Tube-based model predictive control for dynamic positioning of marine vessels

This paper focuses on the design of a robust model predictive control (MPC) law for dynamic positioning (DP) of marine vessels in the presence of actuator saturation and environmental disturbances.

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The proposed solution is a tube-based MPC ensuring robustness and constraint fulfillment. Formulation of the tube-based MPC relies on a sufficient robust invariant set condition, along with a linear matrix inequality (LMI) synthesis procedure, and an efficient analytical Pontryagin set difference computation. Simulation results show the effectiveness and satisfactory behavior of the proposed controller.

Introduction

Autonomous marine vessels have been a subject of substantial recent interest in both the marine industry and the academic control community. On the business side, there is a significant potential to reduce marine accidents and costs connected to human mistakes (Apostol-Mates and Bar-bu (2016)) whereas on the academic side, the dynamic characteristics of marine vessels result in control problems that challenge the state-of-the-art, see, for example, (Fossen and Strand (2001); Johansen et al. (2004); Do and Pan (2009); Caharija et al. (2014)) and references therein.

Among the broad range of control challenges for autonomous marine vessels, dynamic positioning (DP) is a task of particular interest. Traditionally, a marine vessel is said to have DP capability if it is able to automatically maintain a predetermined position and heading angle using active thrusters. The development of DP systems for marine vessels have been widely studied in the literature, using several different control strategies (Pettersen and Fossen (2000); Loria et al. (2000); Sørensen (2011)). Nonlinear control strategies are among the most popular ones, since ship dynamics can be characterized by nonlinear differential equations (Fossen (2011)). The mainstream nonlinear techniques for DP include the Lyapunov-based backstepping (Fossen and Grovlen (1998)) and sliding mode control (Tannuri et al. (2010)).

An important aspect usually neglected on DP control design is to explicitly account for physical constraints on forces and torques generated by thrusters. In general, either such constraints are completely neglected, or the controller is specially tuned so that they are not violated under desired conditions. One of the few techniques in the literature which is capable of handling constraints is model predictive control. MPC is by now an established multivariate control technique for constrained linear systems (Rawlings and Mayne (2009)). In addition, the basic technique can be extended to deal with nonlinear, hybrid, and switched systems (Allgöwer and Zheng (2012)). The viability of using MPC for DP was established in Veksler et al. (2016), who presented compelling advantages over state-of-the-art techniques. To the best of our knowledge, there are no papers which study the DP problem under environmental

disturbances, such as wave, wind and ocean currents, while explicitly enforcing constraints.

In order to address this problem, we develop a tube-based MPC for dynamic positioning of marine vessels. In particular, the controller consists of two terms: a nominal control input, which is the outcome of a finite horizon optimal control problem and is computed offline; and an additive state feedback control law, which is designed offline and implemented online (along with the nominal control input) for vessel control, guaranteeing that the real trajectory of the closed-loop system will belong to a tube centered along the nominal trajectory. We also present an efficient approach for the Pontryagin set difference calculation required for control design. In order to test the performance of the controller, we perform a numerical simulation on a nonlinear vessel model subject to external disturbances.

Problem formulation
Notation: We let $\mathbb{R}$ and $\mathbb{N}$ denote the set of real numbers and natural numbers, respectively. A polyhedron is the (convex) intersection of a finite number of open and/or closed half-spaces and a polytope is a closed and bounded polyhedron. Given two sets $X, Y \subseteq \mathbb{R}^n$, the Minkowski sum is defined by

$$X \oplus Y = \{x + y \mid x \in X, y \in Y\},$$

and the Pontryagin set difference is

$$X \ominus Y = \{x \mid x \oplus Y \subseteq X\}.$$

We describe henceforth in this section vessel's kinematics and dynamics equations and formulate the DP problem.

Ship model
Kinematic equations of a 3-DOF marine vessel model, relating its body-fixed frame and its inertial frame velocities, can be written as (Fossen (2011)):

$$\dot{\eta} = R(\psi)\nu. \tag{1}$$

Here, $\eta = [x, \ y, \ \psi]^\top$ denotes the position and orientation of the vessel, where x and y are the ship's geometric center of gravity and ψ is the heading angle, all of them written with respect to the iner-

tial frame. The vector $\nu = [u, \ v, \ r]^\top$ denotes the velocity of the vessel, where u and v are the surge and sway velocities, and r is the yaw velocity, all of them written with respect to the body-fixed frame. Matrix $R(\psi)$ is the rotation matrix given by

$$R(\psi) = \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 \\ sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Dynamic equations of a 3-DOF vessel model, describing its motions due to forces and torques generated by ship's actuators, can be written as (Fossen (2011)):

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau + w. \tag{2}$$

Here, M is the body inertia matrix, which is the sum of rigid-body mass and hydrodynamic added mass. The $C(\nu)$ matrix contains nonlinear terms due to the Coriolis and centripetal effects. The matrix $D(\nu)$ contains hydrodynamic damping or drag forces. The vector $\tau = [F_x, \ F_y, \ M_z]^\top$, captures forces and moment produced by the actuators, where F_x, F_y, and M_z are respectively the forces and moment that act on the surge, sway and yaw dynamics. We assume that the control input τ is constrained to lie inside the compact set $\mathcal{U}$, *i.e.*,

$$\mathcal{U} = \{\tau \in \mathbb{R}^3 \mid -\tau_{\max} \leq \tau \leq \tau_{\max}\}.$$

Note that $\mathcal{U}$ represents the physical limits of the actuators. At last, w is a term encompassing all external disturbances such as ocean currents, wind and wave; it also includes any model mismatches that the system may have. We assume that $w(t)$ is bounded, *i.e.*,

$$\|w(t)\|_2 \leq w_{\max}$$

for some $w_{\max} > 0$.

Problem statement
Let $\bar{p}^I(t) = [\bar{\eta}(t), \bar{\nu}(t)]^\top$, $p^I(t) = [\eta(t), \nu(t)]^\top$ and $p_{tg}^I = [\eta_{tg}, 0, 0, 0]^\top$ be the nominal (undisturbed), actual and target state vectors respectively, having its three first components written in the inertial frame and its three last components written in the body-fixed frame. The main objective of this paper is to design a control policy such that $p^I(t)$ will be maintained inside an ellipsoid centered

around $\bar{p}^I(t)$ in the presence of the bounded disturbance $w(t)$, and under the input constraint $\tau(t) \in \mathcal{U}$, which is to be satisfied for all $t \geq 0$. In particular, as t goes to infinity, $\bar{p}^I(t)$ will tend to pltg asymptotically, forcing $p^I(t)$ to be contained in an ellipsoid centered around p^I_{tg}. More specifically, the control objective is to make

$$\lim_{t \to \infty} \|p^I(t) - p^I_{tg}\| \leq \epsilon$$

for some $\epsilon > 0$.

Main results

This section proposes a tube-based MPC control law to solve the DP problem.

Coordinate transformation and linearization

Consider the coordinate transformation

$$e = R(\psi)^\top (\eta - \eta_{tg}), \tag{3}$$

which expresses the tracking error $\eta - \eta_{tg}$ in the body-fixed frame (Fossen (2011)). Using (1), the dynamic equation of the body-fixed tracking error e is given by

$$\dot{e} = -S(r)e + \nu, \tag{4}$$

where $S(r)$ is the skew-symmetric matrix defined as

$$S(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

For control design, a standard linearization of (2) and (4) around $e_{\text{eq}} = [0,0,0]^\top$ and $\nu_{\text{eq}} = [0,0,0]^\top$ yields:

$$\dot{p} = Ap + B\tau + B_w w, \tag{5}$$

where:

$$p = [e, \nu]^\top, A = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}D_l \end{bmatrix} \text{ and } B = B_w = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$$

The matrix I denotes the $3x3$ identity matrix, 0 is the zero matrix of compatible size and D_l is the linear part of the damping matrix $D(\nu)$. The linearization adopted in this work can be justified by the assumption that the DP task will be performed in low speed. A linear model is then a reasonable approximation in such cases, as quadratic and higher order components in $C(\nu)\nu$ and $D(\nu)\nu$ become negligible (chapt. 7 - Fossen (2011)).

Tube-based MPC design

We briefly introduce the main ideas behind tube-based MPC. The nominal system of (5) is described by

$$\dot{\bar{p}} = A\bar{p} + B\bar{\tau}, \tag{6}$$

where the bar on top of a variable denotes a system not considering external disturbances. The error between the actual and nominal states is $z = p - \bar{p}$ and satisfies

$$\dot{z} = Az + B(\tau - \bar{\tau}) + B_w w. \tag{7}$$

The basic idea of the tube MPC approach is to decompose the computation of a receding-horizon control law into (i) a deterministic MPC problem which uses the nominal dynamics model in (6) to compute a desired state and control trajectory pair $(\bar{p}(t), \bar{\tau}(t))$ over a finite time horizon $[t, t+T]$, and (ii) a feedback control problem that provides a control policy to keep the actual state $p(t)$ close to $\bar{p}(t)$ (Limon et al. (2010)). More precisely, the tube-based MPC technique will use (6) and (7) in order to construct the following input signal

$$\tau(t) = \bar{\tau}(t) + Kz(t). \tag{8}$$

The first term in (8), is the nominal input $\bar{\tau}(t)$ and the second one is the ancillary feedback input $Kz(t)$. To make sure that $\tau(t)$ respects the original input constraints imposed by thrusters, the total input available to the system will be divided into the two terms above during the control design phase. First, the ancillary feedback input gain K will be designed offline by the solution of an LMI. The input capacity not used by Kz will be made available to the nominal input component which will be calculated by an MPC. This process will rely on the set difference computation, in order to "subtract the worst" possible input set defined by the Kz input usage, from the original input constraints, leaving the remainder input capacity as the constraint defining the available input that can be used by the nominal input.

Ancillary feedback synthesis

We now formulate the design of the ancillary controller gain K as a semidefinite program. Assume that there exists a positive definite matrix X, a non-square matrix Y, and scalars $\lambda, \mu > 0$ such that

$$\begin{bmatrix} (AX + BY)^\top + AX + BY + \lambda X & B_w \\ * & -\mu I \end{bmatrix} \leq 0.$$

Then, according to Lemma 2 in Yu et al. (2013), the set

$$\Omega := \left\{ z \;\middle|\; z^\top P z \leq \frac{\mu w_{\max}^2}{\lambda} \right\} \tag{9}$$

is a robust invariant set for the error system (7), where $P = X^{-1}$ and the ancillary feedback gain is calculated as

$$K = Y X^{-1}.$$

Nominal MPC formulation
Using (6), the nominal MPC problem solved at the discrete time instant t_k is (Yu et al. (2013)):
minimize

$$\int_{t_k}^{t_k+N} \left(\bar{p}(s)^\top Q \bar{p}(s) + \bar{\tau}_k^\top(s) R \bar{\tau}(s) \right) ds$$
$$+ \bar{p}(t_k + N)^\top Q_f \bar{p}(t_k + N)$$

subject to

$$\begin{aligned} \dot{\bar{p}}(t) &= A\bar{p}(t) + B\bar{\tau}(t), & t \in [t_k, t_k + N] \\ \bar{\tau}(t) &\in \bar{\mathcal{U}}, & t \in [t_k, t_k + N] \\ \bar{p}(t_k + N) &\in \mathcal{X}_f. \end{aligned} \tag{10}$$

Here, N is the prediction horizon, Q is the state penalty matrix, R is the input penalty matrix, Q_f is the terminal penalty matrix, and $\mathcal{X}_f$ is the terminal set constraint. The set $\bar{\mathcal{U}}$ is the "reduced" input set constraint, defined as

$$\bar{\mathcal{U}} = \mathcal{U} \ominus K\Omega,$$

where Ω is the invariant set calculated in (9). Note that the matrix Q_f and terminal set $\mathcal{X}_f$ can be designed together to ensure nominal MPC stability (Cannon and Kouvaritakis (2016)). In this work,
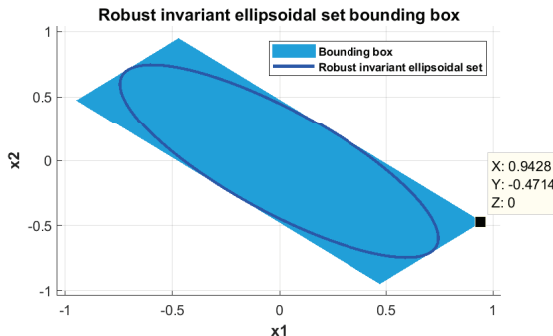
matrices Q and R have been chosen to be diagonal and positive definite. The penalty matrix, due to the DP task, has been chosen to penalize more heading variations about the target and less the remaining states, while R has been chosen to have small entries when compared to Q.

Set difference
Next, we describe how can we efficiently compute the set $\bar{\mathcal{U}}$. Consider the H-representation (half-space representation) of two polytopes V and W, defined as

$$V = \{ x \in R^n \mid A_v x \leq b_v \},$$
$$W = \{ x \in R^n \mid A_w x \leq b_w \}.$$

Then, one efficient approach to compute the Pontryagin set difference of V and W is

$$V \ominus W = \{ x \in R^n \mid A_v x \leq b_v - \mathcal{H}(V, W) \}, \tag{11}$$

where the $\mathcal{H}$ operation is defined as

$$\mathcal{H}_i(V, W) = \max_{x \in W} [A_v]_i x$$

with $[A_v]_i$ being the ith row of the matrix A_v (Borrelli et al. (2017)).

To use this approach for computing $\bar{\mathcal{U}}$, first, we find the H-representation of sets $\mathcal{U}$ and $K\Omega$. It is straightforward to show that the original box input constraint imposed by thrusters can be written as

$$\mathcal{U} = \left\{ x \in R^3 \;\middle|\; \begin{bmatrix} I \\ -I \end{bmatrix} x \leq \begin{bmatrix} \tau_{max} \\ \tau_{max} \end{bmatrix} \right\}$$

Since Ω is an ellipsoidal invariant set, we consider the bounding box of Ω, the set Ω_b as follows

$$\Omega_b = \left\{ x \in R^6 \;\middle|\; \begin{bmatrix} J^T \\ -J^T \end{bmatrix} x \leq \begin{bmatrix} \Lambda_b \\ \Lambda_b \end{bmatrix} \right\}$$

Here, J is the matrix of normalized eigenvectors coming from the decomposition $P = J\Lambda J^T$, Λ is a diagonal matrix assumed to contain the eigenvalues of P, and Λ_b is a diagonal matrix containing the square root of the ratio between $\frac{\mu w_{max}^2}{\lambda}$ and the eigenvalues appearing in the diagonal entries of Λ. For instance, using the decomposition in the ellipsoidal set $x^T \Omega x \leq 1$, with

$$\Omega = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

would produce a bounding box as in Figure 1.

Robust invariant ellipsoidal set bounding box

Legend: Bounding box; Robust invariant ellipsoidal set

X: 0.9428
Y: -0.4714
Z: 0

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Figure 1: Bounding box enclosing the set $x^T \Omega x \leq 1$

It follows from (11) that the reduced input constraint $\bar{\mathcal{U}}$ can be calculated as:

$$\bar{\mathcal{U}} = \left\{ x \in R^3 \;\middle|\; \begin{bmatrix} I \\ -I \end{bmatrix} x \leq \begin{bmatrix} \tau_{max} \\ \tau_{max} \end{bmatrix} - \begin{bmatrix} \max(KV(\Omega_b)) \\ |\min(KV(\Omega_b))| \end{bmatrix} \right\},$$

where $V(\Omega_b)$ is the vertex enumeration operation of Ω_b (Borrelli et al. (2017)). Implementation of this procedure will typically result in the sets as shown in Figure 2.
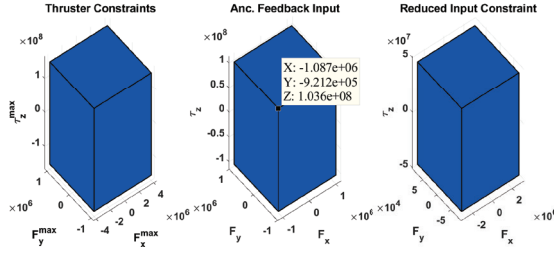
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Figure 2: Left-hand set $(\mathcal{U})$, middle set $(K\Omega_b)$ and righthand set $\bar{\mathcal{U}} = \mathcal{U} \ominus K\Omega_b$
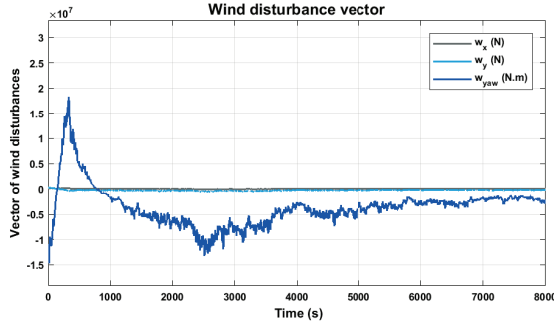
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Figure 3: Wind disturbance generated during simulation
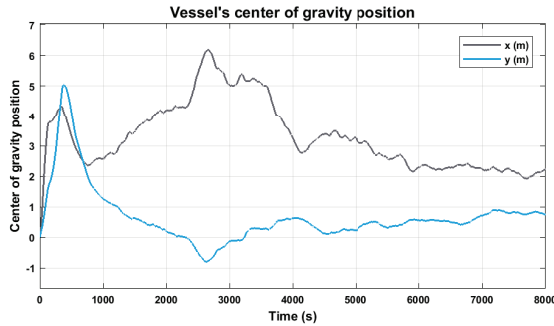
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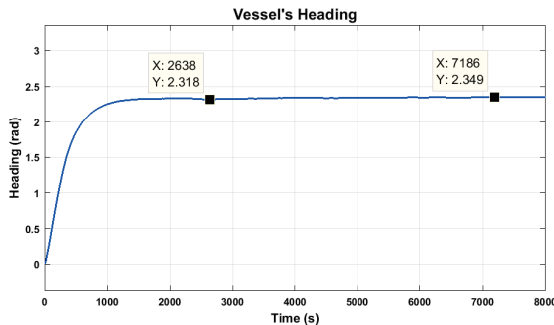Figure 4: Vessel's center of gravity position

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Figure 5: Heading angle of the vessel

The analytical approach above has yielded important benefits for the tube MPC design when comparing to some packages capable of performing set differences. In special, when the eigenvalues of the P matrix differ a lot in order of magnitude, these packages could generate empty sets, when the set difference operation should return a nonempty set. The adoption of this technique could, among other benefits, reduce the number of empty set differences calculated incorrectly.

Simulation results
The method presented will be evaluated by numerical simulation of the technique applied to a non-linear vessel model described by (1) and (2). Simulations done in Matlab/Simulink were performed in a real sized vessel having a length of 294 meter, a beam of 37.9 meter, and a draft of 8 meter. We assume that measurements of all states including position and velocity vectors are available.

For simulation purposes, it was required for the vessel to change its heading angle from 0° to 135° while its center of gravity position should be kept on the same place during the rotation. The limits imposed on the input τ generated by the thrusters can be approximated by the box constraint

$$\begin{bmatrix} -4.10^6 \\ -1.10^6 \\ -1.5.10^8 \end{bmatrix} \leq \begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} \leq \begin{bmatrix} 4.10^6 \\ 1.10^6 \\ 1.5.10^8 \end{bmatrix}.$$

External disturbances consisting of wind in different directions entering the system can be seen in Figure 3. The wind disturbance vector was chosen so that its norm is smaller than 2.10^7, but has three different magnitude components entering the nonlinear vessel. Wind disturbance components w_x and w_y have an order of magnitude of 10^5 while w_{yaw} has an order of magnitude of 10^7.

The vessel's center of gravity position and heading under these disturbances are shown on Figure 4 and Figure 5, respectively. These variables have final values close to the desired target values. The vessel's center of gravity position, for instance, moves according to the external disturbance although is maintained relatively close to the goal during the whole simulation as desired. Heading values, which seem to be constant after reaching steady state, is varying according to the external disturbance as well, albeit in a very contained manner, as seen on two different instants on

Figure 4. This behaviour reflects the ancillary feedback gain solution found by the LMI which penalizes more deviations in heading and yaw when compared to other states.
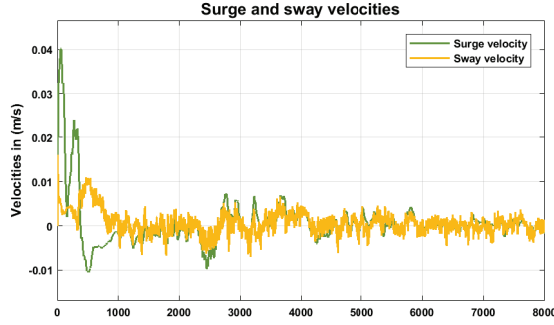

Surge and sway velocities

Yaw velocity

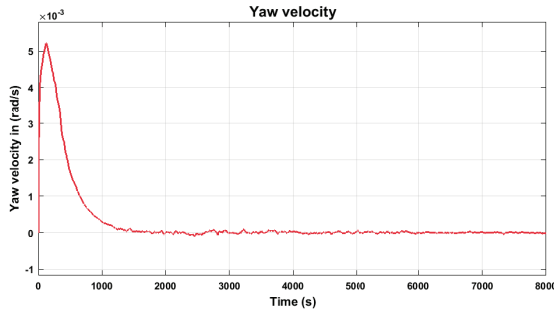Total and nominal forces on x and y directions

Total and nominal inputs on z direction

The settling time (around 1900s on Figure 5) is in part defined by the choice of the weights in the matrices Q, R, so as on the parameter λ. In general the higher the weights in Q and λ and the "cheaper the control", the faster (up to a limit) the system settles. It is important to note, on the other hand, that the faster the system is required to settle, the farther away from the assumption of low speed operation (allowing the linearization procedure) the system will be, producing therefore a trade-off.

On Figure 6 surge and sway velocities are depicted while on Figure 7 the yaw velocity is shown. The assumption of low speed DP has been a reasonable one as can be seen by the graphs.

The total input τ and its nominal contributions $\bar{\tau}$ calculated by the nominal MPC can be seen in Figure 8 for the x and y directions, and in Figure 9 for the z direction. It can be seen that τ respects the imposed limits on the three directions during the whole simulation time. At this point, some remarks are relevant. First, note that the ancillary feedback in Figure 8 and in Figure 9 is the difference between total and nominal inputs. In Figure 8, the nominal forces $\bar{F}_x$ and $\bar{F}_y$ are almost zero, leaving the total forces F_x and F_y to be constructed solely by its ancillary feedback counterparts. On the other hand, M_z in Figure 9, is in its majority constructed by $\bar{M}_z$. This happens due to the structure obtained from the linearized dynamics which describes that a turn in the z direction has no effect on the x and y directions. Thus, the act of turning the vessel will be reflected only on $\bar{M}_z$ whereas the rejection of external disturbances and mismatches between the nonlinear model and the linearized version will reflect upon the ancillary feedback portions in the x, y and z directions.

Conclusion and future direction
In this paper, a robust MPC technique for dynamic positioning of marine vessels has been proposed. A nonlinear vessel model is linearized and used in the tube-based MPC formulation. The designed controller is capable of performing the desired task under bounded external disturbances while respecting input constraints. Simulation results have shown that the controller can successfully drive and maintain the vessel close to the target point under disturbance.

There are some points which can be further strengthened in this type of approach, opening space to interesting future directions. First, under the developed formulation, it would be interesting to better understand the mentioned tradeoff and how to optimally tune the values of Q, R and λ, in order to extract the best vessel behavior. It would also be very useful to compare such method, under some metric, to other types of robust controllers in order to see how they would perform against each other. Finally, in order to relax the assumption of low speed application and the usage of system linearization, it would be interesting to develop a new formulation using a nonlinear tube MPC technique, as presented in Singh et al. (2017) for instance, in order to see how well such task would be performed.

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References

Allgöwer, F. and Zheng, A. (2012). Nonlinear Model Predictive Control, volume 26. Birkhäuser.

Apostol-Mates, R. and Barbu, A. (2016). Human errorthe main factor in marine accidents. Mircea cel Batran Naval Academy Scientific Bulletin, 19(2), 451–454.

Borrelli, F., Bemporad, A., and Morari, M. (2017). Predictive control for linear and hybrid systems. Cambridge University Press.

Caharija, W., Pettersen, K., Bibuli, M., Calado, P., Zereik, E., Braga, J., Gravdahl, J., Sørensen, A., Milovanovic, M., and Bruzzone, G. (2014). Integral line-of-sight guidance and control of underactuated marine vehicles. IEEE Transactions on Control Systems Technology, 24(5), 1–20.

Cannon, M. and Kouvaritakis, B. (2016). Model predictive control-classical, robust and stochastic.

Do, K.D. and Pan, J. (2009). Control of Ships and Underwater Vehicles: Design for Under-actuated and Nonlinear Marine Systems. Springer Science & Business Media.

Fossen, T.I. (2011). Handbook of Marine Craft Hydrodynamics and Motion Control. John Wiley & Sons.

Fossen, T.I. and Grovlen, A. (1998). Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping. IEEE transactions on control systems technology, 6(1), 121–128.

Fossen, T.I. and Strand, J.P. (2001). Nonlinear passive weather optimal positioning control (wopc) system for ships and rigs: experimental results. Automatica, 37(5), 701–715.

Johansen, T.A., Fossen, T.I., and Berge, S.P. (2004). Constrained nonlinear control allocation with singularity avoidance using sequential quadratic programming. IEEE Transactions on Control Systems Technology, 12(1), 211–216.

Limon, D., Alvarado, I., Alamo, T., and Camacho, E. (2010). Robust tube-based mpc for tracking of constrained linear systems with additive disturbances. Journal of Process Control, 20(3), 248–260.

Loria, A., Fossen, T.I., and Panteley, E. (2000). A separation principle for dynamic positioning of ships: Theoretical and experimental results. IEEE Transactions on Control Systems Technology, 8(2), 332–343.

Pettersen, K.Y. and Fossen, T.I. (2000). Underactuated dynamic positioning of a ship-experimental results. IEEE Transactions on Control Systems Technology, 8(5), 856–863.

Rawlings, J.B. and Mayne, D.Q. (2009). Model Predictive Control: Theory and Design. Nob Hill Pub. Madison, Wisconsin.

Singh, S., Majumdar, A., Slotine, J.J., and Pavone, M. (2017). Robust online motion planning via contraction theory and convex optimization. In 2017 IEEE International Conference on Robotics and Automation (ICRA), 5883–5890. IEEE.

Sørensen, A.J. (2011). A survey of dynamic positioning control systems. Annual reviews in control, 35(1), 123– 136.

Tannuri, E., Agostinho, A., Morishita, H., and Moratelli Jr, L. (2010). Dynamic positioning systems: An experimental analysis of sliding mode control. Control engineering practice, 18(10), 1121–1132.

Veksler, A., Johansen, T.A., Borrelli, F., and Realfsen, B. (2016). Dynamic positioning with model predictive control. IEEE Transactions on Control Systems Technology, 24(4), 1340–1353.

Yu, S., Maier, C., Chen, H., and Allg¨ower, F. (2013). Tube mpc scheme based on robust control invariant set with application to lipschitz nonlinear systems. Systems & Control Letters, 62(2), 194–200.