Turbomachinery Aero-Thermodynamics

Aero-Energetics 3D

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Evaluation

Evaluation for sessions 3

QROC II (20 mins)

And now what are the stakes and technologies?

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Electricity production – Improve on existing technologies

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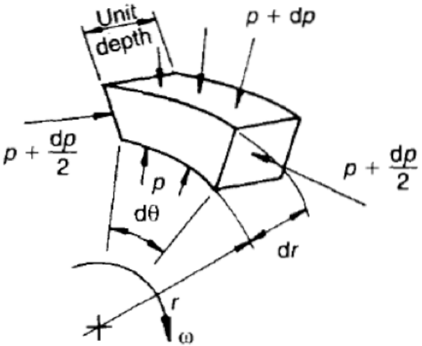
Radial Equilibrium

The Euler equation for turbomachines gives a link between the jump in total enthalpy and the evolution of the velocity vector.
Yet, it provides no information about the evolution of those quantities in the meridional plane.
They can be obtained using the notion of radial equilibrium linking the evolution of aero-thermodynamic quantities in the meridional plane.

Radial Equilibrium – Naive derivation

Let's consider a fluid particule in an axial turbomachine. In the relative frame, there is an equilibrium between the pressure and the centrifugal forces as illustrated below



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Radial Equilibrium – Naive derivation

Let's express those forces more explicitly

- The centrifugal force $d\vec{F}_c$ related to the rotation of the fluid particule. It is a volume force such that $d\vec{F}_c = \rho \underbrace{(drrd\theta dz)}_{dv} \omega^2 r \vec{u}_r$
- The pressure force $d\vec{F}_p$ applying on the fluid particule. It is a surface force such that

$$
\begin{aligned}
d\vec{F}_p &= -(p+dp)(r+dr)d\theta dz\vec{u}_r + prd\theta dz\vec{u}_r \\
&\quad +2\left(p+\frac{dp}{2}\right) sin\left(\frac{d\theta}{2}\right) drdz\vec{u}_r \\
d\vec{F}_p &= -dprd\theta dz\vec{u}_r \cancel{-pdrd\theta dz\vec{u}_r} \\
&\quad \cancel{+pd\theta drdz\vec{u}_r} \\
d\vec{F}_p &= -dprd\theta dz\vec{u}_r
\end{aligned}
$$

Radial Equilibrium – Naive derivation

It the momentum along the radius is steady, both forces equilibrate so that

$$dprd\theta dz = \rho\omega^2 r^2 drd\theta dz$$

As $V_\theta = \omega r$, one can write

$$\frac{dp}{dr} = \rho\frac{V_\theta^2}{r}$$

Radial Equilibrium – Naive derivation

Following Gibbs Equation $\quad dh = Tds + \frac{dp}{\rho},$

and from the definition of h_0, $\quad h = h_0 - \left(\frac{V_z^2}{2} + \frac{V_\theta^2}{2}\right)$

where we assume that $V_r = 0$. Deriving this last relation along r and assuming also that the entropy is independant of the radius, one gets the simplified radial equilibrium equation

Simplified Radial Equilibrium Equation

$$\frac{dh_0}{dr} = \frac{1}{2}\left(\frac{dV_z^2}{dr} + \frac{1}{r^2}\frac{d(rV_\theta)^2}{dr}\right)$$

This equation links the radial distributions of total enthalpy, azimuthal velocity and axial velocity in the flow.

Radial Equilibrium – Formal derivation

As mentioned this equation is simplified but **we have not clearly expressed all the assumption.** Most importantly we have directly considered an axial machine thereby neglecting centrifugal compressors/turbines without justification.
Also, one might want to have a more precise relation for design purposes.

For all these reasons, a more formal derivation is necessary.

Radial Equilibrium – Formal derivation

Let's first express the conservation of momentum:
In the absolute frame we have:

$$\frac{\partial \rho \vec{V}}{\partial t} + \vec{\nabla}.\left(\rho \vec{V} \otimes \vec{V}\right) = \vec{\nabla}.\left(\bar{\bar{\sigma}}\right)$$

In the relative frame we have:

$$\cancel{\frac{\delta \rho \vec{W}}{\delta t}} + \vec{\nabla}.\left(\rho \vec{W} \otimes \vec{W}\right) + 2\rho\vec{\omega} \wedge \vec{W} - \rho\vec{\nabla}\left(\frac{U^2}{2}\right) \cancel{+\rho\frac{d\vec{\omega}}{dt} \wedge \vec{r}} = \vec{\nabla}.\left(\bar{\bar{\sigma}}\right)$$

We assume a stationary flow in the relative frame and a constant angular rotation speed of the relative frame.

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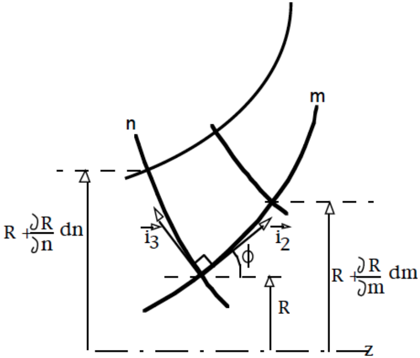
Radial Equilibrium – Formal derivation

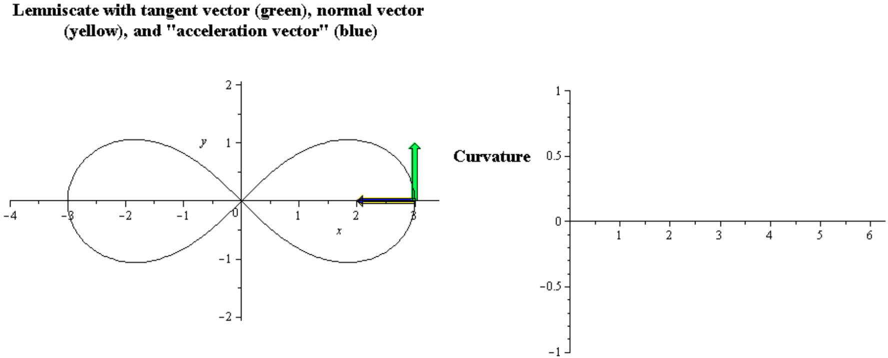
$$\vec{\nabla}.\left(\rho\vec{W}\otimes\vec{W}\right)+2\rho\omega\vec{i_\theta}\wedge\vec{W}-\rho\vec{\nabla}\left(\frac{U^2}{2}\right)=\vec{\nabla}.\left(\bar{\bar{\sigma}}\right)$$

First, it is necessary to express the conservation equations in the local coordinates (θ, m, n) presented in the picture below



Radial Equilibrium – Formal derivation – Curvature

In order to derive the equations in the local coordinates (θ, m, n). One first need to define the curvature $K_x = -\frac{1}{r}\frac{\partial r}{\partial x}$. Let's have a look at the following movie and follow the blue vector



"Source: Urs Hartl – Wikimedia Commons"

The curvature for a variation of m is such that $K_m = -\frac{1}{R}\frac{\partial R}{\partial m}$.

The curvature for a variation of n is such that $K_n = -\frac{1}{R}\frac{\partial R}{\partial n}$.

Radial Equilibrium – Formal derivation

For the component of the momentum along n, if we neglect the viscous stress tensor (away from the blades), the equation writes:

$$\frac{1}{r}\frac{\partial \rho r W_\theta W_n}{\partial \theta} + \frac{\partial \rho r W_m W_n}{\partial m} + \frac{\partial \rho r W_n^2}{\partial n} - \rho \left(W_\theta^2 + \omega^2 r^2 + 2\omega r W_\theta\right) cos(\Phi)$$
$$+2\rho r W_m W_n K_n - \rho r K_m \left(W_m^2 - W_n^2\right) = -r\frac{\partial p}{\partial n}$$

If we average this equation in the θ direction, we get

$$\frac{1}{r}\frac{\partial \rho r \bar{W}_\theta \bar{W}_n}{\partial \theta} + \frac{\partial \rho r \bar{W}_m \bar{W}_n}{\partial m} + \frac{\partial \rho r \bar{W}_n^2}{\partial n} - \rho \left(\bar{W}_\theta + \omega r\right)^2 cos(\Phi)$$
$$+2\rho r \bar{W}_m \bar{W}_n K_n - \rho r K_m \left(\bar{W}_m^2 - \bar{W}_n^2\right) = -r\frac{\partial \bar{p}}{\partial n}$$

having neglected fluctuations in the θ direction and assuming $\bar{\rho} = \rho$ and $\overline{cos(\Phi)} = cos(\phi)$.

Radial Equilibrium – Formal derivation

Away from the blades, one can assume that the azimuthal average flow follows the m streamlines so that $\bar{W}_n \approx 0$ and $\bar{W}_m \approx \bar{V}_m$. In this case we have:

$$-\rho \left(\bar{W}_\theta + \omega r\right)^2 cos(\Phi) - \rho r K_m \bar{V}_m^2 = -r \frac{\partial \bar{p}}{\partial n}$$

Remembering that $\bar{W}_\theta + \omega r = \bar{V}_\theta$, and dropping the $\bar{x}$ signs for convenience, we have

$$\frac{V_\theta^2}{r} cos(\Phi) + K_m V_m^2 = \frac{1}{\rho} \frac{\partial p}{\partial n}$$

Radial Equilibrium – Formal derivation

From Gibbs equation:

$$\frac{\partial h}{\partial n} = T\frac{\partial s}{\partial n} + \frac{1}{\rho}\frac{\partial p}{\partial n}$$

and the definition of the stagnation enthalpy $h_0 = h + \frac{V_m^2}{2} + \frac{V_\theta^2}{2}$, we have

$$2\frac{\partial h_0}{\partial n} = 2T\frac{\partial s}{\partial n} + \frac{2}{\rho}\frac{\partial p}{\partial n} + \frac{\partial V_m^2}{\partial n} + \frac{\partial V_\theta^2}{\partial n}$$

$$2\frac{\partial h_0}{\partial n} = 2T\frac{\partial s}{\partial n} + \underbrace{\frac{2V_\theta^2}{r}cos(\Phi) + \frac{\partial V_\theta^2}{\partial n}}_{\frac{1}{r^2}\frac{\partial (rV_\theta)^2}{\partial n}} + 2K_m V_m^2 + \frac{\partial V_m^2}{\partial n}$$

Radial Equilibrium – Formal derivation

general expression of the radial equilibrium

The general expression of the radial equilibrium writes:

$$2\frac{\partial h_0}{\partial n} = 2T\frac{\partial s}{\partial n} + \frac{1}{r^2}\frac{\partial (rV_\theta)^2}{\partial n} + 2K_m V_m^2 + \frac{\partial V_m^2}{\partial n}$$

If we apply this expression to an axial machine ($m = z,\ n = r,\ K_m = 0$) and assume no radial dependance of the entropy ($\frac{\partial s}{\partial n} = 0$).

Simplified radial equilibrium

The expression for the simplified radial equilibrium writes:

$$2\frac{\partial h_0}{\partial n} = \frac{1}{r^2}\frac{\partial (rV_\theta)^2}{\partial r} + \frac{\partial V_z^2}{\partial r}$$

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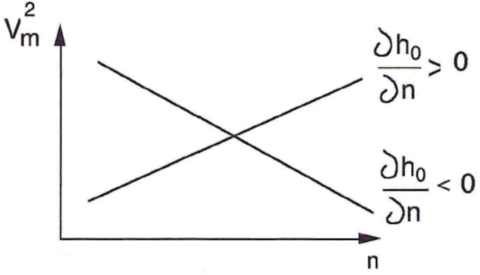
Full Radial Equilibrium – Physical interpretation

Let's now see what is the influence of the different terms on the distribution of the flow stream velocity (V_m) along n.

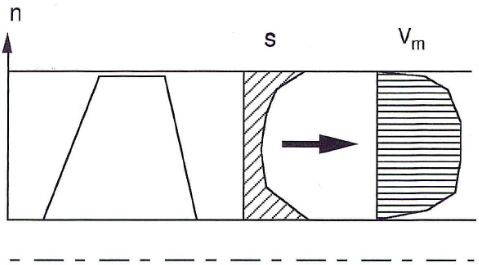
- **Influence of a distribution of total enthalpy**
  If the total enthalpy increases with n, V_m^2 will increase accordingly as displayed below



Full Radial Equilibrium – Physical interpretation

- **Influence of a distribution of entropy**
  The velocity V_m decreases if the entropy increases such as in the boundary layers as shown below.

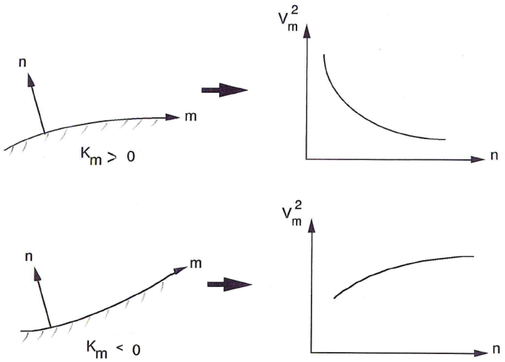
$$\frac{\partial V_m^2}{\partial n} = -2T\frac{\partial s}{\partial n}$$



Full Equilibrium – Physical interpretation

- **Influence of curvature**
  The curvature K_m also influences V_m. If all other terms are zero we have:

$$\frac{\partial V_m^2}{\partial n} = -2K_m V_m^2 \rightarrow V_m^2 = V_{m0}^2 e^{-2K_m n}$$

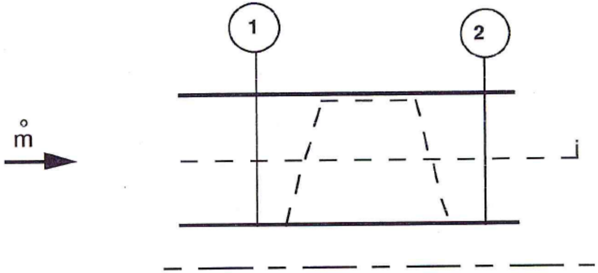


Application of the Simplified Radial Equilibrium

Usage

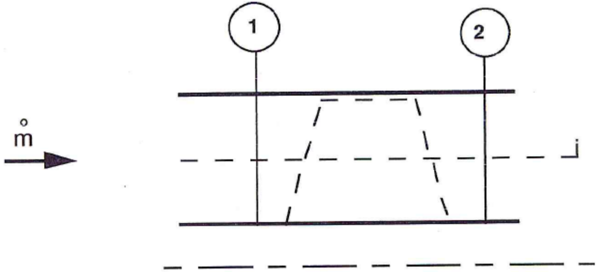
The simplified radial equilibrium equation is most importantly used during the design stage of a turbomachine It relates the three distributions of total enthalpy h_0, of axial velocity V_z and of azimuthal velocity V_θ.

Let's consider the following axial compressor stage



Application of the Simplified Radial Equilibrium

Let's consider the following axial compressor stage



- During conception we place ourselves at the most upstream section (1)
- We assume that all distributions are known in section (1)
- We can now use Euler equation to obtain information about section (2)

Application of the Simplified Radial Equilibrium

Using Euler equation and deriving with respect to r, one gets:

$$h_{02} - h_{01} = \omega r \left(V_{\theta 2} - V_{\theta 1} \right)$$

$$\frac{\partial h_{02}}{\partial r} - \frac{\partial h_{01}}{\partial r} = \omega \left[\frac{\partial r V_{\theta 2}}{\partial r} - \frac{\partial r V_{\theta 1}}{\partial r} \right]$$

By using the simplified radial equilibrium equation at section (2), one obtains

$$\frac{1}{2}\frac{\partial V_{z2}^2}{\partial r} = \frac{\partial h_{02}}{\partial r} - \frac{1}{2r^2}\frac{\partial \left(r V_{\theta 2} \right)^2}{\partial r}$$

$$\frac{\partial h_{02}}{\partial r} = \underbrace{\frac{\partial h_{01}}{\partial r} - \omega \frac{\partial r V_{\theta 1}}{\partial r}}_{\text{Known distributions}} + \omega \frac{\partial r V_{\theta 2}}{\partial r}$$

$$\frac{1}{2}\frac{\partial V_{z2}^2}{\partial r} = \underbrace{\frac{\partial h_{01}}{\partial r} - \omega \frac{\partial r V_{\theta 1}}{\partial r}}_{\text{Known distributions}} + \omega \frac{\partial r V_{\theta 2}}{\partial r} - \frac{1}{2r^2}\frac{\partial \left(r V_{\theta 2} \right)^2}{\partial r}$$

Application of the Simplified Radial Equilibrium

$$\frac{1}{2}\frac{\partial V_{z2}^2}{\partial r} = \underbrace{\frac{\partial h_{01}}{\partial r} - \omega\frac{\partial rV_{\theta 1}}{\partial r}}_{\text{Known distributions}} + \omega\frac{\partial rV_{\theta 2}}{\partial r} - \frac{1}{2r^2}\frac{\partial\left(rV_{\theta 2}\right)^2}{\partial r}$$

This equation relates two unknown distributions. We need to impose one of them. Most of the time designer choices are amongst the following

- **The free vortex distribution** ($rV_\theta =$constant). With this distribution $dh_{02}/dr = 0$ so that the blade "works" the same regardless of the radius.[1]
- **The constant absolute angle distribution** ($tan\alpha = V_\theta/V_z =$constant). With this distribution the stagger angle of the downstream stator is constant with r.

Often the designer will impose $\frac{\partial V_{z2}^2}{\partial r} = 0$ at the design point to prevent the stream flow velocity from reaching dangerously low values when departing from this nominal point.

[1] We also assume here that the flow is uniform in 1.

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Characteristics in axial turbomachines

So far, we have been mostly interested in the aero-thermodynamic properties of the turbomachine at the design point.
Yet, most of the difficulties arise from the fact that the turbomachine must adapt to different mass flow rates because of

- a change in the far field total pressure or temperature
- a change of velocity in the case of aeronautical machines
- a change of mass flow rate due to regime change
- etc...

Characteristics in axial turbomachines

The term characteristics refer to the curves describing the evolution of

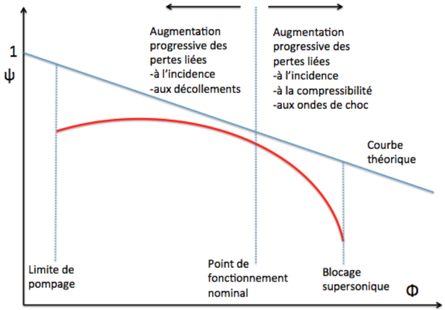
- the loading (Ψ) or pressure (Ψ_{press}) coefficient as a function of the flow coefficient (Φ)
- the pressure ratio (Π) as a function of the reduced mass flow rate ($\dot{m}_{red}$).

Characteristics-Compressor I

As we have seen, the design load coefficient (Ψ_d) is related to the design flow coefficient (Φ_d) by the relation

$$\Psi_d = 1 - \Phi_d(tan\beta_2 + tan\alpha_1) = 1 - A\Phi_d$$

The outlet angles (β_2 for the rotor, $\alpha_1 = \alpha_3$ for the stator) are fixed by design. The theoretical vs realistic evolution is represented below



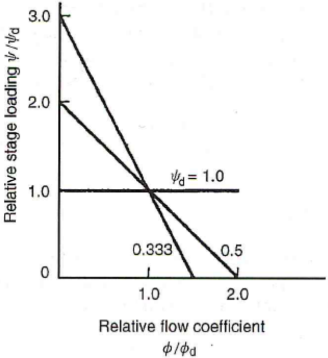
Characteristics-Compressor I

Let's now change the flow coefficient to Φ. We still have

$$\Psi = 1 - A\Phi$$

$$\text{So that } \frac{\Psi}{\Psi_d} = \frac{1}{\Psi_d} - \frac{\Phi}{\Phi_d}\left(\frac{1-\Psi_d}{\Psi_d}\right)$$

Below is represented the evolution of Ψ versus Φ for different values of the design load coefficient.



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As we have seen, the load coefficient (Ψ) is related to the flow coefficient (Φ) by the relation

$$\Psi = 1 - \Phi(tan\beta_2 + tan\alpha_1)$$

If we remember that

$$\Psi = \frac{\Delta h_0}{U^2} = \underbrace{\frac{T_0}{U^2}\Delta s_0}_{\text{Friction loss coefficient if adiabatic}} + \underbrace{\frac{\Delta p_0}{\rho_0 U^2}}_{\text{Pressure coefficient}}$$

we have

$$\Psi_{pres} = 1 - \Phi(tan\beta_2 + tan\alpha_1) - \text{Losses}$$

Characteristics-Compressor II

An other way to represent characteristic curves is to consider pressure ratio (Π) versus reduced mass flow rate $\dot{m}_{red}$. Let's consider the following non dimensional numbers that describe the behavior of a compressor:

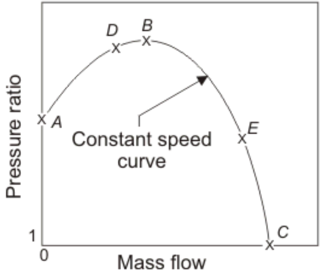
- the pressure ratio $\Pi = \frac{p_{02}}{p_{01}}$
- the isentropic efficiency η_c
- the reduced mass flow rate $\dot{m}_{red} = \dot{m}\frac{\sqrt{T_{01}}}{p_{01}}$
- the reduced rotation speed $N_{red} = \frac{N}{\sqrt{T_{01}}}$

$\dot{m}_{red}$ and N_{red} are not true dimensionless parameters but they are the quantities largely used to describe the behavior of the compressor

Characteristics-Compressor II

Characteristics

Knowing for all different reduced speeds (N_{red}) the evolution with the reduced mass flow rate ($\dot{m}_{red}$) of (i) the pressure ratio (Π) and (ii) the isentropic efficiency (η_c) completely determinate the behavior of a given compressor.



Source: G.Biswas

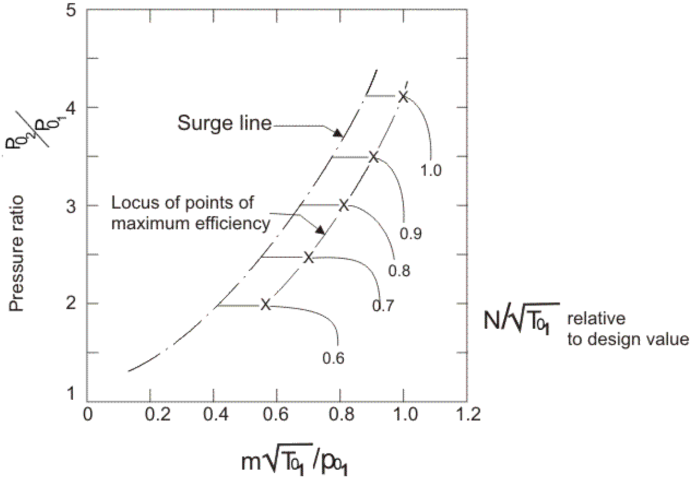
Characteristics-Compressor II

Unfortunately, not all points on the previous curve are physically accessible.

- When the mass flow increases, the Mach number increases in the location having the smallest section. The maximum mass flow rate $\dot{m}_{red\ max}$ is reached when the Mach number in this section is equal to unity. No further increase of the mass flow is possible unless the inflow velocity becomes supersonic.
- When the mass flow is reduced, the adverse pressure gradient will tend to stall the blades at first decreasing the pressure ratio. Beyond a certain mass flow rate, the phenomenon of surge appears inverting the flow direction in the machine.

Characteristics-Compressor II

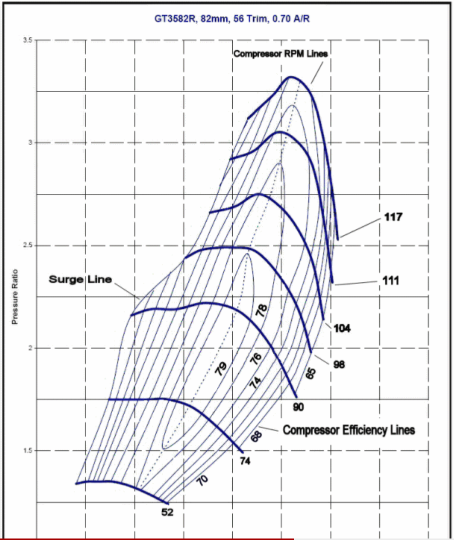
Example of realistic pressure ratio vs. reduced mass flow rate curves



G.Biswas – Indian National programme on technology enhanced learning

Characteristics-Compressor II

In order to determine the operating point, one then superimposes the island of iso-efficiency as represented below.



Garrett GT3582R Compressor Map Source: http://www.epi-eng.com

The dotted line up the center of the map is the peak efficiency operating line: the maximum available efficiency for each combination of airflow and pressure ratio.

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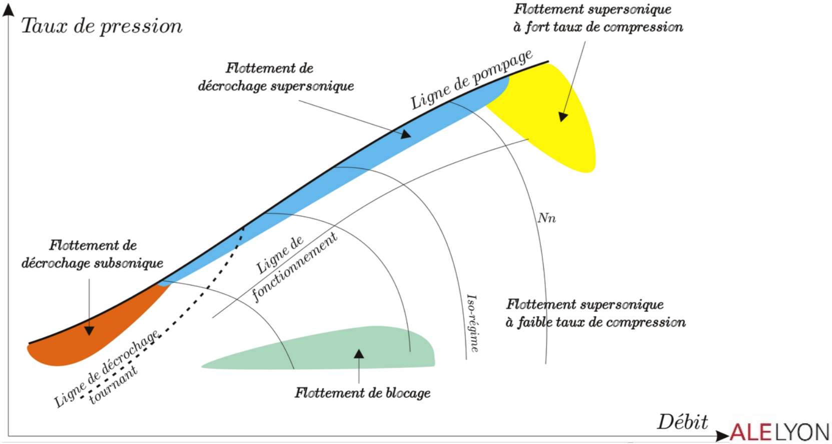
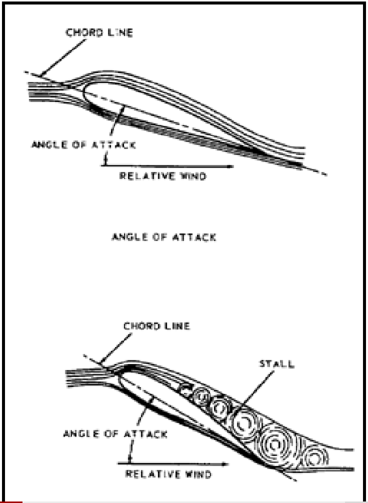
Instabilities in compressors

Let's have a look at the typical types of instabilities hindering the pressure ratio when varying the reduced mass flow rate.



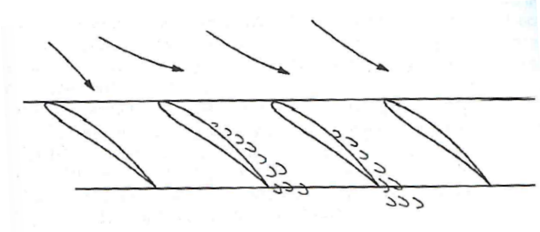
Instabilities-Stall

When the flow coefficient is reduced, the incidence angle on the blade increases. As for a regular wing, beyond a certain value, there is a risk of stall as illustrated below



Instabilities-Stall

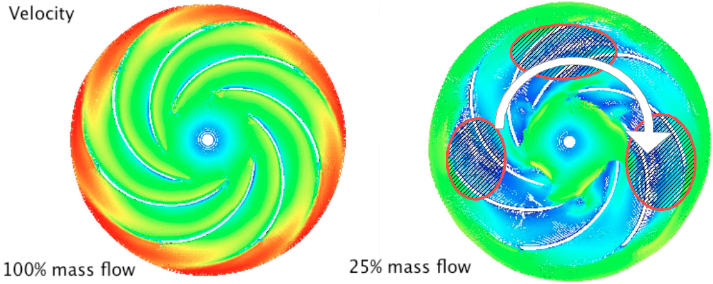
Most of the time stall is a rotating phenomenon.



The reason is that due to the blockage created by the stall, the incidence is further increased behind the stall cell propagating the stall. As a consequence, the rotating frequency of a stall cell is lower than the rotating velocity of the compressor (Usually 20 to 70%)

Instabilities-Stall

Example of stall in a centrifugal compressor



Ruprecht, Ginter and Neubauer; University of Stuttgart

The relative motion of the stall cells in the rotating frame of reference is a clockwise rotation at 30 to 80% of the rotation speed.

Review of stall measurements in axial compressors

Single-Stage Compressors						
Type of Velocity Diagram	Hub-tip Radius Ratio	Number of Stall Zones	Propagation Rate, Stall Speed, abs/ Rotor Speed	Weight-flow Fluctuation during stall, $\Delta\left(\frac{\overline{PV}}{PV}\right)$	Radial Extent of Stall Zone	Type of Stall
Symmetrical	0.50	3	0.420	1.39	Partial	Progressive
		4	0.475	2.14	↓	↓
		5	0.523	1.66		
	0.90	1	0.305	1.2	Total	Abrupt
	0.80	8	0.87	0.76	Partial	Progressive
		1	0.36	1.30	Total	Abrupt
	0.76	7	0.25	2.14	Partial	Progressive
		8	0.25	1.10	↓	↓
		5	0.25	1.10	↓	↓
		3	0.23	2.02		
		4	0.48	1.47	Total	
		3	0.48	2.02	↓	
		2	0.49	1.71	↓	
	0.72	6, 8	0.245	0.71=1.33	Total	Progressive
Free	0.60	1	0.48	0.60	Partial	Progressive
vortex		2	0.36	0.60	Partial	Progressive
		1	0.10	0.68	Total	Abrupt
Solid	0.60	1	0.45	0.60	Partial	Progressive
body		1	0.12	0.65	Total	Abrupt
Vortex	0.50	3	0.816		Partial	Progressive
transonic		2	0.634		Total	Progressive
	0.50	1	0.565		Total	Abrupt
	0.40	2			Partial	Progressive

Instabilities-Surge

Surge is a global aerodynamic instability of the compressor when the mass flow rate decreases below a certain threshold.

- The mass flow rate is too small to counter the adverse pressure gradient
- The flow changes direction globally almost instantly inside the compressor
- When the pressure gradient decreases enough the compressor recovers its normal function
- etc...

Surge is feared by manufacturers as it creates large yaw that even make pilots think a bomb has exploded.

Instabilities-Surge

Consequences of surge can be catastrophic such as

- Compressor blade failure
- Large radial vibrations with destruction of sealing systems in centrifugal compressors
- etc...

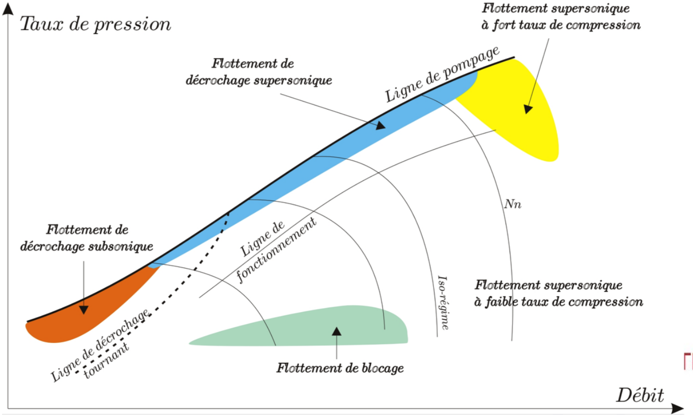
Specific systems are therefore designed to automatically react in case of surge but in any case the most used technique consists in having a "margin to surge" large enough so that surge is never experienced.

Instabilities-Surge

Instabilities-Surge

Instabilities

Most of the time, surge is predated by blade vibrations associated with fluctuations of the flow. Many aeroelastic instabilities limit the compressor map as seen below. In practice, frequencies associated with these instabilities are related to the product of the blade passing frequency (BPF) by the number of blades in the stage concerned.



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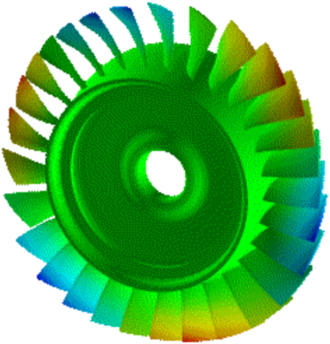
Instabilities-Flutter

Flutter is a dynamic aeroelastic instability wherein at a particular flow speed a self-sustained oscillation of the structure persists.
A further increase in flow speed leads to oscillations of the structure with increasing amplitude. Flutter occurs because the blades can absorb energy



from the airstream.

Structural Dynamics and Vibration Laboratory; McGill University

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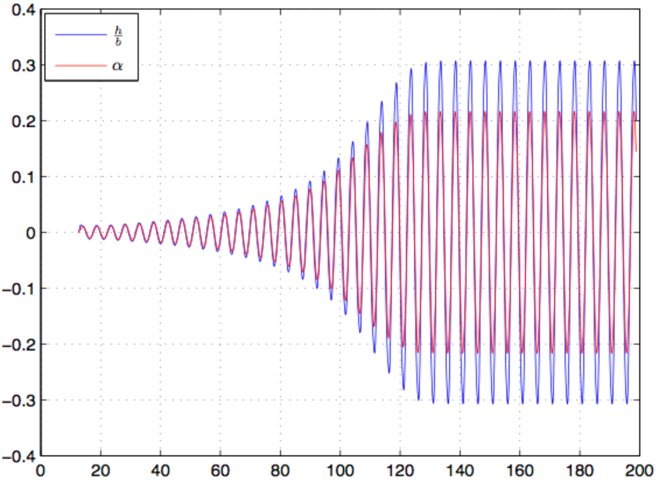
Instabilities-Flutter



Pradeepa, Venkatraman; Indian Institute of Science, Bangalore; ICAS2012

Instabilities-Practical case

An axial compressor (on which a lot of people rely!) is subject to malfunction. Your work is to propose a scenario of anomaly and a remedy

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This compressor has seen 3 of the blades of the 5th stage deteriorate significantly during the last 10 hours. You cannot stop the machine until we are sure to know what is going on, as the cost would be prohibitive... You have access to dynamic pressure sensors on the hub at stages 1,2,3,4,5, and at the shroud in stage 8. These sensors provide the spectrum of the pressure fluctuation at the point of measurement.

Instabilities-Practical case

This table provides you with number of blades for each stage

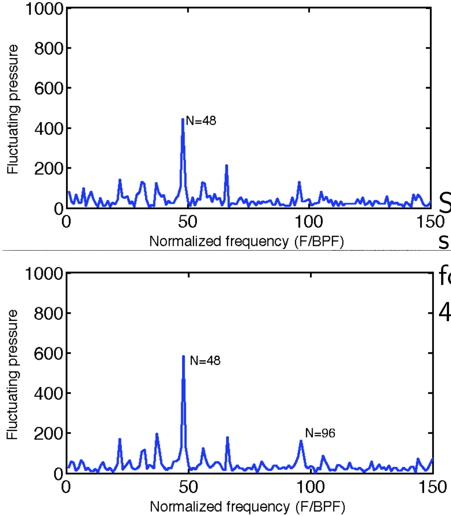
Stage Number	1	2	3	4	5	6	7	8
Number of blades	22	27	31	37	48	57	67	86

Measurements showed no noticeable fluctuations at stages 1,2,3 and 4...

Here are your follow-up measurements...

a)

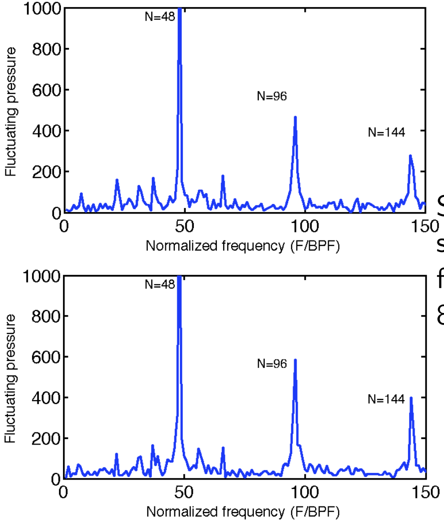
b)



Spectrum of fluctuating pressure at stage 8 (shroud) for two rotation speeds a) 4100rpm, b) 5400rpm.

Instabilities-Practical case

Here are your measurements...

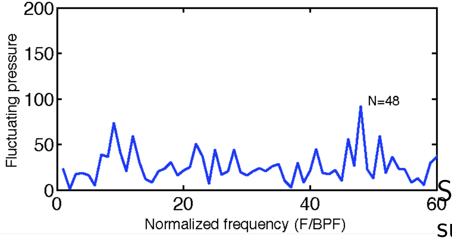


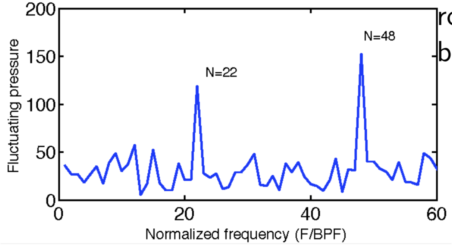
a)

b)

Spectrum of fluctuating pressure at stage 8 (shroud) for two rotation speeds a) 8000rpm, b) 9400rpm.

Here are your measurements...

a) 

b) 

Spectrum of fluctuating pressure at stage 5 (hub) for two rotation speeds a) 5800rpm, b) 6800rpm.

Some questions to put you on track:

- What are the facts that make you think the origin of the problem is in stage 5?
- What are the fact against such a conclusion?
- Which new measurements would you need to be completely sure of your conclusion?
- What do you think most probably causes the malfunction (which stage, which physical phenomenon?)
- What changes would you propose to permanently get rid of the malfunction?