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LEARNING MATHEMATICS THROUGH REASONING AND PROBLEM SOLVING

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An extended vignette demonstrates a new take on problem solving: how to provide multiple entry points for K–grade 2 learners, leverage their intuitive understandings of the world around them to make sense of and solve problems, and engage them in rich mathematical modeling.

TRYING THREE-ACT TASKS WITH PRIMARY STUDENTS

Kendra Lomax, Kristin Alfonzo, Sarah Dietz, Ellen Kleyman, and Elham Kazemi

At mid-morning in Kristen Alfonzo's kindergarten classroom, students are sitting on the carpet with whiteboards and markers, listening to her tell a story. "Jamal is making sandwiches. He makes six sandwiches."

Students obediently draw six sandwiches on their boards.

"Then Jamal eats two of the sandwiches."

Students begin crossing out two of the sandwiches they have drawn.

"How many sandwiches are left?"

Some students quickly shout out, "Four!" causing others to look around at their neighbors' whiteboards and quickly write a 4 also.

In our attempt to engage our kindergartners in solving real-world problems, it seemed they were learning a different lesson: Math is about following directions and getting the right answer quickly.

A new take on problem solving

As primary teachers, coaches, and teacher educators from various school settings in the Seattle area, we discovered we shared common challenges. Experiences like the one in Alfonzo's class prompted us to work together to seek out new approaches to engaging young children in problem solving. The goals of problem-solving activities in the elementary grades often include making sense of story problems, developing a range of strategies, and reaching accurate solutions. These are important mathematical aims, but they do not fully address the demands of *modeling with mathematics* as described in the fourth of the Common Core's eight Standards for Mathematical Practice (SMP 4) (CCSSI 2010, pp. 6–8). Modeling with mathematics also involves identifying mathematical problems in our world, gathering information and determining which details will help us solve a problem, and developing and revising mathematical models of situations. For young children, such models might include diagrams, equations, and using manipulatives to represent the quantities and mathematical relationships in a given situation.

After investigating various approaches and activities, we discovered an exciting problem-solving activity structure specifically designed to engage students in mathematical modeling: three-act tasks (Meyer 2011) designed by Dan Meyer for use in secondary classrooms and adapted by Graham Fletcher and other educators for elementary school classrooms (Fletcher 2016). The activity structure comprises three parts or "acts" (see **fig. 1**).

First, students view an image or video that depicts an interesting real-world situation—such as a child throwing rocks from the beach into the ocean or a "cookie monster" stealing some cookies—and discuss what they notice and wonder. Then students generate mathematical questions about the situation, identify important information they need to answer the questions, and construct mathematical models of the situation. Finally, they discuss their strategies and interpret the results of their modeling in the context of the situation. Students verify how well their modeling helped them answer their mathematical question.

Since learning about three-act tasks, we have engaged our K–grade 2 students in many of them. The purpose of this article is to share our learning with you. To begin, let's step inside Sarah Dietz's second-grade classroom for a closer look.

Act 1: Learning to ask mathematical questions about the real world

It's Friday afternoon following recess, and Dietz's second graders are squirmy, but she knows that the three-act task she has planned will pique their interest and get them engaged in rich mathematical work. "Mathematicians," Dietz addresses the class, "as you know, my daughter Carrie often comes to help me after school. Well, yesterday she was here, and I made a video of what she did. Watch the video and think to yourself, What am I noticing and wondering?"

Students watch intently as Carrie approaches a stack of crayon boxes on the counter. She takes one of the boxes. She also opens another box and removes a few of the crayons. She

FIGURE 1

The authors discovered a problem-solving activity structure specifically designed to engage students in mathematical modeling: three-act tasks designed by Dan Meyer (2011) for use in secondary classrooms and adapted by Graham Fletcher and other educators for elementary school classrooms.

Summary of a three-act task lesson

Act 1

- The teacher shares a compelling multimedia depiction of a situation through a video or photographs.
- Students discuss what they notice and wonder about the video, including mathematical features of the situations.
- Students decide on a mathematical question to answer about the situation.

Act 2

- The teacher provides information or resources that students think they need to work on the focal question.
- Students work to answer the question.

Act 3

- Students discuss their strategies and solutions.
- The teacher may compare and connect students' ideas or "reveal the answer."
- If relevant to the focal problem, students consider why their modeling was different from the real-world resolution.

then puts the partially empty box back and walks away.

Students are engrossed in the video and ask to watch it again. During the second viewing, they start to excitedly comment to one another:

"Oh! I think there are four boxes."

"She took out some crayons before she put it back!"

"She took three! No four!"

These seven- and eight-year-olds have a puzzle to solve.

To launch a three-act task, the teacher shows a short video or an image depicting an interesting situation in the real world and invites students to share what they notice and then what they wonder about the situation. Note that the videos and images do not include explicit mathematical structures like graphs or diagrams. Rather, students are tasked with making sense of the situation and bringing structure to the real-world situations (SMP 1).

Once the video is over, hands shoot into the air, and it is not just the usual hands. *All* the second graders want to share what they noticed. Kaya, who receives special education services in math and who often avoids eye contact during the subject, is up on her knees with her hand in the air, smiling. Edgar, whose English is still developing and who is usually hesitant to say anything, also has his hand up. In fact, so many hands are in the air that Dietz instructs the class to do a quick turn-and-talk to give everyone an opportunity to share before recording their noticings on the board. "Now I want you to take a moment to think about what you are wondering." Once again, as Dietz gives students time to turn and share, the classroom erupts into conversations:

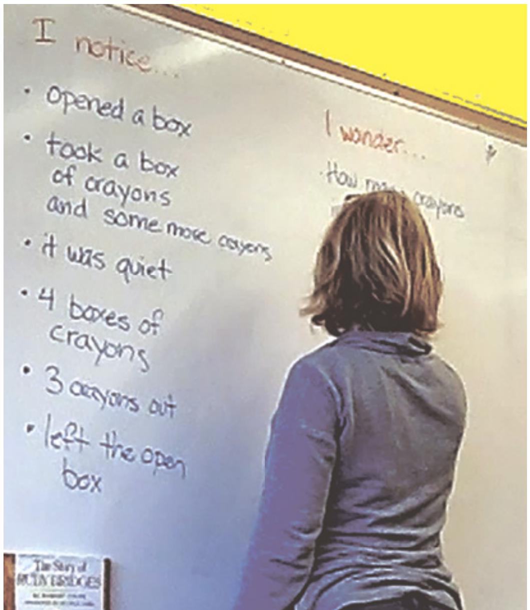
"I wonder how many crayons are in each box?"

"I wonder how many crayons were left in the open box?"

"Why did she take the crayons?"

"How many crayons were left after she took some?"

The first act invites every student in the class to participate in a way that supports the classroom community as intellectually and socially inclusive. Some of the children's observations and questions will be mathematical in nature, and some will not—and that is OK! Learning which types of questions can be answered with



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In Act 1, every idea is recorded, sometimes with words and pictures added to make the information accessible to young learners. After listing students' wonderings, the teacher guides students toward a single mathematical question that they can all investigate.

mathematics and which cannot is an important aspect of modeling, so every idea is recorded. With young learners, teachers can record students' noticings, using words and pictures to make the information more accessible. For example, if these were kindergartners, Dietz could sketch the four boxes or the three crayons next to the written words.

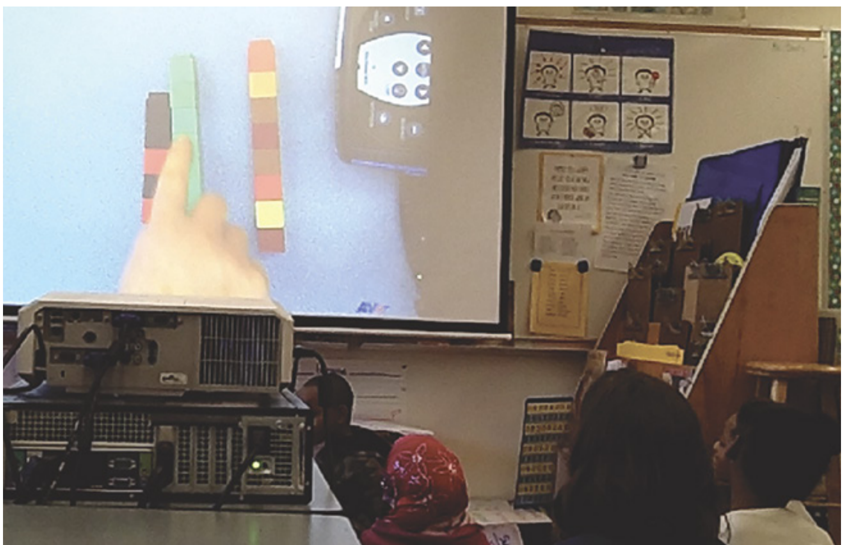
After carefully recording their wonderings, Dietz guides the class toward one mathematical question that all the students can investigate. "We have a lot of great questions here! Let's think about this one: How many crayons were left after Carrie took them?" Together, they have selected a question on which to focus.

In planning for a three-act task, the teacher will anticipate questions that students might ask and, on the basis of the mathematical goal, will identify one to work on (NCTM 2014). However, teachers sometimes revise their plan in the moment if students generate interesting questions that are worth pursuing, even if the teacher did not anticipate them; or the teacher may allow students to choose which question most interests them.

A TEACHER'S PRIMARY ROLE IN ACT 2 IS TO OBSERVE STUDENTS' APPROACHES AND STRATEGIES, TO INFORM THE DISCUSSION IN ACT 3.

Act 2: Learning to identify information needed to answer a mathematical question

In the second act, students consider what information they need to answer the selected mathematical question. The teacher has typically anticipated these needs and will provide some of the requested information before students begin working independently or in pairs. Students should be offered tools, such as cubes and hundred charts, to use as they see fit (SMP 5).



KENDRA LOMAX

Alejandro created four stacks of eight blocks to represent eight crayons in each box. He took away a "whole box" and then removed three blocks, or "crayons," from another stack.

"OK, if we want to figure out how many crayons are left after Carrie took some away, what will we need to know?" A couple of hands pop up, but most students look unsure. Dietz encourages them to turn and talk to their partner, and then she starts collecting questions.

"How many crayons are in a box?"

"How many crayons are left in the open box?"

"How many did she take out of the open box?"

Dietz reveals some of the information they asked for: Eight crayons are in each of the boxes, and Carrie opened one box and took three crayons. Some students express confusion about the number of boxes at the beginning of the video. "Were there four or five crayon boxes?" The class watches the video once more to confirm that there were four boxes to begin with. Conversation erupts; students clearly want to get started.

Students may not immediately identify all the information they will need to model the situation. Or, after getting started, they may want to confirm details of the situation by reviewing the video. These requests for more information are indications that students are working to make sense of the context (SMP 1 and SMP 4). As needed, the teacher can call the class back together to offer additional details.

As the second graders continue working, Dietz notices that some students are figuring out the total number of crayons and then subtracting the number that Carrie took; other students are summing the crayons in the three remaining boxes. Dietz sees a variety of strategies. Some students, such as Alejandro (see **fig. 2a**), are directly modeling the situation using cubes or drawings (Carpenter et al. 2014), but many are using number lines (see **fig. 2b**) or equations (see **fig. 2c**) to add the groups of crayons. Some students are starting their number line at zero, whereas others, such as Semaiah, are starting at eight. "Is it four hops on the number line?" Semaiah wonders. "There are four boxes . . ." Dietz makes a mental note to discuss this representation as a whole class in the next act.

A teacher's primary role in Act 2 is observing how students approach the problem and which strategies they use. These observations will inform the Act 3 discussion. During this

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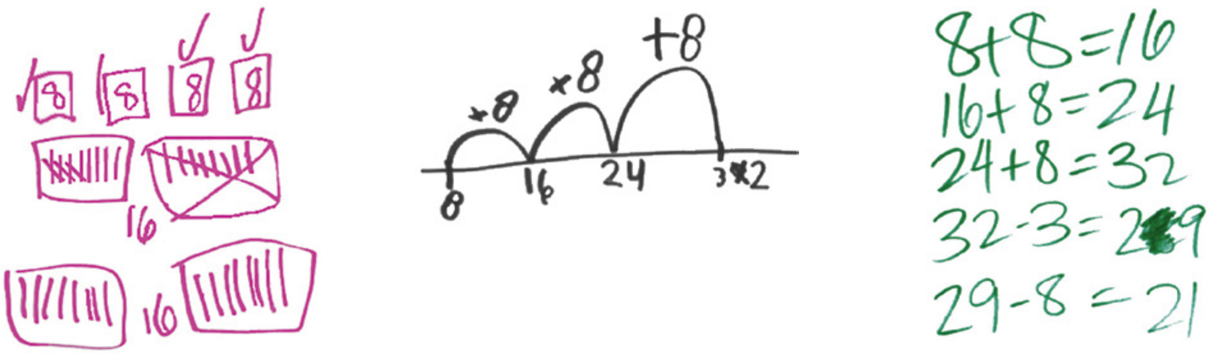
FIGURE 2

Alejandro's clear, visual, direct-modeling approach prepared other students to share their strategies.

(a) Alejandro counted the "crayons" in each box.

(b) Like many students, Semaiah used a number line to find the total number of crayons, but she started hers at the number 8.

(c) After Ramla explained her equations with respect to the crayon video, the class compared her equations to Semaiah's number line.



time, teachers might also support students to productively engage with the problem by helping them work through confusion or get started (NCTM 2014).

Act 3: Learning to examine, connect, and revise mathematical models

In the third act, the teacher gathers students to discuss the different models the class has generated to answer their focal question. On the basis of their goals for the lesson, teachers determine which strategies will be most valuable for the class to examine and discuss—and in which order (Smith et al. 2009). As part of this whole-group discourse, teachers might ask students to discuss how their different strategies or representations match the situation, or students could explore the differences and similarities among their models (SMP 3).

Students gather at the carpet, and Dietz asks Alejandro to show his thinking using the document camera. Alejandro used blocks to create four stacks of eight, representing the eight crayons in each box (see **fig. 2a**). He shows the class how he took away a "whole box" and then removed three blocks, or "crayons," from another stack. Using Alejandro's strategy, the class counts together to determine how many crayons were left. This strategy affords a clear visual model, setting the stage for other students to share their strategies.

Next, Semaiah puts her number line up for the class to see (see **fig. 2b**). Knowing that some students were working on making sense of this representation, Dietz wants to make sure the class has an opportunity to reason through this representation together: What was the meaning of each hop on the number line? If the number line starts at the number 8, what does that mean in the situation context?

Last, Ramla shares her strategy (see **fig. 2c**). After she explains how her equations connect to the crayon video, the class compares her equations to Semaiah's number line. The teacher challenges students to find the four boxes of crayons in each strategy and to think about how to show—on Semaiah's number line—the crayons that were taken away.

Another approach to the discussion of Act 3 is to show an image or video clip that reveals the answer to the focal question or reveals the conclusion to the task. The purpose of revealing the answer is not to ask children to "check their work," but to create an opportunity for students to examine how well their models fit the situation and how they could revise their models to improve their accuracy or efficiency (SMP 4).

What we learned

In our experimentation with three-act tasks, we noticed three important distinctions between this activity structure and other problem-solving activities. These features suggest why three-act tasks are a useful addition to the repertoire of elementary school problem-solving activities:

1. Three-act tasks leverage the wealth of knowledge that young students bring about the world around them. From asking mathematical questions and modeling the story situation, to discussing strategies for answering the question, children's ideas are central to each act.

2. Three-act tasks provide entry points into mathematics for all learners. Three-act tasks use images, action, and sound to convey situations, providing a different set of entry-points for learners than typical story problem solving offers. We have found video and images to be exciting for all students and particularly valuable for our English language learners. All students have access to the story situation and, through structured discourse opportunities, they negotiate meaning and develop increasingly precise language around their shared understanding of the situation. Because students take charge of posing the mathematical questions and modeling the situation, they may choose to work on a "just right" portion of the task. In Dietz's class, some students, such as Semaiah, worked on only one piece of the crayon problem: How many crayons were in the boxes?

3. Three-act tasks engage young children in the work of doing mathematics as described by the SMP. The three-act task lesson structure is designed to engage children in modeling with mathematics (SMP 4), but other important mathematical practices make an appearance as well. Act 1 is all about making sense of contextual problems (SMP 1) and bringing mathematical structure to bear on the situation (SMP 7). In Act 2, children select the information and tools they need to model the situation (SMP 5). In Act 3, they present arguments and analyze the reasoning of others (SMP 3).

Try it out

When we first introduced three-act tasks, our students were confused about what we were asking them to do. What does it mean to ask a mathematical question about the real world? Over time, they learned what to expect and how to analyze their world through a mathematical lens. It may take a few tries, but with repeated opportunities, your students will get into the routine. The routine structure of three-act tasks can be repeated with different tasks, streamlining the teacher's planning efforts. Instead of worrying about how the lesson will unfold, teachers can focus on the mathematical ideas that they want students to consider and how

Tips for getting started with primary-grade three-act tasks

Find tasks or make your own. Such websites as http://www.gfletchy.com and http://www.101Qs.com have great tasks for elementary school students, or you can make your own videos on the basis of your students' interests and the mathematical ideas you are currently working on. Videos and images that show familiar situations with a little mystery work well.

Avoid the temptation to rush students toward *your* noticings and wonderings. Remember, these are opportunities for children to apply their own mathematical thinking to make sense of and solve contextual problems. If we always pose the problem to be solved, offer all the relevant information, and overstructure the problem-solving process, we rob students of the chance to fully engage in problem solving.

Move students gently toward mathematical thinking. If students' noticings and wonderings are not mathematical in nature (e.g., "I have those kinds of crayons!" or "Why did she take them?"), encourage them to focus on the math in the situation. Try this prompt: "One thing mathematicians do is look around the world and ask questions. What kinds of questions might a mathematician ask about this situation?"

Break the activity into smaller parts. Engaging in all three acts is likely to take a full math block. You might find that, depending on their stamina, young students fare better taking a break between acts. For example, they might ask mathematical questions and begin to solve the problem—and then discuss the problem after recess or PE or even on the following day. This has the added benefit of allowing you to carefully look over their work and purposefully select and sequence students' ideas to discuss.

More resources. Check out videos of three-act tasks in action at http://www.TeachingChannel.org, and find more planning resources at http://www.TEDD.org.

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to guide conversations for students to work on these ideas.

For even the youngest learners

Collaborating with one another to better understand our students' mathematical capabilities has been a joyful and productive experience for students and teachers alike. We have found three-act tasks, originally designed for secondary mathematics, to be mathematically engaging and productive for even our youngest learners. If, as we are, you are eager to tap into elementary students' intuitive ability to make sense of problems in a new way that presents entry points for all learners, we invite you to try three-act tasks.

Common Core Connections

SMP 1	SMP 4
SMP 3	SMP 5
2.OA.1	SMP 7

Let's chat about three-act tasks!

On the second Wednesday of each month, *TCM* hosts a lively discussion with authors and *TCM* readers about a topic important in our field. You are invited to participate in the fun.

On **Wednesday, October 11, 2017**, at **9:00 p.m. EDT**, we will discuss "Trying Three-Act Tasks with Primary Students" by Kendra Lomax, Kristin Alfonzo, Sarah Dietz, Ellen Kleyman, and Elham Kazemi.

Follow along using #TCMchat. Unable to participate in the live chat? Follow us on Twitter@TCM_at_NCTM and watch for a link to the recap.

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