Trig. Modeling

Short Answer

1. Mario's bicycle has 42 teeth in the crankset attached to the pedals. It has three sprockets of differing sizes connected to the rear wheel. The three sprockets at the rear have 16, 20, and 24 teeth each.

   a. The diameter of each wheel is 65 cm. What rear sprocket should Mario choose so that he travels the furthest with each turn of the pedal? How far will he travel with one turn of the pedal using that gear arrangement? Show your work.

   b. Suppose that Mario turns the crankshaft at a rate of 50 revolutions per minute and is using the rear sprocket with 20 teeth. How many times per minute does the rear wheel turn?

   c. Using the conditions described in Part b, how far does Mario travel each minute?

2. Use the relationships between circular revolutions, degrees, and radians to complete the following.

   a. 1.25 revolutions = _________ degrees = __________ radians

   b. $\frac{7\pi}{6}$ radians = ___________ degrees = ___________ revolutions

3. Demonstrate how you can determine the value of cos 210° without using technology.

4. The Ferris wheel on Navy Pier in Chicago has 40 equally spaced gondolas and a 70-foot radius. Passengers load the Ferris wheel from a platform above the ground. After loading the passengers, the Ferris wheel moves in a counterclockwise direction.

   a. There are spokes connecting each gondola to the center of the wheel. What is the measure of the angle formed by adjacent spokes that connect each gondola to the center of the wheel if the angle is measured in degrees? In radians? Show your work.

   Angle measure in degrees: __________

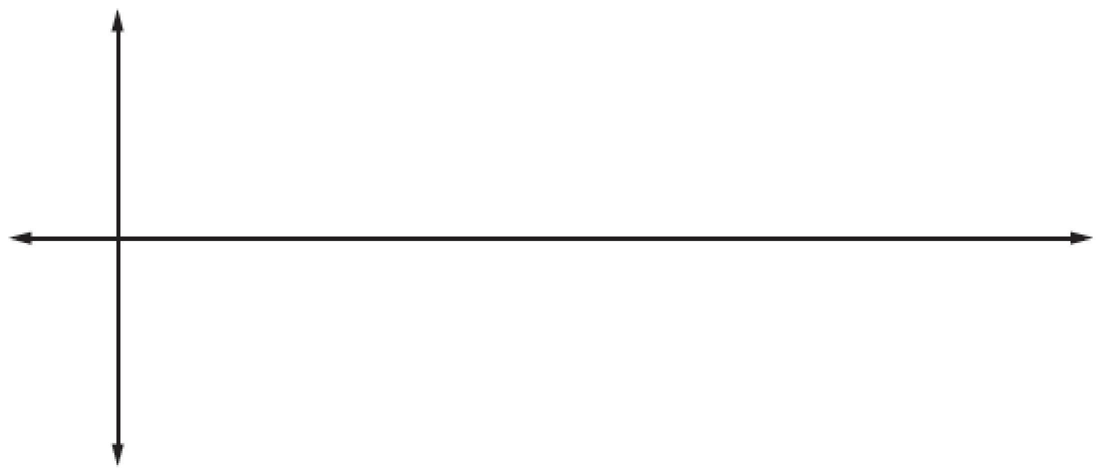
   Angle measure in radians: __________

   b. Sydney begins in a gondola at the "3 o'clock" position of the Ferris wheel. How far must she rotate to reach the lowest position on the Ferris wheel? Give your answer in degrees and radians.

   Degrees: __________

Radians: __________

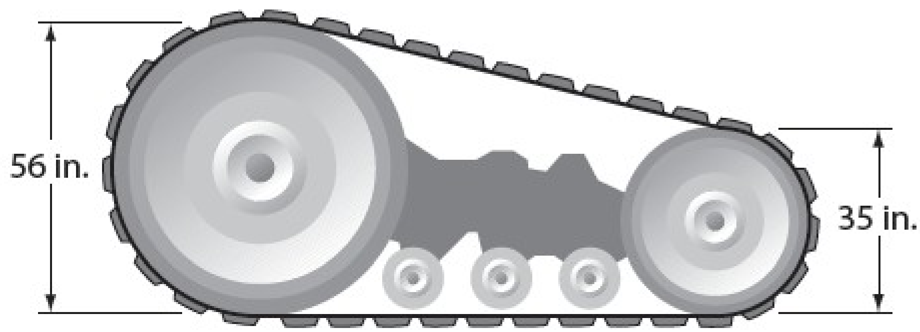
c. Make a sketch showing Sydney's position relative to the horizontal line through the center of the wheel as the wheel makes one complete revolution. Label the x-axis of your sketch using radians or degrees. Label the y-axis using feet.



d. If x is the amount of rotation of Sydney's gondola in radian measure, write a function rule that models Sydney's position relative to the horizontal line through the center of the wheel.

e. The wheel takes 3 minutes to make a complete revolution. Find Sydney's distance above the horizontal line through the center of the wheel after she has been rotating for 1 minute. Show your work or explain your reasoning.

5. Instead of moving on wheels, a track tractor uses a belt that goes around wheels in order to move across the ground. A picture of one such tractor and a drawing of the track system are shown below.



Suppose that for one tractor, the large rear wheel has a diameter of 56 inches and the smaller front wheel has a diameter of 35 inches as shown on the above diagram.

a. If the rear wheel is turning at a constant rate of 6 revolutions per minute, determine each of the following.

i. The angular velocity of the rear wheel in degrees per minute and in radians per minute

Degrees per minute: _____________ *Radians per minute:* _____________

ii. The linear velocity of the rear wheel

iii. The angular velocity of the front wheel in revolutions per minute

iv. The linear velocity of the front wheel

v. The distance the tractor will travel in one hour

b. Write a rule that shows the relationship between the angular velocity of the rear wheel *vr* and the angular velocity of the front wheel v_f.

6. Use the relationships between circular revolutions, degrees, and radians to complete the following. Show any work or computations in the space provided.

a. 480 degrees = _________ revolutions = __________ radians

b. $\frac{17\pi}{6}$ radians = ___________ degrees = ___________ revolutions

7. The minute hand on a clock is five inches long and is pointing directly at the 2.

   a. How far is the end of the minute hand above the horizontal line through the center of the clock?

   b. How far is the end of the minute hand from the vertical line through the center of the clock?

8. Without using technology, explain why each of the following statements is true.

   a. $\cos \frac{\pi}{6} = \cos \frac{13\pi}{6}$

   b. $\sin 25^\circ = \sin 155^\circ$

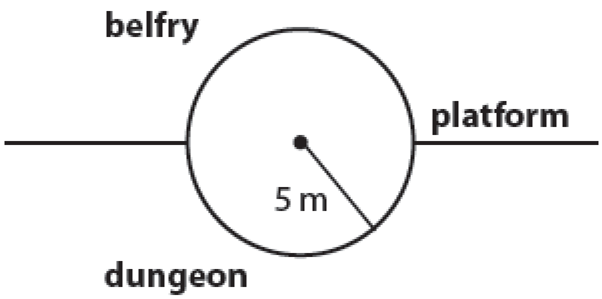
9. Suppose that the height in feet of a Ferris wheel seat changes in a pattern that can be modeled by the function $h(t) = 30 \sin t + 5$, where t is time in minutes since the wheel started turning.

   a. What is the radius of the Ferris wheel?

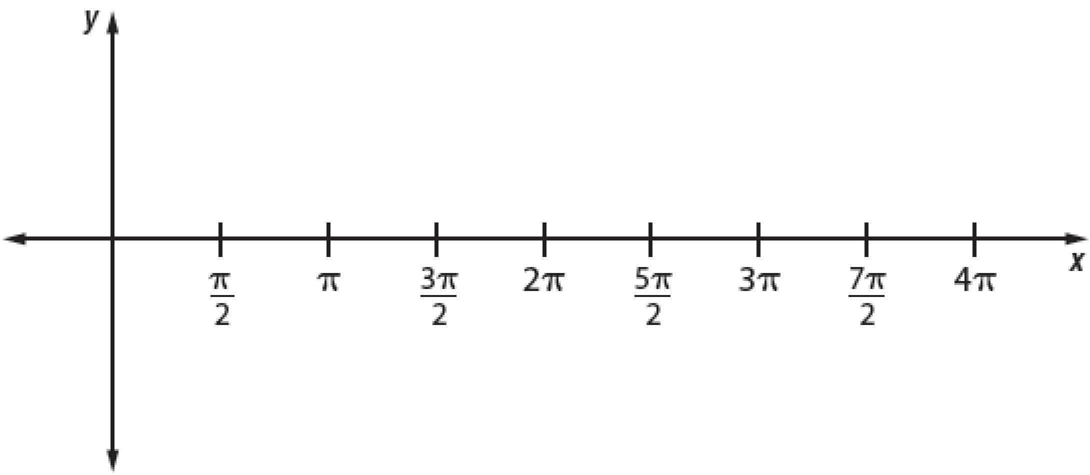
   b. Determine the maximum height of a seat on this Ferris wheel. Show your work.

   c. If the Ferris wheel is operating without stopping, how long will it take a seat to move from the highest point on the wheel all the way around the circle and back to the highest point?

10. The Wheel of Horror carnival ride is like a Ferris wheel that is inside a haunted house. You board the ride from a platform at the height of the center of the wheel. From your perspective, the Wheel of Horror turns in a counter-clockwise direction. When your seat is above the platform level, you are in the "belfry" where you are bombarded with fl ying monsters. When your seat descends below the level of the platform, you are in the "dungeon" where equally scary creatures lurk. The Wheel of Horror has a radius of 5 meters. The main features of a two-dimensional side view are sketched below.



a. You and a friend board the Wheel of Horror. Sketch a graph showing your directed distance y from the level of the platform during 2 revolutions of the wheel (where x is measured in radians). Mark the scale you use on the y-axis.



b. Write a function rule that represents the graph you drew in Part a.

c. Indicate whether you are in the belfry or in the dungeon in each of the following intervals by placing a check in the appropriate place in the table below.

	$0 < x < \pi$	$\pi < x < 2\pi$	$2\pi < x < 3\pi$	$3\pi < x < 4\pi$
Belfry				
Dungeon				

d. How far above or below the platform will you be after the wheel has turned $\frac{\pi}{3}$ radians?

e. Find two other radian measures where you will be at the same height as you were in Part d.

11. The engine sprocket of Gino's go-cart is 3 cm in diameter. It is attached to an 18-cm rear axle sprocket that drives the go-cart.

a. Find the angular velocity of the rear axle when the engine is turning 4,800 rpm.

Answer: ____________________

Work or explanation:

b. Find the speed of the go-cart in km/hr under the conditions in Part a if the rear wheels are 30 cm in diameter.

Answer: ____________________

Work or explanation:

c. On the first curve of the Springdale go-cart track, Gino knows he must reduce his speed to 40 km/hr to avoid sliding. What rate of engine sprocket rotation will result in the desired speed?

Answer: ____________________

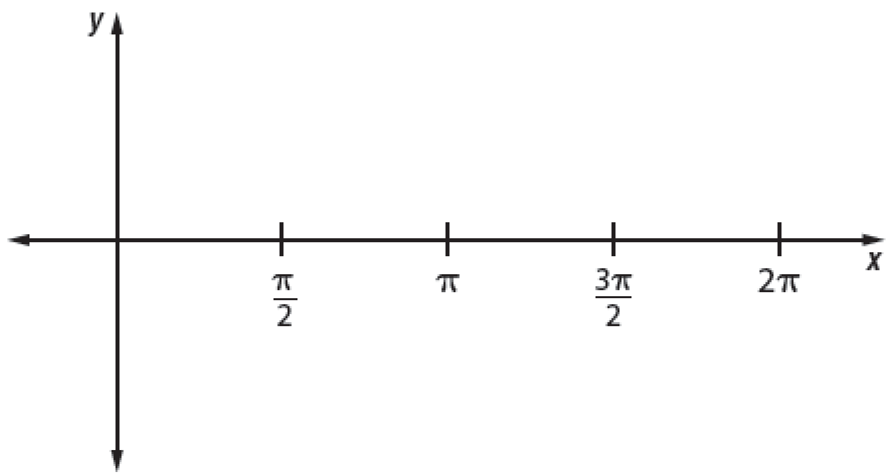
Work or explanation:

12. A function rule in the form $y = A \sin Bx$ has period 2π and amplitude 4.

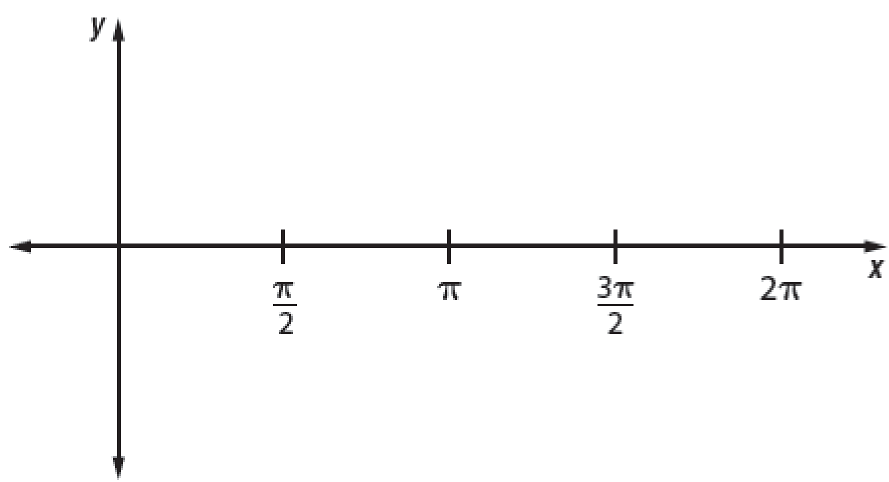
a. Find A and B. Explain your reasoning.

$A =$ $\qquad\qquad$ $B =$

b. Graph the function in Part a. Mark the scale on the y-axis. Explain how you can see from the graph that the period is 2π and the amplitude is 4.



c. Change one number in the above function rule so the period is π. Write the new rule. Sketch the resulting graph.



$y =$

13. Complete each of the following. You should not use technology to help in your explanations.

a. If $\frac{\pi}{2} < \theta < \pi$, then sin θ (*positive, negative*) and cos θ is (*positive, negative*).

Explanation:

b. The measure of an angle is 225°. In radians, the measure of the angle is _______.

Explanation:

c. Find an exact value for sin $\frac{10\pi}{3}$. Show your work.

d. Suppose that m$\angle A$ = 54°.

i. Determine the measure of $\angle B$ if 0° < m$\angle B$ < 360°, m$\angle B \neq$ m$\angle A$, and cos *B* = cos *A*?

m$\angle B$ = _________

Explanation:

ii. Determine the measure of $\angle C$ 0° < m$\angle C$ < 360°, m$\angle C \neq$ m$\angle A$, and sin *B* = sin *A*?

m$\angle C$ = _________

Explanation:

Trig. Modeling
Answer Section

SHORT ANSWER

1. ANS:

a. Mario should use the rear sprocket with 16 teeth. Using that sprocket, he will travel $\frac{42}{16} \cdot 65 \cdot \pi \approx$ 536 cm with one turn of the pedal.

b. $50 \cdot \frac{42}{20}$ = 105 rpm

c. $105 \cdot 65 \cdot \pi \approx$ 21,441 cm per minute ≈ 214 meters per minute

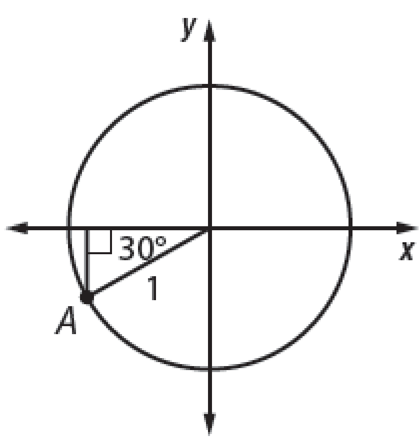
2. ANS:

a. 1.25 revolutions = 450° = $\frac{5\pi}{2}$ radians.

b. $\frac{7\pi}{6}$ radians = 210° = $\frac{7}{12}$ revolutions ≈ 0.583 revolutions

3. ANS:

a.



cos 210° will be the *x*-coordinate of the point *A*. Using the fact that the side opposite the 60° angle in a 30°-60° right triangle is $\frac{h\sqrt{3}}{2}$, where *h* is the length of the hypotenuse, you can determine that *x*-coordinate of a point *A* is $-\frac{\sqrt{3}}{2}$. So, cos 210° = $-\frac{\sqrt{3}}{2}$.

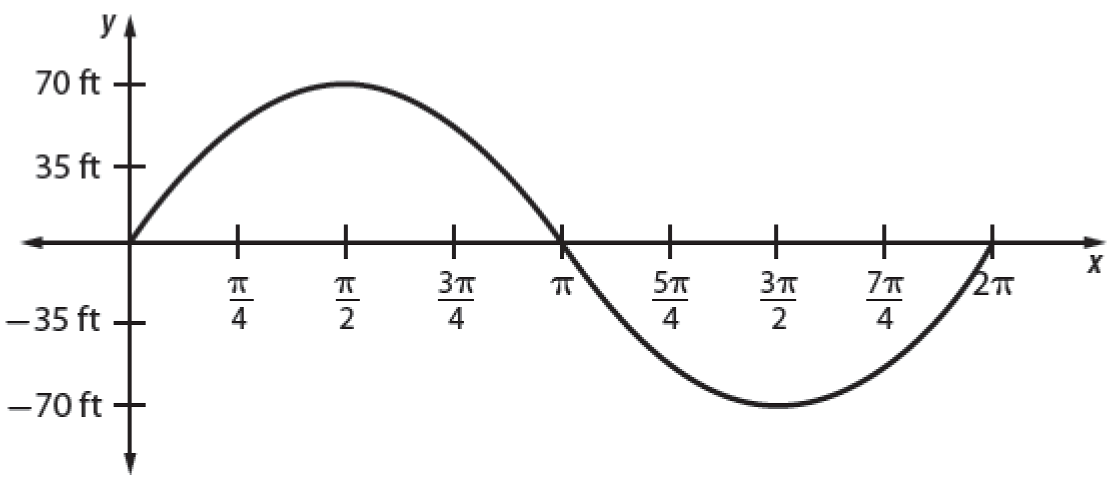
4. ANS:

a. $\frac{360°}{40} = 9°$

$\frac{2\pi}{40} = \frac{\pi}{20}$ radians

b. 270°, $\frac{3\pi}{2}$ radians

c.



d. $y = 70 \sin x$

e. Sydney will have traveled $\frac{1}{3}$ of the way around the circle. This is equivalent to a 120°, or $\frac{2\pi}{3}$ radians, rotation. Thus, 70 sin 120° = $\frac{70\sqrt{3}}{2} \approx$ 60.6 feet above the horizontal line through the center of the wheel.

5. ANS:

a.

i. Degrees per minute: 2,160°/minute
Radians per minute: 12π radians/minute

ii. $(56\pi)(6) \approx$ 1,055.6 inches per minute

iii. $\frac{56}{35} \cdot 6$ = 9.6 revolutions per minute

iv. $(9.6)(35\pi) \approx$ 1,055.6 inches per minute

v. $(1{,}055.6)(60) \approx$ 63,336 inches, or approximately 12 miles

b. $v_f = \frac{8}{5} v_r$

6. ANS:

a. $1\frac{1}{3}$ revolutions = $8\frac{\pi}{3}$ radians

b. 255 degrees = $\frac{17}{24}$ revolutions

7. ANS:

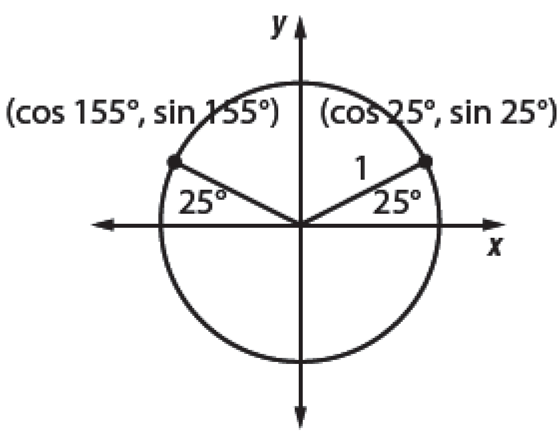
a. 5 sin 30° = 2.5 inches

b. 5 cos 30° ≈ 4.33 inches

8. ANS:

a. Since the period of the cosine function is 2π, $\cos\frac{\pi}{6} = \cos\left(\frac{\pi}{6} + 2\pi\right) = \cos\frac{13\pi}{6}$.

b. The value of sin 25° is the *y*-coordinate of the point where the unit circle intersects the terminal side of a 25° angle in the standard position, similarly for the value of sin 155°. These will be equivalent because of the symmetry of the unit circle.



9. ANS:

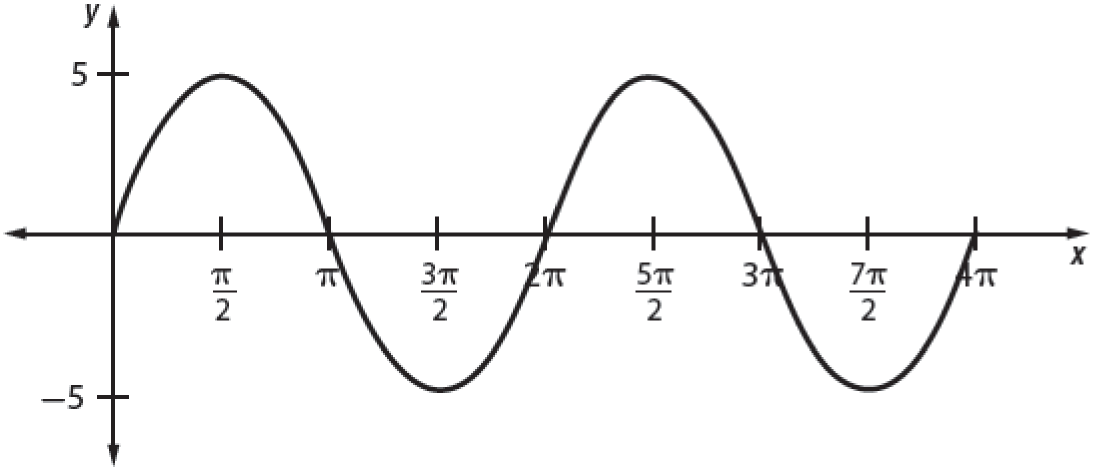
a. The radius is 30 feet.

b. T he maximum height is obtained when $t = \frac{\pi}{2}$. The maximum height is $30 \sin\frac{\pi}{2} + 5 = 35$ feet.

c. It will take approximately $2\pi \approx 6.28$ minutes.

10. ANS:

a.



b. $y = 5 \sin x$

c.

	$0 < x < \pi$	$\pi < x < 2\pi$	$2\pi < x < 3\pi$	$3\pi < x < 4\pi$
Belfry	✓		✓	
Dungeon		✓		✓

d. $5 \sin \frac{\pi}{3} = \frac{5\sqrt{3}}{2} \approx 4.33$ ft above the platform

e. Responses will vary. The height will be the same after turns of $\frac{2\pi}{3}$, $\frac{7\pi}{3}$, and $\frac{8\pi}{3}$ radians.

11. ANS:

a. *Angular velocity* = $\frac{1}{6} \cdot 4{,}800 = 800$ rpm

b. *Speed* = $800 \cdot \pi \cdot 30 \approx 75{,}398$ cm/min = $\frac{(75,398 \text{ cm/min}) \cdot (60 \text{ min/hr})}{100,000 \text{ cm/km}} \approx$ 45.2 km/hr

c. $\frac{45.2}{4800} = \frac{40}{x}$, so $x = \frac{(4,800)(40)}{45.2} \approx 4,248$ rpm

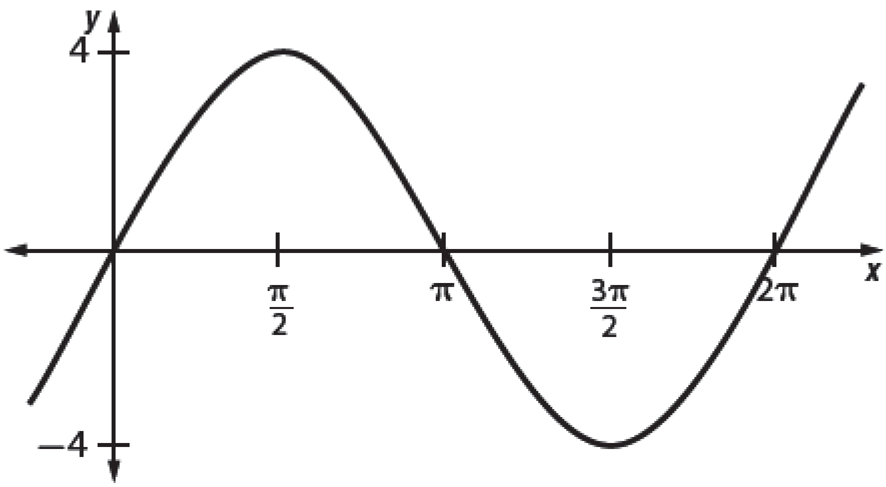
12. ANS:

a. $A = 4$ or -4; $B = 1$

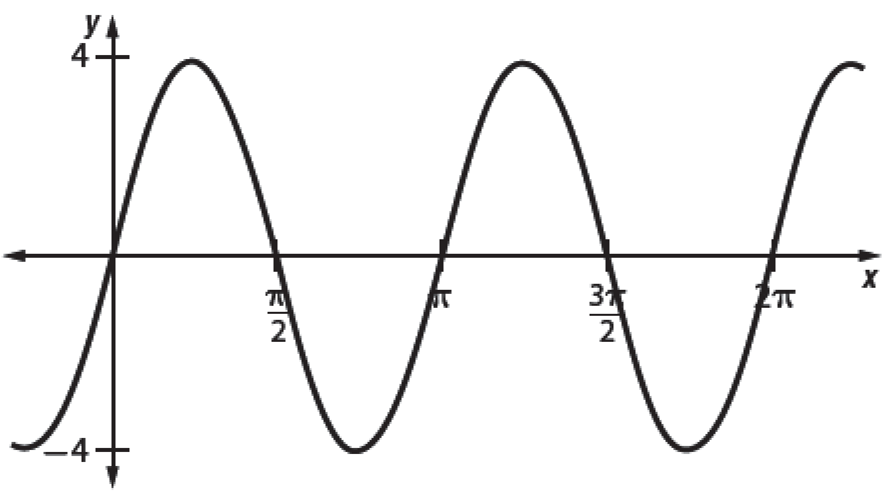
$y = \sin x$ has amplitude 1 and period 2π, $A = 4$ (or -4) will make the y-coordinate reach 4 times the minimum and maximum values. The period is affected by B; and since $B = 1$ in $y = \sin x$, B should remain 1.

b. $y = 4 \sin x$

Between $x = 0°$ and $x = 2\pi$, the graph completes a period, that is, from the x-axis, up to a maximum of $y = 4$, down again to the x-axis and on to a minimum of $y = -4$, and finally back again to the x-axis. The amplitude, in this case, is the maximum value of y, namely 4. Students may also graph and describe $y = -4 \sin x$.



c. Change the value of B so that $B = 2$. The new function is $y = 4 \sin(2x)$. Other possibilities are $y = 4 \sin(-2x)$, $y = -4 \sin(2x)$, or $y = -4 \sin(-2x)$.



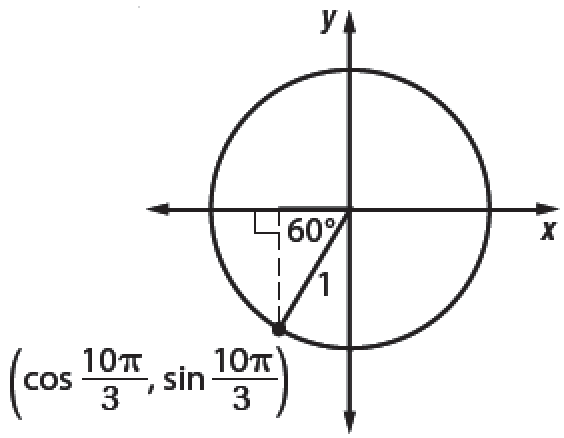
13. ANS:

a . If $\frac{\pi}{2} < \theta < \pi$, then $\sin\theta$ is *positive* and $\cos\theta$ is *negative*.

If $\frac{\pi}{2} < \theta < \pi$, then the terminal side of the angle is in the second quadrant. In the second quadrant, the x-coordinate of a point on the unit circle is negative and the y-coordinate is positive. Thus, $\cos\theta$ is negative and $\sin\theta$ is positive.

b. The measure in radians is $\frac{225}{360} \cdot 2\pi = \frac{225}{180} \cdot \pi = \frac{5}{4}\pi$.

c. By considering the unit circle and periodicity of sin θ, sin $\frac{10\pi}{3}$ = sin $\frac{4\pi}{3}$. The value of sin $\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$, so sin $\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$.



d. **i.** m$\angle B$ = 306°

Since cos 54° is positive in the first and fourth quadrants, $\angle B$ must have terminal side in the fourth quadrant and make a 54° angle with the positive *x*-axis. Thus, m$\angle B$ = 360° - 54° = 306°.

ii. m$\angle C$ = 126°

Since sin 54° is positive in the first and second quadrants, the terminal side of $\angle C$ must be in the second quadrant and make a 54° angle with the negative *x*-axis. Thus, m$\angle C$ = 180° - 54° = 126°.