2009-01-0307

Turbomachinery Performance Modeling

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ABSTRACT

Analytical modeling of turbomachinery components and systems has been used for more than a century to develop new machines and understand internal flow states. Flow modeling basics are reviewed in this survey including a summary of the flow processes observed in nature. The development of a variety of different loss, blockage, and deviation models is reviewed, and the complexity of mathematical data processing and model development is presented. Examination of different modeling philosophies is given with critique of the consequences. Examples of data matching, modeling for design work, and modeling uncertainty are given. Suggestions for future improvements are offered.

INTRODUCTION

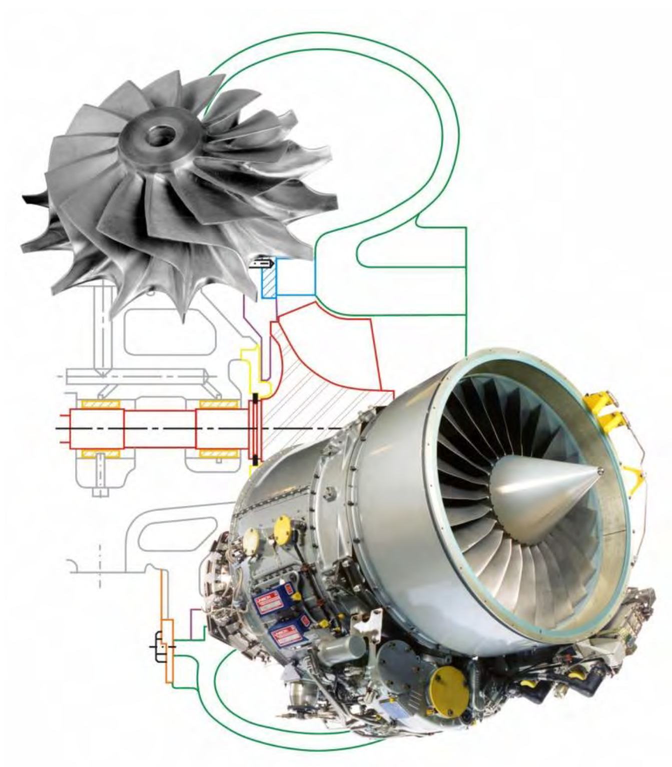

Figure 1. At the heart of transportation: *turbomachinery*!

WHAT IS A TURBOMACHINE? - The world relies on power machinery for most aspects of daily living. Turbomachinery and positive displacement equipment are the two essential types of mechanical power machines. One need only look at the formulation of Newton's laws, extended to describe mechanical work, to see the difference. When work transfer is most easily described by multiplying force times applied distance, F x L, then one is working in a Cartesian or curvi-linear coordinate system and is describing a positive displacement machine (even when L is along a curved path). When work transfer is most easily described by multiplying torque times angular velocity, $\tau \times \omega$, then one is working in a cylindrical coordinate system (for a process that is mostly axisymmetric, at least nearly so at its design condition) and is describing a turbomachine (turbo – Latin for „to spin or rotate'); see Figs. 1-5.

Figure 2. New competitive turbomachinery is optimized with the best extant performance models. Market survival demands breakthroughs in efficiency, stability, durability, and cost – all requiring conceptual models.

THE WORLD OF TURBOMACHINERY - The world of turbomachinery is vast. Most electricity is generated by an electrical generator driven by a turbine (gas, steam, or hydraulic). Even when an internal combustion (IC) engine (positive displacement) is employed, it is apt to utilize a turbocharger in order to meet modern performance criteria, and hence, turbomachinery is again involved. Most electric power generation uses turbomachinery, and generation today demands high-performance aerodynamics. However, vast amounts of power are generated without electricity appreciably being involved: IC engines drive most cars, trucks, ships, and various construction, mining, and transportation equipment, and, once again, nearly all of it is turbocharged except for some millions of cars each year, but even that is changing (about 20 – 30 million turbochargers are manufactured each year).

Quite apart from the power production side of the energy equation, it has been estimated by EPRI (Electric Power Research Institute) that about 3% of all power in the US is used just to drive turbo pumps (see Figs. 3, 5), and to this we must add fans, blowers, process compressors, and refrigeration compressors, all of which account for billions of units worldwide.

Figure 3. Billions of pumps meet the needs of global society; many are very common in style; some, such as this marine firefighting pump, require advanced modeling to achieve aggressive targets.

Finally, do not forget the aircraft industry (see Figs. 1, 4) with turbofan engines and ships employing turboshaft engines that consume over 5% of all liquid fuels.

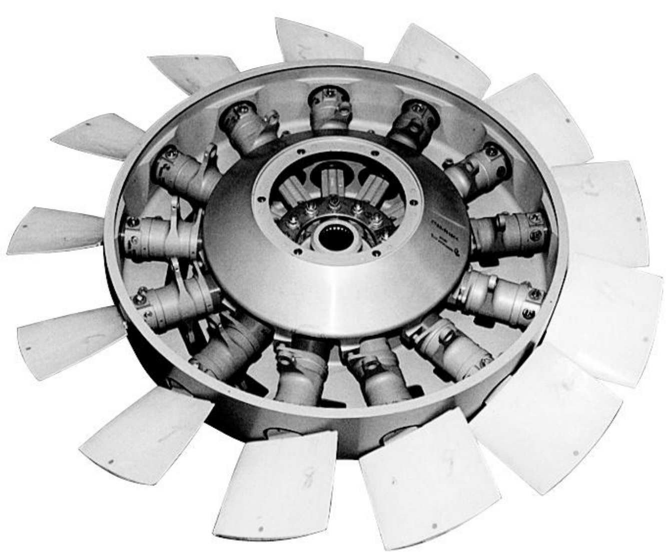

Figure 4. Aerospace engineering is a key to world commerce and defense and has a clear energy impact; this compressor for a no-tail rotor (NOTAR) helicopter is a result of careful system modeling.

ENERGY IMPACT OF TURBOMACHINERY – Considering the machinery world outlined above, imagine what an improvement of just 1% in efficiency would do to the economy! This might be hard to do on many aircraft engines and a few power turbines, but 5% or more can be achieved in many other cases. Such a number is very hard to calculate, but just thinking about it staggers the imagination! Significant parts of the industrial turbomachinery world place little emphasis on efficiency even today.

IMPACT ON WORLD ECONOMY - Impact is not hard to see, but hard to guide and quantify. Pumping is essential for civilization: bringing clean water to people and taking away and processing waste must be done. Lacking this, disease is rampant and death follows, as frequently observed when a natural disaster strikes and wipes out infrastructure. Managing storm water is also important, but less so by comparison except for low land areas such as Holland, New Orleans, and Bangladesh.

Transportation without turbomachinery would take us back to about 1920 or so, with low compression Diesel and gasoline IC engines and steam locomotives; we would only produce a small fraction of today's electric power and would fly only at low altitudes with piston engines.

In short, *turbomachinery is essential for life as we know it today: for clean water, sanitation, transportation, petrochemicals…the count is endless!*

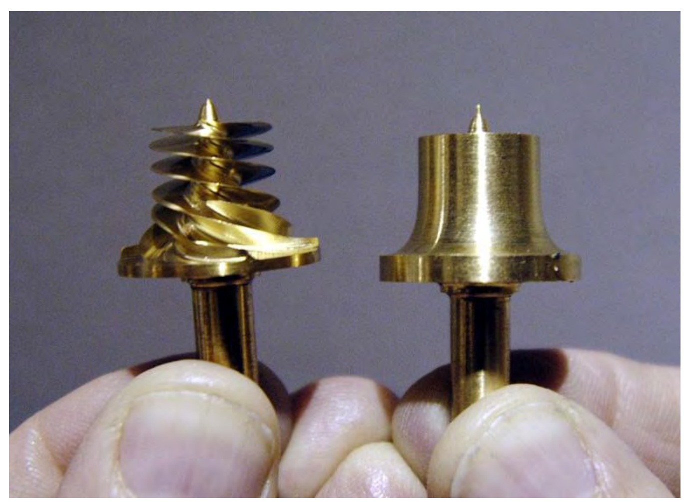

Figure 5. This miniature inducer pump may potentially impact many industries; when translating designs to very small scales, modeling takes on critical issues.

WHY MODEL THESE MACHINES? - So how do we design better machines? How do we extend water handling (clean drinking and sanitation) to all parts of the world? How do we sustain travel, freight transportation, agriculture, construction, and so on with ever more efficient machinery? We do it by using good math and science (engineering) based on solid principles of turbomachinery performance (and we teach new engineers this process, also a major issue but not part of this review).

Analytical and empirical models of nature result from engineering studies with the application of science and mathematics to specific processes. If these models have integrity, then we can explore a useful domain of nature and attempt profitable optimization studies. If our models lack integrity, then any such attempts at design risk wasteful confusion or outright failure (which happens from time to time).

Such performance models can be put together in various ways, and these ways rarely have been critiqued. Hence, we know little about the relative strengths and weaknesses of such schemes. It is the objective of this study to focus the types of modeling procedures, explore the underlying mathematics, contrast the work of various recognized authorities, and offer some detailed comparisons. It is hoped that discussion will be started that will yield better designs and design procedures in the future.

MODELING OVERVIEW

FUNCTIONS SERVED - Performance models are utilized by engineers in multiple ways. Principal ones are design, redesign, mapping, and scoping.

Design - When used for design (see Fig. 6), modeling tools must be flexible to handle a variety of different modalities. Sometimes a designer is afforded the opportunity to pursue a clean-sheet original design, and for this, the performance models must be highly generalized and in rigorous keeping with basic principles of thermodynamics and classical fluid dynamics. The designer must explore a large design space and carry out sensitivity studies to assess the impact of a wide variety of performance variables.

Redesign – The modeling task is quite different if the designer is asked, for example, to create a new stage for a multistage pump or compressor (Fig. 7). In this case, the design is highly constrained and the designer may have only a limited degree of freedom with respect to the actual blade angle distribution, as the „eye' and discharge of the impeller may be tightly constrained. The performance models are still important, but may be used only to assure that enough head or pressure rise will be available, and that power requirements are acceptable. Standard product designs are similar to the multistage design option, but may offer a bit more freedom to the designer. For example, many pumps are redesigned for improved performance by using a new impeller in an old casing (volute) due to cost. The impeller exit is still highly constrained, but the eye or inlet may have more freedom of choice for the designer. In this case, the flexibility of the performance models may again be exploited.

Since redesign of a turbomachine is usually restricted to a tight geometric envelope, geometry optimization for best efficiency is often a matter of low importance. Nonetheless, estimating the expected efficiency, stability, and pressure rise may be a great challenge, since the operating conditions may be anything but standard or common, and the redesign operation may push into extreme areas for performance (e.g., an old compressor rerated for more flow and power will drive Mach numbers up, perhaps into a region of real concern for the product). To do this, reliable performance models are needed.

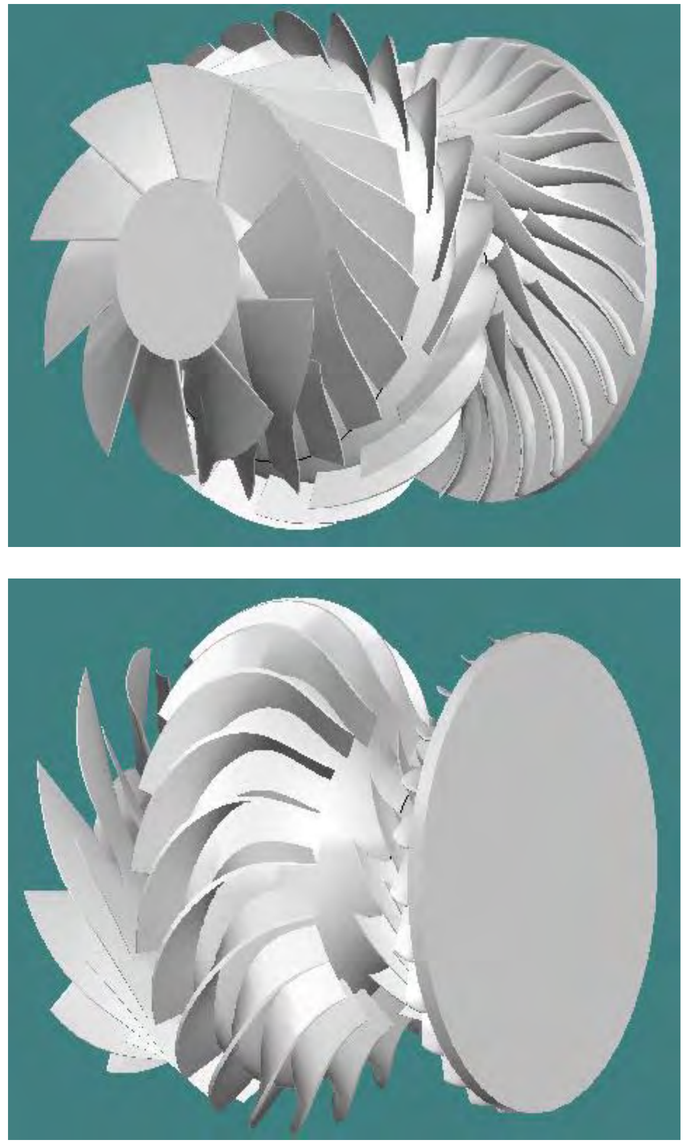

Figure 6. A 700 SHP gas turbine compressor design; good modeling is needed at each step of the design process. Conceptually, every step in the design process is open to scrutiny until optimum choices are made.

Mapping - Whether a new machine is a clean-sheet design, a replacement product redesign stage, a rerate, or even just an existing stage that needs review, it is always necessary to be able to model (predict) the entire performance map of a machine stage, even when all details of blading shapes may not be available (see Figs. 8 & 9 blind prediction before construction; client overlay). Therefore, the analytical process of map prediction from performance models is essential for all designers and many users of turbomachinery stages.

Figure 7. A rerated process compressor redesign; modeling demands are highly constrained: Impeller diameter and passage widths are specified *a priori*.

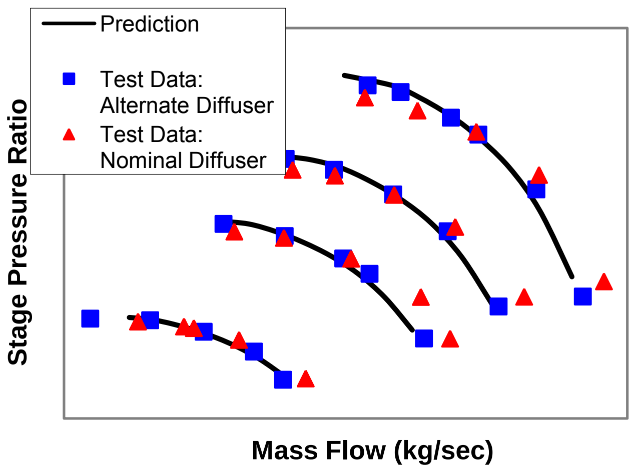

Figure 8. Mapping of new design calculations (1D) and final stage measurements of pressure rise.

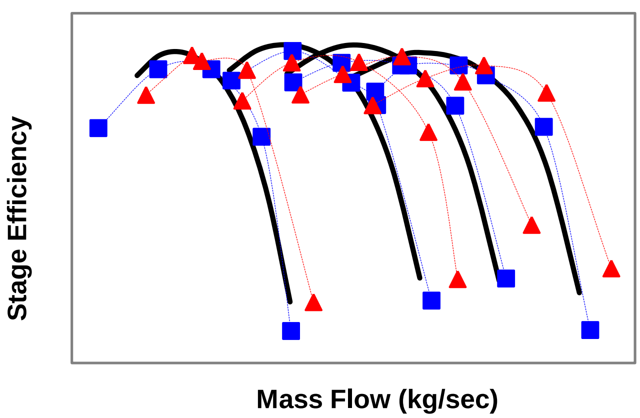

Figure 9. Mapping of new design calculations (1D prediction) and final stage measurements of efficiency.

Scoping - Scoping studies must also be mentioned. Even before a new stage or machine is seriously committed to the design process it is not uncommon for multiple scoping calculations to be made in order to set wise design criteria for subsequent work (see Fig. 10 example). Such scoping studies demand strong performance models to guide the study tradeoffs.

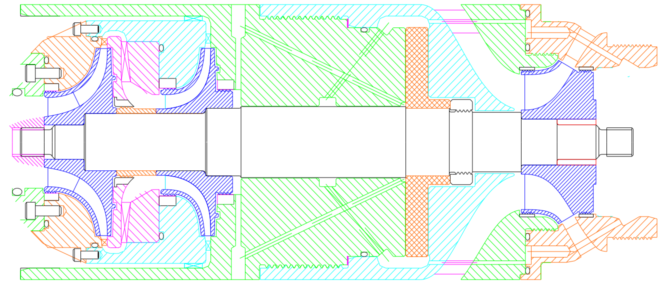

Figure 10. An *avant garde* conceptual design of a compact, turbine-driven, gas compression system; excellent modeling of each component is imperative just to decide conceptual feasibility (scoping studies).

Note - These examples cover the world of preliminary design, always to be followed by detailed design to assure that each component is properly optimized with detailed codes. The examples given in this and the next sections utilize one-dimensional (1D) or meanline models of performance; at the level implied in these examples, this is the correct modeling type.

MODELING LEVELS - Performance modeling is effected at multiple levels.

1D - Much of the optimization design work described above is best achieved with one-dimensional (1D) models of machine performance, usually steady-state or time-averaged. They are fast, general, and easy to use. Determining the power level for a given stage is essentially a 1D problem: the Euler equation shows that one must get the flow angles correct (deviation or slip) to set power levels properly.

2D - Two-dimensional (2D) models are usually used for a hub-to-shroud or a blade-to-blade analysis (per the Wu approach of 1952 [1]) or for two-dimensional boundary layer (viscous) analysis. These performance modeling tools are valuable for optimizing blade and duct shapes, but do not predict performance unless they are viscous (and then they only give a slice of the machine). Empirical loss models may be applied on a layer-by-layer approach through the flow field; the simpler 1D models are usually adapted for this purpose. Such 2D tools are still quite helpful, but often do not add much original content to the performance modeling/prediction process *per se*.

3D - Computational Fluid Dynamics (CFD) or viscous three-dimensional (3D) modeling tools are readily available, fast, and cheap to use today. They are a great help for studying the structure of a flow field and deducing detail concerning the creation of losses, blockage, and deviation. Nonetheless, they are best used later in the design process after suitable trial designs are created, and they are still subject to modeling concerns (e.g., there is no universal turbulence model to be used for all flow situations today). Calibration to specific tasks is still recommended. CFD is discussed more later in this paper.

MODELING OPTIONS - Performance models have been developed along differing lines of thought and following different norms or standards. Two description methods are used herein. Referring to how empirical data is used in the modeling, there are heuristic models, data-driven physical models, and data-driven global models. One can also distinguish the way conservation equations are written by referring to single-zone and two-zone modeling. We shall look at these and also consider how information from related but different disciplines may influence good model development.

Heuristic models (definition: *heuristic - using or arrived at by a process of trial and error rather than set rules; procedure for arriving at a solution but not necessarily a proof* – Encarta N.A. Dictionary) – are very common in turbomachinery development. Historically, it worked something like this: engineers had a need for understanding a process, postulated a possible set of relationships to describe the machine, set a few constants, and a new model was born. If it seemed to work out in design practice, then it became more and more entrenched over time. Usually, such models are built on concepts that cannot be taken apart element by element and tested in the laboratory on the merits of each element (examples are evaluated below). Their value rides on their apparent overall utility. They are very common.

Data-driven physical models - are restricted to a specific domain, defined either by a specific set of performance parameters (data) or by a particular geometry class, and by basic principles of physics. They can be very detailed about the specifics of a particular process and can yield considerable fidelity. They are built on well-accepted physical models of fluid mechanics and thermodynamics (physics-based).

Data-driven global models - are overall process models based explicitly on overall performance data and correlated according to statistical methods. They offer little or no fine-scale detail of the process modeled, and the physics is usually limited to the choice of governing parameters, often with good underlying mathematics (statistics).

Single-zone models – are common calculations of mass, energy, and momentum on a station-by-station basis treating the entire flow process between stations as a single, average flow process. This is also referred to as meanline, pitchline, plug flow, or slug flow analysis in various disciplines.

Two-zone models – are station-by-station calculations of mass, energy, and momentum using an isentropic stream (core flow) and a non-isentropic stream (average of all loss bearing streamtubes) between two stations with full mixing calculations at each end station. This approach evaluates the level of fluid dynamic blockage in each element and distinguishes between the deviation of the primary and secondary zones just before mixing.

There are important related physical processes that should be considered when constructing or evaluating performance models. Turbomachinery is a class of machinery that is very closely related to various classical aspects of fluid mechanics. Diffusers, nozzles, bends, flat plates, and ducts (stationary or rotating) all have points of relevance in modeling turbomachinery performance. In some cases, these basic elements actually exist in a turbomachine, and in all cases, they reflect basic fluid dynamic performance that will be evidenced at least asymptotically while modeling a turbo flow problem. Consider a rectangular cross-section bend for example: this is just a simplified version of an axial turbine guide vane or a centrifugal compressor or pump return channel vane. Even a refined classical model such as Couette flow has its counterpart in film seals and in journal bearings, and the boundary layer analysis of flow on a rotating disk is at the heart of all disk friction models used today. All the knowledge from these classical studies must be brought to bear on turbomachinery modeling to test asymptotes, to check exponent relationship trending, and to test for overall coherence.

NATURE'S PROCESSES - We must, however, be specific about the particular processes under evaluation in turbomachinery modeling. These have been well-identified and broadly accepted over the past years, and common textbooks differ little on the following listing (the illustrations used below are taken mostly from axial turbines, but apply equally well to all classes of turbomachinery):

Profile (Boundary Layer Shear) loss – The thin viscous shear layers along flow passage surfaces gives this loss. The frictional loss along the blade surfaces is usually called the profile loss (see Figure 11).

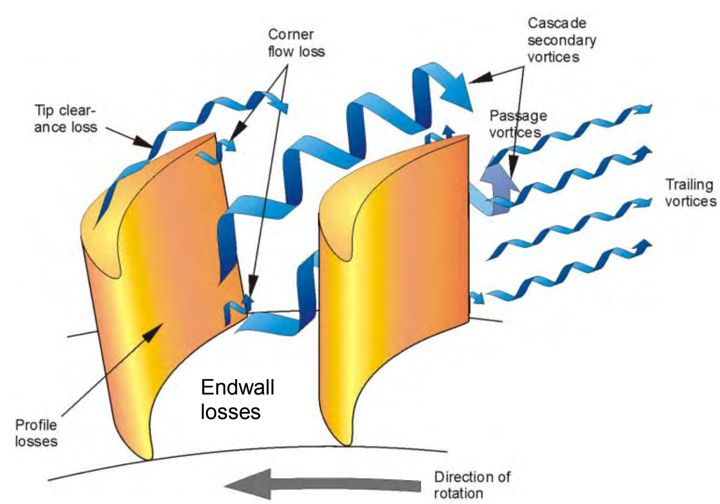

Figure 11. Illustration of profile losses (surface boundary layer development) plus elements of secondary flows.

Secondary flow loss - The end wall flows in a bladed passage develop vorticity due to the transport of fluid elements of different velocity levels through the turning pressure gradient (loading) and cause this loss; see

Figure 12. This gives a highly-structured flow field, but not one that can be used effectively to derive useful work in the subsequent flow elements. Hence, much of the kinetic energy associated with such processes is lost (but may have a secondary benefit in triggering earlier transition in various downstream regions, hence, reducing losses a bit later in the process). The area at blade row exit taken by these secondary flows is on the order of 20% to 40% for axial blading and 30% to 60% for radial blading near design point operation.

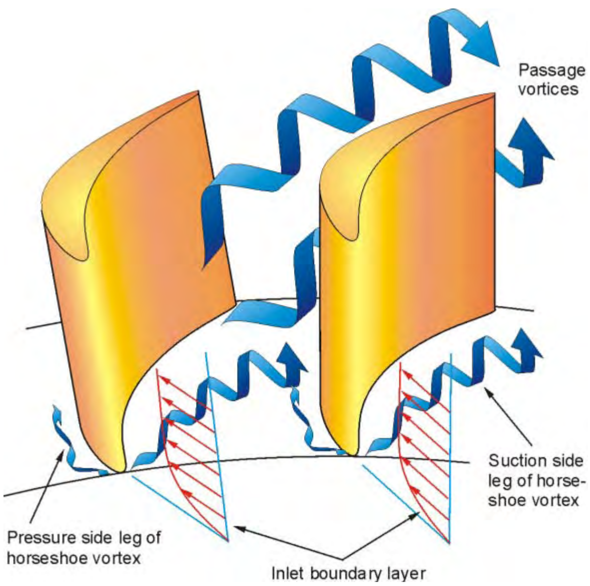

Figure 12. The development of secondary flows by introducing profiles into a turning flow field.

Clearance loss - In many modeling systems clearance effects are book-kept separately. It is a separate process, and it may involve tip clearance spillage for open blades or leakage through seals and back into the flow path, with a subsequent mixing loss, for covered wheels. In practice, this might better be considered as part of a secondary flow loss, as open wheel clearance flow is swept into the larger vortex already present and might be more a variable influencing the secondary flow (Figure 13).

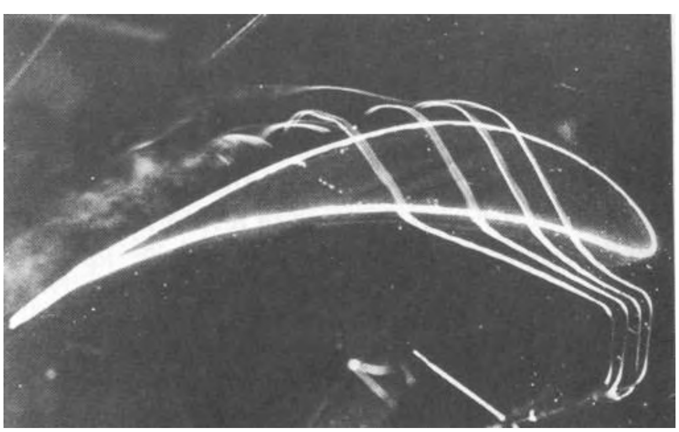

Figure 13. Illustration of tip clearance leakage. Notice how the clearance flow is swept into the passage vortex or secondary flow.

Exit mixing loss; base pressure loss - A proper mass, momentum, and energy exit mixing calculation should be made at the exit of every bladed row so that penalty of momentum (profile) deficit is charged to the respective row, and so that there is no misrepresentation about the level of useful total pressure available for subsequent rows. Warner Stewart seems to have introduced this approach in 1955 [2], although the broad concept was clearly known earlier. For many early correlations, this was done somewhat casually by having a rough estimate of exit wake size and the level of associated loss (usually a heuristic model). If the exit profile variations are strong, including secondary flows, they should be fully mixed out through a proper calculation using the full mass, momentum, and energy equations (two-zone modeling). However, the problem is even more complex. For thick trailing edge shapes, such as one finds for cooled axial turbines and many industrial centrifugal compressors and pumps, the flow physics can get quite involved even in a very small region (see Figures 14 through 16). For the latter cases, it is mostly a matter of getting deviation (or slip) correct for the power calculations. For the axial turbine under highly-loaded operating conditions, however, the base pressure problem is more than just an exit loss matter. In this case, the structure of shock waves, exit flow angle, and even exit blockage and vorticity can be interconnected and difficult to model.

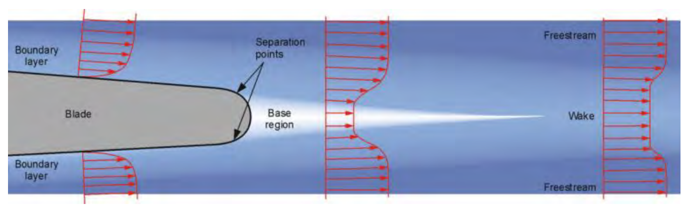

Figure 14. The development of the exit mixing process for surface shear layers.

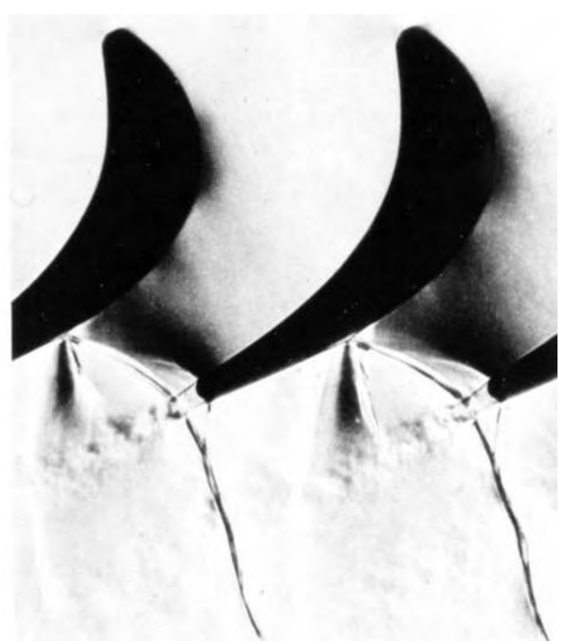

Figure 15. Exit shock system for an axial turbine cascade M = 0.98.

Figure 16. Exit shock system for an axial turbine cascade M = 1.33.

Shock losses - For highly accelerated flows, local compression waves coalesce into shock waves, and entropy is generated. The associated losses must be correlated and included in competent flow models. Detailed analysis requires sophisticated CFD solutions, and the flow fields occasionally may be unsteady. The shock wave patterns are complex and vary with stage back pressure, as shown in Figures 15 and 16.

Leakage loss - This loss may be the same as the clearance loss. For some studies, leakage through seals may be book-kept separately under this name. When complex cavities or thrust balance devices are involved, multiple leakage paths may require evaluation.

Disk friction - Disk friction on rotating disks must not be neglected. It is a classical boundary layer flow problem but can be studied well with CFD, especially when the associated cavities are also modeled. For low specific speed pumps and compressors, disk friction can emerge to be the largest loss of all the mechanisms!

Separated flow losses - Ideally, this loss would be built into the mechanisms outlined above as a natural part of off-design performance. Usually this is difficult to do and is frequently an add-on model element. This is especially true for the off-design performance of radial flow machines (recirculation loss) which can perform quite stably even with considerable backflow or recirculation. The effect should be included before a proper mixing calculation is made. Turbine partial admission losses are another example of this loss.

Incidence effects - Some investigators have created an incidence loss model to stand alongside all the other loss items listed above. The *implicit* presumption is that incidence is inherently non-isentropic, and therefore, a loss results; but this is false. Incidence is just a turning or bending of streamlines as they enter a bladed row. It is certainly true that high levels of incidence lead to higher levels of profile and secondary losses, and should therefore be included in such models. In fundamental loss model development, it should not be necessary to build a separate incidence model, but this is often done in heuristic model applications.

Cavity losses - Such losses are invariably minimized in the design process, but cannot be eliminated. They tend to defy simple, general correlation but have received some attention in the book by Seleznev, *et al.* (1986) [3] dealing with cavity inflow and out flow problems.

Annulus Loss – Also known as interstage duct loss, this loss accounts for shear stresses along the duct between stages; it may be coupled to cavity losses as well.

Part Span Loss – Part span shrouds and lacing wires also introduce entropy and must be modeled by empirical correlations.

Wetness and Condensation – A number of machines operate with two phase flow (e.g., steam turbines) and losses associated with phase change, entrainment, and liquid or gas removal must also receive attention.

Note - The radial turbine field, see below, uses an incidence loss model. It is a heuristic model with little or no internal data extant to support higher level modeling. The incidence model works with a major correction and is essential, but it is probably true that if sufficient data were ever recorded to build detailed data-driven, physics-based models, the current incidence loss model would be replaced by a strong secondary flow loss model with incidence as a very strong independent variable. Our problems with such incidence loss models may stem back to Spannhake (1934) [4] who introduced the concept and it needs a careful review; we would do well today to concentrate as much as possible on physics-based, data-driven models focusing on each component of the flow process whenever possible.

PROBLEMS ENCOUNTERED – Several major model development problems can be recognized across all types of turbomachinery. Key issues are given below.

Coupling of effects - The first problem encountered with some models is the lack of coupling of all pertinent physical processes which must include losses, diffusion, blockage, deviation, and unique profiles (the latter including velocity, turbulence, and vorticity). Many models consider just one of these processes even when it reasonably may be expected that coupling should be involved. A clear example of this fact was presented by Japikse in 1986 [5] when diffusers and nozzles were mapped for loss, recovery, and blockage *together,* and the simple relationship used already for decades, $K = C_{pi} - C_p$ was finally corrected as $K = C_{pi} - C_p \pm \Delta$, where Δ can be 0.05 or even 0.10 (a non-negligible correction in various turbomachinery applications);

examples were given where this difference would have large impact in a design. Δ is the correction for velocity profile differences from case to case. Turbulence and vorticity were absent from that study. For turbomachinery, these effects are more important and must be considered when accurate modeling is desired.

Exit mixing – Proper treatment of exit mixing has only rarely been pursued. To understand this problem, albeit indirectly, one should study the work of Osterwalder and Hippe (1982) [6]. These authors studied a large collection of centrifugal pump data sets in order to learn more about pump performance. They made important insights about frictional effects (see later discussion) and about the split in hydraulic losses, finding that the latter is *often comprised of large non-frictional effects and less than half are frictional*. These non-frictional effects exclude cavity losses, but would include secondary flows, exit mixing, and leakage. This will also be true in the detailed axial turbine example given below, although arrived at by a different approach. A key point must be made: simple frictional effects give rise to thin or moderate boundary layers with low exit blockage; secondary flows, separated flows, and other distortions give much greater blockage which is often overlooked. Although not widely recognized, the exit blockage from a full length, optimum divergence angle, diffuser (and hence, one that is well into transitory stall) is on the level of 35 to 65%, depending on inlet conditions. Mixing losses from such exit distortions are appreciable.

Model limits - Stall (or surge), choking, cavitation, and limit loading are limit flow problems which also must be considered. They must enter into performance models so that rational operational bounds can be set. This topic is very important in its own right, but goes beyond what can be considered in this presentation; it is hoped that the author may extend comments in a later study.

Irrational summations - Irrational summations occur when one pieces together less than perfect performance models that have not balanced loss, recovery, blockage, and deviation all together, especially when approaching any of the limits given in the paragraph above. A good example is found when operating a steam (or just gas) turbine to very high pressure ratios where Mach numbers may reach 2 or 3 or even higher at discharge. For these cases, deviation must be forced away from traditional correlations and made to follow the states dictated by one-dimensional, compressible flow conservation relationships. Another example occurs when single zone models are compared to measured row static pressures; to get agreement some engineers add an arbitrary level of blockage to the calculation and then do not mix out the blocked flow region, violating the basic control volume boundaries.

MODELING STATUS IN MAJOR TURBOMACHINERY DISCIPLINES

PART 1: GENERAL SURVEY – Four major classes of turbomachinery are identified below, and a short description of principal efforts at modeling is given. The reader is also directed to earlier surveys, Dunham, 1970 [7] and Denton, 1987 [8], in which 1) secondary flow loss data sets are reviewed and 2) a physical understanding of the process of loss generation is outlined, respectively in the works cited. By contrast, this survey compares model building efforts across major turbomachinery areas and it attempts to consider comparative strengths and weaknesses (with an eye towards rigorous but practical design methods). Focus is on the historical roots of modeling; by scope, covering fine scale subsequent refinements is impractical.

Axial Turbines - Axial turbine research has enjoyed a lengthy and well-published history of performance modeling. Starting with a key paper by Ainley and Mathieson (1951) [9], nearly six decades of publishing and updating of axial turbine modeling has continued and has led to a good starting point for the loss modeling (only) of axial turbines. Subsequently, Dunham and Came (1970) [10] revised the secondary flow loss model substantially; this, in turn, led to more revisions by Kacker and Okapuu (1982) [11], Moustapha, Kacker, and Tremblay (1990) [12], and Benner, Sjolander, and Moustapha (1995) [13]. This continuous string of investigations is widely used and respected today (sometimes it is referred to as the AMDCKO system, although it could already be extended). There are, however, alternative loss modeling systems for axial turbines, and these need to be considered carefully as well. These would include Traupel (1962) [14], Craig and Cox (1970) [15] and various eastern models. Since the AMDCKO system is widely recognized and of very decent pedigree, it will be used for one of the examples given below. All of these correlation systems are partly heuristic, partly data-driven, and single-zone in their modeling mathematics. Deviation modeling is weak in these studies, and no attention is given to passage aerodynamic blockage. The methods are unable to distinguish between the deviation of the core flow and that of the secondary flow and are weak on exit mixing.

Axial Compressors and Pumps - The axial compressor field evolved differently from the axial turbine field. It preceded the turbine side, since work was conducted on cascade tests resulting in a large selection of pedigree design cascade options. Many of these sets were accompanied by measurements of downstream velocity defect and flow angle, hence giving profile losses and turning, and therefore, deviation. Nonetheless, these early studies did not yield durable models of loss, deviation, recovery, and blockage, all of which are necessary for general design and analysis work.

Comprehensive modeling first appeared in 1975, with the work of Koch and Smith (KS) [16], decades after the start of such modeling on axial turbines. These authors took a fundamental look at compressor performance and followed the lines of classical fluid mechanics in setting out models for profile loss and profile exit mixing. The profile effects were computed with a pedigree boundary layer code assuming fully turbulent flow (and they introduced a correction for low Re flows, i.e., laminar operation). With a realistic calculation of exit momentum thickness for both the suction and pressure blade sides, they made valid exit mixing calculations following the method of Stewart [2]. Consequently they largely avoided the degree of controversy (see discussion below) surrounding the Re treatment in the AMDCKO turbine loss modeling. They also introduced models for end wall or secondary flow loss, shock loss, and part-span shroud losses. The secondary flow loss was an extension of their prior work (Smith [17]) and concentrated on establishing end wall boundary layer displacement thicknesses under the influence of local leakage. It appears that these secondary losses were taken independent of Re effects and without exit mixing and it appears that they did not treat the issues of overall blockage and flow deviation. (Note: Dring [18] followed a similar boundary layer calculation process for the axial turbine, but it never seemed to catch on as it did in the compressor field.) This method is, at its core, a data-driven physical model, but also single-zone in its mathematical description.

This work was extended to modeling the stall pressure rise of compressors by Koch [19], again following the lines of classical fluid mechanics, but now using classical diffuser theory as their basis. Koch was able to develop a stage-averaged pitchline correlation using the recovery coefficient of annular diffusers with 9% inlet aerodynamic blockage. Corrections were introduced for the effects of Re, tip clearance, axial spacing, camber and stagger while also modifying the reference inlet dynamic head for effects of extreme velocity triangles. The model tested out very well against extensive machine data.

Additional work is reported by Miller and Wasdell (MW) [20] and Wright and Miller (WM) [21]. MW present a somewhat more simplified full loss system, compared to KS, which seems to function well, but without direct comparison to KS. They give a good treatment of deviation. In the WM work, the deviation model was further improved using both data and time-marching flow field calculations. Also, a blockage model was developed. In contrast to KS, their Re correction was applied to both the profile and secondary flow losses. These corrections were based on thin shear layer flows on flat plates, sensibly chosen for this problem. Extensive emphasis was placed on calculating the end wall boundary layer blockage development and understanding its impact on blade row performance. This method is a data-driven physical model and single-zone in its mathematical description. An excellent survey of axial modeling is given by Casey (1977) [22].

<u>Centrifugal Compressors and Pumps</u> - Various models for the radial flow work input machines have been used over the past several decades, although most of these have not been published and remain the special domain of individual companies. One example is the work of Gulich (1999, 2008) [23] focusing on centrifugal pumps. His work, drawing on a career at Sulzer, is quite complete, even though the numerous elements presented do not flow into a simple overall design process without interpretation and thorough checking. One alternative, however, has evolved systematically over 50 years, and it does tie losses, diffusion, blockage, and deviation tightly together. This is the so-called two-zone or jet-wake modeling of Dean *et al.* (1965, 1970) [24, 25] and the work of Japikse *et al*. (1985, 2005) [26, 27]. The latter method is one of the few rigorous data-driven physical models and is always two-zone in mathematical description; hence it handles passage aerodynamic blockage, deviation of both the core flow and also the secondary flow, and sensible exit mixing. When necessary, mixing at rotor exit can be suppressed and allowed to occur through the next element.

<u>Radial Turbines</u> - The radial turbine field is a fascinating counterpoint for the above examples. Little money (by comparison) has been spent for radial turbine model development. These turbines are used extensively for small turbocharger drives, cryogenic expansion, other industrial gas expansion, some hydro turbines, and some small gas turbine drives. By comparison to the other fields, this is a small area of activity, and it has not sustained the detailed level of interest and investigation that the other fields have. Nonetheless, some very valuable and quite interesting models have been developed. These models are heuristic and global in nature with single-zone modeling mathematics in all applied cases. The base concepts for loss modeling fall into two equations (with variations):

$$K_{inc} = \rho W_2^2 (sin^2 i_2)/2, \qquad \text{where } \ i_2 = \beta_2 - \beta_{2,opt} \qquad \text{[a]}$$

$$K_p = k\rho(W_2^2 cos^2 i_2 + W_3^2)/2 \qquad \text{[b]}$$

The first equation is a Spannhake type „shock' loss wherein the normal component of the relative kinetic energy entering the blade rotor is taken as lost but corrected by an offset angle $\beta_{2,opt}$ recognizing the unusual impact of strong inlet loading, which skews the inlet streamlines greatly. $\beta_{2,opt}$ has a magnitude of about 30 degrees but can exceed 50 and is empirical (no good design correlations to guide in the choice of $\beta_{2,opt}$). K_p is the passage loss and a variety of authors have refrained from making this a passage friction relationship and instead just use a simple fraction of passage kinetic energy. Two-zone modeling was successfully tested for radial inflow turbines by Japikse 1987 [28] but not developed for common usage. Except for the last example, no attention is given to passage aerodynamic blockage and the methods are unable to distinguish between the deviation of the core flow and that of the secondary flow and are weak on exit mixing.

PART 2: AXIAL TURBINE CRITIQUE - The discipline of axial turbine modeling has been chosen for the first example of this review. An examination of the basic equations is given first, followed by a critique of how they came about, and then followed by an examination of the trends that can be evidenced by these equations.

Modeling Process - Equations [d] through [u], given below, present the AMDCKO system as widely used by a variety of axial turbine designers (presented in a form that closely follows Moustapha, *et al.*, 2003 [29]).

Ainley and Mathieson first described the losses in a turbine as a simple summation of each individual effect listed above, and they used an arc cosine rule for flow angle (hence deviation) plus an overall machine Reynolds number correction. Their specific loss models are based on sensible physical concepts using global data; their choice of profile loss (eq. [e] below) is a mixture of guide vane and impulse stage data and is both inspired (it seems to have worked well for six decades with little criticism) and heuristic.

Dunham and Came modified the secondary loss model (hence called AMDC) and suggested that the losses be combined as:

$$K_T = \left(K_{P,AM}(1 + 60(M-1)^2) + K_S\right)(Re/2^* 10^5)^{-1/5} + K_{clr} \quad \text{[c]}$$

where the Reynolds number is now recommended for the stage, not for overall correction. Transonic drag rise was introduced with the Mach number correction (employ this term only for M > 1). This modeling is still single-zone, quite heuristic, and had sparse data available to guide the selections.

Kacker and Okapuu (KO) modified the earlier assumption of joining the loss elements (linear supposition of the losses) as shown in equation [d] below. Equation [e] for the profile loss comes directly from Ainley and Mathieson: a simple combination of two extremes – an impulse stage and an inlet guide vane. These authors used more data than their predecessors and carefully studied diverse turbine applications. They introduced acceleration [eqns. h, k, l] and transonic corrections [eqns. g, i, j] to the profile loss models based on trial and error (heuristic) studies.

Secondary-flow treatment by Kacker and Okapuu remains similar to Dunham and Came, giving equations [o] through [u]. The principal differences that they introduced are aspect ratio (h/c) corrections, equation [s], and acceleration effects, equation [t]. They also removed trailing edge loss treatment (hence, the 1.2 modification in equation [o]) from the profile and secondary losses and made it a linearly independent term as given by equations [v, w]. Kacker and Okapuu indicated that they were not at all sure of the Mach number correction, but went with it anyway. For supersonic operation, they assume that the trailing edge loss goes with the drag rise relationship which is back to profile loss again. All of these authors also dealt with tip clearance leakage, but this comparison will leave those details for further review by the reader from the original sources; the profile, secondary, and so-called trailing edge losses are sufficient to support a useful study of some details given below.

There is no blockage, deviation, or static-state change model (e.g. suction side pressure recovery) which accompanies these loss systems and no explicit exit mixing calculation. To a limited degree, the trailing edge

$$K_T = K_p f_{Re} + K_s + K_{TE} + K_{clr} \quad \text{[d]}$$

Uncorrected loss

$$K_p^* = \left\{K_{p(\alpha_{1b}=0)} + \left|\frac{\alpha_{1b}}{\alpha_2}\right|\left(\frac{\alpha_{1b}}{\alpha_2}\right)\left[K_{p(\alpha_{1b}=\alpha_2)} - K_{p(\alpha_{1b}=0)}\right]\right\}\left(\frac{t_{max}/c}{0.2}\right)^{(\alpha_{1b}/\alpha_2)} \quad \text{[e]}$$

$$K_p = 0.914\left(\tfrac{2}{3}K_p^* K_{accel} + K_{sh}\right) \quad \text{[f]}$$

$$K_{sh} = \left(\frac{\Delta p_0}{q_1}\right)_{sh}\left(\frac{p_1}{p_2}\right)\frac{1-\left(1+\frac{k-1}{2}M_1^2\right)^{k/(k-1)}}{1-\left(1+\frac{k-1}{2}M_2^2\right)^{k/(k-1)}} \quad \text{[g] [h]}$$

$$K_{accel} = 1 - K_2(1-K_1)$$

$$K_1 = \begin{cases} 1.0 & \text{for } M_2 \le 0.2 \\ 1 - 1.25(M_2 - 0.2) & \text{for } M_2 > 0.2 \end{cases} \quad \text{[i] [j]}$$

$$\left(\frac{\Delta p_0}{q_1}\right)_{sh} = \left(\frac{r_h}{r_t}\right)\left(\frac{\Delta p_0}{q_1}\right)_h \quad \text{[k]}$$

$$K_2 = (M_1/M_2)^2$$

$$\left(\frac{\Delta p_0}{q_1}\right)_h = 0.75(M_{1h} - 0.4)^{1.75} \quad \text{[l]}$$

$$f_{Re} = \begin{cases} \left(Re_{ref}/2\times10^5\right)^{-0.4} & \text{for } Re_{ref} \le 2\times10^5 \\ 1.0 & \text{for } 2\times10^5 < Re_{ref} < 10^6 \\ \left(Re_{ref}/10^6\right)^{-0.2} & \text{for } Re_{ref} > 10^6 \end{cases} \quad \text{[m]}$$

$$CFM = 1 + 60(M_2 - 1)^2 \quad \text{[n]}$$

Used per equation '*c*' for *M* > 1

$$K_S = 1.2K_S^* K_{CS} \qquad [o]$$

$$K_S^* = 0.0334 f_{AS}\left(\frac{\cos\alpha_2}{\cos\alpha_{1b}}\right)\left(\frac{C_L}{S/C}\right)^2 \frac{\cos^2\alpha_2}{\cos^3\alpha_m} \qquad [p]$$

$$\text{where} \quad \frac{C_L}{S/C} = 2(\tan\alpha_1 + \tan\alpha_2)\cos\alpha_m \qquad [q]$$

$$\text{and} \quad \alpha_m = \tan^{-1}\left[\frac{1}{2}(\tan\alpha_1 - \tan\alpha_2)\right] \qquad [r]$$

$$f_{AS} = \begin{cases} \dfrac{1 - 0.25\sqrt{2 - h/c}}{h/c} & \text{for } h/c \le 2 \\ \dfrac{1}{h/c} & \text{for } h/c > 2 \end{cases} \qquad [s]$$

$$K_{CS} = 1 - K_3(1 - K_{accel}) \qquad [t]$$

$$\text{where} \quad K_3 = (b/h)^2 \qquad [u]$$

$$\Delta\emptyset_{TET}^2 = \Delta\emptyset_{TET(\beta_1=0)}^2 + \left|\frac{\beta_1}{\alpha_2}\right|\left(\frac{\beta_1}{\alpha_2}\right)\left[\Delta\emptyset_{TET(\beta_1=\alpha_2)}^2 - \Delta\emptyset_{TET(\beta_1=0)}^2\right] \qquad [v]$$

$$K_{TE} = \frac{\left[1 - \frac{k-1}{2}M_2^2\left(\frac{1}{1-\Delta\emptyset_{TET}^2} - 1\right)\right]^{-\frac{k}{k-1}} - 1}{1 - \left(1 + \frac{k-1}{2}M_2^2\right)^{-\frac{k}{k-1}}} \qquad [w]$$

loss models of the previous authors gave some correction for blade row boundary layer exit mixing, but not explicitly. When the AMDCKO system is implemented, it may be necessary to modify the approach for very high exit Mach numbers, limit loading, etc. It should be mentioned that other systems are also available that are worthy of equal attention (such as Craig and Cox, *op cit.*), perhaps as part of a future extended study.

Studying the form of the above equations reveals a number of insights. From the introductory comments, it is clear that this is a partial system (no blockage, deviation, etc.), even though it has been widely researched and used. An assumption has been made that the basic core components are linearly related (K_T is a simple sum of four components), and that the Reynolds number dependency is only through the profile losses (KO but not AMDC); a natural question arises: why would the secondary flows be independent of Reynolds number, and what does laboratory data tell about this question? The treatment of Reynolds number has a large flat or dead zone in equation [m], but not in other systems; why would nature do this? (See Re discussion below.) Next, one should take a hard look at the full system of equations given above: the subordinate parts of the system are each highly nonlinear. The non-linearities show up in the turning angles, thickness effects, and Mach number. How would this come about in nature for a process where the key loss elements are assumed to be independent? Also, if a large base of performance data over a wide range of Mach numbers were available, would we ever deduce a Mach number dependency from the data corresponding to that of the above equations?

It may be fair to assume that part of the apparently good success that the AMDCKO system has achieved may be due to the comparatively short chord of axial turbine blades. With a short chord, the flow transit time is comparatively short, possibly implying that the time to truly intertwine the effects of secondary flow with leakage, etc., is reduced compared with other problems such as radial inflow turbines or centrifugal compressors. Hence, the absence of non-linear coupling of major effects, i.e., the appearance of a degree of linear supposition of effects, may have a factual basis.

Other contributions to this field include Binder and Romey [30], Okapuu [31], Denton and Xu [32], Gregory-Smith and Okan [33], Perdichizzi and Dossena [34], Li *et al.* [35] and others. Most of these provide vital refinements of detail but not a shift in the pattern given above. Okapuu's work shows a mild shock-boundary layer interaction that modifies the transonic regime and would mildly impact Figure 17 given below.

Ideally, one would like to turn to a large experimental database to see what nature is really doing in these turbines, but this poses a real problem: what data are available to support any conclusions concerning modeling? Interestingly, not much of a database has emerged for public comparisons. The original study, plus each succeeding study, used several dozen test cases, but the data sets were never shared publicly. Each successive investigator used new data, and was unable to incorporate much or any of the original data to check relationships (with the possible exception of the last two studies). It is intriguing to think what might have been learned if all data used at any point in the studies had been carefully organized and preserved in a database for each subsequent investigator. It is possible that we would not have kept the same formulation given above. Additional non-linearity or additional grouping of variables might have been discovered. We could be looking at trends that are different from those commonly accepted today if a larger data base were available.

<u>Application Example</u> - To study this matter in greater depth, a sample axial turbine design case was evaluated. The sample is taken from Japikse and Baines [36], pages 6-40 to 6-48. This is a single-stage axial turbine redesign problem with the goal of achieving increased power from the stage. The classic single-

zone or station-by-station meanline equations are used to compute the thermodynamic and kinematic variables subject to just the losses shown above in equations [d] through [w]. The intent is to show trends and variations in computed losses across a range of design variables.

Figure 17 shows the computed losses (design point) for this problem as a function of the rotor exit Mach number.

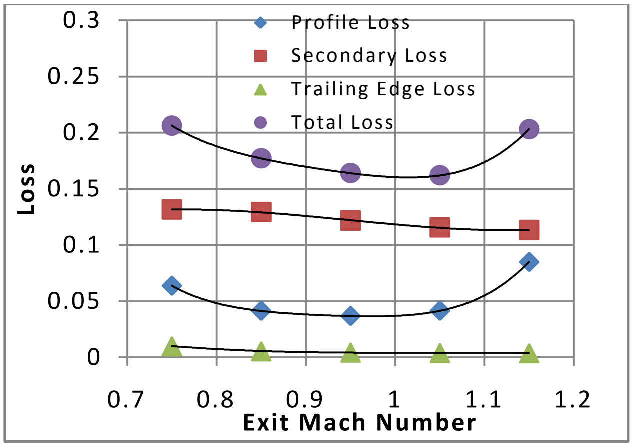

Figure 17. Turbine rotor losses vs. M_{exit}

Clear trends are noted and hopefully the variations are realistic. However, the Mach number variation is just that which is shown in equations [e – w], and the originators (KO) were openly unsure about the last part of the modeling. These Mach functions are sensible, but do they accurately model the blade row? Lacking the ability to query a suitable database of pertinent measurements, we may never know (see discussion by Dubitsky, *et al.*, [37] who comment on the Okapuu *ibid.* correction.)

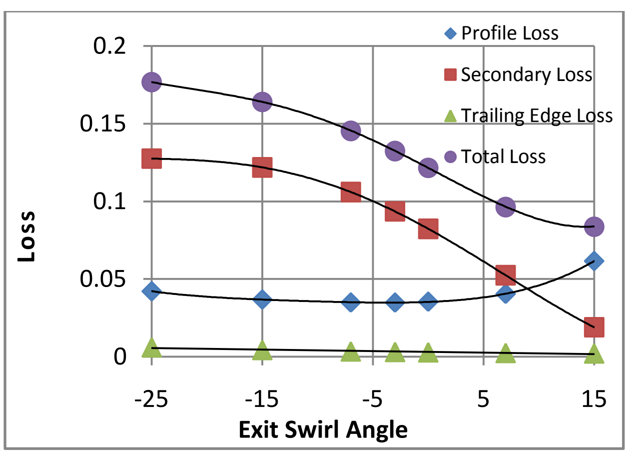

Figure 18. Turbine rotor losses vs. exit swirl

Figure 18 continues the design evaluation by looking at the exit swirl angle: the total loss is a minimum at about 15 degrees of swirl. Should this be the preferred design point? Figure 19 shows that the best power is at about -2 degrees of exit swirl, even though the Figure 18 losses are included. The same result was obtained even using a fixed loss of just 0.1; the dominant factor here is just the velocity triangle variation. The redesign exercise suggests an exit Mach number of about 0.95 and an exit swirl angle of -2 or zero degrees. How much this is supported by the real physics of the problem is unclear.

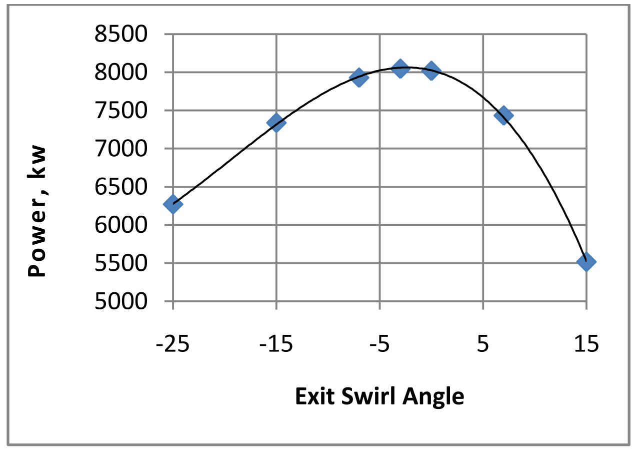

Figure 19. Turbine rotor power vs. exit swirl

This design result will be compared numerically with a two-zone model as introduced below.

PART 3: RADIAL PUMP (& COMPRESSOR) CRITIQUE - The second trial study was based on two models for centrifugal pumps.

<u>Modeling Process</u> - First, the work of Gulich [23] is used, and then the two-zone model of Japikse [26, 38] was employed. Emphasis will be placed only on impeller internal losses.

Gulich recognizes two types of loss:

Passage friction: $K_f = 4C_d[L/D_h][W_{av}/U_2]^2$ [x]

Where: $C_d = \left(c_f + 0.015\right)(1.1 + 4b_2/d_2)$ [y]

And: $K_{sh,l} = 0.3\left(\left(W_{1m} - W_{1q}\right)/U_2\right)^2$ for $W_{1q}/W_{im} > 0.65$ [z]

The first equation is pipe flow friction, and the second is a blade inlet „shock' loss (via Spannhake, not a compressible flow shock, but rather it appears to be a kinetic energy parameter). This application is unusual because it depends only on the level of approach velocities and not incidence, which is the more common application. This is a single-zone heuristic model.

The second approach uses the two-zone control volume modeling equations from Japikse [26, 38]. The equations will not be repeated here but they provide:

1. Mass, momentum, energy, and deviation for an isentropic zone;
2. Mass, momentum, energy, and deviation for a non-isentropic zone; and
3. The full set of equations for the mixed-out state.

These control volume equations are simply a rigorous application of conservation principles to a flow problem; there is nothing inherently unusual, but when done on a two zone basis, blockage, and hence mixing, is modeled, which is critically important to sort out the different loss mechanisms.

Additionally, the Two-Elements-in-Series (TEIS) diffusion model Japikse [38, 39] is used to describe the diffusion/loss process within the rotor. TEIS started as a heuristic notion, but became a hypothesis within hours after it was tested against pedigree data [39]. In the ensuing decades, it became a validated theory based on solid physics, as it was tested without fail against many hundreds of data samples. Enhancements to the TEIS modeling have been made by Pelton *et al.* [27]. Their equations describing the diffusion, blockage and deviation processes were employed as follows:

Impeller inlet effectiveness (loss):

$$\eta_a = k_1\sqrt{\mathrm{Re}_{R1t}}\left(B_1/R_{1t}\right)^{1.5} + k_2\cdot OC - k_3\cdot L/D - k_4\cdot Ro_{W2} - k_5\left(Clr_R/B_2\right)^2 - k_6\cdot\cos(I_{1t}) - \frac{k_7}{AK\cdot\cos(\beta_{1t}-I_{1t})} - k_8\cdot Z_R + k_9 \qquad \text{[aa]}$$

Impeller passage effectiveness (loss):

$$\eta_b = k_1\sqrt{L/D} - k_2\sqrt{A2/A1} - k_3\frac{\left(Ro_{W2i,R2}\right)}{\left(B2/R2t\right)^{k_4}} - k_5\left(\frac{(L/D)}{(A2/A1)^{k_6}}\right) + k_7\cdot\ln(k_8 + Ro_{C2i}) - k_9\cdot ClrR/B2 - k_{10}\cdot\frac{\left(Ro_{W2i,R2}\right)}{\left(B2/R2t\right)}\left(\frac{(L/D)}{(A2/A1)}\right)^{k_{11}} - k_{12} \qquad \text{[bb]}$$

Primary zone deviation at rotor exit (deviation for secondary zone taken as 0):

$$\delta_{2p} = -k_1\exp\left[-k_2\cdot(-\sin(\beta_{2b}))^{k_3}\right] - k_4\exp(-k_5\cdot S_2) - \exp\left(k_6\cdot DR_2^5\right) + 2 \qquad \text{[cc]}$$

Secondary flow mass fraction off-design:

$$\chi = \chi_{BIP} + k_1(I_{1t} + \Delta I_{BIP})^2 - k_2\left(\exp\left(k_3\cdot DR_2^5\right) - 1\right) \qquad \text{[dd]}$$

Secondary flow mass fraction at <u>B</u>enign <u>I</u>ncidence <u>P</u>oint (BIP), essentially the design point:

$$\chi_{BIP} = \frac{1}{k_1\sqrt{Rot_{W1}} + k_2\sqrt{Ro_{W2}} - k_3\cdot AR_{12}^{-0.2} + k_4} - \frac{k_5\left(\frac{A2}{A1}\right)^{k_6}}{\ln\left(1.1 + NS^{k_7}\right)} + k_8\left(\frac{Clr}{B2}\right) - k_9 \qquad \text{[ee]}$$

Experienced fluid dynamicists can readily appreciate the basic physics evident in the parameters above, all of which were indicated by the data processing procedure, with experiential guidance, and not essentially heuristic in style. These equations give a unique description of basic stage characteristics. With these, one can calculate the exit mixing directly. The development of these equations is based on extensive data (160 builds) and complex mathematics (see below). These are data-driven physical models. They introduce a wide variety of geometric and fluid dynamic variables such as passage area ratio (A2/A1), L/D for dimensionless passage length, Rossby number Ro, Rotation number Rot, Reynolds number, incidence, diffusion ratio DR_2, clearance, pitch $S_2 = Z_R/2\pi\cos\beta_{2b}$, and so forth. Little heuristics were used; the variables were gleaned from nature as illustrated later.

<u>Application Example</u> - To contrast these two loss modeling systems, a series of pump designs was run over the range of Ns = 2000, 2700, 3400, and 4500. For each case, off-design performance was also computed. Figure 20 shows a comparison of the two modeling systems. The actual loss levels in this figure (normalized) have no comparative meaning, as either could be larger or smaller than the other.

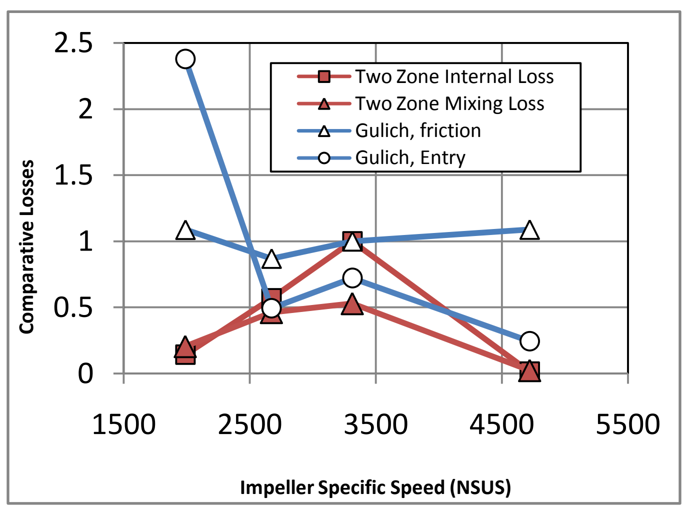

Figure 20. Pump comparative design point loss study

The trends are the important part, and there are obvious differences. The „shock' entry loss of Gulich *ibid.* (open circles) seems ill-behaved (note the high point on the left), and it is clear that the exit mixing loss from the two-zone modeling (solid triangles) is an important variable. The solid symbols are from the two-zone model; open are from Gulich, *ibid*.

Figures 21 and 22 reveal the off-design characteristics from this comparative study. Again, the actual loss levels of one model against the other are not scaled in this study, but the scale between components within each model is correct. The two zone model shows a classical loss bucket form as would be expected; the Gulich models increase monotonically from the lowest flows to the highest flow levels, which is unusual in character.

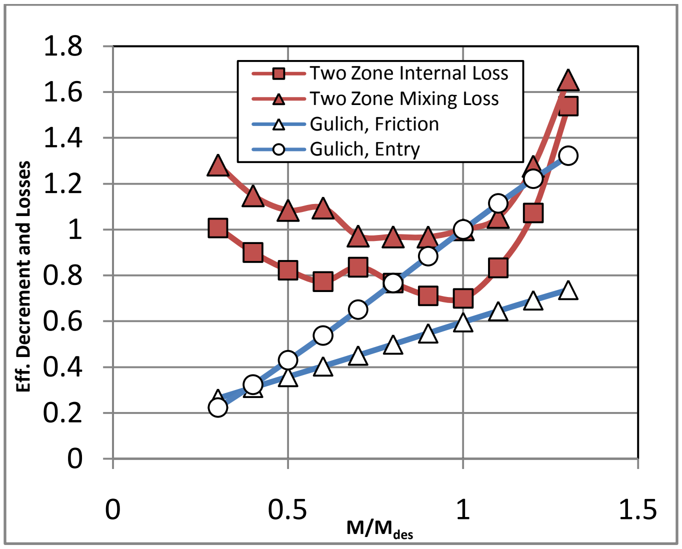

Figure 21. Pump off-design loss comparison, NS = 2000

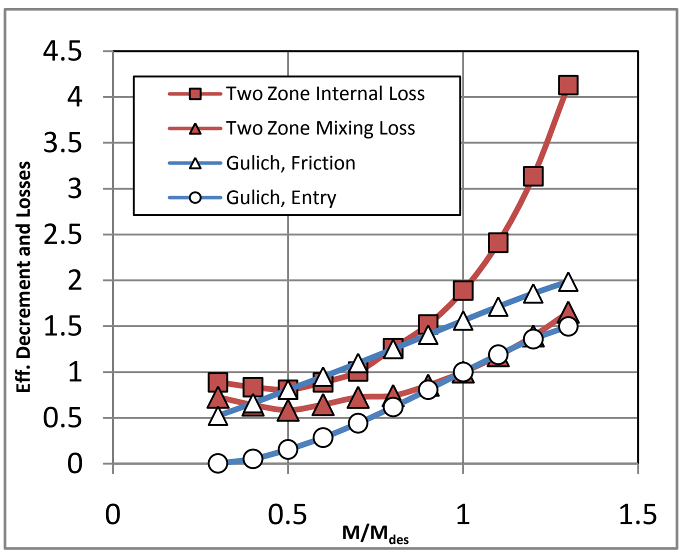

Figure 22. Pump off-design loss comparison, NS = 3200

PART 4: CRITIQUE OF EXAMPLES - The preceding examples give opportunities for thoughtful examination.

Two-zone Axial Models - For the case of the axial turbine, we can apply the two-zone model as just used for the pump examination, but now in compressible flow form. The table below gives a careful match of the kinematic and thermodynamic variables computed via the AMDCKO loss modeling with the conventional single-zone mass, momentum, and energy modeling approach, and the table also shows a near-perfect match using two-zone modeling.

Control volume analysis of the problem makes it clear that the traditional, single-zone calculations as used above *must* end in what is thought of as the mixed-out state. Hence, we match, in Table I, those results with a real two-zone calculation with explicit exit mixing calculations. To do so, one must find a choice of relative velocity ratio within the passage, secondary mass flow fraction, and deviation for the primary and secondary zones. This has been done as shown in the following table:

TABLE I. DESIGN POINT COMPARISON OF SINGLE- AND TWO-ZONE AXIAL TURBINE MODELING

Variable	Units	1-zone AMDCKO	2-zone equiv.
Rotor Inlet			
$p_{02,rel}$	pa	185962.33	
T_{02}	K	1144.00	1144
T_2	K	987.96	987.84
p_2	pa	159445.64	159372
ρ_2	kg/m^3	0.5623	0.56
C_{m2}	m/s	271.7718	271.87
$W_{\theta 2}$	m/s	123.0655	124.36
W_2	m/s	298.3371	298.97
$M_{2,rel}$	-	0.4822	0.48
C_2	m/s	587.7625	588
$C_{\theta 2}$	m/s	521.1572	521.4
Rotor Exit			
T_3	K	887.93	888.199
W_3	m/s	557.2116	557.629
C_{m3}	m/s	369.0970	369.228
$C_{\theta 3}$	m/s	-19.3436	-19.6301
C_3	m/s	369.6035	369.749
M_3	-	0.6301	0.630296
T_{03}	K	949.63	949.949
$p_{03,rel}$	pa	176052.66	
ρ_3	kg/m^3	0.3924	0.3922
p_3	pa	100000	100001
Output			
m	kg/s	37.5480	37.55
Power	kW	8079.15	8093
β_3	deg	-48.5168	-48.7

NB: The exceptional agreement between all numbers in the last two columns confirms the equivalency of the two-zone model; see text.

The reader can observe that the agreement between the two models is very close (about 0.05%; ranging from 0.01% to 1.5%). From this, we can compute the mass-averaged total pressure at rotor exit (p_{02a} = 130555.27pa), and the mixed-out total pressure (p_{02m} = 129663.46pa), and the difference of the two is 892pa lost in the mixing process; this is about 9% of the 9910pa lost on the rotor by the AMDCKO calculation. Just where did AMDCKO account for this part of the loss? Such accounting is not directly in anything, but by the heuristic model building, it has to be rolled up into excessive coefficients for the other terms. These calculations used a mass fraction for the non-isentropic flow of χ =0.18, which resulted in the corresponding area fraction ε = 0.23 while using δ_{2p} = 0 and δ_{2s} = 0 as a first trial set.

In fact, there are a multitude of two-zone parameter combinations that match the AMDCKO calculations as shown in Figures 23 and 24. Each point in these figures is equally as good as the case in Table I, but they will each have slightly different exit mixing losses.

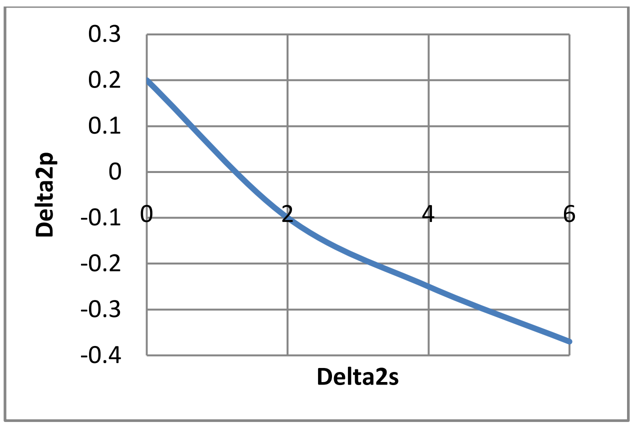

Figure 23. Combinations of primary and secondary zone deviation which match all AMCDKO calculations for the sample problem. Positive values are likely valid ones.

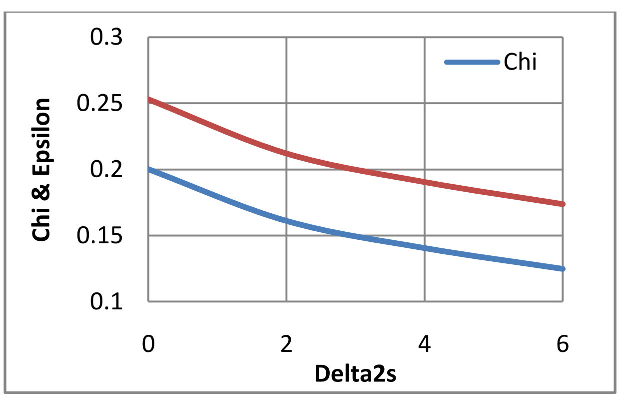

Figure 24. Resultant secondary zone mass and area fractions from the modeling for the sample problem.

An additional test was made for an axial fan and again the two zone model can readily match the single zone model of test data, again to an accuracy of about 0.5% (zero to 3% with angles to better than 0.4deg. for the selected case). These calculations used a mass fraction for the non-isentropic flow of χ =0.125, which resulted in the corresponding area fraction ε = 0.30 while using δ_{2p} = 1.7 and δ_{2s} = 4 and MR_2=1.145 as a first trial set. For both of these examples, the computed blockage calls for an explicit exit mixing calculation, following Stewart's work. This is missing from past axial models.

In the work of Pelton *et al.,* a thorough accounting is given of detailed development of two-zone modeling for centrifugal pumps and compressors based on a large database of test results; a similar approach for axial turbines and compressors may now be suggested. Indeed, explicit exit mixing may be the most overlooked part of proper stage modeling.

<u>Reynolds number variations</u> – Problems with modeling Reynolds number effects plague the industry. The effect has been introduced differently in all the cases considered. In general, there is a tendency by many workers to use equations that, in part, conform to the classic Moody friction chart (the AMDC example above essentially does this and many others, too). There are two serious problems with this approach.

First, the Moody chart was developed for fully-developed (pipe) flow; turbomachinery blading never fully meet this criterion. Most blading has an L/D_h that is on the order of 1 to 10, which is far short of the 50 to 80 required for fully-developed pipe flow. Instead, we are always dealing with classical inlet or developing flows. In this case, wall shear stress starts at infinity at the leading edge of a sharp plate, and then drops down quickly with length, and asymptotically approaches the fully-developed level far along the plate. This is just for open flat plates with no adverse pressure gradients; for the latter, greater shear is certain.

Secondly, the Moody chart is in conflict with original data concerning transitions, both laminar-to-turbulent and smooth-to-rough, and we do operate in and out of these régimes with many turbomachinery stages. Osterwalder and Hippe [6] demonstrated that any one problem may have many modes of flow requiring multiple Reynolds numbers with different properties to describe the various parts of the viscous flow processes.

So, one must ask, just what is a safe Reynolds number scaling to use, and textbooks are full of variations around the Moody chart. By contrast, ASME struggled with this topic with the last two issues of the PTC10 codes and could progress but little further. Ideally, it is desirable to learn what nature indicates from real machine performance, but these studies are rare.

The axial compressor field treated the Re problem better by consistently relying on basic theory: boundary layer calculations were made for turbulent flows and corrections were introduced for occasional laminar operation. Nonetheless, even this area was just as vague about Re effects for secondary flows: KS avoided the effect whereas WM included Re corrections.

Equation [aa] above is one example where a Reynolds number dependency unfolded directly from a study of a large database (160 builds) of machine performance. Another example is given in Dubitsky and Japikse [40] where entry type skin friction was deduced for a group of radial compressor vaneless diffusers. A Reynolds number to the -0.3 power seemed to work a bit better than to -0.2. Presumably, these trends are reasonable. The Re number in equation [aa] is not a boundary layer value per se, but a generic formulation to encompass all viscous effects in the inlet region. Hence, there is no real problem with the direct correspondence to $\sqrt{Re}$. For the vaneless diffuser case, the Re variation is of common form for a flat surface, but the power is a little high for a turbulent flow. This is likely correct, however, since the first part of the surface may well have a laminar boundary layer, followed by a short transition régime, and then fully turbulent flow following, so a power other than -0.2 is not unreasonable.

It appears that much more work needs to be done before we will have really good viscous scaling (Reynolds number) relationships for most turbomachinery problems.

Loss, Recovery, Blockage and Deviation – Emphasis was placed above on including all four of these effects together in any detailed modeling study. In fact, all of the previous studies are incomplete in this respect, except for the system shown for equations aa – ee. The AMDCKO work gave losses, with little attention to blockage and deviation; no attention was given to static state change, even on a localized surface recovery basis. KS concentrated on losses and looked at the effect of profile blockage and recovery (static state change or velocity changes) but not net blockage or deviation. WM considered blockage of both the profile and secondary flows, but seems to treat mixing only for the profile losses. Japikse *et al.* deduced recovery trends (and hence loss via the modeling conservation equations), deviation, and secondary flow blockage with subsequent mixing of the secondary flow regime. There is strong contrast in these different approaches; it is suggested that future investigators ought to carefully consider these differences.

Data Domains and Consequences – The reader cannot yet fully appreciate the type of data employed by the different investigators and the impact that this may have on the modeling process. AM used overall multistage turbine data and then adjusted the coefficients for a row-by-row treatment until the overall results agreed well. Their Re number was based on an average of turbine inlet and exit conditions. KO used a similar approach but added some stage data as well and used Re based on a given stage; great emphasis was again placed on the overall matching of a wide set of turbines. KS used multiple levels of data starting with incompressible testing in air of 41 configurations, each having three or four repeating stages; hence, both a sort of local and overall matching was done. It was discovered that a fixed level of loss had to be added to the profile loss in order to achieve the overall match. WM focused principally on the characteristics of each blade row. None of these studies (AM, KO, KS & WM) mentioned using integrated traverses of total and static pressure and flow angle. The Japikse *et al.* studies were developed solely for individual impellers based on row measurements including various traverse data.

Accuracy – To the extent that different authors discussed expected accuracy, it can be fairly stated that the historical expectations are that modeling accuracy is as good as several points of stage efficiency. In many cases, this might be ±2 points, in other cases, it may be no better than ±3 or more points of stage efficiency. Occasionally, one can refine a model with additional experience and small-domain calibration to get better predictions. However, the amount of data available in any area of turbomachinery is just not great enough to allow truly precise generalized models. Equations [aa–ee] are a step in that direction, but also need further evolution with more quality data. Their accuracy is similar to the rest.

BUILDING MATHEMATICAL MODELS

OVERVIEW – Developing mathematical models for turbomachinery performance is challenging, even though it seems that it ought to be straightforward based on college training. Thinking back to college: what was the most complex *empirical* modeling equation (correlation) presented? It probably only had two independent variables, such as the equation for heat transfer on a flat plate, which was nonlinear and built around readily identifiable variables from the Buckingham Pi theory [41]. Taking the log of each side of that equation, one gets: $\log(Nu) = \mathrm{Log}(0.023) + 0.8\mathrm{Log}(Re) + 0.33\mathrm{Log}(Pr)$, which shows that the equation is linear in logarithmic space and easy to correlate with data to set the coefficients by Log-Log graphing. So these equations were not complex, and subsequently, we learned that the actual heat transfer problem involved transitional boundary layers for many applications, with a dependency on freestream turbulence and other parameters such as vorticity shed from upstream obstructions, as well as the effects of velocity profiles in accelerating and diffusing flows (such as cooled turbine blades). That equation, when applied to real world problems, was often no better than ±25% accurate. When an engineer makes the step from classical training into the design world of turbomachinery, a problem of much greater complexity is at hand (e.g., equations [aa-ee]). How does one deal with this?

We must first focus on the type of approach taken. For the heat transfer problem cited above, there are fundamental principles (Buckingham Pi Theorem) to set the choice of parameters, available appropriate data (e.g., for a very specific process – heat transfer on a well-defined flat plate), and appropriate mathematics (just algebra in this case). For the radial turbine mentioned above, the same theorem applies, some data exist, and the mathematics is available (again, algebra).

In this case, however, the data are overall data for the complete stage, and the models are suggestive ones for the impeller and for a stator element, with no way to rigorously separate the two. Hence, for the radial turbine, we step into heuristic modeling using overall global data (and, in fact, it works well, and this author uses these models whenever needed). However, we have just two elements to model (impeller and stator), so the problem is bounded to some degree. Similar elementary attempts have been made in all other areas of turbomachinery.

For the axial turbine system, however, one historically finds designers making many „improvements' on earlier models, and the process has evolved for six decades. We are dealing with multiple effects on the row or stage losses. The Buckingham Pi Theorem leads us to many of the variables in the models, but not all of them (there are at least 8 fluid dynamic and 6 geometric basic variables used in the turbine model development given above). Some of the loss model elements can be isolated and studied alone in the laboratory, at least in part. Profile losses are one such example. However, secondary flow losses are not easily separated from the rest, nor can incidence be said to affect only the profile loss. The base pressure model is clearly weak compared with basic physical understanding, but the impact is usually mild (not a big loss most of the time for axial turbines). The model supposition of terms is clearly heuristic in nature and is validated only tangentially as additional row or stage data are added for subsequent studies. Furthermore, no effort has been made to study loss, local surface pressure recovery, fluid dynamic blockage, and deviation as a complete set of interacting processes. The database has never been brought together from all prior studies and examined as one macro set. Additionally, all available data have been used to build the models, and all validation has been with the same data, as far as initial publication is concerned. Subsequent testing by other professionals has clearly found defects, and hence, more additions have been published, and so on, seemingly forever.

The examples considered so far have been comparatively easy to create, just as the first example of heat transfer suggests: we can try different power law relationships and different additive combinations until something seems to pick up the trends rather well. However, *if we did have the entire historical empirical database assembled and were to try a generalized model development exercise, we would need different mathematical modeling tools.* This is exactly the problem encountered when the TEIS and two-zone parameter models were created for the pump example given above and may be expected in other areas as research moves forward.

LARGE-MODEL DEVELOPMENT TOOLS (LARGE EMPIRICAL DATA SETS) – The preceding examples showed that there is merit in using large databases to get a better understanding of the flow processes in turbomachinery. This brings a further complication, however; namely, how does one work with a large database and create models (e.g., equations [aa-ee]) from it? Several options are explored below. (Note: This section draws on earlier observations [42]).

Genetic algorithms were first developed as a method for solving problems where a specific solution form was not known. Genetic algorithms are based on the natural phenomenon of survival of the fittest. First, an initial group of randomly generated models, called a population, is developed. Then, each proposed solution, or individual in the population, is represented in a form that can be manipulated automatically. GEP (Genetic Expression Programming) is a form of genetic algorithm where the individuals are expressed as an alphanumeric string. This is a simple and powerful method that does not require converting individuals to and from a binary form, as required in genetic algorithms. Next, each equation in the population is evaluated based on a predefined objective function to determine how well it models the data. In this analysis, the objective function is defined as the mean square error (MSE). The MSE is a basic measure of the residual error between the model and the data and is commonly used in many regression techniques. Typically, in the first generation, none of the equations in the population can model the data with any degree of accuracy, which is expected since they were randomly generated. Although none of the equations in the initial population will be a suitable final model, some will be identified as better than others at predicting the data.

The best performing models are then selected from the initial population to contribute to a second generation in a process called reproduction. During reproduction, elements of the best models are rearranged and mutated to form new models to populate the next generation. Through many generations, traits that produce good models, based on the objective function, are emphasized, and those that do not are removed from consideration. These traits are simply specific arrangements of variables and constants that have a statistically significant correlation in tracking the variance in the data. As more and more generations are created and evaluated, higher quality models will be identified. Following this procedure for many generations will eventually result in a model that best represents the data using the supplied variables. The final result will be the best performing model from the final generation tested. The resulting equation will typically be non-linear and include several coupled terms. Although the resulting models may accurately represent the data that it was built upon, further scrutiny is necessary. By their very nature, genetic algorithms may produce results that are *very* complicated. Some simplification may be possible through careful manual evaluation of the terms in the resulting equation. This process is often referred to as Genetic Expression Programming or GEP.

Like genetic algorithms, neural networks are based on nature.

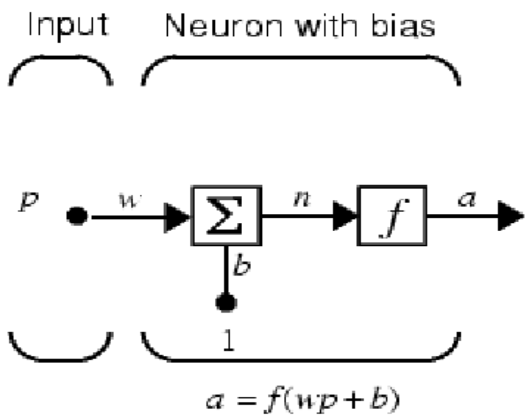

Figure 25. A schematic of a single neuron with input, p, weight, w, and a bias, b, used to calculate the output, a.

Artificial neural networks operate in a manner very similar to that of biological neural networks that exist in the human brain. A graphical representation of a single neuron is shown in Figure 25.

The input to the neuron, p, is altered by a weight, w, and a bias, b. The sum of the weighted input and the bias is the argument of the transfer function, f, usually a step or a sigmoid function, which results in the output, a. The basic function of a neuron is to receive a weighted signal, alter it according to an applied bias and mathematical function, and return the output. A neural network is composed of many layers of these single neurons connected together in parallel – one output could lead to the input of many different neurons. Once a neural network is created, it then needs to be trained before the tool can be used.

Generally, neural networks are adjusted, or trained, so that a particular set of inputs leads to a specific target output by adjusting the weights and biases of the individual neurons. The author's colleagues have successfully trained networks to model compressor performance using a database of about 150 stages; routine design usage has not yet been adopted.

This is quite a different approach to predicting modeling values than is traditionally performed. The major difference with using neural networks is that it does not provide closed-form equations of physics. It provides an unseen process in obtaining an analytical answer. Because the equations developed in the neural networks cannot be described in a physically meaningful manner, many input/target pairs must be used to ensure the network has been trained on a statistically sound sampling of expected input parameter values. After the network is trained, it is a generally-accepted practice to validate and test the neural network with independent sets of data in order to quantify its performance on previously unseen inputs. Using GEP or neural networks is not the same as just fitting data. It is model building with resulting empirical equations. Only a part of the data is used to build the model; the rest is used for validation and testing. For GEP, truly fundamental models can be built if sound choices of independent parameters (e.g., similitude parameters based on the Buckingham Pi theorem) are used. For neural networks, one only obtains simplistic systems, perhaps only linear, which are not fundamental in any real physical sense.

A third option for handling large databases is multi-variable linear regression. In linear regression, the engineer must supply the basic model format, and the analysis adjusts the model coefficients to match the supplied data. The model coefficients are selected to reduce the SSE (sum of square errors) between a user-defined model and the data set. In linear regression, only the model coefficients are varied. Some degree of nonlinearity can be accounted for by allowing the exponents on the different terms to also vary. Linear regression cannot automatically capture the complex interactions that a non-linear method, such as GEP, would identify. However, the user has much more control over the form of the final model and can tailor it to match the current understanding of the physics behind the problem. This approach often allows better defined model asymptotes and avoids singularities more easily.

MODEL INTEGRITY ISSUES - Eleven different issues may be identified which concern the quality of model building. These are reviewed below. While it is not possible to prepare a definitive position on some issues, it is still possible to identify rational approaches and to follow these as carefully as possible during the process of model development.

1. Mathematical Characteristics

Classical education in engineering mathematics teaches the importance of uniqueness, convergence, and stability in dealing with mathematical equations. For equations to be useful in an engineering modeling process, the equation must yield a unique value (assuming a quasi-steady fluid dynamic process, hence, eliminating bi-valued, or time-dependent solutions), the equations must converge smoothly to the unique value, and the system of equations must be stable. Although the equations developed for advanced turbomachinery fluid dynamic models often are far more complex than equations studied in classical engineering mathematics, the principles still apply. The models must give unique values for any set of independent variables. A system of equations must be numerically stable (see items 8-10, below). These three principles can be rigorously imposed by carefully studying the modeled equations and with sufficient testing.

Care must also be taken to select a set of dependent modeling variables that are independent of the parameter that is being modeled. In the author's experience, a case arose of model building that was focused on using sets of non-dimensional variables to generate a model of the independent variable. The preliminary models developed performed well numerically, but when they were used to make an iterative prediction in a performance code, they would not converge. The cause was traced to non-dimensional variables that had been selected which were composed of parameters that were directly coupled to the independent variable. For example, specific speed should be avoided when modeling internal losses and recovery, since it is defined using an overall head rise parameter, which is directly determined by the setting of the internal losses or recovery. Other such dependencies exist, and they must be avoided in equation building or the resulting models will not converge in a predictive application.

2. Statistical Accuracy

Statistical accuracy must be achieved in developing any model. The coefficient of multiple determination, R2, seems well-suited to the present work, as it is for many other numerical processes. R2 varies from 0 to 1, and represents the amount of variation in the dependent variable that is modeled with the independent variables (e.g., equations [aa-ee]). A refinement, however, may well be considered because of the different levels of data quality reflected in Table II below (Japikse *et al*. 2005C) [40]. Hence, a weighted R2 error may be helpful and has been considered in the present work. This is achieved by multiplying the residual for each data point by an appropriate weighting factor. This results in greater emphasis being placed on the data that come from the highest level investigations (higher quality data) and less emphasis placed on the data of lower-valued investigations (essential process data, but of less rigor).

TABLE II. LINEAR REGRESSION CORRELATION STATISTICS

Cross Correlation	*DELTA Pin*
β_{2b}	-0.865
Ro_{W2}	-0.756
LEB	-0.699
$AS_{1,tip}$	-0.637
AS_1	-0.633
Flow Coefficients, ϕ_0	-0.503
Ro_{CA}	-0.502
Exit Leaving KE	-0.500
r_2	-0.508
clr	-0.470
C_{m2s}	-0.463
Z_i	-0.426
Re_{r1t}	-0.392
b_2	-0.372
C_{m2m}	-0.358
C_{p25}	-0.352
DR_2	-0.350
S_2	-0.347
$L/D_{hydro,ave}$	0.264
$C_{pb,i}$	0.252
b_2/r_2	0.059

3. Singularities

Singularities are always a problem. They can be readily detected by examining the equations with only modest experience. Thus, thoughtful examination is the best way to search for and detect singularities. In the case of neural networks, implicit singularities cannot be easily identified since the equations cannot be examined. (The transfer equations can be set to eliminate this problem, but this also limits one to an elementary relationship.)

4. Asymptotic Behavior

Asymptotic behavior must also be carefully reviewed. Trends must be rational as each parameter approaches its asymptotic limits. For example, any fluid dynamic process that depends upon L/D cannot have rapid change as the length of the impeller increases to larger and larger values. A smooth approach to an asymptote would be expected. Likewise, high Reynolds number performance is different from low Reynolds number performance, and the basic trend in these asymptotic limits can well be anticipated. Again, with neural networks, it is problematic to work with these limits.

5. Levels of Non-linearity and Coupling

Ascertaining the appropriate level of non-linearity and the appropriate level of coupling is difficult. There are no analytical guidelines or rules by which the degree of non-linearity or the degree of coupling can be established *a priori.* Nonetheless, it is reasonable to expect that the lowest levels of non-linearity and coupling which yield the highest R2 values should be preferred; any higher order effects would probably be overworking the existing data and likely not be well-supported in practice. Thus, a principle of minimum coupling and minimum exponent levels for the non-linear terms should be used regularly throughout the model building exercise, but not at the expense of correlation accuracy (R2). It should be mentioned that orthogonalization of input parameters (i.e., finding the pivotal combinations) is usually essential to good dependency determination. It can also reduce stiffness problems with the resulting modeling equations.

6. Database Voids

Voids will exist in the database. These voids simply reflect the fact that not enough laboratory testing has been conducted to cover all of the variables that are important. Short of careful observation, there are no specific tools available to identify data voids. Certainly, a survey should be made through the database with an attempt to detect these voids. Once detected, one must be very careful about reaching conclusions where such a void could be present, and one should look to supplement the database with additional testing.

7. Noise

Data error provides a distinct noise level. Inescapable data errors may be found in all investigations and such error is part of any engineering enterprise. It is quite probable that overall statistical methods could be applied to the global database to systematically reduce the impact of distributed random error through the process by second order refinements to all modeling parameters. This is an area for future investigation.

One possible means for refinement is the consideration of hierarchical dependencies between correlations in a computational modeling system which may help to form operating models with reduced noise and improved overall accuracy. This requires deriving correlations hierarchically – by sequential removal of the strongest factors by adding correlations for these parameters into the modeling system and then considering lower level correlations for other parameters to minimize the obtained residual.

8.-10. Training, Testing, and Validation

Training, testing, and validation are important steps in developing models. The subject is treated in a number of references including Hassoun (1995) [43] and Bishop (1995) [44]. In short, a significant portion of the database must be used to train or develop and build the appropriate performance models. Another portion of the database must be employed for continuous testing of the models that are built. In fact, data can be swapped back and forth between these data groups. When this is complete, it is appropriate to use a completely impartial set of data to validate how accurate the resulting models are. The designer working on a new advanced stage always wants to have the maximum utility from every previous test *and* the best possible validation of the tools which are to be employed. This is a conflict, because we would prefer to have the models built on 100% of prior test data (no validation data left), and yet we would like to have good validation (substantial data set aside for validation) with potentially insufficient data for model building! This trade-off requires evaluation and re-evaluation of the process before a proper balance is struck, serving the greatest usefulness for future design work. In fact, any given database can be sequentially repartitioned to give various sets for training, testing, and validation until the best overall modeling is achieved. Great care must be taken to eliminate any chance of over-training a model with any data set. This will likely lead to a stiff mathematical system where two terms fight each other (one term positive, the other negative, each with large coefficients) in order to attempt to match the last (overworked) data points. This problem is prevalent when building neural networks but has its counterpoint in building closed-form algebraic modeling equations.

Stiff systems have occurred in a number of CFD computer programs where the complicated equations for turbulence modeling could not be well-integrated with the rest of the CFD computational process. The same has happened in the meanline modeling of this investigator during early modeling attempts. When sufficient data are not available, equations can result where terms for different variables can fight against each other, yielding large swings in possible values of the dependent variable. This invariably indicates an unstable computational process with, in the extreme cases, no convergence possible at all.

11. Trending

An additional process called "trending" may be used. By running numerical tests on the resulting equations, one independent variable at a time, the basic trends can be tested for physical reasonableness (e.g. Figs. 17-24 herein). This is closely related to the matter of asymptotic behavior. However, through studying the trending results, one can also test further for possible singularities, for data voids, and for unexpected behavioral or trend relationships. For example, the basic variation of performance with Reynolds number can be anticipated from decades of prior experience, and if a radically different trend relationship were implied by the resultant equations with all the diverse independent variables, then one would have good reason to doubt whether the mathematical tools had been used properly.

These eleven principles must be reviewed and followed for all model development work. Some of them have specific analytical strictures which are followed (e.g., R2 statistical evaluation); others have simple fulfillment objectives (e.g., testing for singularities), whereas others reflect general principles (e.g., minimizing the level of non-linearity and coupling in the final model).

TURBULENCE MODELS - Turbulence models are just as complex and encounter all the issues presented above. The preceding modeling remarks focused on meanline loss, recovery, deviation, and blockage modeling, with occasional usage for 2D or quasi-3D studies which are very important in their own right; but turbulence models are absolutely critical when comprehensive modeling of the details of a flow field is necessary. Turbulence models, by contrast, cannot readily be built using stage performance data; these data can only be used to compare the net result of the CFD modeling/calculation process. Such models must be built using detailed measurements of the structure of flow phenomena from select pedigree study cases. This, however, is still problematic and will likely be so for some time to come. Classic fluid dynamic research, with sufficient detailed measurements to aid in turbulence model development, has been prepared on flat plate surfaces with some degree of adverse pressure gradients. Some work has also been done with curved surfaces and a bit for swirling flows. However, there are no comprehensive data for highly-loaded curved surfaces (strong adverse pressure gradients) with strong swirl and surface rotation. Hence, our models still leave room for research and further development. Today, modern, economical CFD tools are available that can calculate the pressure change across a stage and the work transferred, and distribute the losses rather sensibly through the row or stage. The greatest risk now seems to be misinterpreting the onset of stall, which is a challenge even for the best fluid dynamicists when using such codes.

DESIGN MODELING

INTRODUCTION - The previous sections show that useful performance models are available but should be employed with care, since certain historical weaknesses persist in all of these models. Most systems were *not* built with attention to all four key criteria: loss, recovery, blockage, and deviation, yet attention must be given to all criteria when conducting an actual design! Hence, individual designers and design groups are forced to make certain *ad hoc* extensions to the published models in order to complete a calculation. These extensions are rarely presented in public and remain an obstacle to coherent development. The matter is complicated with transonic flow at low back pressures (high Mach number), choke, limit loading, and stall. To model or compute under these conditions requires even further *ad hoc* treatment. It is likely a fair representation to suggest that design modeling today is reasonably accurate (estimated to be good to within a couple of points of stage efficiency) and rational for conventional designs, near the design point, and at higher and lower flows of some ± 25% to 50% from the design condition. Beyond this point, we have enhanced risk of modeling error, and there is a need for further investigation.

MODELING EXAMPLE – To tie together many of the points made herein, a modeling example for a centrifugal compressor is presented. Data for a modern centrifugal compressor impeller are given in Figures 26 to 29. The calculations were all done blind: no data matching!

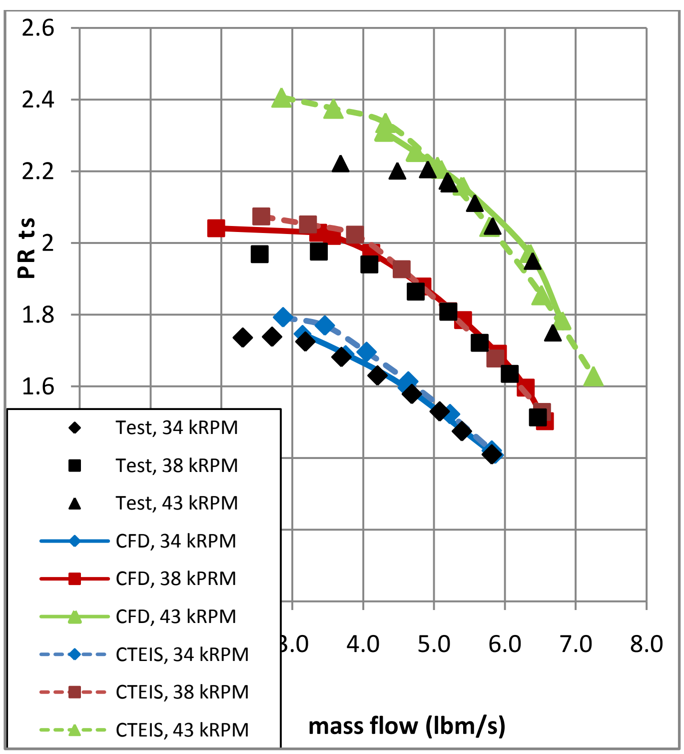

Figure 26. Stage pressure ratio vs. flow; laboratory data, 1D and CFD modeled results.

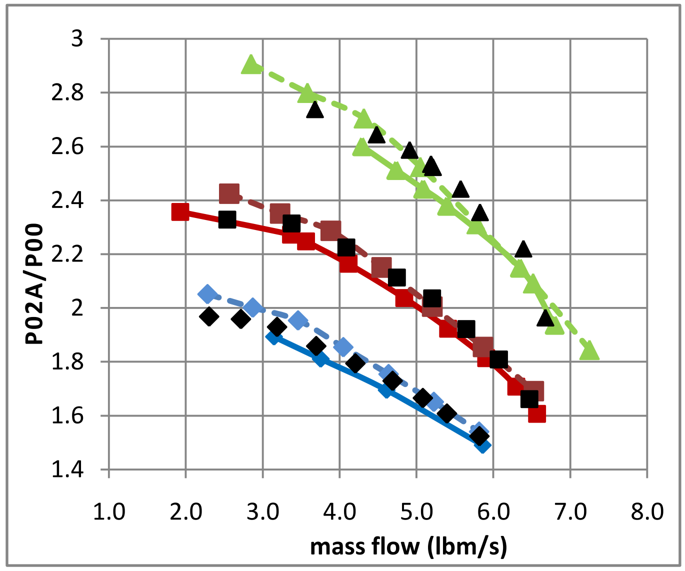

Figure 27. Impeller pressure ratio vs. flow; laboratory data, 1D and CFD modeled results. Symbols per Fig. 26

The data comparison (individual symbols) of stage pressure rise in Figure 26 shows good meanline modeling (dashed lines; CTEIS refers to modeling equivalent to equations [aa–ee], but for compressors) and good CFD (solid lines) compared to the data. A similarly good comparison is shown for the impeller-alone pressure rise, Figure 27. Each model has worked quite well and is design-worthy. The stage power is shown in Figure 28, and again, the comparison is appropriate for design work; in this case, the CFD captured the characteristic droop at high flow a bit better than the meanline model for the two lower speed lines. The resulting stage efficiencies are given in Figure 29; both meanline and CFD have reasonable agreement to the data (a couple of points variation at different flows).

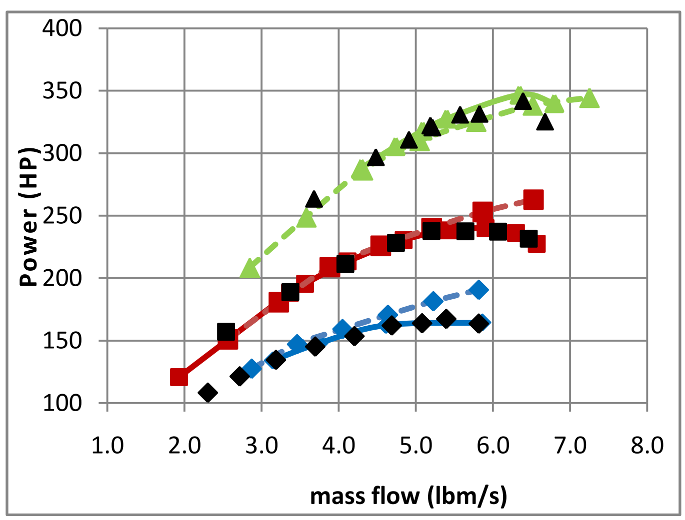

Figure 28. Impeller power vs. flow; laboratory data, 1D and CFD modeled results. Symbols per Figure 26.

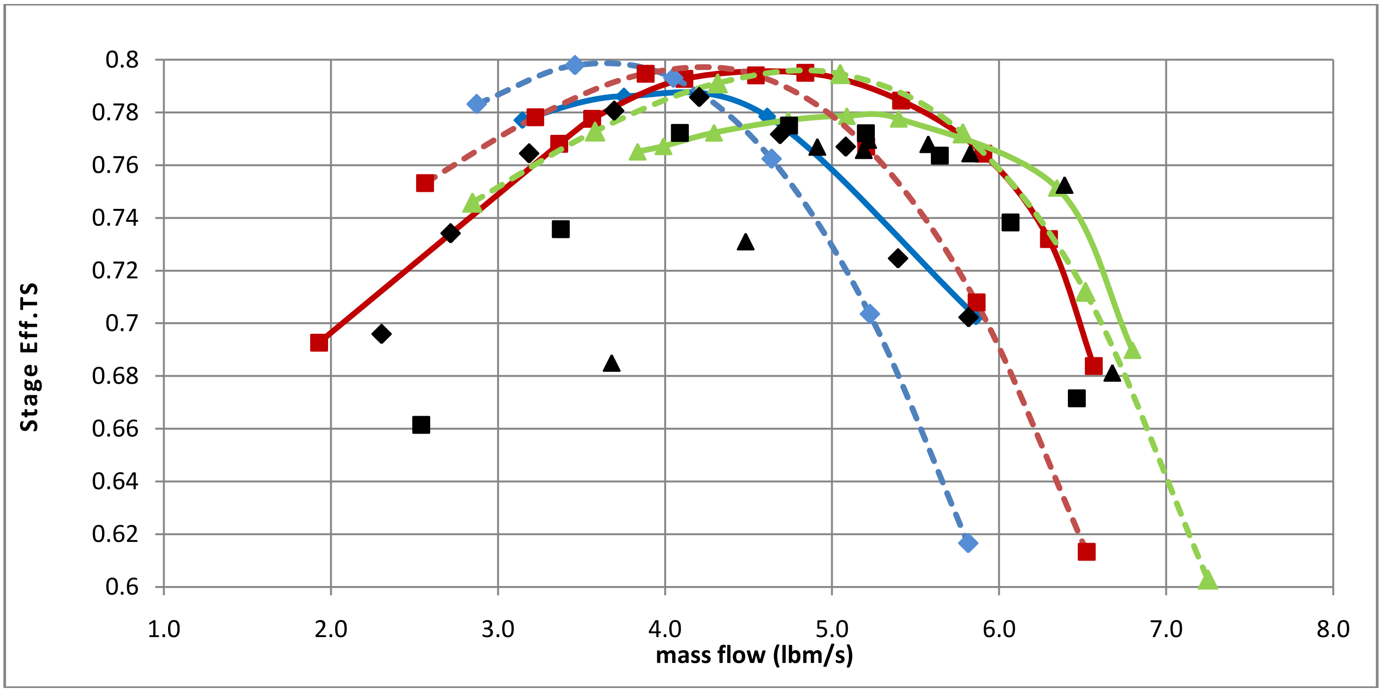

Figure 29. Stage efficiency vs. flow; laboratory data, 1D and CFD modeled results. Symbols per Figure 26.

This modeling example suggests the utility of both meanline and CFD modeling techniques. Much attention has been given to the integrity of meanline models; some remarks concerning CFD may also be useful.

CFD MODELING – Three-dimensional CFD modeling solves the fluid dynamic equations directly, giving maximum detail. Referred to as Full Navier-Stokes (FNS) methods, they have grown significantly in popularity with the explosion of inexpensive computing power and memory. Historically, computational time and costs were the limiting factor in CFD. More recently, the dominant issue has become the human time for problem setup and interrogation. Specifically, generating the solution grid and post-processing solution results for meaningful data have become the most significant drivers. Software manufacturers have realized this and have invested significant resources to reducing this aspect of cost, but much more work remains to be done. The calculations just shown were set up and run in a day using an o-grid and a Spallart-Allmaras algebraic turbulence model. CFD modeling is valuable at many points in the design process: trend confirmation as shown in Figures 26 through 29, detailed trade-off studies of complex phenomena and development of data for structural analysis.

CFD PROBLEMS ENCOUNTERED – Theoretically, CFD can account for nearly all fluid dynamic phenomena that can potentially affect the performance of turbomachinery. In practice, many issues come into play that can compromise solution results. These issues include: turbulence modeling, numerical schemes, grid resolution, convergence criteria, and others. Virtually all of them can be reduced or even eliminated with sufficient time and computational resources. The trick is to minimize these effects within the practical limits of a real design problem.

CFD MATHEMATICAL MODELING CHALLENGES – The computing power available today has somewhat reduced the variation seen in the past from one turbulence model to the next. While this can be broadly stated for attached flow nominally near the design point, there remains substantial uncertainty for significantly separated flows typically found well off-design or nearing stall. Progress on this front may well come more from systematic study and comparison to test data and “tuning” the approach rather than through any revolutionary breakthrough in turbulence modeling. In this respect, the approach comes full circle back to heuristic methods pioneered in meanline modeling.

ACCURACY – best practice CFD and meanline models each achieve useful accuracy; it is not a question of one versus the other in most design problems. Rather, it is a matter of using the correct tool at the correct time to deal with the level of detail required.

CONCLUSIONS AND RECOMMENDATIONS

OBSERVING THE DESIGN WORLD - Four types of design problems were outlined at the beginning. Some discussions about best design practice fail to recognize how different the actual demands are in each area, and hence, lose relevancy. A wise designer will be aware of the differences and choose his/her models carefully. Likewise, the designer will judiciously apply a variety of modeling methods from 1D to 3D, including both fluid dynamic and structural analysis methods.

MODEL PEDIGREE AND IMPLICATIONS FOR THE FUTURE - The examples presented above show definite variation in prediction capability and some real uncertainty with respect to functional dependencies within the modeling parameters. This is particularly true for the heuristic-based models, but remains also true, perhaps to a more limited level, for the data-driven physical models. In each case, the ultimate problem is the same: a lack of sufficient empirical data to tease out the trends and tendencies of nature.

OBSERVATIONS FOR BETTER MODELS - Application of two-zone modeling to an axial turbine and an axial compressor sample problem was very successful and is an original part of this survey. It may be suggested that the historical AMDCKO and other models perhaps be reformulated on the two-zone basis with explicit mixing calculations and then be tested for potentially better predictive fidelity. This matter is left for future study.

NEW MODEL BUILDING - Several examples of model development have been referred to throughout this text. It may be expected that more of the same will be published in the years ahead; but which types are likely to help us develop truly better machines in the future? With the potentially high cost of energy and the highly competitive world market, we can settle for nothing less than the best models to guide us in future designs. Heuristic models have taken us only so far and perhaps we are suffering their limitations; hopefully, nature can express itself through data-driven models. Such efforts will require ever larger empirical databases, supported by more and more complex mathematical protocols. Neural networks have been shown to work well, but fail to give the physical insights needed for basic design thinking. GEP procedures are now changing rapidly and will be a useful tool sometime in the near future; presently, they often serve to introduce more new questions than answers to earlier questions. A combination of methodologies is needed when building useful new data-driven physical models.

PROSPECTS FOR FUTURE DATA SHARING - We have seen the results of model building based on data and techniques available to individual contributors, and, while they are quite good, they have distinct limits due to a lack of good data shared on a broad basis. There is a need to access more quality data if better modeling, and hence, better designs are to be made in the future. Tragically, much of the data of the past is either lost or badly compromised when a piece of key data is lost from the set. In all cases, data upon which published models are built really must be faithfully recorded, with a detailed discussion of the experimental set-up, including full coordinates of all the hardware used, and accurate build records, including the operating clearances. In this regard, a suggestion to professional societies might be offered: require that all data used to support a published model/study is archived in a safe mode with assured proprietary control for a given period of time. Three years might be suitable for some cases, seven years should meet almost all industrial concerns, and something like eleven years should be a safe upper limit. After the prescribed date, all data should be fair game for further investigations by the community at large, except for extreme cases concerning *bona fide* national security. Only when the base of vital empirical data is expanded will there be any significant breakthroughs in enhanced modeling of turbomachinery stages.

ACKNOWLEDGMENTS

The author wishes to acknowledge the contribution of colleagues who have been instrumental in developing and testing a variety of modeling systems over the past years, including Oleg Dubitsky, Dr. Xuwen Qiu, Mark Anderson, Dr. Nick Baines, Rob Pelton, and Kerry Oliphant. They are a wonderful team of star contributors. Figures in this paper are from the author's textbooks, including numerous extracts from Ref. [29, 37, and 38]. All other illustrations were developed for this paper or are from Concepts NREC. The assistance of Jane Waks and Jan Johnston for reference work and text preparation is greatly appreciated.

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NOMENCLATURE

A	Area
AOA	Angle of attack
AR; AS	Area ratio; Aspect ratio
B	Blockage
b, B	Blade height
C	Absolute velocity
c	Chord
C_D	Discharge coefficient; dissipation coef.
C_f	Friction coefficient
C_p	Specific heat at constant pressure
D	Diameter
DR	Diffusion ratio
h	Enthalpy, blade height
i	Incidence angle
K	Total pressure loss coefficient
k_i, k	Constant; specific heat ratio
L	Length
M	Mach number
m	Mass flow rate
N	Rotational speed
N_s, *NS*	Specific speed
o	Throat
P	Power, pitch
p	Pressure
pr	Pressure ratio
R	Gas constant, radius of curvature
r	Radius
Re	Reynolds number
RN, Rot	Rotation number
Ro	Rossby number
s	Blade spacing
S	Solidity
T	Temperature
t	Blade thickness
U	Blade speed
W	Relative velocity
Z	Blade number

Greek

α	Absolute flow angle
β	Relative flow angle
Δ	Difference
δ	Deviation angle; displacement thickness
ε	Fluid dynamic blockage
η	Efficiency; Effectiveness
θ	Camber angle; turning angle
ρ	Density
τ	Torque
χ	Secondary flow mass fraction
ω	Angular velocity

Subscripts

0	Stagnation state
0, 1, 2 …	Stations
av	average
b	Blade
clr	Clearance
D	Diameter
des	Design
f	Friction
h, H	Hydraulic
h	Hub
L	Length, leakage
LE	Leading edge
m	Meridional component
p, P	Profile; Passage
rel	Relative
ref	Reference value
s	Isentropic, secondary flow
T	Turbine, total
t	Tip
TE	Trailing edge
ts, tt	Total to static; Total to total
θ	Tangential component

Overbar

Mean value