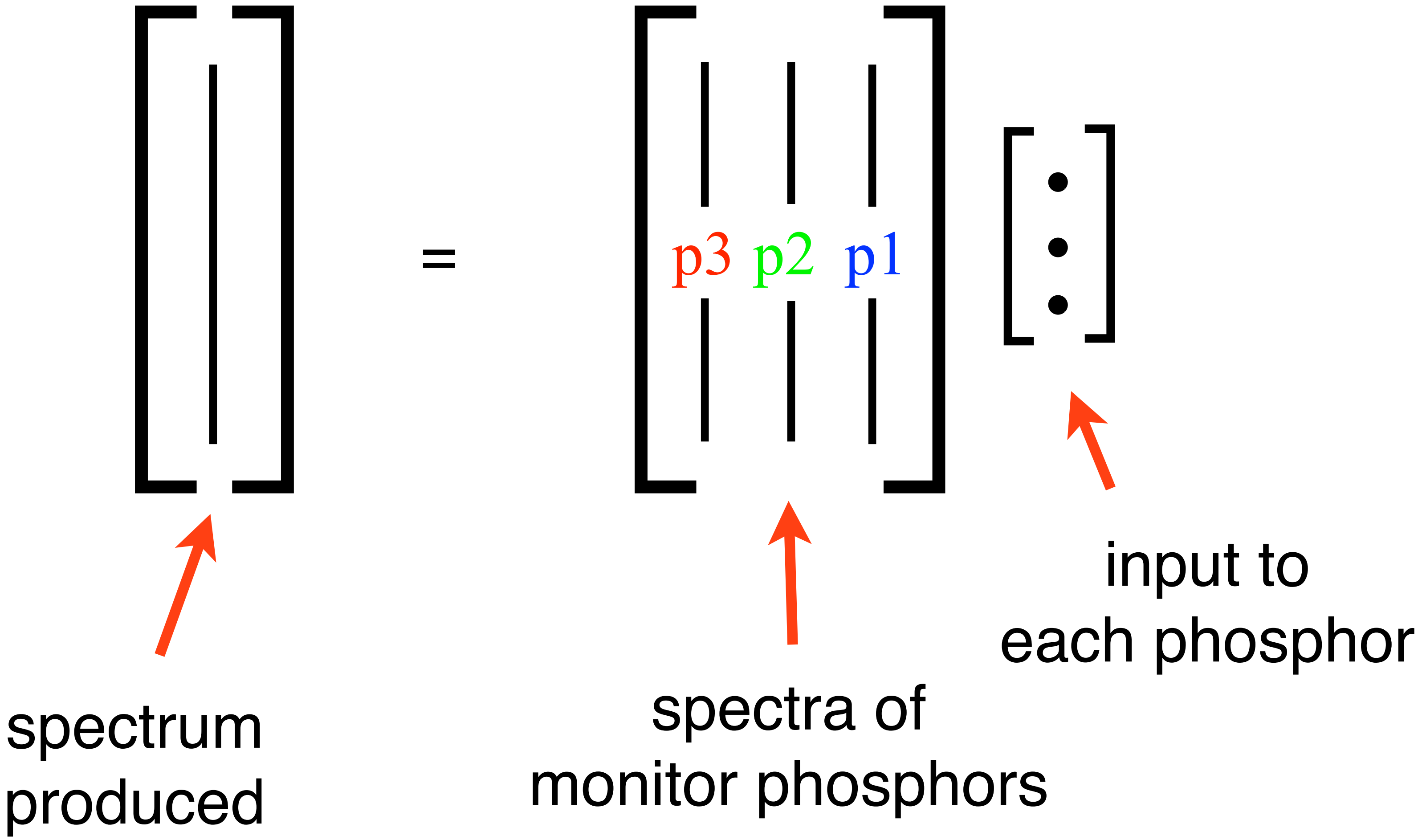


Spectra of typical CRT monitor phosphors



Written as a linear algebra problem

$$\vec{x} = P\vec{z}$$



This wouldn't be the case if we had more cone classes.

hyperspectral marvel:
mantis shrimp (stomatopod)

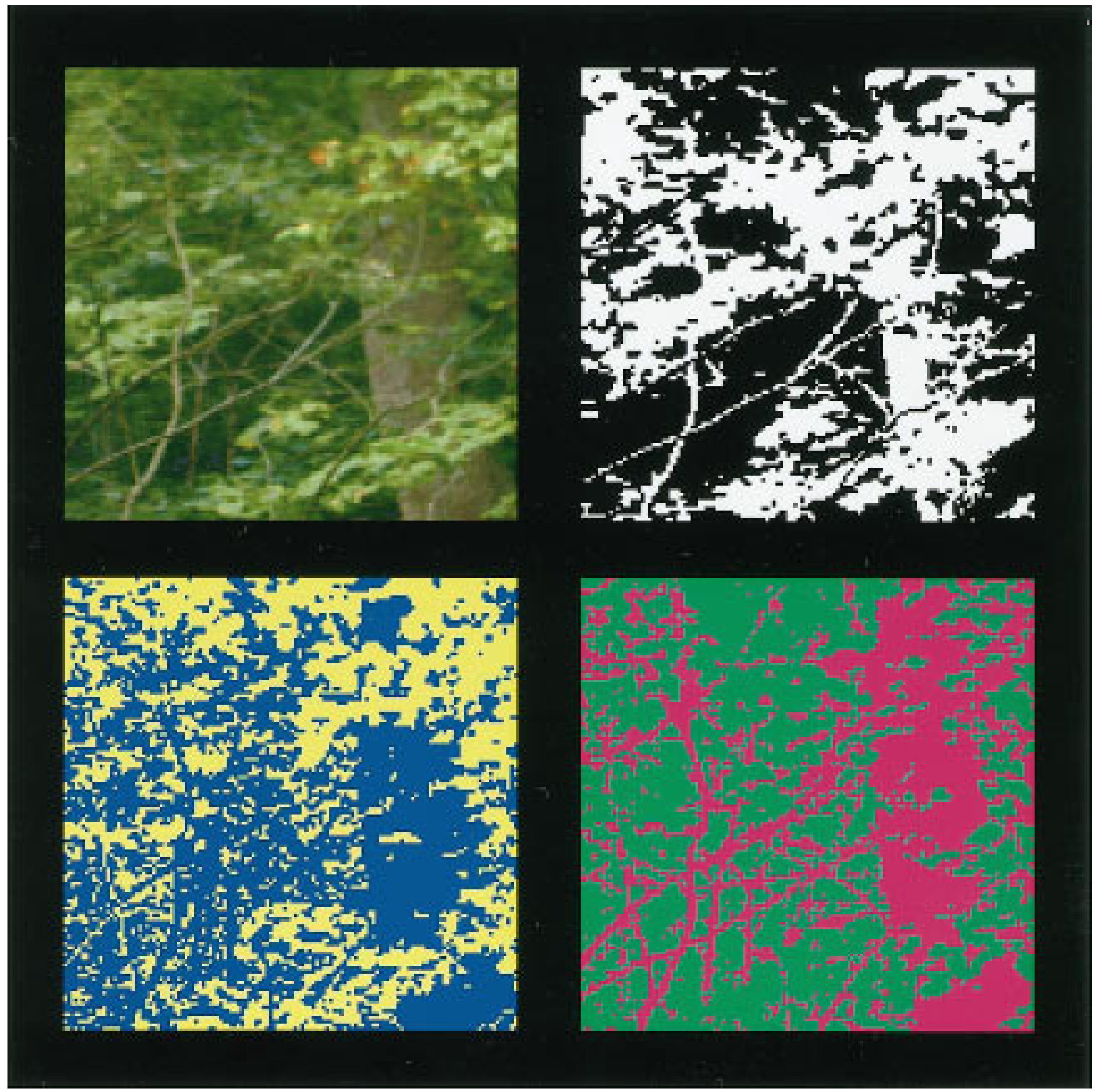
- 12 different cone classes
- sensitivity extending into UV range
- 12-dimensional color vision space

Why these three cone classes?

• "efficient coding" of natural spectra: preserve most of the variability present in hyper-spectral images

projection of a natural image onto first 3 principal components

Ruderman et al 1998



(let's revisit this when we discuss PCA)

color blindness

- About 8% of male population, 0.5% of female population has some form of color vision deficiency: Color blindness
- Mostly due to missing M or L cones (sex-linked; both cones coded on the X chromosome)

Types of color-blindness:

dichromat - only 2 channels of color available
(i.e., color vision defined by a 2D subspace)
(contrast with “trichromat” = 3 color channels).

Three types, depending on missing cone:

	Frequency: M / F
• **Protanopia**: absence of L-cones	2% / 0.02%
• **Deuteranopia**: absence of M-cones	6% / 0.4%
• **Tritanopia**: absence of S-cones	0.01% / 0.01%

includes true dichromats and color-anomalous trichromats


Scene Viewed by Protanope
Same Scene Viewed by Normal Trichromat


Scene Viewed by
Deuteranope
Same Scene Viewed by
Normal Trichromat

Scene Viewed by Tritanope

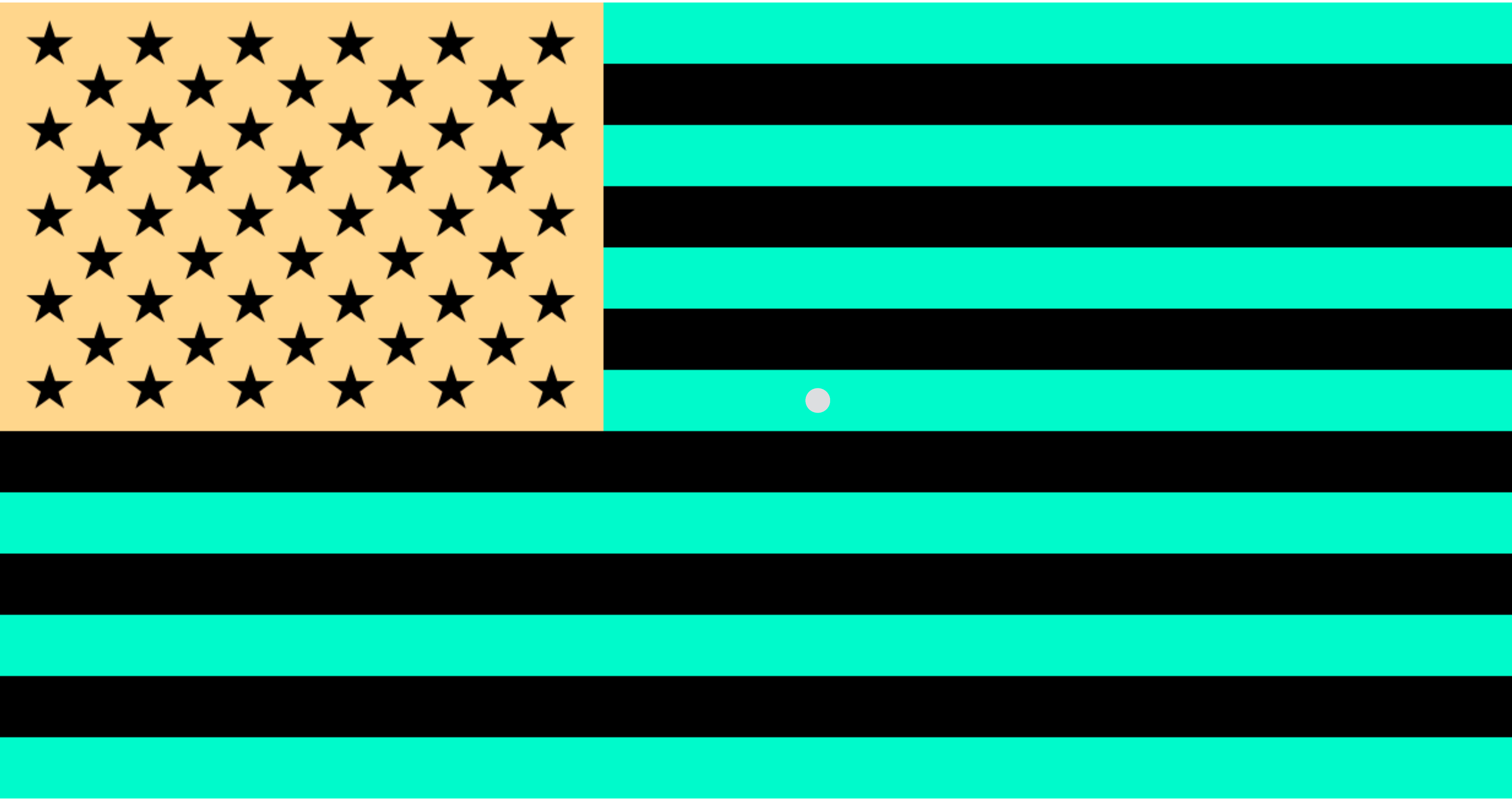
Same Scene Viewed by Normal Trichromat

So don’t call it color *blindness*.

instead: “Dude… I’m living in a 2D subspace.”

Color Vision in Animals

- most mammals (dogs, cats, horses): dichromats
- old world primates (including us): trichromats
- marine mammals: monochromats
- bees: trichromats (but lack "L" cone; ultraviolet instead)
- some birds, reptiles & amphibians: tetrachromats!



Opponent Processes

Afterimages: A visual image seen after a stimulus has been removed

Negative afterimage: An afterimage whose polarity is the opposite of the original stimulus

- Light stimuli produce dark negative afterimages
- Colors are complementary: Red produces green afterimages, blue produces yellow afterimages (and vice-versa)

color after-effects:

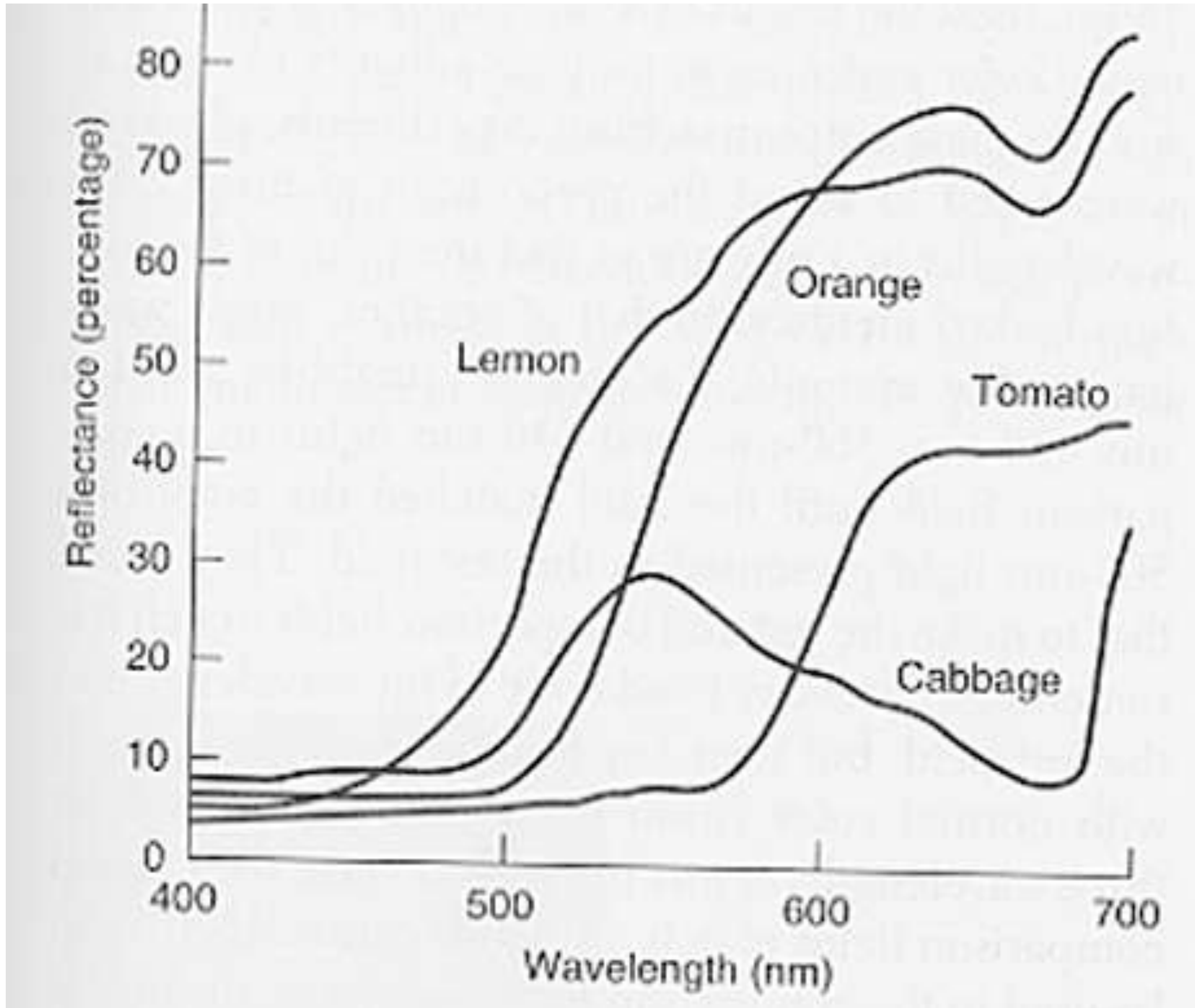
lilac chaser:

http://www.michaelbach.de/ot/col-lilacChaser/index.html

last piece: *surface reflectance function*

Describes how much light an object reflects, as a function of wavelength

Think of this as the *fraction* of the incoming light that is reflected back



By now we have a complete picture of how color vision works:

Illuminant — defined by its power or “intensity” spectrum

amount of light energy at each wavelength

↓

Object — defined by its reflectance function

certain percentage of light at each wavelength is reflected

↓

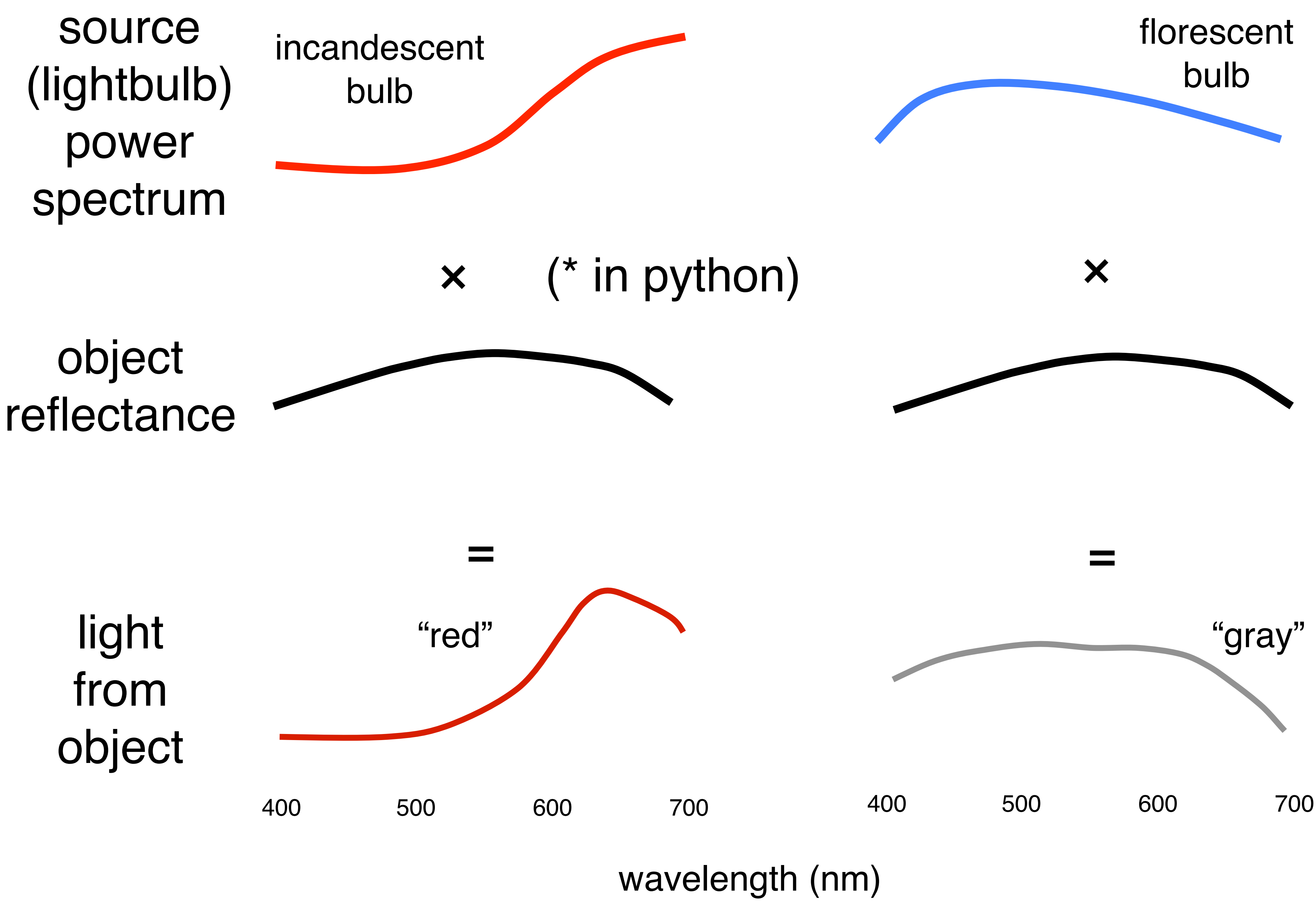
Cones — defined by absorption spectra

each cone class adds up light energy according to its absorption spectrum

↓

cone responses — three spectral measurements

convey all color information to brain via opponent channels


source
(lightbulb)
power
spectrum
incandescent
bulb
florescent
bulb
×
(* in python)
×
object
reflectance
=
=
light
from
object
“red”
“gray”
400
500
600
700
400
500
600
700
wavelength (nm)

But in general, this doesn't happen:

We don't perceive a white sheet of paper as looking reddish under a tungsten light and blueish/grayish under a halogen light.

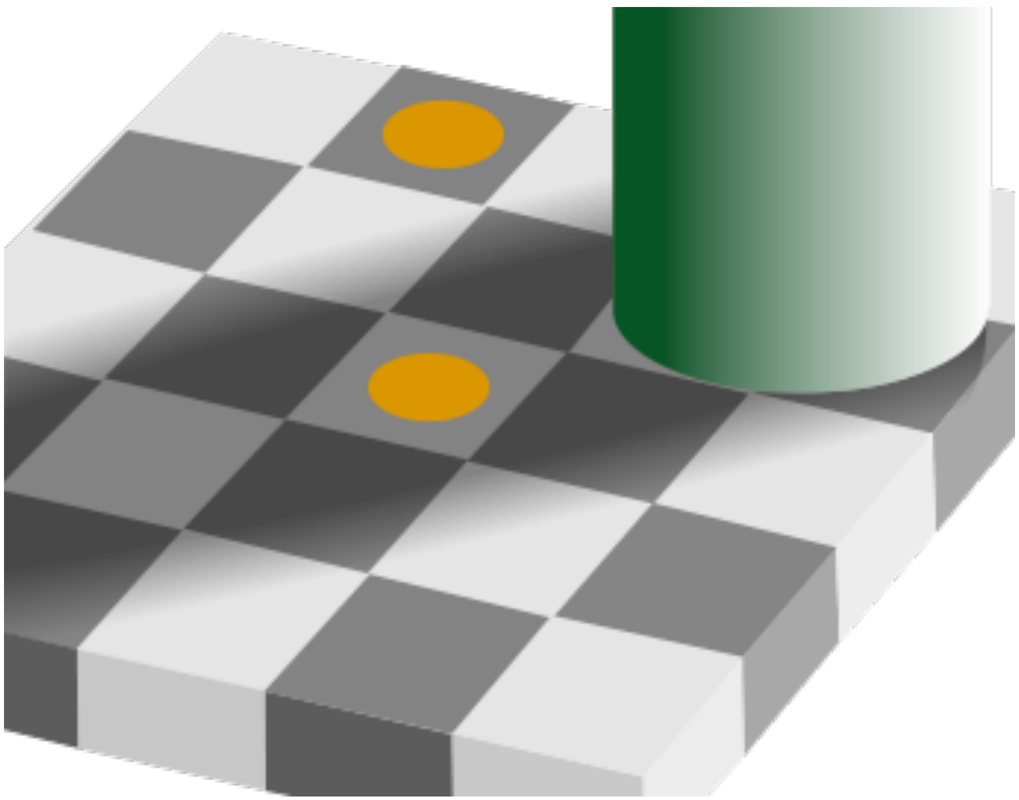
Why?

Color Constancy

The visual system uses a variety of tricks to make sure things look the same color, regardless of the illuminant (light source)

- **Color constancy** - tendency of a surface to appear the same color under a wide range of illuminants
- To achieve color constancy, we must discount the illuminant and determine the surface color, regardless of how it appears

Illusion illustrating Color Constancy



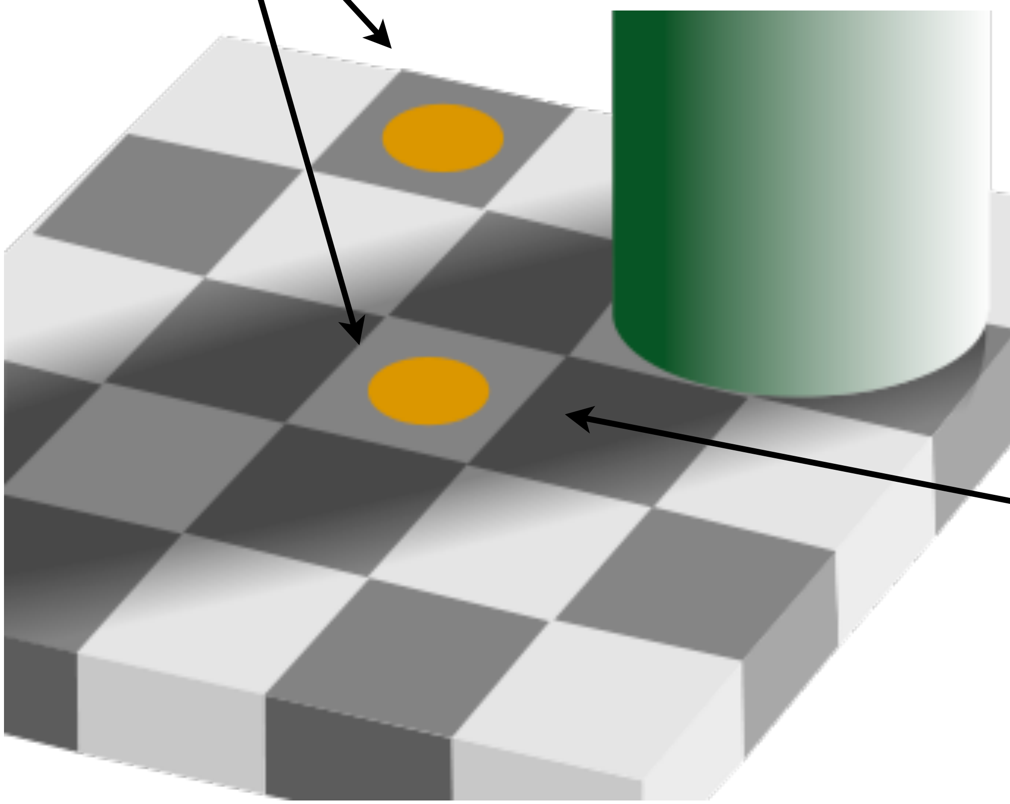
Same yellow in both patches

Same gray around yellow in both patches

(the effects of lighting/shadow can make colors look different that are actually the same!)

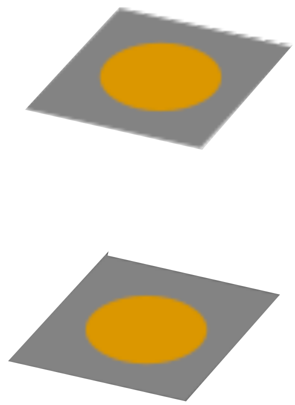
Bayesian Explanation

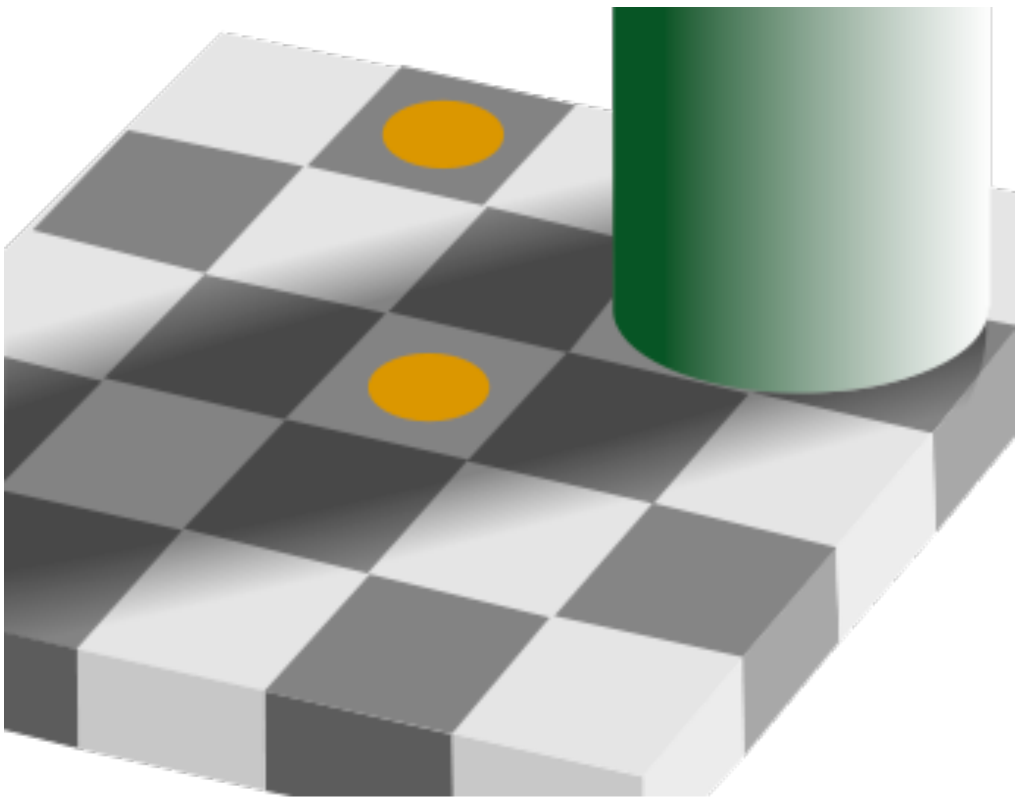
Exact same light hitting emanating from these two patches

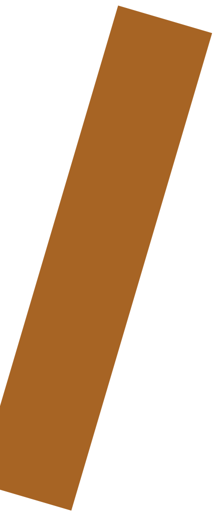


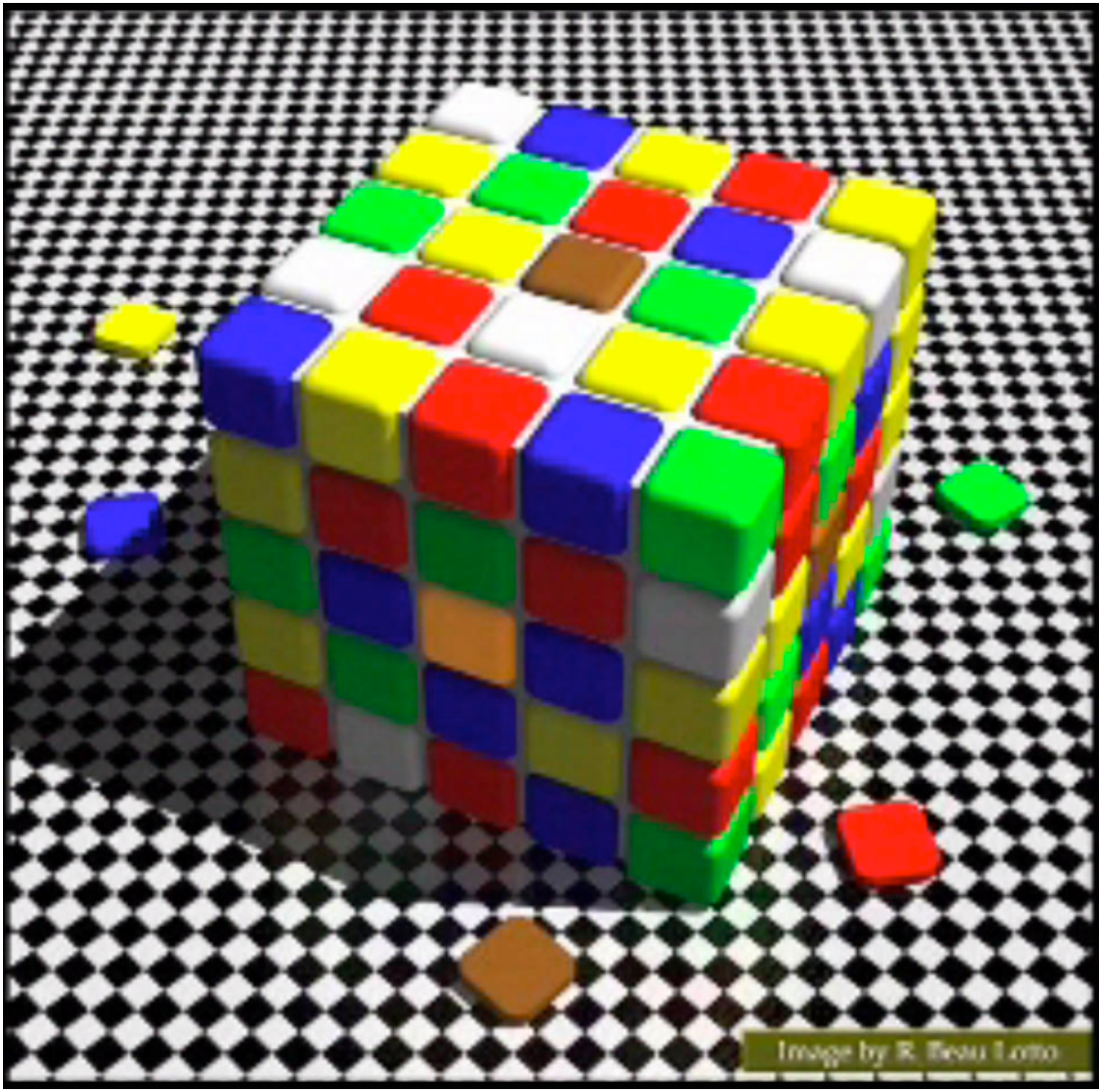
But the brain infers that less light is hitting this patch, due to shadow

CONCLUSION: the lower patch must be reflecting a higher fraction of the incoming light (i.e., it’s brighter)









Beau Lotto

- Visual system tries to discount the effects of the illuminant: it cares about the properties of the *surface*, not the *illuminant.*

- still unknown how the brain does this: believed to be in cortex (V1 and beyond).

- *but*: color-constancy is not perfect

- possible to fool the visual system:

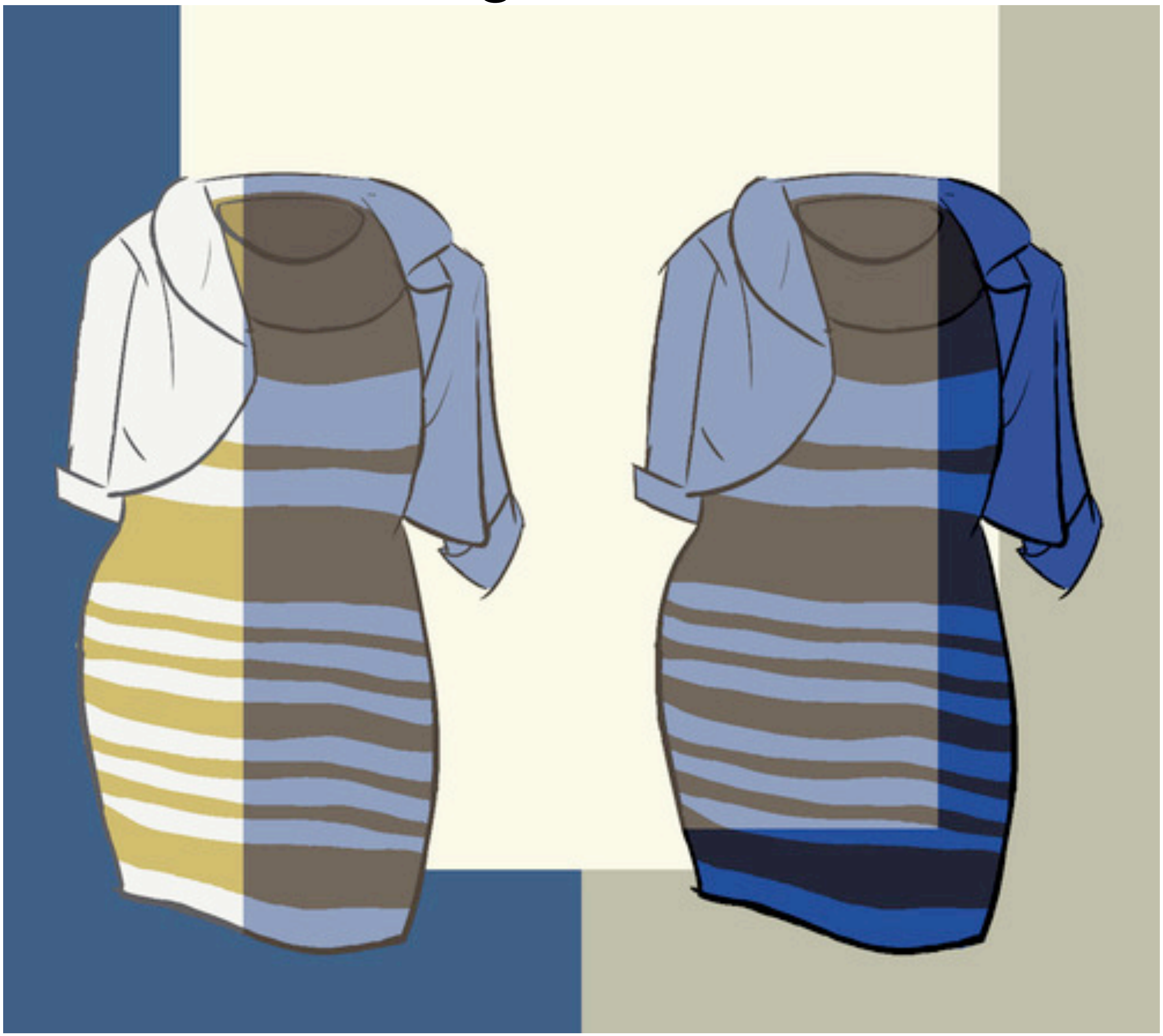
  - using a light source with unusual spectrum (most light sources are broad-band; narrow-band lights will make things look very unusual)

  - showing an image with little spectral variation (e.g., a blank red wall).

the dress incident

"guys please help me - is this dress white and gold, or blue and black? Me and my friends can't agree and we are freaking the fuck out."

Percept depends on inferences about the light source!



https://nathanfairbairn.tumblr.com/post/112246531934/about-that-dress

Percept depends on inferences about the light source!

Rosa Lafer-Sousa

But: we have no idea (so far) why people are making such radically different inferences about light

Color vision summary

- light source: defined by *illuminant power spectrum*

- Trichromatic color vision relies on 3 cones: characterized by *absorpotion spectra* ("basis vectors" for color perception)

- Color matching: any 3 lights that span the vector space of the cone absorption spectra can match any color percept

- *metamer*: two lights that are physically distinct (have different spectra) but give same color percept (have same projection)
  - this is a very important and general concept in perception!

- *surface reflectance function*: determines reflected light by pointwise multiplication of spectrum of the light source

- adaptation in color space ("after-images")

- color constancy - full theory of color vision (unfortunately) needs more than linear algebra!