Turbo Machines

Ramesh K

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1 Introduction to Turbo Machines

1.1 Introduction

Turbomachine is important class of fluid machine, which has as its characteristic the ability to transfer energy continuously between a dynamic fluid and a mechanical element rotating around a fixed axis. The definition of turbo-machine as given by different authors

- The Turbo-machine is a device in which the energy exchange is accomplished by hydrodynamic forces arising between moving fluid and the rotating and stationary elements of the machine - **Daily**.

- A Turbo-machine is characterized by dynamic energy exchange between one or several rotating elements and a rapidly moving fluid - **Wislicenus**.

- A Turbo-machine is characterized by dynamic action between a fluid and one or more rotating elements - **Binder**.

As we can see that the study of turbo-machine comprises of fluid motion in relative to a moving mechanical elements. When ever some body/fluid is in motion either forces act on them or the motion is the result of forces being acted upon the body/fluid. The resultant of forces acting result in energy transfer to/from the fluid to the machine. Hence the essential prerequisite of this course is knowledge/awareness in

- Fluid Dynamics (Incompressible and Compressible fluid flows)

- Vector Algebra and Calculus

- Theory of Machines (Kinetics and Kinematics)

- Basic and Applied Thermodynamics

Some of the application areas where in we use turbo-machines

- Propulsion systems - Aircraft,Marine, Space(Liquid rockets) and Land propulsion systems

- Power generation - Steam ,gas and hydraulic turbines

- Industrial pipe line and processing equipments such as gas, petroleum and water pumping plants

- Heart assist pumps, industrial compressors and refrigeration plants

So what do we do in this course?
We will try to

- Unleash the basic terminology of the turbo-machine.

- Understand the basic working of a turbo-machine irrespective of its use.

- Understand the performance parameters of its effects on a turbo-machine.

2   Classification of Turbo Machines

There are many categories in which a turbo-machine is classified

2.1   Direction of Energy Conversion

- **Turbine**- It is a machine where the energy in the fluid in what ever the form (Kinetic Energy, Potential Energy or Internal Energy) is converted to mechanical energy by rotating a element(rotor) of the machine.
  The energy is being extracted from the fluid in the form of shaft power by decreasing its enthalpy, hence they are also called power generating machines.

- **Pump** - In these kind of machines, mechanical energy in the rotating member is transferred to the fluid raising its energy (enthalpy) in the form of (KE, PE or IE). Since energy is gained by the fluid they are called power consuming machines.
  Fans, blowers, compressors etc also fall in this category.

2.2   Components of Turbo-machine

- Casing - Outer enclosing of the machine used to stop spill over of the fluid and also serves as guide.

- Runner - Also called "Rotor", where in actual energy transfer from the fluid or to the fluid takes place. It changes the stagnation enthalpy,Kinetic energy and stagnation pressure field of the fluid.

- Blades - Also called as "Vanes", which are use to extract or take energy from the fluid.

- Draft tube - A pipe at the exit of the machine used to maintain the energy loss and continuity at the exit of the machine.

If the turbo-machine doesn't have shroud or annulus wall near the tip, then the machine is called extended. Examples of such machines are aircraft and ship propellers, wind turbines, fans. Other way around if they have shroud or enclosure then they are called enclosed machines.
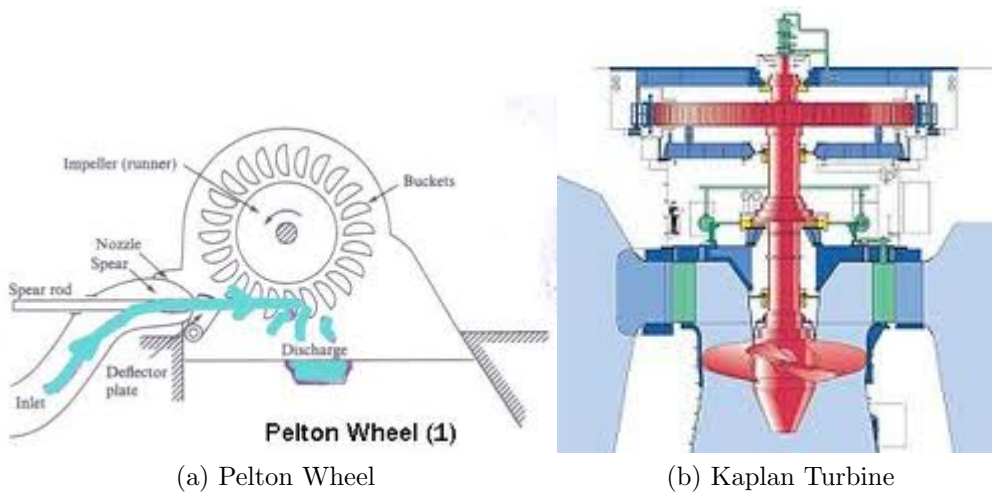
(a) Pelton Wheel   (b) Kaplan Turbine

Figure 1: Turbo machines, Images courtesy World Wide Web

2.3   Principle of Operation

- **Positive Displacement Machine** - As the name suggests, the functioning of these machines depends on the physical change in the volume of the fluid within the machine.

  The name positive displacement is given as the fluid in the machine forms a closed system and the boundary of the system is physically displaced as in piston cylinder arrangement as shown in the Piston cylinder Figure (2a), Gear pump Figure (2b), screw pump Figure (2c). Since the motion of the piston is reciprocating (to and

(a) Reciprocating pump/machine   (b) Gear Pump   (c) Screw Pump
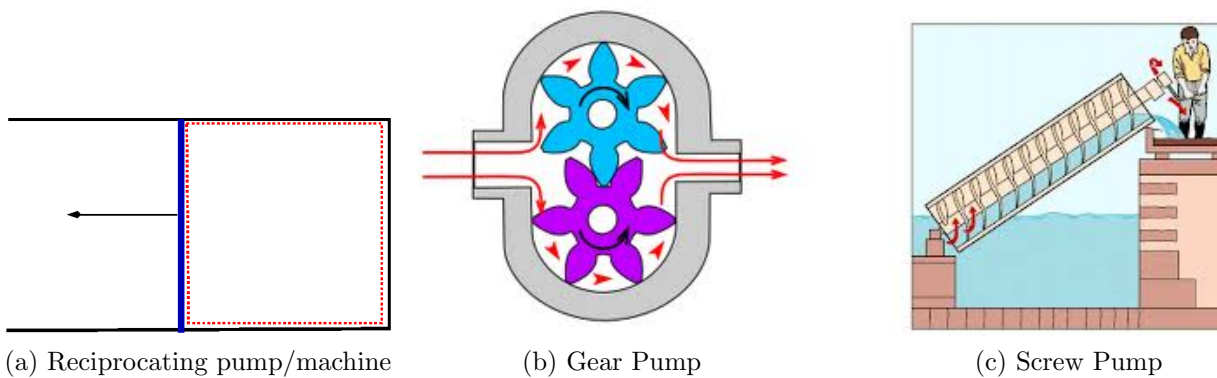
Figure 2: Positive Displacement machines, Images b &c courtesy World Wide Web

fro linear motion), these machines are also known as reciprocating engines. If the machine used is for generating power then it is called as Internal combustion engine other wise if used for compression of gas it is called reciprocating pump. Gear pumps and screw pumps also fall in the category of positive displacement pumps though there is no reciprocating action involved.

- **Roto Dynamic machine** - In this kind of machine both thermodynamic and dynamic interaction between the flowing fluid and the rotating element takes place and involves energy transfer with change in both pressure and momentum. These machines are distinguished from positive displacement machines in requiring that

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there exists a relative motion between the flowing fluid and rotating element. The rotating element usually consists of blades/vanes which are used to transfer the energy and also aids the fluid to flow in a particular direction.

2.4 Direction of flow

- Radial - Fluid is in the direction perpendicular to the axis of the rotating element and leaves radially.

- Axial - Flow enters parallel to the axis of the rotating element and leaves axially.

- Mixed - Fluid enters radially or axially and leaves axially or radially.

2.5 Types of Fluids used

- Gas or Vapour - Air,Argon,Neon,Helium,Freon,Steam,Hidrocarbon gas etc,.

- Liquid - Water, Cryogenic liquids ($O_2, H_2, F_2, NH_2 etc, .$), Hydrocarbon Fuels, Slurry (Two Phase liquid/solid mixture),Blood, Potassium,Mercury etc

2.6   Examples where Turbo-machines are used

| Field Name	Turbine	Pump or Compressor	Application area
Aerospace Vehicle application	Gas Turbines	Compressors, Pumps,Propellers	- Power and propulsion of aircrafts - Helicopters, UAV,V/STOL aircrafts Missiles - Liquid rocket engines
Marine applications	Gas Turbines and Turbines	Compressors, Pumps,Propellers	- Power and propulsion for submarines - Hydrofoil boats, Naval surface ships, Hovercrafts - Underwater vehicles
Land Vehicle application	Gas Turbines and Turbines	Centrifugal compressor and radial turbine	Trucks,cars and high speed trains
Energy application	Gas,Steam and wind Turbines	Compressors,Pumps	- Hydraulic turbines in hydro-power plants - Gas turbine power plants
Industrial applications		Compressors, Pumps	- Compressors - Transport of Petroleum and other processing applications - Pumps - Fire fighting, water purification, pumping plants - Refrigeration - High speed miniature turbo expanders
Miscellaneous		Pumps	Heart assist devices(artificial heart pump), automotive torque converters, swimming pools, hydraulic brakes

Looking at wide area of application of turbo-machines, any small gain in performance and efficiency of a turbo-machine would impact the economy globally.

2.7   Nature of flow field

It is complex 3D flow field that exist in any turbo-machine. A typical example of flow field that exist is as shown in figure

2.7.1   Fluid Used

By the nature of the fluid used they can be classified as compressible and incompressible
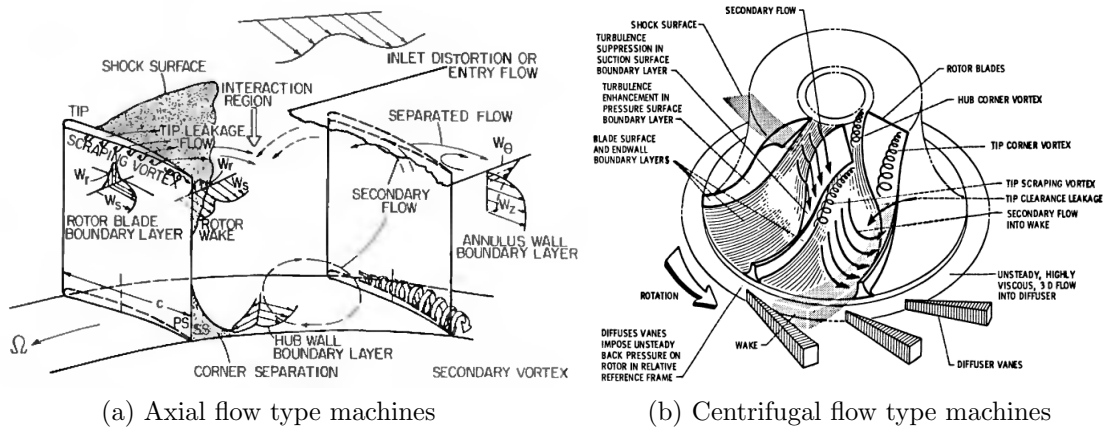
(a) Axial flow type machines

(b) Centrifugal flow type machines

Figure 3: 3D Flow Field in any turbo-machine-Image courtesy - Budugur Lakshminarayana(Fluid dynamics and heat transfer in turomachinery)

- Compressible - If the changes in the pressure caused by the fluid motion results in changes in density of the fluid then such type of fluid is known to be a compressible fluid. In a given turbo-machine since pressure changes are inevitable and if accompanied by density changes then it is under the category of compressible flow turbo-machine. Gas turbines, air compressors etc fall in this category. The flow in compressible machines may be further classified into
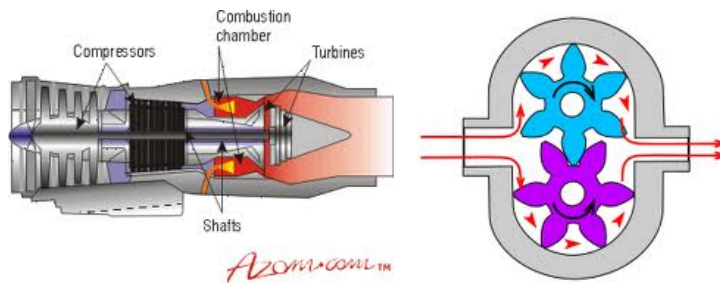
(a) Compressible Type Turbo-machine

(b) Incompressible type Turbo-machine

Figure 4: Turbo machines, Images courtesy World Wide Web

- Subsonic - $M \leq 0.7$
- Transonic -$0.7 \leq M \leq 1.0$
- Supersonic -$M \geq 1.0$

- Incompressible -If the changes in the pressure doesn't affect much of the density changes then they are considered to be incompressible flow turbo-machines. Steam turbine and all hydraulic turbines fall in this category.

2.7.2 Kinds of flow

- Steady - If the fluid properties doesn't vary in time.
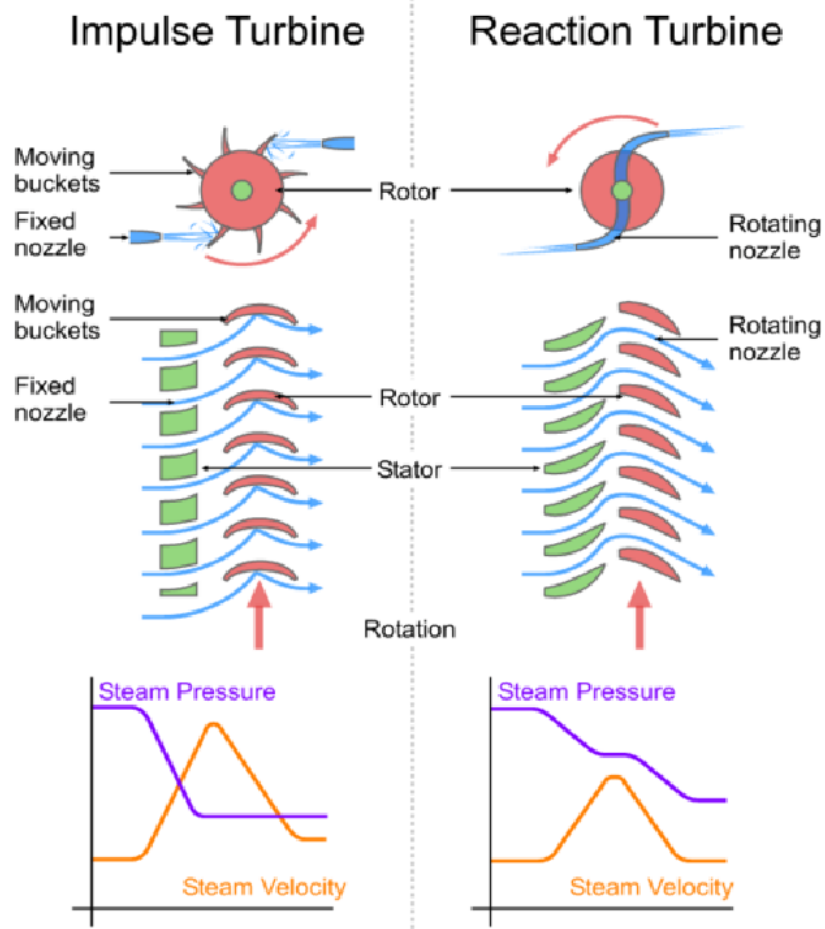- Unsteady- Fluid properties vary in time.

Figure 5: Turbo machines- Impulse and Reaction types, Images courtesy World Wide Web

2.8 Type of forces acting

- Impulse type - Static Pressure change is zero in the rotor.

- Reaction type - Static Pressure drop occurs in the rotor.

2.9 Comparison between Positive displacement machine and Turbo maching

	Positive Displacement Machine	Turbo Machine
Action:-	Involves change in volume or a displacement	No Positive confinement of fluid at any point in the system
Operation	- Reciprocating motion and unsteady flow - Fluid gets trapped in the machine when stopped and remains in that state - Rotary type with steady state motion is possible	- Purely rotory motion and steady flow - Fluid does not get trapped in the machine when stopped - Unsteady state is also possible during starting and stopping
Mechanical Features	- Low speed and complex mechanical design - Heavy per unit weight - Employ valves which open only partly - Requires heavy foundations because of vibration problems	- High rotational speeds and simple design - Light in weight per unit of power output - No valve to operate
Efficiency of Conversion	- Positive Confinement of fluid - Nearly Static Energy transfer - Higher efficiency of energy conversion	- No Positive Confinement of fluid - Dynamic process and high speed flow - Relative lower efficiency due to dynamic compression process
Volumetric efficiency	- Involves opening and closing valves - Less volumetric efficiency	- Doesn't involve opening and closing valves - High volumetric efficiency - High fluid handling capacity per kg wt of the machine
Fluid Phase change and surging	- Non Smooth flow operation - Surging or Pulsation	- Smooth flow operation - No Surging

3 Basics of Fluid Mechanics and Thermodynamics

It can be viewed that this subject of turbo-machinery is truly interdisciplinary as depicted in the following picture. In this section the basic governing laws of fluid mechanics and thermodynamics
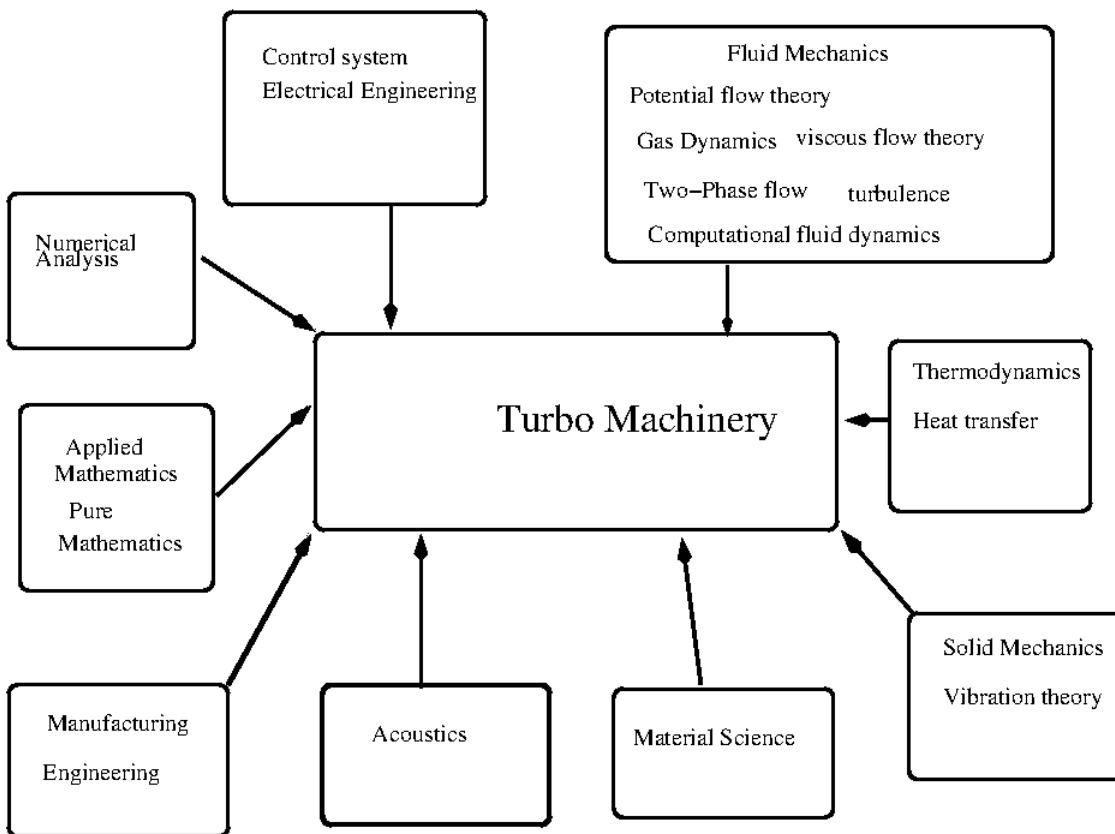

Figure 6: Inter diciplenary nature of the subject Turbomachinery

as applied to a turbo-machine is explained. Since the governing equations are Navier-Stokes equations and they are non-linear in nature, the complete 3D solution to the equations are out of the scope of this introductory course. A general form of the governing equations are explained in this section and further reductions to 1D cases are followed for each case.

Let us consider a turbo-machine enclosed in a control volume (Open system in thermodynamic sense) of volume Ω as shown in Figure 7, let "dS" be elemental surface area with unit outward normal to the surface $\hat{n}$. Let $\vec{V}$ be the velocity with which the fluid is coming out of the surface element "dS". Let $\dot{m}_{in}$ & $\dot{m}_{out}$, be the mass flow rates in and out of the control volume, also $\dot{Q}_{in}$ & $\dot{Q}_{out}$ be the rate at which heat is transferred into or out of the control volume.

The properties which we are interested in analyzing a turbo-machine vary both in space and time (i.e, as the fluid moves through the turbo-machine). If we denote $\phi(x, y, z, t)$ as the property
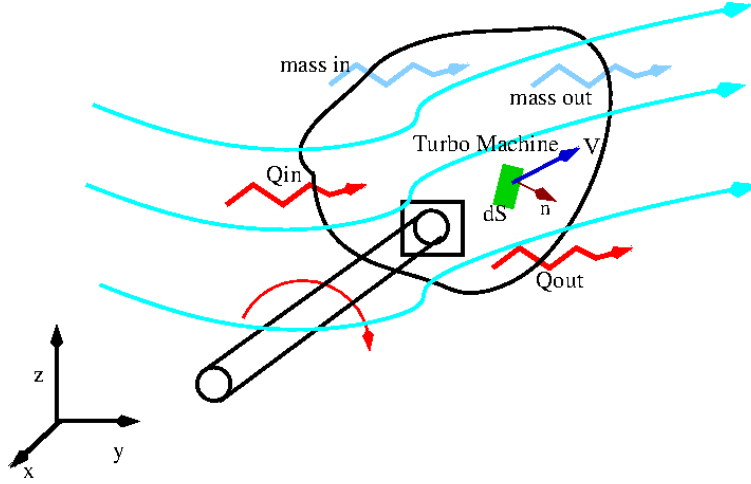
Figure 7: Control volume enclosing a turbo-machine

which is varying with both Cartesian spatial coordinates (x, y, z) and "t", then

$$d\phi = \frac{\partial \phi}{\partial t}dt + \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz \tag{3.1}$$

$$d\phi = \frac{\partial \phi}{\partial t}dt + \nabla\phi \cdot d\vec{r} \tag{3.2}$$

$$\nabla = \frac{\partial()}{\partial x}\hat{\imath} + \frac{\partial()}{\partial y}\hat{\jmath} + \frac{\partial()}{\partial z}\hat{k} \tag{3.3}$$

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \tag{3.4}$$

$$d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k} \tag{3.5}$$

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \nabla\phi \cdot \vec{V} \tag{3.6}$$

where equation (3.3) represents the gradient operator being operated on any property. The property ϕ can be a scalar, vector or a tensor. Total rate at which the property changes is given in equation (3.6), which has two parts the fist part indicating the local rate at which the property is changing with respective to time and the second term represents the rate at which the property is changing from one point to other inside the control volume, also called as convective part.

This gradient when applied to a scalar results in a vector, when applied to a vector results in a second order tensor and when applied to a second order tensor results in a higher order tensor. Let us consider the gradient operator applied to a velocity vector $\vec{V}(\vec{r}, t) = u(\vec{r}, t)\hat{\imath} + v(\vec{r}, t)\hat{\jmath} + w(\vec{r}, t)\hat{k}$, all the individual components of this vector are varying w.r.t space as well as time.

$$\nabla(\vec{V}) = \frac{\partial(\vec{V})}{\partial x}\hat{\imath} + \frac{\partial(\vec{V})}{\partial y}\hat{\jmath} + \frac{\partial(\vec{V})}{\partial z}\hat{k} \tag{3.7}$$

$$= \frac{\partial(u(\vec{r},t)\hat{\imath} + v(\vec{r},t)\hat{\jmath} + w(\vec{r},t)\hat{k})}{\partial x}\hat{\imath} \quad + \quad \frac{\partial(u(\vec{r},t)\hat{\imath} + v(\vec{r},t)\hat{\jmath} + w(\vec{r},t)\hat{k})}{\partial y}\hat{\jmath}$$

$$+ \quad \frac{\partial(u(\vec{r},t)\hat{\imath} + v(\vec{r},t)\hat{\jmath} + w(\vec{r},t)\hat{k})}{\partial z}\hat{k}$$

$$= \left(\frac{\partial u}{\partial x}\hat{\imath}\hat{\imath} + \frac{\partial u}{\partial y}\hat{\imath}\hat{\jmath} + \frac{\partial u}{\partial z}\hat{\imath}\hat{k}\right) \quad + \quad \left(\frac{\partial v}{\partial x}\hat{\jmath}\hat{\imath} + \frac{\partial v}{\partial y}\hat{\jmath}\hat{\jmath} + \frac{\partial v}{\partial z}\hat{\jmath}\hat{k}\right) \tag{3.8}$$

$$+ \quad \left(\frac{\partial w}{\partial x}\hat{k}\hat{\imath} + \frac{\partial w}{\partial y}\hat{k}\hat{\jmath} + \frac{\partial v}{\partial z}\hat{k}\hat{k}\right)$$

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equation(3.7) contains "9" components as shown in equation(3.8), and each component has 2 directions associated with it. Thus it can be observed that gradient operator increases the order by one. The other operator of importance is the divergence operator given by $\nabla() \cdot (\vec{A})$, it is the dot product or scalar product of the gradient operator with any vector or a tensor. As it is known that the dot product of two vectors results in a scalar since the $\nabla()$ is a vector quantity, when taken a dot product with any vector or a tensor will result in a scalar or a vector reducing the order.

$$\nabla() \cdot \vec{V} = \left(\frac{\partial()}{\partial x}\hat{i} + \frac{\partial()}{\partial y}\hat{j} + \frac{\partial()}{\partial z}\hat{k} \right) \cdot \left(u\hat{i} + v\hat{j} + w\hat{k} \right) \tag{3.9}$$

$$= \left(\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} + \frac{\partial(w)}{\partial z} \right) \tag{3.10}$$

The cross product of the gradient operator applied to the velocity vector results in a vector which signifies the linear and angular strain rates of the fluid element as it moves through the fluid

$$\nabla() \times \vec{V} = \left(\frac{\partial()}{\partial x}\hat{i} + \frac{\partial()}{\partial y}\hat{j} + \frac{\partial()}{\partial z}\hat{k} \right) \times \left(u\hat{i} + v\hat{j} + w\hat{k} \right) \tag{3.11}$$

$$= \left(\frac{\partial(w)}{\partial y} - \frac{\partial(v)}{\partial z} \right)\hat{i} - \left(\frac{\partial(w)}{\partial x} - \frac{\partial(u)}{\partial z} \right)\hat{j} + \left(\frac{\partial(u)}{\partial y} - \frac{\partial(v)}{\partial x} \right)\hat{k} \tag{3.12}$$

Equation 3.12, gives the rotation of the fluid element as it traverses through the control volume. The Gauss-Divergence theorem gives the relation between the volume integral of a divergence and the surface integral of the vector as given by

$$\int_\Omega \left(\nabla \cdot \vec{A} \right) d\Omega = \oint_S \vec{A} \cdot \hat{n} dS \tag{3.13}$$

This equation (3.13) represents the divergence of the vector quantity inside the control volume to the flux of the quantity passing through the surface.

3.1 Conservation of Mass or Continuity equation

"Mass is neither created nor destroyed", or in other words,"The sum of rate of change of mass inside the control volume and the net mass flux from the control surface is zero".

$$\frac{d}{dt}\int_\Omega \rho d\Omega + \oint_S \rho\vec{V} \cdot \hat{n} dS = 0 \tag{3.14}$$

$$\frac{d}{dt}\int_\Omega \rho d\Omega + \int_\Omega \nabla \cdot \left(\rho\vec{V} \right) d\Omega = 0 \tag{3.15}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_i}{\partial x_i} = 0, i = 1, 2, 3 \tag{3.16}$$

Equation (3.14) represents the conservation of mass as applied to a control volume (open system), using Gauss-Divergence theorem, converting the second term in (3.14), surface integral to volume integral results in eq (3.16) which is valid at any given point inside the control volume including the boundaries. The first term in the above equations is the time rate of change of mass in sided the control volume or at a point and the second term the net mass flux (mass flux coming in and mass flux going out) crossing the boundaries. For 1D case with constant control volume equation (3.14) reduces to

$$\frac{d\rho}{dt}\Omega + \sum_i \left(\rho A(v_1 \cdot \hat{i}) \right)_i = 0 \tag{3.17}$$

$$\frac{d\rho}{dt}\Omega - \sum_{inlet} \left(\rho A v_1 \right)_{inlet} + \sum_{outlet} \left(\rho A v_1 \right)_{outlet} = 0 \tag{3.18}$$

In the case of steady flow the first term in eq (3.17) is zero, meaning that the the "mass flux entering the control volume is equal to mass flux leaving the control volume". The equation for continuity or mass conservation for 1D stead flow is given by eq (3.18)

$$\sum_i \left(\rho A(v_1 \cdot \hat{\imath})\right)_i = 0 \tag{3.19}$$

$$-\sum_{inlet} (\rho A v_1)_{inlet} + \sum_{outlet} (\rho A v_1)_{outlet} = 0 \tag{3.20}$$

$$\sum_{inlet} (\rho A v_1)_{inlet} = \sum_{outlet} (\rho A v_1)_{outlet} \tag{3.21}$$

3.2 Conservation of Linear Momentum

Newtons second law of motion says, "The rate of change of linear momentum is equal to the net force acting on a body". From engineering mechanics or physics point of view for particle bodies, the Newtons second law is given by

$$\frac{d(m\vec{\mathbf{V}})}{dt} = \sum_i \vec{F_i} \tag{3.22}$$

where the summation indicates all the forces acting on the body and the right hand side indicating the time rate of change of linear momentum. In the integral form as applied to a control volume the equation (3.22) takes the form as given in equation (3.23)

$$\frac{d}{dt}\int_\Omega \rho\vec{V}\ d\Omega + \int_{\partial\Omega} (\rho\vec{V})\vec{V}\cdot\hat{n}\ dS = -\int_{\partial\Omega} P\hat{n}\ dS + \int_\Omega \rho\vec{g}d\Omega + \int_{\partial\Omega} \tau.\hat{n}dS \tag{3.23}$$

$$\frac{d}{dt}\int_\Omega \rho\vec{V}\ d\Omega + \int_\Omega \nabla\cdot\left((\rho\vec{V})\vec{V}\right) d\Omega = -\int_\Omega \nabla.(P\hat{n})d\Omega + \int_\Omega \rho\vec{g}d\Omega + \int_\Omega \nabla\cdot\tau d\Omega \tag{3.24}$$

In the equation (3.23) the surface integrals are converted to volume integrals by using Gauss-Divergence theorem. The forces considered in the present context are pressure forces (surface force 1st term), gravity force (body force 2nd term) and viscous forces (surface force 3rd term). If the surface is further divided into discrete elemental surfaces then the equation (3.23) can be rewritten as given in equation (3.25)

$$\frac{d}{dt}\int_\Omega \rho\vec{V}\ d\Omega + \sum_i (\rho\vec{V}S)(\vec{V}\cdot\hat{n})_i = -\sum_i (P\hat{n}S)_i + \rho\vec{g}\Omega + \sum_i (\tau.\hat{n}S)_i \tag{3.25}$$

$$\frac{d}{dt}\int_\Omega \rho\vec{V}\ d\Omega + \sum_i (\dot{m})(\vec{V}\cdot\hat{n})_i = -\sum_i (P\hat{n}S)_i + \rho\vec{g}\Omega + \sum_i (\tau.\hat{n}S)_i \tag{3.26}$$

The negative sign in the 1st term on RHS indicating that the force due to the pressure is of compressive in nature. The second term on LHS is the momentum flux that is crossing the surface, the momentum that is carried away by the fluid from the control volume. The summation indicates the momentum flux that is entering the control volume and the also leaving the control volume. This aspect of inlet or exit of momentum form the control volume is taken care by the the $\left(\vec{V}\cdot\hat{n}\right)$ term in the above equation. If this term is positive $\left(\vec{V}\cdot\hat{n} > 0\right)$ then the flux is moving out and if its negative $\left(\vec{V}\cdot\hat{n} < 0\right)$ it is flowing in.

For a steady state, inviscid with negligible body forces, the momentum equation (3.26) reduces to

$$\sum_i (\dot{m})(\vec{V}\cdot\hat{n})_i = -\sum_i (P\hat{n}S)_i \tag{3.27}$$

which is the summation of momentum flux on the control volume equals the net pressure force acting on the control volume and the momentum variation inside the control volume is fixed.

3.3 Conservation of Angular Momentum

The equation for the conservation of angular momentum equation is obtained by taking the moment of the momentum by the position vector.

$$\vec{r} \times \frac{d(m\vec{\mathbf{V}})}{dt} = \vec{r} \times \sum_i \vec{F}_i \tag{3.28}$$

$$\frac{d(\vec{r} \times (m\vec{\mathbf{V}}))}{dt} = \sum_i \left(\vec{r} \times \vec{F} \right)_i \tag{3.29}$$

$$\frac{d(H)}{dt} = \sum_i T_i \tag{3.30}$$

From the control volume analysis of the linear momentum equations (3.23 & 3.24), by taking the moment w.r.t the position vector $\vec{r}$, results in the integral form of the angular momentum equation as applied to a control volume.

$$\frac{d}{dt} \int_\Omega \vec{r} \times \rho\vec{V} \, d\Omega + \int_S \vec{r} \times (\rho\vec{V})\vec{V} \cdot \hat{n} \, dS = -\int_S \vec{r} \times P\hat{n} \, dS + \int_\Omega \vec{r} \times \rho\vec{g}d\Omega + \int_S \vec{r} \times \tau.\hat{n}dS \tag{3.31}$$

$$\frac{d}{dt} \int_\Omega \vec{r} \times \rho\vec{V} \, d\Omega + \sum_i \left(\vec{r} \times (\rho\vec{V}) \left(\vec{V} \cdot \hat{n} \, S \right) \right)_i = -\sum_i (\vec{r} \times P\hat{n}S)_i + \vec{r} \times (\rho\vec{g}\Omega) + \sum_i (\vec{r} \times \tau.\hat{n}S)_i \tag{3.32}$$

for steady state cases with only pressure forces being acting on the control volume the equation (3.32) reduces to

$$\sum_i \left(\vec{r} \times (\rho\vec{V}) \left(\vec{V} \cdot \hat{n} \, S \right) \right)_i = -\sum_i (\vec{r} \times P\hat{n}S)_i \tag{3.33}$$

3.4 Conservation of Energy or First law of Thermodynamics

"Energy is neither created nor destroyed, but is converted from one form to another", valid when relativistic effects are not taken into consideration and no nuclear reactions occur in the process.

$$E_t = e + \frac{V^2}{2} + gz \tag{3.34}$$

$$\frac{d}{dt} \int_\Omega (\rho E_t)d\Omega + \oint_S (\rho E_t)(\vec{V} \cdot \hat{n}dS) = -\oint_S (P\vec{V} \cdot \hat{n}dS) + \oint_S \vec{q} \cdot \hat{n}dS + \oint_S (\tau \cdot \vec{V}) \cdot \hat{n}dS \tag{3.35}$$

In equation (3.35) is valid for a 3D arbitrary fixed control volume, the 1st is the rate of change of total energy inside control volume, 2nd is the energy flux crossing the boundary and also the work done by the pressure at the boundary, 3rd term represents the heat transfer by conduction at the boundaries and the last term represents the work done by viscous stress at the boundary. For steady state conditions the first term in the equation (3.35) becomes zero and the net energy balance is due to the energy flux transfer across the boundaries and the amount of heat and work transfer across the boundaries. If this equation (3.35), is rewritten for a 1D steady state case

$$\oint_S (\rho E_t + P)(\vec{V} \cdot \hat{n}dS) = \oint_S \vec{q} \cdot \hat{n}dS + \oint_S (\tau \cdot \vec{V}) \cdot \hat{n}dS \tag{3.36}$$

$$\sum_i (\rho E_t v_1 S + P v_1 S)_i - \sum_i (q_1 S)_i - \sum_i \tau_{ij} v_j S_i = 0, j = 1, 2, 3 \tag{3.37}$$

$$\sum_i (\rho E_t v_1) = \sum_i (q_1)_i - P v_1 + W \tag{3.38}$$

$$\sum_i (\rho E_t v_1) = \sum_i (q_1)_i - \sum_i W_i \tag{3.39}$$

$$\Delta U = \delta Q - \delta W \tag{3.40}$$

4 Static and Stagnation Properties

The properties (like pressure, temperature) that are measured by the instruments which move with the velocity of the fluid in motion are called static properties. If we consider an adiabatic system with no shaft work done on/by the system and zero potential then from the first law per unit mass applied to the control volume results in eq(4.5)

$$dq = dE_t + dw \tag{4.1}$$

$$dq = dE_t + dw_s \tag{4.2}$$

$$dE_t = 0 \Rightarrow E_{t2} - E_{t1} = 0 \tag{4.3}$$

$$E_t = h + \frac{V^2}{2} + gz \tag{4.4}$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \tag{4.5}$$

If we now assume that at any given point, if we hypothetically bring the fluid particle to rest i.e.,(V_2 is made zero) under the assumptions that, there is no transfer of heat and no work done, also further assumed that there are no irreversibilities that arise during this process (meaning to say that, the hypothetical process is isentropic) then we can deduce from eq(4.5) that

$$h_1 + \frac{V_1^2}{2} = h_2 \tag{4.6}$$

$$h = c_p T \Rightarrow c_p T_1 + \frac{V_1^2}{2} = c_p T_2 \tag{4.7}$$

$$T_1 + \frac{V_1^2}{2c_p} = T_2 \tag{4.8}$$

This state obtained is called stagnation property. Remember that this is an hypothetical process which may or may not occur in the real processes. Also this process is assumed to occur at all the points inside the flow field. Hence at every point in the flow field if we can measure a static property then corresponding to that point it is possible to obtain a corresponding stagnation property for all the measurable properties. Stagnation properties are also called total properties and are designated by the symbols subscripted with "o" or "t" as given P_o, T_t, P_t, T_o. It is the convenience that the use of "o" or "t" is used.

5 Application of Conservation Laws

5.1 Flow through nozzles and diffusers

Understanding flow through a nozzle and diffuser will be of great help in understanding the flow through turbo-machine. Let P_o be the atmospheric pressure that is acting on the nozzle as shown in figure 8, similarly the pressure can assume the pressure distribution on the diffuser also. The forces acting are the pressure, viscous forces and intertial forces. Body forces due to the gravity are neglected. Assuming steady flow through the nozzle or diffuser and applying the conservation equations of mass and momentum to the control volume abcd we obtain the
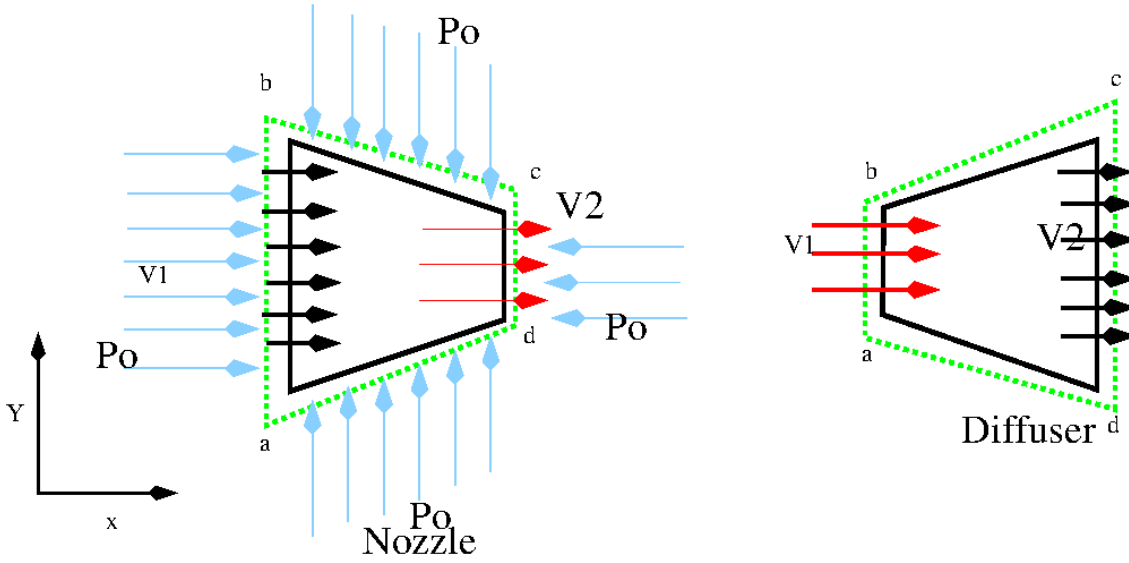
Figure 8: Control volume enclosing a Nozzle and a Diffuser

following equations. Mass Conservation equation as applied to nozzle or diffuser results in

$$\int_S \rho \vec{V} \cdot \hat{n} dS = 0 \tag{5.1}$$

$$\int_a^b \rho \vec{V} \cdot \hat{n} dS + \int_b^c \rho \vec{V} \cdot \hat{n} dS + \int_c^d \rho \vec{V} \cdot \hat{n} dS + \int_d^a \rho \vec{V} \cdot \hat{n} dS = 0 \tag{5.2}$$

$$\int_a^b \rho \vec{V} \cdot \hat{n} dS + \int_c^d \rho \vec{V} \cdot \hat{n} dS = 0 \tag{5.3}$$

$$-(\rho V_n S)_{inlet} + (\rho V_n S)_{outlet} = 0 \tag{5.4}$$

$$(\rho V_n S)_{inlet} = (\rho V_n S)_{outlet} \tag{5.5}$$

$$\dot{m}_{inlet} = \dot{m}_{outlet}, \ Mass \ flow \ rate \tag{5.6}$$

$$Q_{inlet} = Q_{outlet}, \ Discharge\text{-} \ Volume \ flow \ rate \tag{5.7}$$

where $V_n = \vec{V} \cdot \hat{n}$ is the normal Velocity vector at the surface under consideration. From the geometry shown in Figure 8 it can be observed that $S_{ab} > S_{cd}$, further if ρ remains constant or there is no considerable change then the $V_1 < V_2$ for the nozzle and the case would be quite opposite in the case of diffuser. The above statements are valid if the flow is subsonic case.
The momentum conservation equation eq(3.23) as applied for the flow through nozzle with no body and viscous forces is given as

$$\int (\rho \vec{V})(\vec{V} \cdot \hat{n}) dS = -\int (P \hat{n} dS) + \vec{F} \tag{5.8}$$

$$\int_a^b (\rho \vec{V})(\vec{V} \cdot \hat{n}) dS + \int_b^c (\rho \vec{V})(\vec{V} \cdot \hat{n}) dS + \int_c^d (\rho \vec{V})(\vec{V} \cdot \hat{n}) dS + \int_d^a (\rho \vec{V})(\vec{V} \cdot \hat{n}) dS =$$

$$-\int_a^b (P \hat{n} dS) - \int_b^c (P \hat{n} dS) - \int_c^d (P \hat{n} dS) - \int_d^a (P \hat{n} dS) + \vec{F} \tag{5.9}$$

$$\int_a^b (\rho \vec{V})(\vec{V} \cdot \hat{n}) dS + \int_c^d (\rho \vec{V})(\vec{V} \cdot \hat{n}) dS = -\int_a^b (P \hat{n} dS) - \int_c^d (P \hat{n} dS) + \vec{F} \tag{5.10}$$

The Contribution from the atmospheric pressure and the momentum exchange gets cancelled out on the surfaces b-c and d-a of the nozzle as can be seen from eq (5.9) because the pressure forces are equal and in opposite in direction and the velocity through the surfaces are zero.

Though the pressure on the surfaces a-b and c-d is equal to the atmospheric, since the area is not the same, force exists because of this difference. The $\vec{F}$ in the above equation eq(5.9) can be cosidered as the one which results in holding the nozzle or the diffuser. Equation (5.9) can be rewritten in component form as

$$-(\rho V_n S_{ab})u_{ab} + (\rho V_n S_{cd})u_{cd} = P(S_{ab} - S_{cd}) + F_x \qquad (5.11)$$

$$-(\rho V_n S_{ab})v + (\rho v)(V_n)S_{cd} = F_y \qquad (5.12)$$

$$-\dot{m}_{ab}u_{ab} + \dot{m}_{cd}u_{cd} = P(S_{ab} - S_{cd}) + F_x \qquad (5.13)$$

The v-component of the velocity is zero the force in y direction is zero. This kind of flow can be seen in the case of fluid flowing through the blades in a turbine or compressors or pumps.

5.2   Flow on Blade passages with out blade motion

Consider a case where in the fluid flows on a blade passage. The following assumptions are considered in this case,

- Steady flow

- Fluid is inviscid and incompressible

- No Body forces

- Inlet and exit areas are equal

The force that exist on the blade by mere deflection of the fluid is to be evaluated. Though Pressure forces are acting on the Control volume it can be proved that they gets cancelled out. Applying the mass and momentum equations to the control volume as shown in Figure 9.
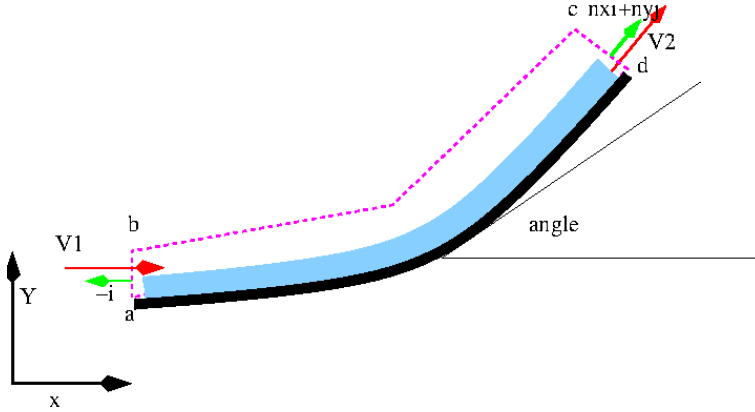

Figure 9: Control volume enclosing a Blade

$$\int_S \rho \vec{V} \cdot \hat{n} dS = 0 \qquad (5.14)$$

$$\int_a^b \rho \vec{V} \cdot \hat{n} dS + \int_b^c \rho \vec{V} \cdot \hat{n} dS + \int_c^d \rho \vec{V} \cdot \hat{n} dS + \int_d^a \rho \vec{V} \cdot \hat{n} dS = 0 \qquad (5.15)$$

$$\int_a^b \rho \vec{V} \cdot \hat{n} dS + \int_c^d \rho \vec{V} \cdot \hat{n} dS = 0 \qquad (5.16)$$

$$-(\rho V_n S)_{a-b} + (\rho V_n S)_{c-d} = 0 \qquad (5.17)$$

$\rho_{a-b} = \rho_{c-d}$ is because of incompressible, $S_{a-b} = S_{c-d}$ is because the area is constant, these two conditions results $V_n|_{a-b} = V_n|_{c-d}$, which means the normal velocity remains constant

across the area. The normals at the sections a-b and c-d are given as $-\hat{i}$ & $n_x\hat{i} + n_y\hat{j}$, where $n_x = cos(\theta)$ & $n_y = sin(\theta)$. The velocities at inlet and exit are $V_1 = u\hat{i}$ & $V_2 = u\hat{i} + v\hat{j}$ where $u = |\vec{V}|cos(\theta)$ and $v = |\vec{V}|sin(\theta)$. Application of Momentum equation for the above conditions results in

$$\int (\rho\vec{V})(\vec{V}\cdot\hat{n})dS = -\int (P\hat{n}dS) + \vec{F} \qquad (5.18)$$

$$\int_a^b (\rho\vec{V})(\vec{V}\cdot\hat{n})dS + \int_b^c (\rho\vec{V})(\vec{V}\cdot\hat{n})dS + \int_c^d (\rho\vec{V})(\vec{V}\cdot\hat{n})dS + \int_d^a (\rho\vec{V})(\vec{V}\cdot\hat{n})dS =$$

$$-\int_a^b (P\hat{n}dS) - \int_b^c (P\hat{n}dS) - \int_c^d (P\hat{n}dS) - \int_d^a (P\hat{n}dS) + \vec{F} \qquad (5.19)$$

$$\int_a^b (\rho\vec{V})(\vec{V}\cdot\hat{n})dS + \int_c^d (\rho\vec{V})(\vec{V}\cdot\hat{n})dS = \vec{F} \qquad (5.20)$$

It can be observed that the pressure force on the surface of the control volume becomes zero as the areas are all equal contrast to the flow through the nozzle. In the component form eq(5.19) is written as

$$-(\rho V_n S_{ab})u_{ab} + (\rho V_n S_{cd})u_{cd} = F_x \qquad (5.21)$$
$$-(\rho V_n S_{ab})v + (\rho v)(V_n)S_{cd} = F_y \qquad (5.22)$$
$$-\dot{m}_{ab}u_{ab} + \dot{m}_{cd}u_{cd} = F_x \qquad (5.23)$$
$$-\dot{m}_{ab}v_{ab} + \dot{m}_{cd}v_{cd} = F_y \qquad (5.24)$$
$$\dot{m}|\vec{V}|(cos(\theta) - 1) = F_x \qquad (5.25)$$
$$\dot{m}|\vec{V}|sin(\theta) = F_y \qquad (5.26)$$

It can be observed from equations 5.25 and 5.26 the force on the blade depends upon the turning angle "θ", and the mass flow rate of the fluid. This aspect is used in designing the blades of impulse turbine (Pelton Wheel). If the blade passage were to be closed similar to the nozzle we get additional force components from the variation of the pressure, if viscous effects and gravity are also considered then it can be seen that the force components gets added up to the RHS of eq(5.25) and eq(5.26).

5.3   Force on a moving blade

If we assume that the blade shown in the Figure 12, moves with a velocity **U**, then what would be the force that the blade conceives as the fluid flows through it. Its obvious that from the elementary physics that the fluid that is approaching or leaving the blade is not the absolute velocity $\vec{V}$. The corresponding changes that occur in the momentum and mass conservation are the changes in the velocity that is being considered in the governing equations. Mass conservation for a moving blade is given by

$$\int_S \rho\vec{V_r}\cdot\hat{n}dS = 0 \qquad (5.27)$$

$$\int_a^b \rho\vec{V_r}\cdot\hat{n}dS + \int_b^c \rho\vec{V_r}\cdot\hat{n}dS + \int_c^d \rho\vec{V_r}\cdot\hat{n}dS + \int_d^a \rho\vec{V_r}\cdot\hat{n}dS = 0 \qquad (5.28)$$

$$\int_a^b \rho\vec{V_r}\cdot\hat{n}dS + \int_c^d \rho\vec{V_r}\cdot\hat{n}dS = 0 \qquad (5.29)$$

$$-(\rho V_{r_n}S)_{a-b} + (\rho V_{r_n}S)_{c-d} = 0 \qquad (5.30)$$

$$(\rho V_{r_n}S)_{a-b} = (\rho V_{r_n}S)_{c-d} \qquad (5.31)$$

The relative velocities that occur at inlet sections (a-b) and the exit sections (c-d) are obtained by the vector addition of absolute and the velocity of the blade at the respective sections. Equation
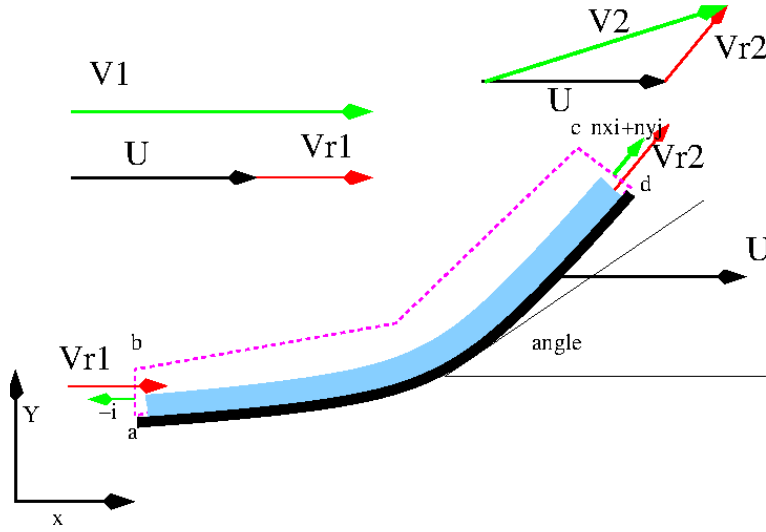
Figure 10: Control volume enclosing a Moving Blade

(5.31) reflects that the relative velocity has to maintain constant $((V_{r_n}|_{a-b}) = (V_{r_n}|_{c-d}))$ $\|$ $V_{r1} = V_{r2}$, if the area variation and the density variation is not present inside the control volume. Application of linear momentum on the moving control volume we obtain

$$\int (\rho\vec{V_r})(\vec{V_r}\cdot\hat{n})dS = -\int (P\hat{n}dS) + \vec{F} \qquad (5.32)$$

$$\int_a^b (\rho\vec{V_r})(\vec{V_r}\cdot\hat{n})dS + \int_b^c (\rho\vec{V_r})(\vec{V_r}\cdot\hat{n})dS + \int_c^d (\rho\vec{V_r})(\vec{V_r}\cdot\hat{n})dS + \int_d^a (\rho\vec{V_r})(\vec{V_r}\cdot\hat{n})dS =$$

$$-\int_a^b (P\hat{n}dS) - \int_b^c (P\hat{n}dS) - \int_c^d (P\hat{n}dS) - \int_d^a (P\hat{n}dS) + \vec{F} \qquad (5.33)$$

$$\int_a^b (\rho\vec{V_r})(\vec{V_r}\cdot\hat{n})dS + \int_c^d (\rho\vec{V_r})(\vec{V_r}\cdot\hat{n})dS = \vec{F} \qquad (5.34)$$

Using the mass conservation equation and the relative velocities we obtain the forces on the blade in horizontal and vertical directions as

$$-(\rho V_{r_n} S_{ab})u_{ab} + (\rho V_{r_n} S_{cd})u_{cd} = F_x \qquad (5.35)$$
$$-(\rho V_{r_n} S_{ab})v_{ab} + (\rho V_{r_n} S_{cd})v_{cd} = F_y \qquad (5.36)$$
$$-\dot{m}_{ab}u_{ab} + \dot{m}_{cd}u_{cd} = F_x \qquad (5.37)$$
$$-\dot{m}_{ab}v_{ab} + \dot{m}_{cd}v_{cd} = F_y \qquad (5.38)$$
$$\dot{m}|\vec{V_r}|(cos(\theta) - 1) = F_x \qquad (5.39)$$
$$\dot{m}|\vec{V_r}|sin(\theta) = F_y \qquad (5.40)$$

It can be observed that the force on the blade becomes zero for two conditions a) for the $\theta = 0$ b) $V_r = 0$. The first case is obvious that the inviscid fluid flowing on a flat plat doesn't exert any force, the second case imposes a limit on the speed with which the blade has to move in order to perceive force on the body. This aspect is being used in the design of the turbo machinery blade design and the speed with which the rotor of the turbine or a compressor or pump should rotate in order to extract work form the machine. From the second condition we observe that the blade velocity should be less than the absolute velocity with which the fluid approaches the blade $\mathbf{U} < \vec{V}$.

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5.4 Application of angular momentum equation - Euler Turbine Equation

Application of angular momentum to a turbo-machine is described here. The general form of angular momentum equation is described in section 3.3, will be applied to turbo-machines with the control volumes as shown for radial flow machine figure (11a) and mixed flow machine figure (11b).
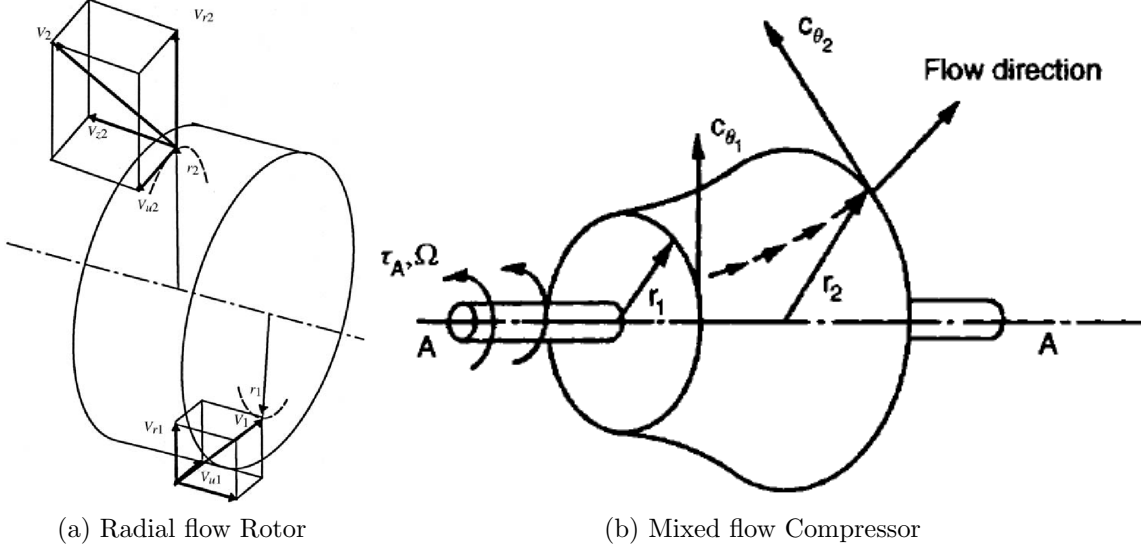

(a) Radial flow Rotor   (b) Mixed flow Compressor

Figure 11: Control Volume for Turbo-Machines

$$\vec{M} = \int_A (\vec{r} \times \rho\vec{V})(\vec{V} \cdot \hat{n})dA \tag{5.41}$$

Equation (5.48) represents the angular momentum equation for steady flow through a turbo machine. The velocity vector is given by $\vec{V} = v_r\hat{e}_r + v_\theta\hat{e}_\theta + v_z\hat{e}_z$ with magnitude $|\vec{V}| = \sqrt{v_r^2 + v_\theta^2 + v_z^2}$. In the above figure (11), consider $v_u = v_\theta$ and $c = v$ where

- v_r - Radial Velocity

- v_θ- Tangential Velocity

- v_z - Axial Velocity

for 1D inlet and exit the equation can be re-written as

$$\vec{\tau} = (\vec{r} \times \rho\vec{V})(V_n)A|_{outlet} - (\vec{r} \times \rho\vec{V})(V_n)A|_{inlet} \tag{5.42}$$
$$\vec{\tau} = (\dot{m}(\vec{r} \times \vec{V}))_{outlet} - (\dot{m}(\vec{r} \times \vec{V}))_{inlet} \tag{5.43}$$

$\vec{r} = R\hat{e}_r + 0\hat{e}_\theta + 0\hat{e}_z$ represents the position vector. Using the position vector, evaluating the cross product in eq(5.43) results in the torque exerted by the fluid flowing through the rotor. If the mass flow rate $\dot{m}$ remains constant from inlet to outlet of a turbo-machine then it requires that the torque depends only on the change in the product of radius vector to the tangential velocity. This component of the torque is used to evaluate the amount of work done by the fluid or on the fluid depending upon whether the turbo-machine is a turbine or a compressor or pump.

$$\tau_z = (\dot{m}(Rv_\theta))_{outlet} - (\dot{m}(Rv_\theta))_{inlet} \tag{5.44}$$
$$\tau_z = \dot{m}((Rv_\theta)_{outlet} - (Rv_\theta)_{inlet}) \tag{5.45}$$
$$\tau_z = \dot{m}R((v_\theta)_{outlet} - (v_\theta)_{inlet}) \tag{5.46}$$

In the case of an axial flow machine the position vector remains constant from inlet to outlet $R_{inlet} = R_{outlet} = R$, then the torque equation reduces to eq(5.46). For mixed flow machine and radial flow machine since $R_{inlet} \neq R_{outlet}$, eq(5.45) gives the torque generated by the machine. As the rotor is rotating at an angular velocity of ω, the amount of work done as the fluid flows through the machine is given by $\tau_z * \omega$ which results in

$$\dot{W} = \tau_z\omega = \dot{m}\omega((Rv_\theta)_{outlet} - (Rv_\theta)_{inlet}) \tag{5.47}$$

$$\dot{W} = \tau_z\omega = \dot{m}((Uv_\theta)_{outlet} - (Uv_\theta)_{inlet}) \tag{5.48}$$

$$\dot{W} = \tau_z\omega = \dot{m}U((v_\theta)_{outlet} - (v_\theta)_{inlet}) \tag{5.49}$$

$$\dot{w} = \tau_z\omega = (U(v_\theta)_{outlet} - (Uv_\theta)_{inlet}) \tag{5.50}$$

Taking the time rate of the from equation (4.1), for adiabatic flow per unit mass through turbo-machine results in

$$q = \delta h_t + w \tag{5.51}$$

$$0 = \delta h_t + w \tag{5.52}$$

$$\delta h_t = -w = (UV_\theta)_{outlet} - (UV_\theta)_{inlet}) \tag{5.53}$$

$$\delta h_t = -w = U((V_\theta)_{outlet} - (V_\theta)_{inlet}) \tag{5.54}$$

where δh_t represents the change in enthalpy of the turbo-machine from inlet to outlet. Equation (5.54) gives a relation between the enthalpy change to the change in velocities in a turbo-machine. The relation between the Uv_θ can be replaced in the equation (5.54) by the absolute velocity(V), the relative velocity (V_R) and blade velocity (U) with the help of velocity triangles. From the general velocity triangle for any turbo-machine as shown in figure

$$v_r^2 = V^2 - v_\theta^2 = V_R^2 - (U - v_\theta)^2 \tag{5.55}$$

$$V^2 - v_\theta^2 = V_R^2 - (U - v_\theta)^2 \tag{5.56}$$

$$Uv_\theta = \frac{V^2 + U^2 - V_R^2}{2} \tag{5.57}$$

obtained from sine law of the triangle. The equivalent form of work done in terms of velocity components can be obtained by replacing eq(5.57) in eq(5.50) resulting in

$$\dot{w} = (UV_\theta)_{outlet} - (UV_\theta)_{inlet} \tag{5.58}$$

$$\dot{w} = \left(\frac{V^2 + U^2 - V_R^2}{2}\right)_{outlet} - \left(\frac{V^2 + U^2 - V_R^2}{2}\right)_{inlet} \tag{5.59}$$

Equation (5.59) is also called alternate form of energy equation. It represents the change in enthalpy or rate at which work done in a machine with respect to the velocity components in the machine.

6 Frame of Reference

- Absolute frame of reference - The Coordinate system as seen by an fixed observer

- Relative frame of reference - The Coordinate system fixed to the rotor and moving with the angular velocity of the rotor

Terminology used in drawing the velocity triangles

- $\vec{V}$ or $C-$ Absolute velocity of the fluid between blade passages (Either stator or rotor).

- $\vec{V_R}$ or $W-$ Relative velocity of the fluid moving between the blades (Rotor).

- $\vec{U}-$ Velocity with which the blade moves. Since the blades are mounted on the rotor rotating at constant angular velocity, it is given by $R\omega\hat{e}_\theta$. Here "R" denotes the radius from the axis of the shaft.

- Rotor Blade Angle, (β) - It is the angle made by the relative velocity with the axis of the rotor. In other sense, it is the angle between the with which the flow enters the rotor of any machine.

- Stator Blade Angle, (α) -It is the angle made by the absolute velocity with the axis of the rotor.

In the stator case since the movement of the blades is zero ($\vec{U} = 0$), results in $\vec{V_R} = \vec{V}$. In the case of rotors for any kind of machines (axial, radial or mixed flow), the following relation for velocity is found at the inlet and exit of the blades.

$$\vec{V} = \vec{U} + \vec{V_R} \qquad (6.1)$$

This velocity relation for different kinds of machine is given as shown below. A typical Velocity triangle for a turbo-machine is shown in the figure
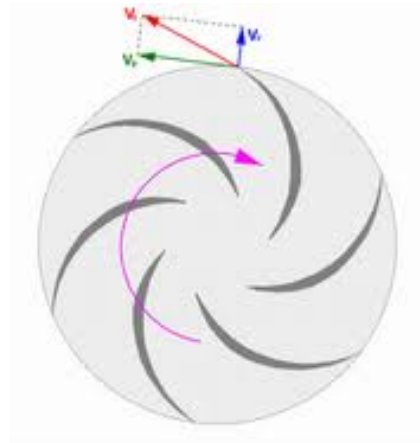

Figure 12: A typical Velocity Triangle

7 Axial flow Machines

7.1 Velocity Triangle for Axial flow Machine

In an axial flow machine fluid flow is parallel to the axis of the shaft, it is also assumed that there will be no variation in the radial velocity. The tangential velocity contributes to the torque and the axial velocity change will contribute to the thrust developed and will be absorbed by the thrust bearing.
Figure (13a) shows a stage axial flow turbine comprising of a stator followed by a rotor, the stator blades acts as nozzles and deflects the flow by an angle (also called as nozzle angle) as required by the rotor blades. Once the fluid leaves the nozzle either a reaction types of blades or impulse type of blades or a combination of reaction and impulse type of blading are used in the subsequent stages.
  Figure (13b) shows a stage axial flow compressor comprising of a rotor followed by a stator, the stator blades acts as diffuser further converting the kinetic energy to the pressure energy .
In figure (15) for a turbine, α_1 is the angle at which the fluid enters the tangent to the stationary
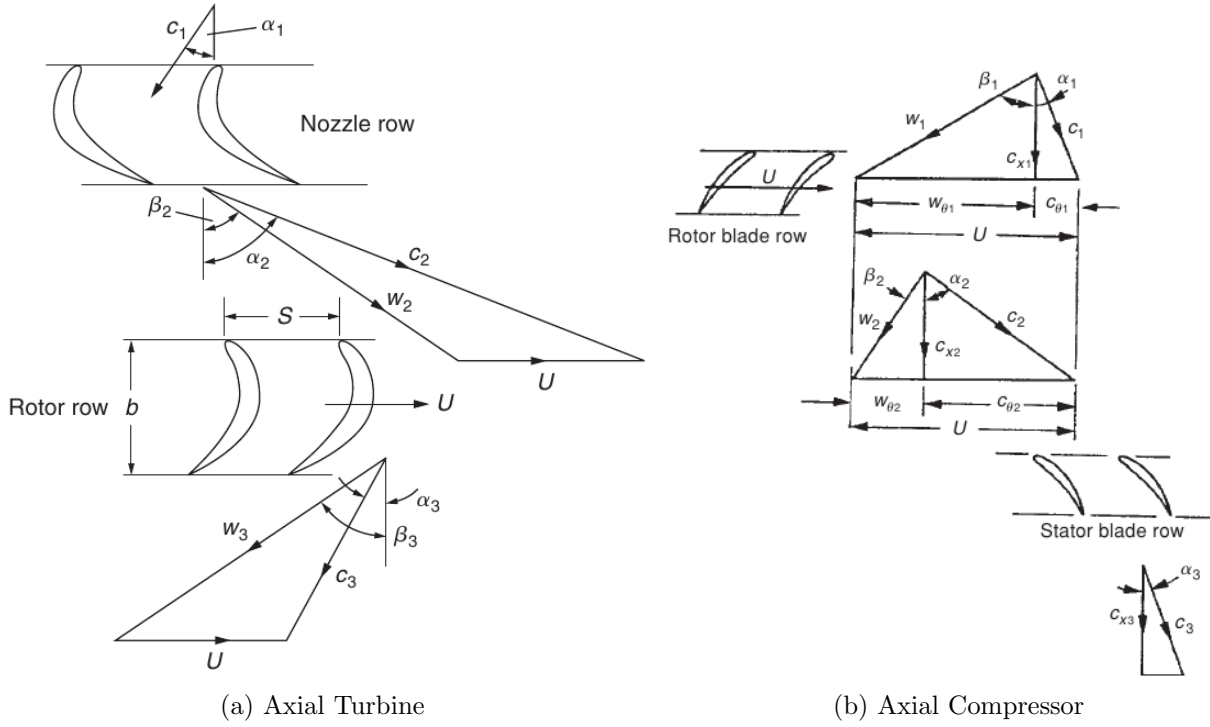
Figure 13: Velocity Triangles for Axial flow machines, angles being measured based on the axial direction(Image Courtesy S.M.Yahya -Turbines,Compressors and Fans)

blade with absolute velocity C_1 *or* V_1 and leaves with a velocity C_2 *or* V_2 at an angle α_2 (Nozzle angle for impulse turibne). Since the stator acts as nozzle $V_2 > V_1$, as there is no work and heat transfer to or from the fluid in these blades from the first law of thermodynamics the enthalpy of the fluid is converted to kinetic energy associated with pressure drop.

The rotor runs at an angular velocity of $\omega = \frac{2\pi N}{60}$, blades mounted on the rotor will have a blade tip velocity or tangential velocity $U_b = R\omega = \frac{\pi DN}{60}$. Since the blades are moving the velocity U_b the velocity with which the flow enters the rotor blade with relative velocities denoted as W_1 *or* V_{R1} and leaves the blade with velocities W_2 *or* V_{R2} at an angle of β_2 at inlet and β_3 at the exit of the rotor. These angles $\beta_2 \& \beta_3$ are also known as blade angles for the rotor blades. Similar terminology can be used for compressors, blowers and fans

From the inlet velocity triangles of rotor as given in Figure (13a) for axial flow turbine,

$$V_{f_2} \ or \ V_{a_2} = V_2 cos(\alpha_2) = V_{R_1} cos(\beta_2) \tag{7.1}$$

$$V_{\theta_2} = V_2 sin(\alpha_2) = U_b + V_{R_1} sin(\beta_2) \tag{7.2}$$

$$\frac{U_b}{sin(\alpha_2 - \beta_2)} = \frac{V_2}{sin(90 + \beta_2)} = \frac{V_2}{cos(\beta_2)} \tag{7.3}$$

$$\frac{U_b}{V_2} = \frac{sin(\alpha_2 - \beta_2)}{cos(\beta_2)} \tag{7.4}$$

Similarity from the exit triangle

$$V_{f_3} \ or \ V_{a_3} = V_3 cos(\alpha_3) = V_{R_2} cos(\beta_3) \tag{7.5}$$

$$V_{\theta_3} = V_3 sin(\alpha_3) = V_{R_3} sin(\beta_3) - U_b \tag{7.6}$$

from equations (7.2) and (7.6) we obtain

$$V_{\theta_2} + V_{\theta_3} = V_{R_1} sin(\beta_2) + V_{R_2} sin(\beta_3) \tag{7.7}$$

$$V_{\theta_2} + V_{\theta_3} = V_{R_{\theta_2}} + V_{R_{\theta_3}} \tag{7.8}$$

if the axial components were to remain constant i.e, $V_{f_1} = V_{f_2} = V_{f_3}$ or $V_{a_1} = V_{a_2} = V_{a_3}$ in a given stage, then

$$V_a \ or \ V_a \ = \ V_1 cos(\alpha_1) = V_2 cos(\alpha_2) = V_3 cos(\alpha_3) \tag{7.9}$$
$$= \ V_{R_1} cos(\beta_2) = V_{R_2} cos(\beta_3) \tag{7.10}$$

using equations (7.9&7.10) in equation equation (7.8) results in

$$tan(\alpha_2) + tan(\alpha_3) = tan(\beta_2) + tan(\beta_3) \tag{7.11}$$

Since the machine under consideration is an axial flow machine, and the flow analysis is along a stream line, the blade velocity remain constant from the inlet ofthe rotor to the exit of the rotor $U_b = U_{b_2} = U_{b_3}$. The work done by the fluid on the rotor is given by

$$w = U_b \left[V_{\theta_2} - V_{\theta_3} \right] \tag{7.12}$$
$$w = U_b \left[V_{\theta_2} + V_{\theta_3} \right] \tag{7.13}$$

since the direction of the tangential velocities at the inlet and exit or the rotor are opposite the "+" appears in the equation (7.13).
Rewriting the equation (7.13) in terms of velocity ratio $\frac{U_b}{V}$ (The ratio of absolute velocity at inlet to the blade velocity) gives some insight to the energy transfer and losses that occur in the rotor.

$$w = U_b^2 \left[\frac{V_{\theta_2}}{U_b} + \frac{V_{\theta_3}}{U_b} \right] \tag{7.14}$$

The first term in eq(7.14) depends upon the nozzle angle and the velocity ratio $\sigma = \frac{U_b}{V_2}$. From the velocity triangle it can be observed that V_{θ_3}, tangential component at the exit of the rotor is small compared to the inlet tangential component V_{θ_2}. Hence its contribution towards work done is also negligible. If V_{θ_3}, is large then the kinetic energy of the fluid leaving the rotor is large and hence huge losses, if it were to be zero then the work done on to the rotor decreases. To minimize the losses from the stage the exit from the stage is to be axial $V_{\theta_3} = 0$ which results in the work depending only on the nozzle angle α_2 and the blade velocity ratio σ.

$$w = U_b^2 \left[\frac{V_{\theta_2}}{U_b} \right] \tag{7.15}$$

$$w = U_b^2 \left[\frac{V_2 sin(\alpha_2)}{U_b} \right] \tag{7.16}$$

$$w = U_b^2 \left[\frac{sin(\alpha_2)}{\sigma} \right] \tag{7.17}$$

7.2   Blade efficiency or Utilization Factor ϵ

The work done by any turbine (axial flow or radial flow or mixed flow) occurs only in rotor, hence work done in a given stage equals the amount of work done in the rotor of that stage. But the efficiency of the stage does not only depend upon the rotor, stator also plays its role in the entire stage efficiency. Hence the blade efficiency and stage efficiency are not the same. Stage efficiency takes into consideration the aerodynamic losses occuring the rotor as well as stator. The blade efficiency also known as utilization factor is an index which measures the capability of the rotor blades in utilization of the energy.

$$\epsilon = \eta_b = \frac{Rotor \ blade \ work}{Energy \ supplied \ to \ rotor \ blades} = \frac{w}{e_i} \tag{7.18}$$

Work done by any turbo machine and its equivalent form in terms of the velocity components are given in eq(5.50) and eq(5.59). The energy supplied in the entire rotor is given by

$$e_i = \frac{V_2^2}{2} + \frac{U_{b_2}^2 - U_{b_3}^2}{2} + \frac{V_{R_3}^2 - V_{R_2}^2}{2} \tag{7.19}$$

$$\epsilon = \eta_b = \frac{\frac{V_2^2 - V_3^2}{2} + \frac{U_{b_2}^2 - U_{b_3}^2}{2} + \frac{V_{R_3}^2 - V_{R_2}^2}{2}}{\frac{V_2^2}{2} + \frac{U_{b_2}^2 - U_{b_3}^2}{2} + \frac{V_{R_3}^2 - V_{R_2}^2}{2}} \tag{7.20}$$

Since for axial flow machines $U_b = U_{b_2} = U_{b_3}$ we get the blade efficiency or utilization factor as

$$\epsilon = \eta_b = \frac{\frac{V_2^2 - V_3^2}{2} + \frac{V_{R_3}^2 - V_{R_2}^2}{2}}{\frac{V_2^2}{2} + \frac{V_{R_3}^2 - V_{R_2}^2}{2}} \tag{7.21}$$

7.3   Single Stage Axial flow Impulse turbine (Steam or Gas Turbine)

If the pressure in the rotor remains constant it is called Impulse turbine. In a single stage turbine the entire pressure drop is converted to increase in velocity at the end of the stator, and further the pressure remains constant in the rotor. The first ever single stage turbine is de-Laval turbine running at 30,000 rpm.

7.3.1   Staging of Impulse turbines

The following reasons are the main cause for turbine stages

- Total pressure drop results in higher absolute velocities, which inturn results in higher blade speeds and large friction losses.

- Reduction gear boxes with large efficiencies are essential to reduce the higher shaft speeds to the speeds desired.

- Higher speeds of the shaft results in higher centrifugal stresses resulting in structural failures.

Owing to the above restrictions the blade speed is limited to 400 $\frac{m}{s}$. With single stage, the exit kinetic energy loss is about 10% to 12%.
Compunding of turbine is utilized in order to reduce the blade speed and effectively utilize the energy in the steam or gas. This compounding is done classified as

7.3.2   Velocity Compounding - Curtis Turbine

Convert the total ennthalpy into velocity in a single stage and then conversion of Kinetic energy of steam into mechanical energy of the wheel in different stages. Curtis Turbine.

- Employs lower blade speed ratio and higher utilization of KE of steam.

- Number of Nozzles depends upon the load on the turbine.

- Two or more number of moving blades with intermediat stages of fixed blades are used. Fixed blades are only steam deflectors.

- There may occur slight variation of velocity in the velocity in the fixed blades due to fluid friction. This can be neglected for the current analysis.

- Velocity reduces in the moving blades. Moving blades are arranged on the same shaft so no power loss occurs.

- The velocity at the exit of the blade is axial, which ensures minimum loss and maximum efficiency.

- Volume remains constant, second and third blades rows may need little bit higher blades.

Advantages of Velocity compounding turbines are

- Fewer number of stages hence less initial cost.

- Arrangement requires little space

- System is reliable and easy to operate.

- Pressure fall in the nozzle is considerable, so the turbine itself need not work in high pressure surroundings and the turbine housing need not be very strongly made.

Disadvatages of Velocity Compounding turbines

- Due to high steam velocity in nozzles, frictional losses are higher.

- Efficiency is low, because the ratio of blade velocity to steam velocity is not same for all the wheels.

- Efficiency decreases with number of stages.

- Power developed in the later rows is fraction compared to the power developed in the first row. Usage of Velocity Compunding turbines

  - Drives for centrifugal compressors, pumps, small generators and for small units working with high pressure steam.s

7.3.3 Pressure Compounding- Rateau Turbine

The pressure drop occurs in different stages. This is equivalent to successive impulse stages put togather. Rateau Staging turbine.

- Small pressure drop takes place in Nozzles and hence the enthalpy drop resulting in small raise in Kinetic energy.

- Rotating blades have typical symmetric blades.

- Fixed blades not only changes the direction but acts as nozzles. Hence KE in the first stage is not lost but carried to further stages.

- For all rotating blades the velocity diagrams are same and power output is same.

- Pressure gradually goes down, volume increases and therfore the blade height has to increase.

7.4 Reaction Turbines

DeLaval built the first pure reaction turbine based on the principle of Hero. This turbine had as speed of $180 \frac{m}{s}$ at 42,000 rpm, because of inefficiency this turbine could not get into operaton. Impulse reaction turbine called as reaction turbine uses partial impulse and partial reaction force. It is extension of pressure compounded turbine.

1. Pressure drop occurs both in the rotor and the stator. Stator passages acts as nozzles (similar to pressure compounded turbines).

2. All the turbine stages are 50% reaction, Blades are symmetrical. Both the stator and rotor blades are similar kind.

8 Radial Flow Machines

8.1 Velocity Triangle for Radial flow Machine

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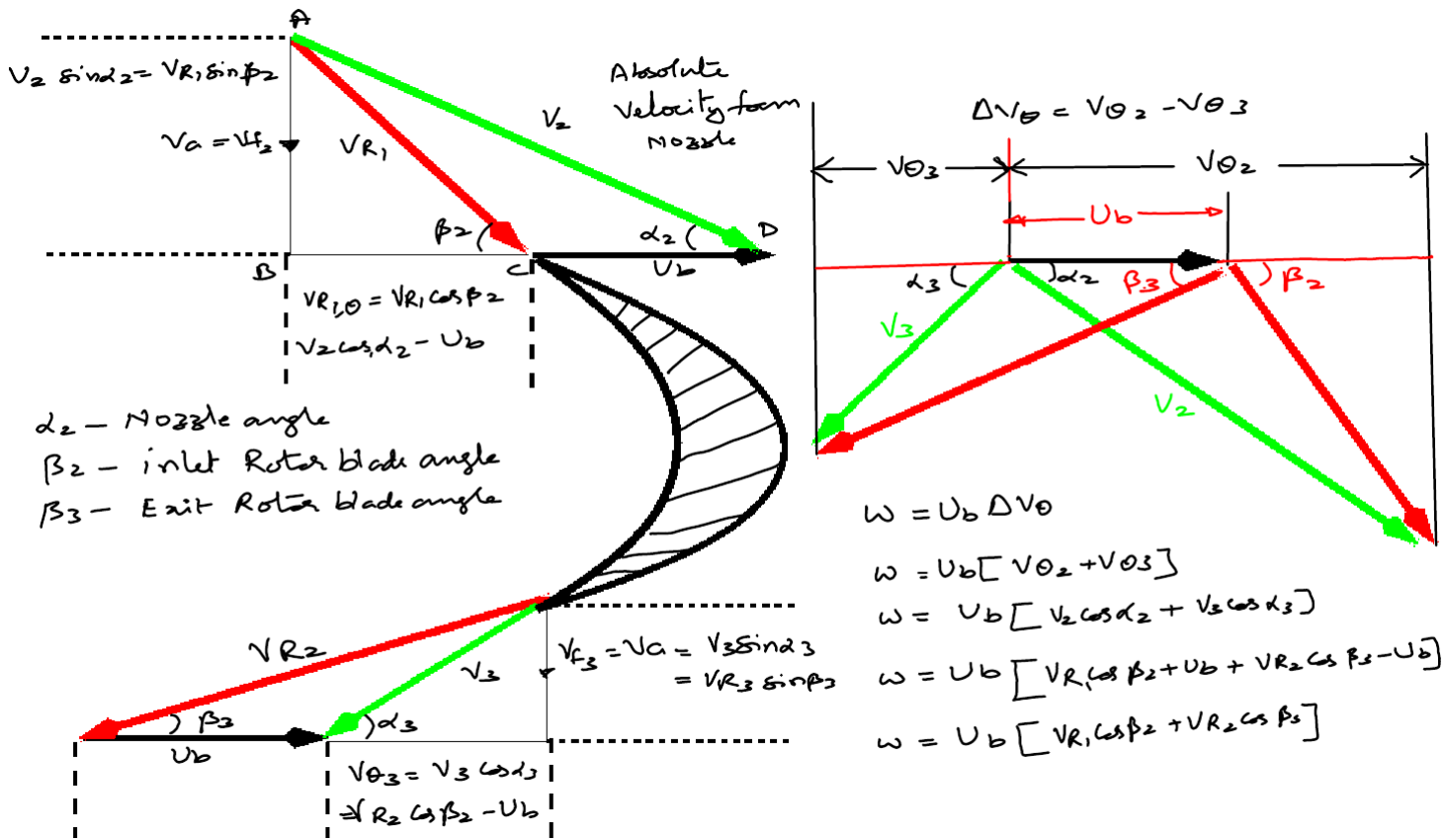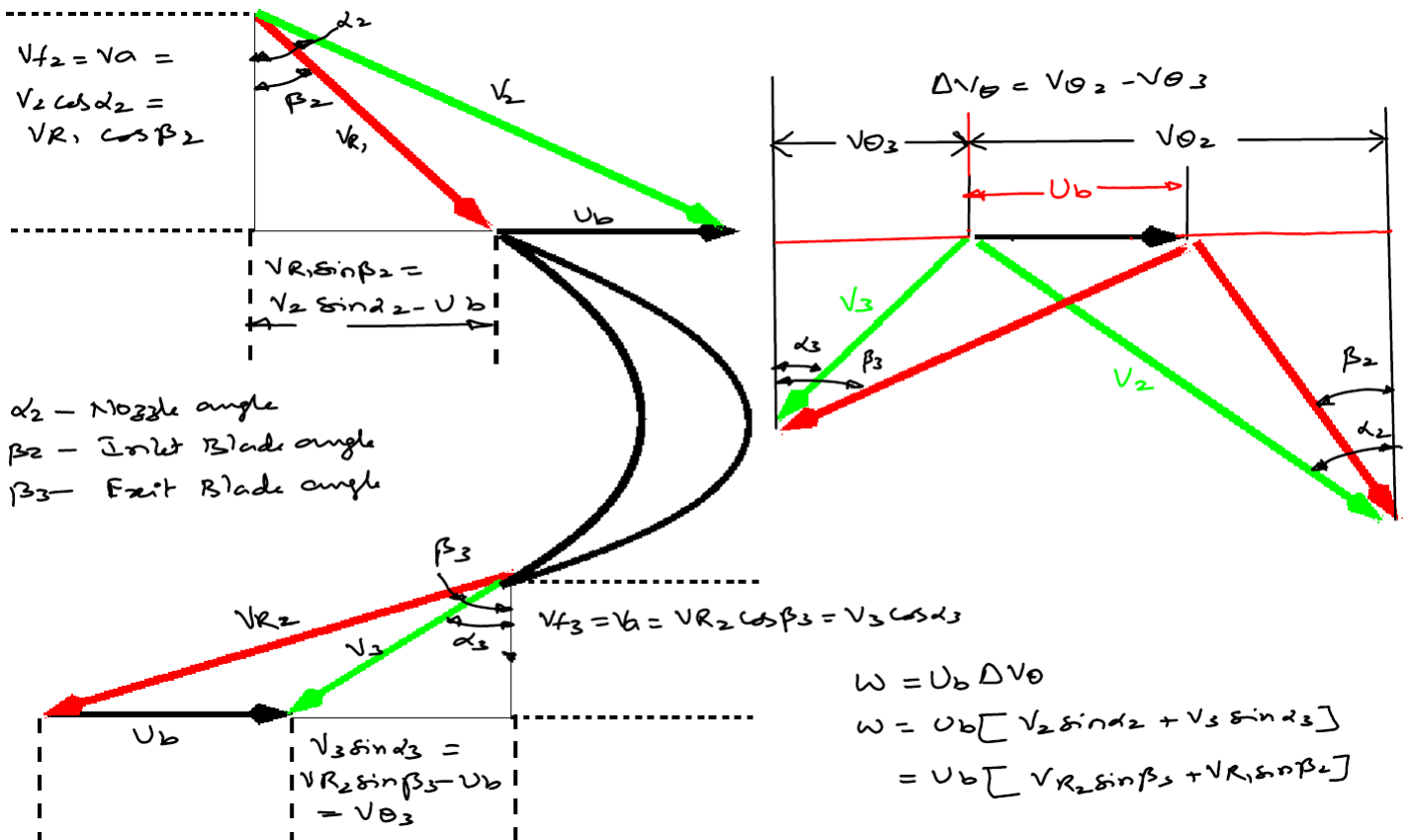
Figure 14: Single stage Impulse Turbine, Velocity Triangles at Rotor w.r.t Horizontal and vertical reference frames

(a) Horizontal Reference

$V_2 \sin\alpha_2 = V_{R_1}\sin\beta_2$

$V_a = V_{f_2}$

V_{R_1}

V_2

Absolute Velocity from Nozzle

$V_{R_1\theta} = V_{R_1}\cos\beta_2$

$= V_2\cos\alpha_2 - U_b$

α_2 — Nozzle angle

β_2 — inlet Rotor blade angle

β_3 — Exit Rotor blade angle

V_{R_2}

V_3

$V_{f_3} = V_a = V_3\sin\alpha_3 = V_{R_3}\sin\beta_3$

U_b

$V_{\theta_3} = V_3\cos\alpha_3 = V_{R_2}\cos\beta_2 - U_b$

$\Delta V_\theta = V_{\theta_2} - V_{\theta_3}$

V_{θ_3} V_{θ_2}

U_b

V_3 V_2

$\omega = U_b \Delta V_\theta$

$\omega = U_b[V_{\theta_2} + V_{\theta_3}]$

$\omega = U_b[V_2\cos\alpha_2 + V_3\cos\alpha_3]$

$\omega = U_b[V_{R_1}\cos\beta_2 + U_b + V_{R_2}\cos\beta_3 - U_b]$

$\omega = U_b[V_{R_1}\cos\beta_2 + V_{R_2}\cos\beta_3]$

(b) Vertical Reference

$V_{f_2} = V_a = V_2\cos\alpha_2 = V_{R_1}\cos\beta_2$

V_2

V_{R_1}

U_b

$V_{R_1}\sin\beta_2 = V_2\sin\alpha_2 - U_b$

α_2 — Nozzle angle

β_2 — Inlet Blade angle

β_3 — Exit Blade angle

V_{R_2}

V_3

$V_{f_3} = V_a = V_{R_2}\cos\beta_3 = V_3\cos\alpha_3$

U_b

$V_3\sin\alpha_3 = V_{R_2}\sin\beta_3 - U_b = V_{\theta_3}$

$\Delta V_\theta = V_{\theta_2} - V_{\theta_3}$

V_{θ_3} V_{θ_2}

U_b

V_3 V_2

$\omega = U_b \Delta V_\theta$

$\omega = U_b[V_2\sin\alpha_2 + V_3\sin\alpha_3]$

$= U_b[V_{R_2}\sin\beta_3 + V_{R_1}\sin\beta_2]$

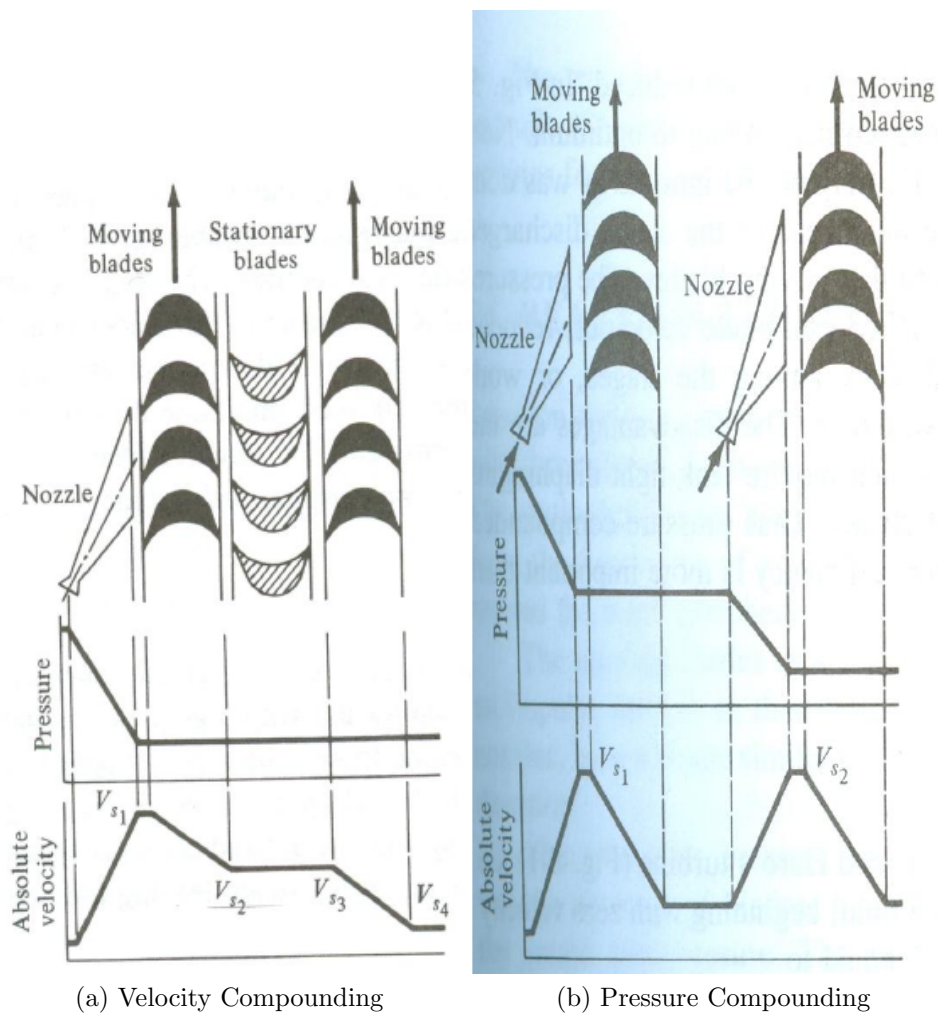(a) Velocity Compounding (b) Pressure Compounding

Figure 15: Compounding of Impulse Turbines(Courtesy:-World Wide Web)