##  <br> <br> TWIST AND TESS <br> <br> TWIST AND TESS <br> Miguel A. Gañán

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Dedicated to my wife, Gabriela and our daughter Sol, for their unconditional support from my beginnings in the world of origami, their help and encouragement in preparing this book, and for allowing me to transform our home into a small museum for origami.
And a very special thanks to the rest of my family in Spain and Argentine, for admiring my models and being the first ones to encourage me to create new designs.

## PROLOGUE

Miguelさんの作品は端正だ。この本に掲載されている美しい写真をご覧になれば，それはすぐにわかる。
丁寧でわかりやすく解説されたこの本は，テセレーション
（Tessellation）の面白さと美しさを皆と分かち合いたい，という Miguelさんの思いと人柄が現れている。

Tessellationsを始めたい人にこの上ない手引書となるだろう。 すぐにも購入して，そういう友人にプレゼントしたい。

Tomoko Fuse

Miguel Gañan＇s works have the beauty of precision．
If you observe the beautiful photographs in this book，you will see it clearly．
With simple and detailed explanations，the book shows the author＇s personality and his desire to share the beauty and all the interesting aspects of the art of Tessellation．
For those who wish to start dabbling in this art，this material will be of great use．
I would like to acquire it right now and gift it to all my friends who have an interest in said art．

Tomoko Fuse

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Miguel A. Gañán

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Alejandra Zapico

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Several steps, "Hexagons and triangles" model.

## LET'S GET THE PAPER

## Why a book about tessellations?

When I first got my hands on Eric Gjerde’s book in October 2011, a new world of possibilities opened up for me in origami.

My beginnings, back in 2007, had been with modular origami and then with "snapology"; always origami related to mathematics (and with my background as an aeronautical engineer).

With Gjerde's book, and then Shuzo Fujimoto with his "hydrangea", my first tessellations couldn't have had better masters.

And so that's how I entered this wonderful world of the tessellation, with the first model "Five and four". After gradually folding the rest of the models and uploading my first photos to Flickr in the "Origami tessellations" group, I met an amazing group of origamists related to tessellations (Robin Scholz and Lydia Diard were who initially caught my attention, and they served as teachers with their models and advice).

The tessellations virus entered my veins and has yet to come out...
At the end of 2017, I can't remember the exact moment, while I was playing with a square grid and twist, my first design, "The Hanoi tower" appeared in my hands. That was the breaking point, the "click" that made me realise I could create my own tessellation models.

Little by little, the rest of the models started appearing, and after showing them to the Buenos Aires origami group and then at the Origami Argentina conventions, a lot of people asked me to give workshops.

In 2018, going over the bibliography related to tessellations, the few existing books were only in English. After talking with some of my origamist friends, and a talk with Laura Rozenberg, I decided that the best way of passing on my knowledge was by preparing a book (which would be the first one in Spanish) with some of my original designs. This english edition is the literal translation of the spanish one.


My first Eric Gjerde tessellation


My first Shuzo Fujimoto tessellation


Arabic Mosaics


Cathedral "Vitreaux"

And so, I got to work. My idea was to create a book with diagrams (not photographs) with the folding instructions. I contacted several origamists that did diagrams, until I found someone from our group. She had never done tessellations before, but, when presented with the challenge of drawing tessellation diagrams, accepted without hesitation.

At first, the idea was to publish Alejandra's drawings directly, but when we saw that the time to finish the book would be too long, we decided to transfer these drawings to Inkscape, to avoid having to redraw the clean diagrams by hand.

It's oriented to origamists who want to get started in the tessellation world, who have seen models folded with this technique, and who feel curiosity to know how to start and everything related to this technique.

## How to use the book

Starting with a brief introduction to tessellations (from their origin in ancient times) and their relation with mathematics, it includes basic information about the knowledge required to start folding tessellations, recommended materials and advice based on my own experience.


Roman Mosaic

After explaining how to prepare the grids in detail, the necessary techniques for folding (twists and pleats) are described, before moving on to the models, all of them my original designs.

The models are classified in accordance to the type of twists used in them, starting with the triangular twists, followed by the rhombic and hexagonal, and ending with some uncommon ones (parallelograms, trapezes, teardrops).

And as last models, those that use stacked twists (they are part of my analysis' published in OrigamiUSA).

Lastly, there are two additional chapters, related to the same tessellations technique, but with a different final result:

- Boxes with tessellations in their lid.
- Omiyage (Tato) with tessellations that appear when opened.

In all the models, the basic molecule is shown with an explanation on how to fold it and how to unite/connect it to others in order to create the final tessellation. The CP with a normal grid is also included (32 or 48 divisions). In those cases in which the model can be extended and its CP is different from the original, said CP is shown.

Included at the end:

- A small English-Spanish dictionary with the usual terms used in tessellations.
- A table with the symbols used in the book.
- References to other authors, books and websites.


## What is a tessellation?

Many of you may have seen tessellations in your daily life, not knowing that they are. If we go back to ancient times, we can observe tessellations in the Roman age constructions.

The origin of the word tessellation goes back to the Roman age. The "teselas" are the small square rocks that were used to create those tiles or mosaics we see in floors and walls.

Later, in the Arab era and especially in Spain, in the mosques, we can see wonderful examples of tessellations created by Arab artisans.


Shuzo Fujimoto's tessellations


Tessellation 3.3.3.3.3.3


Tessellation 4.4.4.4


Tessellation 6.6.6

We can also find them in the stained-glass windows of cathedrals. In the contemporary age, some artists (mainly painters) have also made use of the tessellations in their wok (M.C. Escher for example).

## Tessellations and origami (a little bit of history)

Entering our world of origami, in the 70's of the last century, a great Japanese origamist, Shuzo Fujimoto, published the first books that made use of those designs.

Closer to us, Ron Resch, Chris Palmer, Robert Lang, Eric Gjerde and Eric Joisel have been a few of the origami artists that have heavily influenced the development of this technique.

In origami, what we call tessellations are the geometric designs folded from a single paper, repeating patterns around the paper, (in the book we will call them molecules) by twists and pleats.

They can range from simple models, with very few twists and pleats to complex models such as Eric Joisel's three-dimensional masks.

## A bit of mathematics

For those interested in the connection between tessellations and mathematics, we will go over some basic concepts that help us comprehend the designs.

In mathematics, a tessellation is a repetition of a pattern (usually several polygons, identical or different), that allows the filling of a surface without overlaps and holes between the polygons.

There are three types of tessellations: regulars, semi-regulars and irregulars.

In origami, we mainly use the regular and semi-regular ones.

- Regular tessellations: they are achieved by repeating regular polygons. There are only 3 regular tessellations, repeating triangles, squares and regular hexagons.

One of the vertices and the number of sides of the polygons that surround said vertex are marked in the figure. This is the way to name a tessellation, starting by one of the polygons that has less sides.


Tessellation 4.8.8


Tessellation 3.4.3.3.4

Semi-regular tessellations: they are achieved by repeating 2 or more regular polygons. The pattern must be the same in all vertices. There are 8 semi-regular tessellations:

Of all the above, the ones that most commonly used in origami are the following: 3.3.3.3.6, 3.4.6.4, 3.6.3.6 and 4.8.8.


Tessellation 3.3.3.4.4


Tessellation 3.6.3.6


Tessellation 4.6.12


Tessellation 3.4.6.4


Tessellation 3.12.12


Tessellation 3.3.3.3.6



$$
\begin{equation*}
\text { Abri y plegar al } \Delta S \tag{1}
\end{equation*}
$$


(4)

(3)

Several steps, "One ring to rule them all" model.

## MATERIALS

## Papers

Origami (Ori/fold, Kami/paper)
Papiroflexia: (Papiros/paper, Flexus/fold)
Even though we can argue about which one should be the name or when and where this art started, what is not up for debate is the essential material: paper.

And if in many origami techniques we can use any type of paper, in tessellations, the choice of adequate paper is fundamental, not only for the folding of said paper, but also for the possible transparencies of the final result.


Tessellation with Elephant Hide paper

Not all papers are fabricated in the same way. Given that for tessellations a lot of folds are necessary and sometimes they have to be done and undone, the chosen paper has to be able to hold them without breaking. It has to meet three criteria:

- It must be flexible to be able to work easily.
- Memory: the paper maintains the folded pleats.
- The paper doesn't break when folded repeatedly.

Besides, we have to choose the size we are going to work with. The papers we find in stores normally come in a plain square format or in the form of a roll.


Elephant Hide paper

## SUGGESTION

- To begin folding tessellations my recommendation is to use a normal printer paper (in size A4 or A3). It is not the best for tessellations, but it is more affordable and easier to find. And for initial tests or practices it turns out a lot cheaper.


Tessellation with Tant paper

- The square papers, in formats from $15 \times 15 \mathrm{~cm}$ to $35 \times 35 \mathrm{~cm}$ are the most common.
- They exist in $50 \times 50 \mathrm{~cm}$ or in a larger format, but they are only used in very special works.
- For the models in this book, my recommendation is to initially use papers of $30 \times 30$ or $35 \times 35 \mathrm{~cm}$, given that we will fundamentally use grids of 48 divisions.

Once acquired a certain amount of skill in the folding of tessellations, we can move on to more specific papers.

Between all available papers, the ones I normally use are the following:

## Elephant Hide (110 gr)

It is heavier than other origami papers and very resistant to breaking. It doesn't have a wide range of colours, even though you can sometimes find special colours (yellow, jade green). It's one of the best papers for tessellations.

## Tant (78 gr)

It is very good for tessellations, almost perfect. It allows the folding of very precise grids. Reversing a fold is simple and the pre-creasing doesn't present difficulties.

It is not as strong as Elephant Hide, which allows for folding with less effort. It also has a really big range of colours, which allows work with transparencies.

Glassine ( 40 gr)
It is a very thin and transparent paper. Very hard to use, it has a memory that allows no mistakes. Once the pleat is done, it can't be undone.

However, it allows transparent tessellations folding, very impressive when backlit.

However, it allows transparent tessellations folding, very impressive when backlit.

## Alios Kraft (28 gr)

It is a very thin paper, but very resistant, in several colours. I normally use the white almost transparent one, easier to work with than the Glassine, and with spectacular results.

## Necessary tools

Clips, tweezers
Metal or wood, in diverse
formats. They must not leave marks on the paper.


Glassine paper

## Paper adhesive tape

To maintain the paper in its place while we fold other parts of it. It has to be proven in advance that it does not damage the paper we are using.

## Folders

They are very useful when applying pressure on the folds, both in the pre-creasing and the collapse. There are bone folders, plastic, wood, etc.

## Adhesive points

Small in size ( 5 mm ), they are very useful to mark where certain important pleats go in the model so as to not get lost during folding.

## Hexagonal templates

As we will see later on, for the preparation of triangular grids it is necessary to start from a regular hexagon.
As the paper comes in a square format, to cut a regular hexagon, my recommendation is to prepare several cardboard, plastic or metal templates with the size adequate to the paper measures we normally use (the usual 15, 20, 24, 30, 35 cm ).
Cardboard and plastic ones, with use, tend to lose their shape, especially in the corners, due to the cuts of the blades used.


Tessellation with glassine paper


Tessellation with Alios Kraft paper


Mini stickers


Hexagonal templates


Paper with printed grids


Tessellations tools

On the internet you can find a few small metallic ones, from 15 to 20 cm (see references chapter at the end of the book). Another option is to order them to measure in a workshop that has laser cutting to obtain a regular hexagon. In the photo we see 3 metallic stainless steel templates ( 1.5 mm thick) and one cardboard made ( 2 mm thick).

The templates prevent us from having to generate a regular hexagon for every tessellation with the geometry procedure (such as drawing a regular hexagon inside a square, with a compass).

## Sheets with printed grids

They are used in the process of creating tessellations. They are sheets that can be printed from some internet websites, where you can draw the tessellations CP's to analyse the model and the diverse steps of folding.

## HOW TO FOLD A TESSELLATION

## Reading a CP (crease pattern)

What is a CP?
It is the graphic representation of every fold needed to create a model (be it a tessellation or any other figurative model).
In a CP all the folds can be seen in valley and mountain, drawn on a base grid, which allows us to identify the molecules that will form the tessellation.
To interpret a CP, we have the following elements:

- Mountain fold: black continuous line or with dots and stripes.
- Valley fold: red discontinuous line.
- Valley or mountain folds outside the main grid.
- Molecule: group of pleats that will repeat inside the model.

Figure 1 shows a molecule from a tessellation, with its folds in valley and mountain.

Figure 2 shows folds outside the grid (a triangular closed twist, another open back and a rhombic twist).
Figure 3 shows the folded molecule, based on the CP from figure 1.


Figure 1


Figure 2


Figure 3

## SUGGESTION

## - As a general

recommendation, it is convenient to do tests in a normal paper, folding the molecule of every model, to be sure of the way to fold or collapse every step. - Once sure of every step of the model, we can use the definitive paper to fold the entire model. - In my case, I use 70 / 80 gr printing paper, or even 80 gr wood paper (kraft) with a smooth finish on both sides (non-rough).

## Preparing the grid

The grids are the base of any tessellation. It is the division of the paper in equal divisions, which will serve us as a guide for the folding of the model.

Making a grid with exact dimensions is the beginning of a good tessellation. We will use two types of grids, square and triangular.

## Add the pleats outside the grid (pre-crease)

Once the grid is done, it is necessary to add the folds outside the grid. There are tessellation models that do not need these folds, but in almost all of the models in the book they are fundamental.

Making these folds beforehand makes the subsequent work easier, making the paper twist and move in the direction we want.

There are some origamists that prefer to mark those pleats with a scoring tool (or a pen without ink), without previously doing the fold. In that case, when the fold is a valley, it will be marked at the front of the paper and if it is a mountain, it will be marked at the back.

## Collapse the model

It is the process of real folding of the model. Depending on the type of grid and model, we will start the folding at the centre of the paper (in the tessellations with a hexagonal base) or at one of the corners (in the ones with a square base).

To collapse the model, a series or techniques that we must know and practice until we master them beforehand are used. The mastering of these techniques is a necessary requirement in order to fold a tessellation. Said techniques are:

- Twists
- Pleats


## GRIDS

## Basic concepts

The grids are the base of the tessellations, because they allow us to fold the paper easily, as well as being of great help in the design of the tessellation.

The CP's are drawn on the grids, which allow us to see the folds that make up the tessellation and that we will have to do, in a very clear way.

Preparing a grid consists in folding the paper in a certain number of divisions in each one of the directions (2 in a square grid and 3 in a triangular grid).

The pre-creasing of all the folds that go outside the grid will be made on the grid.
Various methods for preparing grids exist an every origamist chooses the one that adapts more to their way of folding, after having tried the diverse methods.

In the book we are going to show the most widely used method and that produces the best results (the most precise).

## Grid sequences

To prepare a grid, it is necessary to define the number of divisions that we are going to do on paper.

Normally, when you talk about grid division sequences, we are referring to the sequence " $2,4,8,16,32,64,128$ ". This sequence is the easiest one to fold, because as we will see later on, it consists of folding the paper in half in all its directions ( 2 in the square paper and 3 in the hexagonal). And repeating this exact process, folding in half between each two pleats, until we have the necessary number of divisions.

But there are other divisions that could be useful in certain models.

Sequence 3, 6, 12, 24, 48, 96

## SUGGESTION

- The grids should be as accurate as possible. If at the end of the grid, we see that the lines do not intersect at exact points, it will be almost impossible to fold.
- It is advised to turn all the folds of the grids in neutrals (without mountains or
valleys). This will facilitate the folding


## SUGGESTION

- It is essential that the first pleats are as precise as possible, seeing as the rest of the divisions depend on them.
- Verify that after each sequence of pleats, the lines are crossed at a single point and are not offset.



correct grid


Incorrect grid

This sequence can be achieved by initially dividing the paper in 3 equal parts in each of their directions. And once that is done, following the same process of dividing in half between each two divisions.

- Sequence 5, 10, 20, 40, 80

It can be achieved by dividing the paper in 5 equal parts, repeating the previous process of folding in half afterwards.

To divide the paper in 3 or 5 parts, there are some methods based on origami pleats, but I do not recommend their use in grids, because they usually imply a waste of paper and aren't as precise as we need for tessellations. It is better to use a ruler and do the precise divisions in each side of the paper.

## Square grids

Since the process to make a square grid is similar to the one used to prepare a triangular grid from a hexagon, we will explain said process in the following chapter.
In the following figure we can see a square grid of $16 \times 16$ divisions.


## Square Grid

## Triangular grids (in hexagonal paper)

They are the most used in tessellations, because of the versatility that this type of grid brings us, given that the triangles and hexagons can be jointly tessellated in a very easy way.

The triangular grids can be prepared from two paper formats:

```
- Square paper.
- Hexagonal paper.
```

We will start with the grids from a hexagonal paper, using the same method that would work for a square grid.
The hexagonal paper will have been pre-cut from a square paper using the templates previously mentioned.

## Hexagonal Paper


(1) We will start by folding the paper in half in one of the directions. We twist the paper and repeat in the second direction. Another twist and we will finish the first division ( $2 \times 2 \times 2$ grid).


3 In the following figures we can see the detail on a $4 \times 4 \times 4$ grid (to be converted to $8 \times 8 \times 8$ ) and another of $8 \times 8 \times 8$ (to be converted to $16 \times 16 \times 16$ ).

## SUGGESTION

- DO NOT make all the folds in one direction and then in the others.
- In small grids (until 16 divisions, the difference won't be noticed, but in big grids (mostly 64, 80, 96), if we make all the folds in one direction first, we will notice that the paper loses size because of the own folding process.
- Depending on the type of paper, the size of it can be reduced in more than $2 \%$. And in that case, when making the divisions in the other directions, we will see that the grids are unusable because the junctions of the various lines do not match at the same point.

To continue with the rest of the divisions, we will explain the most commonly used method. It consists of creating a valley fold in the middle of two mountain folds. For that, we take one of the folds to the previous one and flatten the paper, generating the valley fold.
This method is very precise and can be done with any size of paper and any number of divisions.

It is recommended to make all folds neutral at the end of the sequence in each direction.




Several steps, "Bouquet" model.

## TESSELLATIONS WITH TRIANGULAR TWISTS

As we discussed in the previous chapter, we will explain at the beginning of this chapter the particular techniques applied to these models.

## Closed Triangle Twist

There are two ways to do this twist. The first one is pre-creasing the triangle's sides that are outside the grid. The other way is without this pre-creasing. We will start with the first one:

(1) Pre-crease the 3 sides from the triangle outside the grid.


(2) Pinch the 3 folds ang twist them in the chosen direction.

(3) Force the twist to flatten the model.

## Without Pre-creasing

Now, the second way to make this twist, which is the common one in the complete models because it avoids the pre-creasing. It starts with the same CP.


Twist the 3 folds trying to keep them flat. With three fingers, squeeze at the 3 ends of the pyramid.

(2) Keep squeezing in those points, forcing the paper to form the flat triangle.

## Open Back Triangle Twist

It is similar to the previous one, with a little variation. The 3 triangle vertices are moved 1 additional space in every direction of the grid. That will leave, in the final model, a central triangle that can be appreciated in the back or when backlit.

(1) Pre-crease the 3 sides from the triangle outside the grid.

(2) The process is similar to the one from the closed triangle.

(3) In process.
(4) Finished twist.

## Double Sized Closed Triangle Twist

Similar to the closed triangular twist, but with the sides of the triangle double-sized, that is, they occupy 4 grid spaces.
The process is identical to that of the closed twist, in its version with pre-creasing of the folds outside the grid.


CP with the folds.
(2) In the process of twisting.
(3) Finished twist.

## SMALL DANCING COUPLES

The name derives from the base molecule of this tessellation, which is composed of 3 pairs of closed triangle twists.


The basic molecule is composed of 12 closed triangle twists, grouped in pairs, around a central hexagon.

## TRIANGULAR TWISTS

## Folding Process

Pre-crease all the triangles. Being closed triangle twists, it could be done without the pre-creasing, depending on the type of paper.


In the centre of the paper, pinch the 6 mountain folds that come out of the vertices of the hexagon and collapse, bringing the sides of the hexagon towards the centre.


Flat paper, with the next triangles pre-creased.


2Twist all the pinched folds in the same direction. That sense of rotation is maintained throughout the whole model.


Open the paper and fold the two triangle twists simultaneously, each in its direction.


Finished pleat. To the right, the next pair of triangles. We twist the paper to handle it better.


The lines in blue and red indicate the next folds. Open the paper forming the mountain and valley folds.


Twist the two folds and flatten. Fold in the indicated order and make the triangle twist.


Repeat on all sides of the hexagon. Central ring finished.


The centre of the next hexagon (with a circle) and the two folds to be made next are indicated, opening the paper and flattening it.


Fold two triangle twists. Opening the paper on the other sides of the hexagon, fold the twists and form the hexagon.

Keep opening the paper and folding the remaining triangle twists, as well as the $120^{\circ}$ pleats that complete every pair.


- "Small dancing couples" finished model, on a 48 divisions grid.


## TESSELLATIONS WITH RHOMBIC TWISTS

Explanation of the particular techniques applied to the models of this chapter. Some of the previous ones can be used as well.

## Rhombic Twist


(1) Pre-crease the 4 sides of the rhombus outside the grid.


[^0]

Pinch the 4 folds and twist them in the chosen direction.


[^1]
## Rhombic Twist (Variation)

It is a variation of the previous twist that allows hiding part of the paper underneath the rhombus, which also allows to make other different designs with the same polygon.


1
Pre-crease the folds of the rhombi outside the grid.


(2) Pinch the 4 mountain folds and keep them that way. In the next step, $A$ will swith to valley and $B$ to mountain.
(3) While twisting, push downwards towards the designated point in the previous graphic, creating $A$ and $B$.
4. Complete the twist of the 4 folds.

## RHOMBUS FLOWERS

The name is due to the shape of the molecule, in which 6 petals (the rhombic twists) come out from under the centre of the flower.


- The basic molecule is composed of 6 closed triangle twists surrounded by 6 rhombic twists that come out from the centre.


## Folding Process

Pre-crease all the triangles and rhombi that go outside the grid. The closed triangle twists can be done without pre-creasing, if you have a lot of practice.


Start at the centre of the paper. Fold the two triangle twists and continue until all six are complete.

Open the paper in the indicated zones and fold the two rhombi.

$\uparrow$

Hide the tips of the rhombi under the triangle twists. Repeat on the other sides.

Open the paper and fold the first two triangle twists of the group of 6 .


Fold the remaining 4 twists until the set is complete. Repeat on the other two sides.


Folding complete. Continue with the groups of six triangle twists until the model is finished.

First open and fold the two inner rhombi and then the outer ones. Repeat on the other two sides.


View of the back of one of the molecules.


- "Rhombus flowers" finished model, on a 48 divisions grid.


## TILED PYRAMIDS

The original name comes from Eric Gjerde's tessellation (Tiled hexagons). The final result, when backlit, is similar to said tessellation, even though the molecule is completely different.


The basic molecule is composed of 1 closed triangle twist, on top of another double sized triangle twist.

## Folding Process

The steps to fold the 6 pyramids that compose the centre of the model will be shown. If you wish to fold the rest of the pyramids, we have to consider the following:


CP with the initial twists.


3
First triangle twist (in grey the pre-creasing of the exterior triangle twist). Complete the rest of the twists.

- Pre-crease all the triangle twists that we will make on the paper (this step is optional if you have a lot of practice).
- Fold those twists; adjacent twists go in opposite directions.






Fold the 6 triangle twists.


Pre-crease the 6 outer triangles (in blue).


Simultaneously fold the two pyramids, using the indicated valley and mountain folds.


0
It is shown how the paper of 32 divisions would look like, having made in the second step all the triangle twists.


Repeat the twist process for the rest of the pyramids.


Pre-crease and fold the rest of the exterior triangles. Finished model.

- "Tiled pyramids" finished model, on an 80 divisions grid. Two versions of the model can be made, with the internal and external twists in the same or opposite direction.

BOXES

## FOLDING SEQUENCE

As part of my analysis related to tessellations, I decided to make boxes based on the same designs.

The lid of these models has a part of a tessellation, while the box is made with only one paper, adjusting the size to the corresponding lid.

First, how to fold the box is explained and the recommendations when sizing and folding it.

And then the folding sequence of the lid is detailed in each model, similar to tessellations.

Regarding the papers used, we can point out the following:

- Types of paper: the same as the ones for tessellations.

The transparent papers such as the Alios Kraft produce very good results, and in opaque, I recommend Tant or EH.

- Paper size:
- I generally use $24 \times 24 \mathrm{~cm}$ paper for the lid, cutting the inside hexagon.
- For the box, I recommend $20 \times 20 \mathrm{~cm}$ paper, cutting the inside hexagon.
- Grid: depending on the model, 32 or 40.

For the folding of these models it is advisable to use clips or tweezers in the final phase of collapsing and folding of the flaps.


Rhombus twist flower I


Rhombus twist flower II


Tiled pyramids


## OMIYAGE (TATO)

Continuing with the analysis of the tessellations, in this chapter we will explain other models that use the same technique to fold "Omiyage" ("tato" in origami terminology).

The beginning of the folding of these "omiyage" has its origin in my trip to Japan with my family in 2017. Knowing the Japanese custom of exchanging gifts in meetings (both business as well as family and between friends), it occurred to me that the best gift I could offer was somethings folded, that it was an original design and that it could be easily carried on the trip.

The "Tato" are origami models made from one paper, that fold over themselves, forming an envelope that can take on diverse forms.

Taking advantage of the tessellations' technique, the same way that we create boxes folding the corners downwards (in mountain), we can achieve this type of models folding the corners upwards (in valley).

As an introduction to this type of designs, we will explain the folding of two models. The majority of these come from a previous tessellation, same as the boxes.

Regarding the papers used, we can point out the following:

- Type of paper: the same as for tessellations. The transparent papers such as the Alios Kraft produce very good results, and in opaque, I recommend Tant.
- Paper size: I generally use $15 \times 15$, cutting out the interior hexagon.
- Grid: depending on the model, 16, 20, 24, 32.




## BASIC VOCABULARY ENGLISH/SPANISH

| ENGLISH | SPANISH |
| :--- | :--- |
| Tessellation | Teselado |
| Corrugation | Corrugado |
| Grid | Trama / red / grilla |
| Pre-crease | Pre-doblado / Pre-marcado |
| Collapse | Plegado final / Colapsado |
| Score | Plegador de hueso |
| Bone folder | Pliegue |
| Pleat | Giro |
| CP (Crease pattern) | Intersección de pliegues |
| Twist | Inversión de pliegue |
| Pleat intersection | Pliegue en montaña |
| Reverse fold | Pliegue en valle |
| Mountain fold | Sentido de las agujas del reloj |
| Valley fold | Sentido contrario a las agujas del reloj |
| Clockwise | Pliegue en fuelle |
| Counterclockwise |  |
| Crimp |  |
| Pocket |  |

## SYMBOLS USED IN THE BOOK

Folley fold
Fold and unfold
Frimp
of the arrow (M, V$)$

## ABOUT THE AUTHOR



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Born in Madrid (Spain) in 1955. I studied aeronautical engineering in ETSIA Madrid University, and after working for 5 years in an engineering company, I met a group of friends with whom I founded a software company, in which I remain a partner.

In 1997 I travelled to Argentina to open our first office out of Spain. Here I met my wife, whom I married in 1998. I have been living here since then and our daughter Sol was born on 2000.
I entered the origami world in 2007 and since the beginning I was mainly interested in geometry and math related origami. That is why I mainly folded modular origami in its different forms.
I designed several modular models based on Tomoko Fusés units and "snapology".
In 2011 I discovered the tessellation world and since then, I started folding other authors' models in order to learn the process. Around the end of 2017, my firsts tessellation designs came to life. At first, I only designed plain tessellations but then I developed some additional models based on the same techniques (boxes, closed cubes and tatos).
During these last few years, I have taken part in several exhibitions:

- Argentine Japanese Cultural Foundation:
- Permanent exhibition in the Japanese Garden, Buenos Aires.
- Related events in several provinces: Japan in Mendoza, Puerto Madryn, Formosa, Bahía Blanca, Catamarca, and so on.
- Exhibition in the Origami Argentine conventions of 2012, 2014, 2016 and 2018.
- Exhibition in the Spanish Papiroflexia Association convention (2019).
- Since 2017, I take part every year in the Art Fair Expoartistas, at Centro Cultural Borges, Buenos Aires.
- In December 2019, the Museum of Origami in Colonia del Sacramento (Uruguay), founded by Laura Rozenberg, opened its doors and chose some of my tessellations to be exhibited besides some of the greatest origami artists in the world.
- In February 2020 I participated in CfC (convention for creators) in Zaragoza for the first time.


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[^0]:    (3) Twist in process.

[^1]:    4. Finished twist.
