

Trigonometry

Targets:

1. To identify the basic characteristics of the 6 trig functions.
2. To use basic trig identities
3. To solve trig equations.
4. To use trig formulae (Compound, Double and Half Angle).
5. To use the form $a\cos\theta + b\sin\theta$

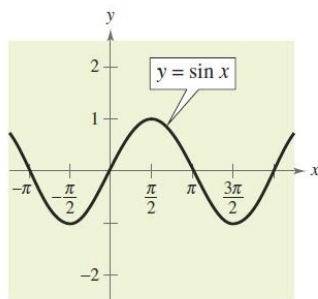
Key Terms or Formulas:

- Refer to the Formula Sheet

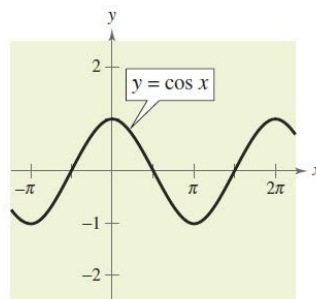
Assessments:

- 3 marks on Short Question and 6 marks on Short Test 2

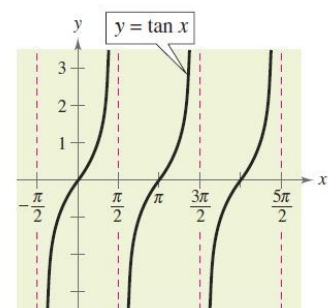
Time Period: 3.5 weeks



DOMAIN: $(-\infty, \infty)$
RANGE: $[-1, 1]$
PERIOD: 2π



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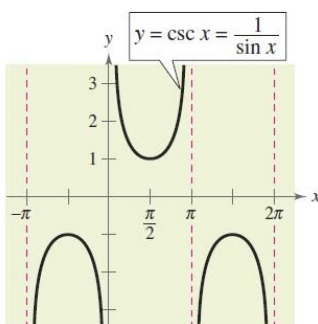
DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, \infty)$
PERIOD: π

Range can also be written as:

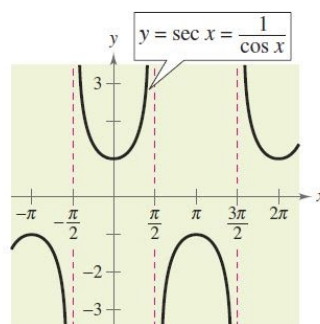
$$-1 \leq y \leq 1$$

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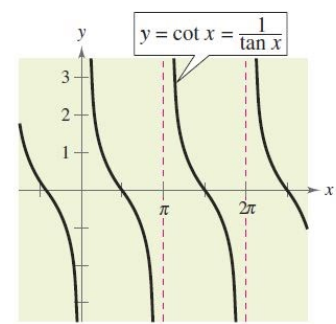
$$-\infty < y < \infty$$



DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
PERIOD: 2π



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Range can also be written as:

$$y \leq -1 \text{ and } y \geq 1$$

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$$-\infty < y < \infty$$

Trigonometry Lesson Video Problems

Section 1: Basic Characteristics

1. Graphing $y = \operatorname{cosec}\theta$, $0^\circ \leq \theta \leq 360^\circ$
2. Graphing $y = \sec\theta$, $0 \leq \theta \leq 2\pi$
3. Graphing $y = \cot\theta$, $0 \leq \theta \leq 2\pi$
4. State the Domain, Range and Period of $y = \operatorname{cosec}\theta$.
5. State the Domain, Range and Period of $y = \sec\theta$.
6. State the Domain, Range and Period of $y = \cot\theta$.
7. Which Trig Functions are Even and Which are Odd?
8. State the Period of $y = -4\sec 3(\theta - 60^\circ)$.
9. State the Period of $y = 2\cot 4(x - \pi)$.

Section 2: Trig Identities

10. Identities #1

$$\begin{array}{ll} \sin\theta = & \operatorname{cosec}\theta = \\ \cos\theta = & \sec\theta = \\ \tan\theta = & \cot\theta = \end{array}$$

11. Identities #2

$$\begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ 1 + \cot^2\theta = \operatorname{cosec}^2\theta \\ \tan^2\theta + 1 = \sec^2\theta \end{array}$$

12. Identities #3

$$\begin{array}{ll} \sin(90^\circ - \theta) = & \operatorname{cosec}(90^\circ - \theta) = \\ \cos(90^\circ - \theta) = & \sec(90^\circ - \theta) = \\ \tan(90^\circ - \theta) = & \cot(90^\circ - \theta) = \end{array}$$

13. Using Identities and the Calculator

If $y = 2\operatorname{cosec}4\theta$ and $\theta = \frac{\pi}{2}$, find the value of y .

14. Using Identities to Simplify

$$\sqrt{\operatorname{cosec}^3\theta \cot\theta \sec\theta}$$

15. Using Identities to Find Other Trig Functions

If $\tan\theta = \frac{3}{4}$ and $180^\circ < \theta < 270^\circ$, find the exact value of $\sec\theta$.

16. Using Identities to Eliminate a Variable

Given that $p = 2\sec\theta$ and that $q = 4\cos\theta$, eliminate the variable θ and express p in terms of q .

17. Using Identities to Prove #1

Prove that $\frac{1-\cos^2\theta}{1-\sin^2\theta} = \sec^2\theta - 1$

18. Using Identities to Prove #2

Prove that $\sec^2\theta - \cos^2\theta = \sin^2\theta(1 + \sec^2\theta)$

19. Using Identities to Prove #3

Prove that $\frac{1-\cos\theta}{\sin\theta} = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

Section 3: Solving Trig Equations

20. Solving with Basic Identities

$\sec\theta = \sqrt{2}$ when $0^\circ < \theta < 360^\circ$.

21. Solving when θ is Not Alone

$\cot(2\theta + 10^\circ) = \sqrt{3}$ when $-180^\circ < \theta < 180^\circ$.

22. Solving with Pythagorean Identities

$\tan^2x - 2\secx + 1 = 0$ when $-\pi < x < \pi$.

23. Solving with Quadratic Forms

$4\sin A + \operatorname{cosec} A = 4$ when $-\pi < A < \pi$.

Section 4: Compound, Double and Half Angle Formulae

24. Compound (addition/subtraction) Formulae for Sin
 $\sin(30^\circ + 60^\circ)$

25. Compound (addition/subtraction) Formulae for Cos
 $\cos\left(\frac{\pi}{3} + \frac{\pi}{3}\right)$

26. Compound (addition/subtraction) Formulae for Tan
 $\tan(45^\circ + 30^\circ)$

27. Using Compound Formulae

Given that $\sin A = \frac{8}{17}$ and A is acute and that $\cos B = \frac{-4}{5}$ and B is obtuse, find

a) $\sin(A - B)$ b) $\cos(A - B)$ c) $\cot(A - B)$

28. Double Angle Formulae for Sin

$\sin(A + A)$

29. Double Angle Formulae for Cos
 $\cos(A + A)$

30. Double Angle Formulae for Tan
 $\tan(A + A)$

31. Using Double Angle Formulae
If $\cos 2x = \frac{1}{8}$, find the value of $\sin x$.

32. Half Angle Formulae for Sin and Cos
 $\cos 2B = 2\cos^2 B - 1$ $\cos 2B = 1 - 2\sin^2 B$

33. Using Half Angle Formulae
If $\sin \theta = \frac{5}{13}$ and $90^\circ < \theta < 180^\circ$ find the value of $\cos \frac{\theta}{2}$.

34. Using Formulae to Solve Trig Equations
 $\cos 2x + \cos x = -1$ when $0^\circ < x < 360^\circ$

35. Using Formulae to Prove
 $\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \theta$

Section 5: To use the form $a\cos\theta + b\sin\theta$

36. Find the value of R and $\tan \alpha$ in the identity:
 $7\cos\theta + \sin\theta \equiv R\sin(\theta + \alpha)$

37. Express $4\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$.

38. If $3\sin x - 4\cos x \equiv R\sin(x + \alpha)$ when $R = 5$ and $\tan \alpha = \frac{-4}{3}$, then find the maximum value of $3\sin x - 4\cos x$.

39. Solve $5\sin\theta - 12\cos\theta = 6$ when $0^\circ \leq \theta \leq 360^\circ$.

Answers to Lesson Problems

1. See First Page
2. See First Page
3. See First Page
4. See First Page
5. See First Page
6. See First Page
7. Even: Cos and Sec
Odd: Sin, Tan, Cosec, Cot

8. $P = \frac{2\pi}{3}$

9. $P = \frac{\pi}{4}$

10.

$$\sin\theta = \sin\theta$$

$$\cos\theta = \cos\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

11. See Video

12.

$$\sin(90^\circ - \theta) = \cos\theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

$$\tan(90^\circ - \theta) = \cot\theta$$

$$\operatorname{cosec}(90^\circ - \theta) = \sec\theta$$

$$\sec(90^\circ - \theta) = \operatorname{cosec}\theta$$

$$\cot(90^\circ - \theta) = \tan\theta$$

13. See Video

14. $\operatorname{cosec}^2\theta$

15. $\frac{-5}{4}$

16. $p = \frac{8}{q}$

17. See Video

18. See Video

19. See Video

20. $45^\circ, 315^\circ$

21. $-170^\circ, -80^\circ, 10^\circ, 100^\circ$

22. $\frac{\pi}{3}$ and $-\frac{\pi}{3}$

23. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

24. See Video

25. See Video

26. See Video

27.

a. $\frac{-77}{85}$

b. $\frac{-36}{85}$

c. $\frac{36}{77}$

28. See Video

29. See Video

30. See Video

31. $\frac{\pm\sqrt{7}}{4}$

32. See Video

33. $\frac{\sqrt{26}}{26}$

34. $90^\circ, 120^\circ, 240^\circ, 270^\circ$

35. See Video

36. $R = 5\sqrt{2}$ and $\tan \alpha = 7$

37. $5 \cos(\theta + 36.9^\circ)$

38. Max value is 5.

39. $94.9^\circ, 219.9^\circ$