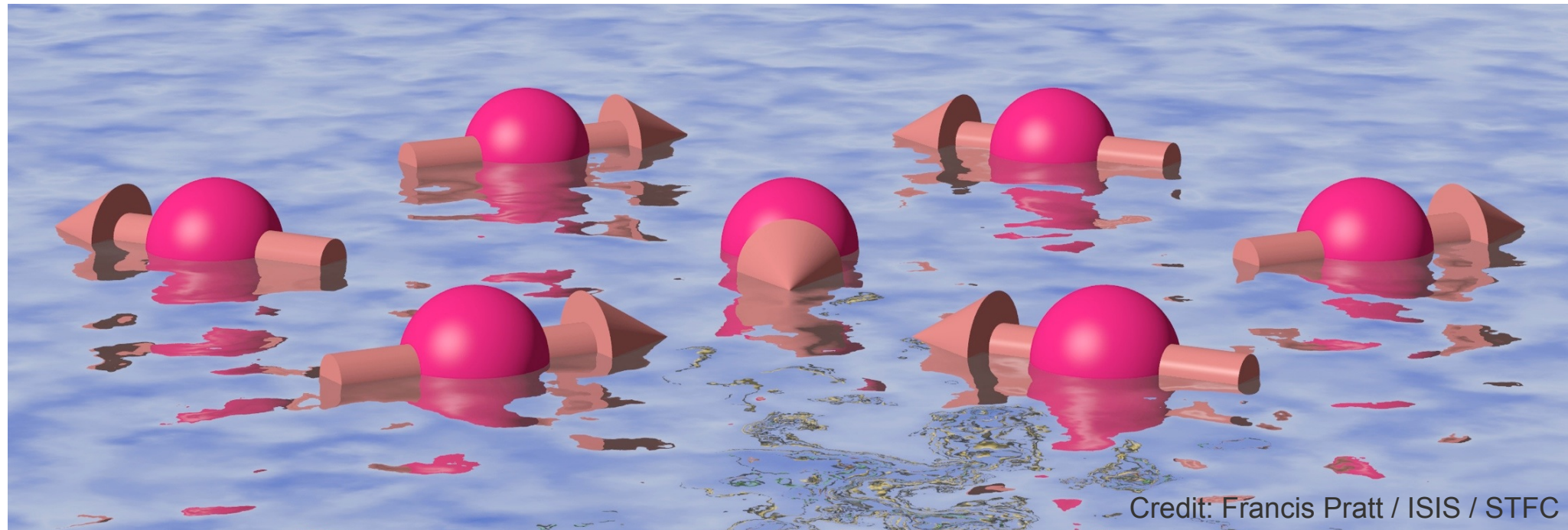


# Two-magnon instabilities and other surprises in magnetized quantum antiferromagnets



***Oleg Starykh***  
University of Utah

Andrey Chubukov, U Wisconsin



*Conference on Field Theory Methods in Low-Dimensional Strongly Correlated Quantum Systems,  
August 25-29, 2014, ICTP, Trieste, Italy*

# Outline

- Emergent Ising orders - a very brief history
- UUD magnetization plateau and its instabilities
- High-field phase diagram of a triangular antiferromagnet

## Discrete-Symmetry Breaking and Novel Critical Phenomena in an Antiferromagnetic Planar (XY) Model in Two Dimensions

D. H. Lee, J. D. Joannopoulos, and J. W. Negele

*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

and

D. P. Landau

*Department of Physics, University of Georgia, Athens, Georgia 30602*

Order parameters: continuous  $\vec{\psi} = \begin{pmatrix} \psi_{\parallel} \\ \psi_{\perp} \end{pmatrix} \equiv \frac{1}{N} \sum_{i=1}^N \exp(-i\vec{q} \cdot \vec{R}_i) \vec{s}_i$  and discrete **spin chirality**  $\chi = \sum_{\text{triangle}} \vec{S}_i \times \vec{S}_j$

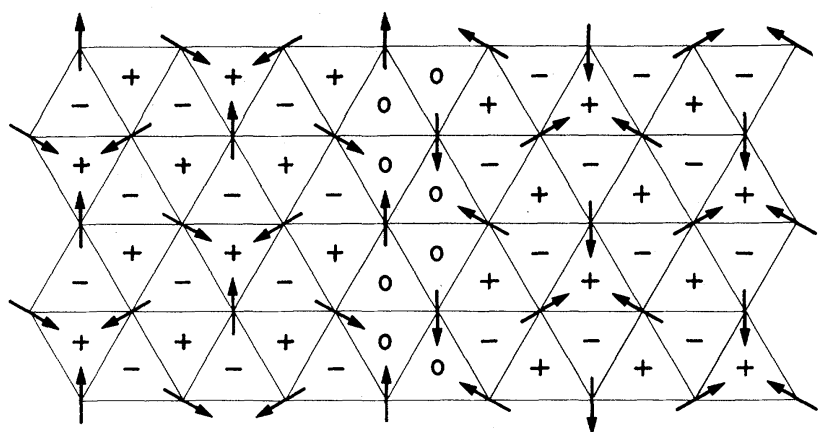
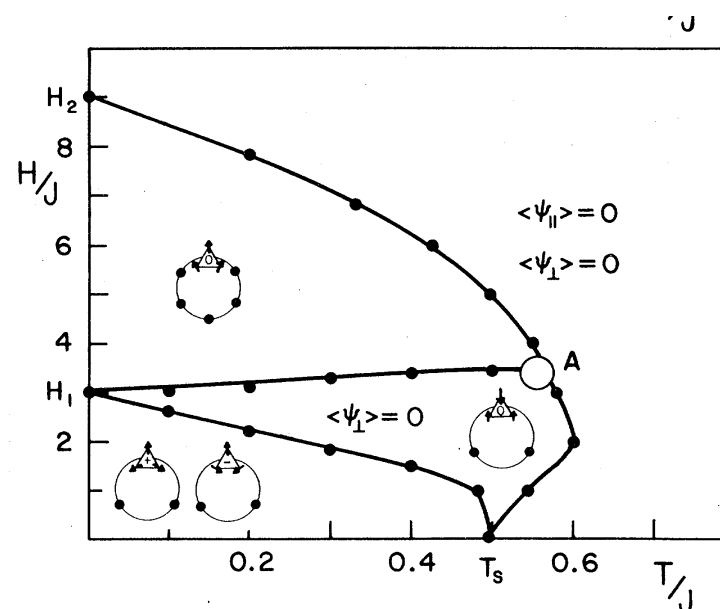


FIG. 1. Two ground states of opposite staggered helicity separated by a domain wall. The helicity of each triangle is indicated.



Two possibilities:

- *single* transition: both **spins** and **chiralities** order,
- *two* separate transitions: **Ising** (**chirality**) transition is followed by the **BKT** (spins)

“An exceedingly simple model leads to a surprising richness of phases and critical behavior.

The underlying triangular lattice and the associated degeneracy play a crucial role in this physics.”

[unfortunately, incorrect identification of spin configurations]

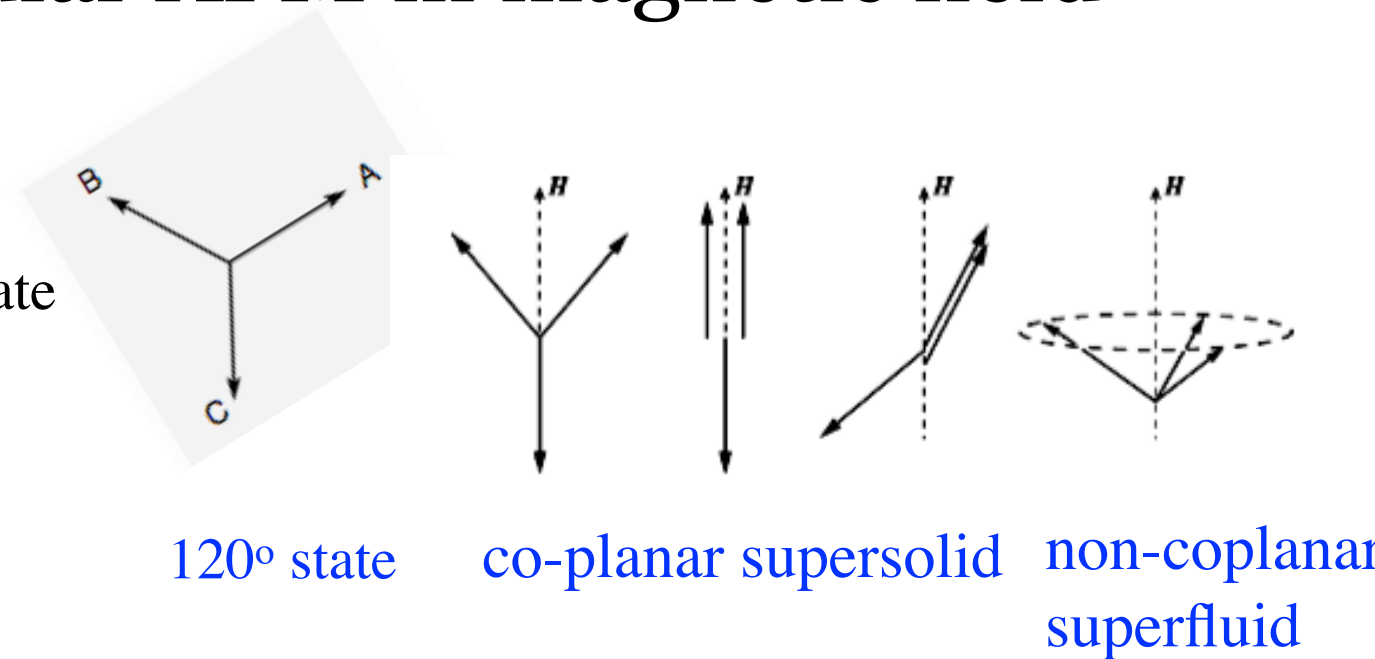
# Classical isotropic triangular AFM in magnetic field

- Zero field: co-planar spiral (120 degree) state
- Magnetic field: **accidental degeneracy**

$$H = J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j - \sum_i \vec{h} \cdot \vec{S}_i$$

$$H = \frac{1}{2} J \sum_{\Delta} \left( \sum_{i \in \Delta} \vec{S}_i - \frac{\vec{h}}{3J} \right)^2$$

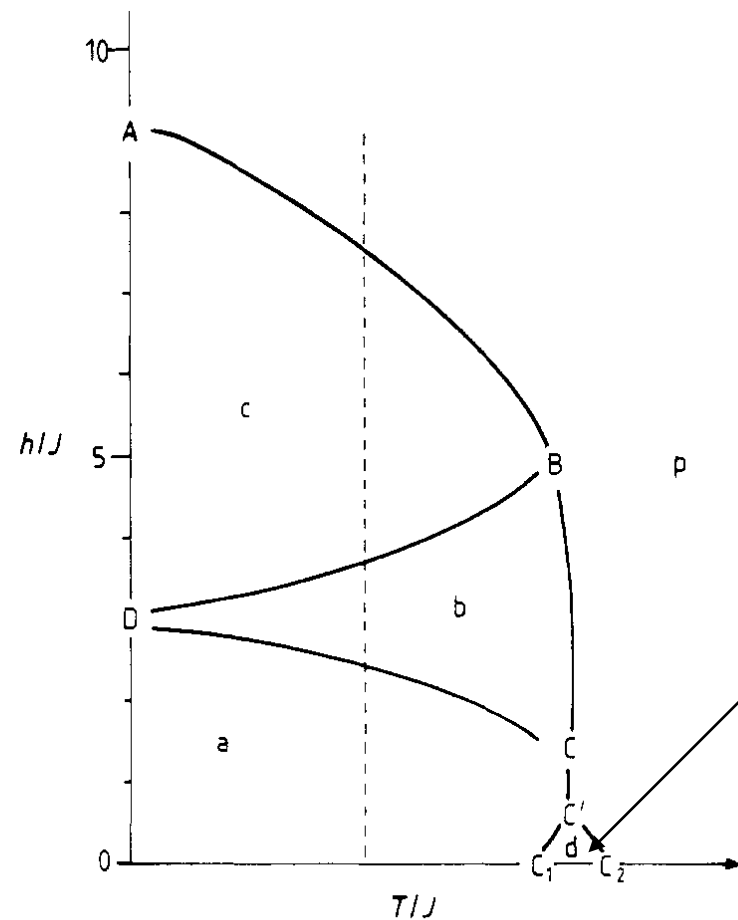
- all states with  $\vec{S}_{i1} + \vec{S}_{i2} + \vec{S}_{i3} = \frac{\vec{h}}{3J}$  form the lowest-energy manifold
- Accidental degeneracy
  - O(2) spins: 3 angles, 2 equations  $\Rightarrow$  1 **continuous angle undetermined**
  - O(3) spins: 6 angles, 3 equations  $\Rightarrow$  2 **continuous angles** (upto global U(1) rotation about **h**)



## Phase diagram of the antiferromagnetic XY model with a triangular lattice in an external magnetic field

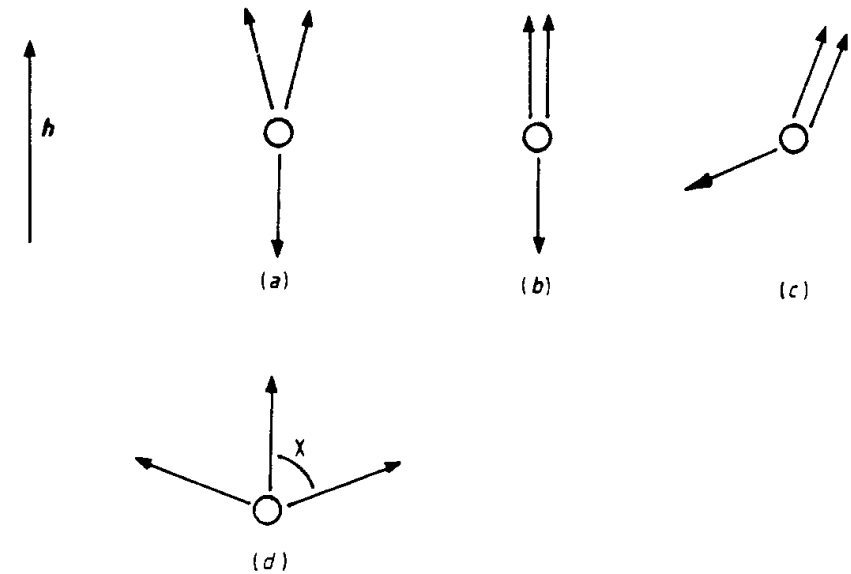
S E Korshunov

L D Landau Institute for Theoretical Physics, Academy of Sciences of the USSR,  
Kosygina 2, 117940 Moscow, USSR



Order of helicities (chiralities) only

**Figure 4.** Phase diagram of the AF XY(t) model in an external magnetic field. In phases a, b and c the mean magnetic moments of different sublattices form configurations similar to those shown in figures 3(a), 3(b) and 3(c) respectively. In phase d only the long-range order with respect to helicities is retained. Phase p is paramagnetic.



**Figure 3.** Configuration of spins of the three sublattices with the minimal spin wave free energy: (a),  $0 < h < h_{c1}$ ; (b),  $h = h_{c1}$ ; (c),  $h_{c1} < h < h_{c2}$ ; (d), in the case of the opposite sign of the anisotropic part of the free energy.

# Phase diagram of the Heisenberg (XXX) model in the field

Journal of the Physical Society of Japan  
Vol. 53, No. 12, December, 1984, pp. 4138–4154

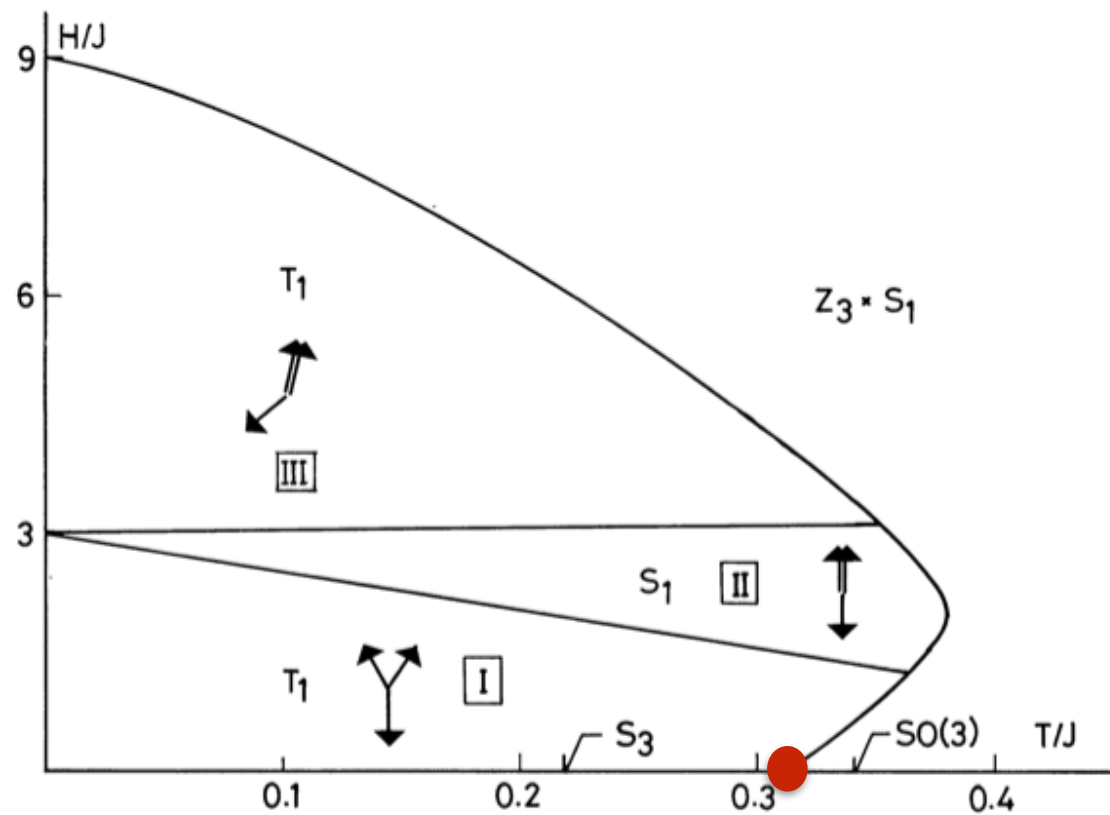
## Phase Transition of the Two-Dimensional Heisenberg Antiferromagnet on the Triangular Lattice

Hikaru KAWAMURA and Seiji MIYASHITA†

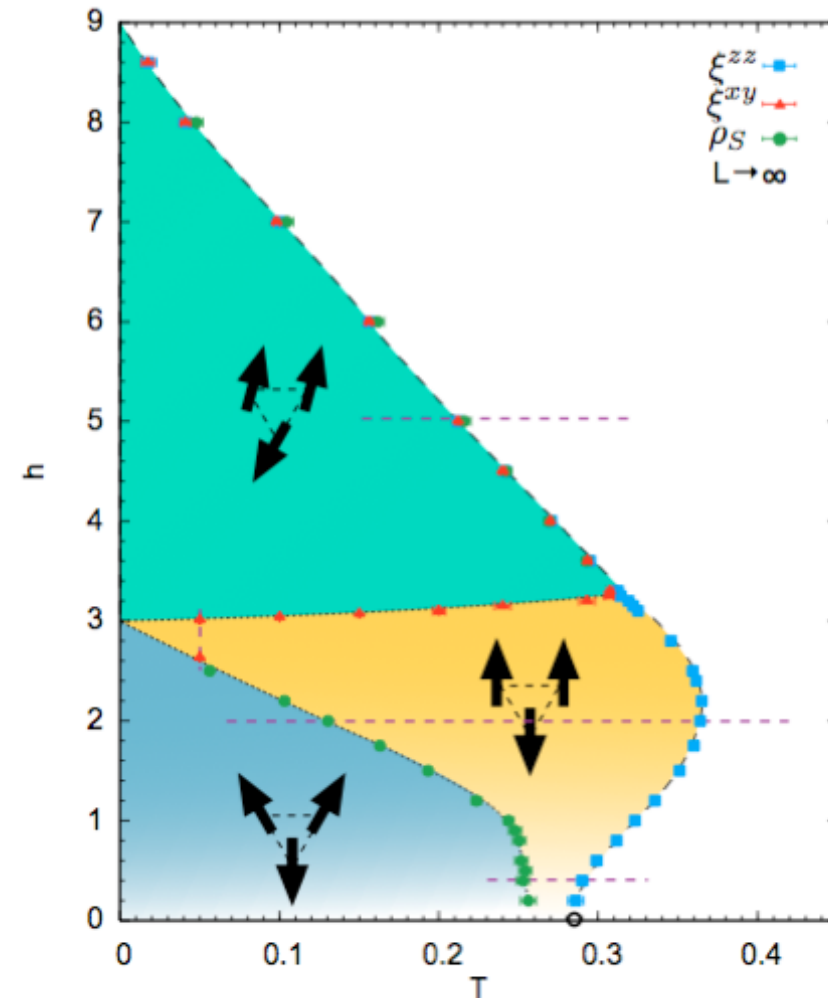
Journal of the Physical Society of Japan  
Vol. 54, No. 12, December, 1985, pp. 4530–4538

## Phase Transition of the Heisenberg Antiferromagnet on the Triangular Lattice in a Magnetic Field

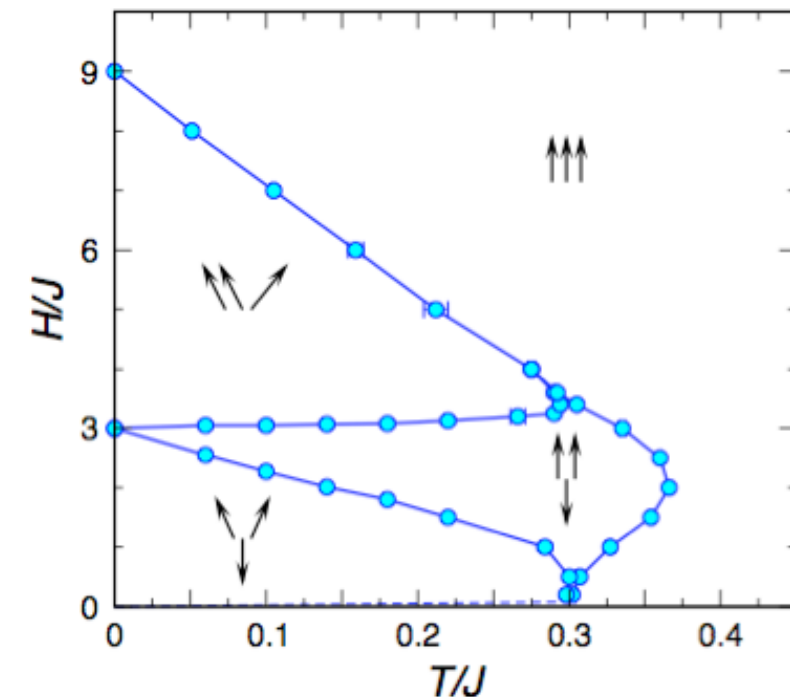
Hikaru KAWAMURA and Seiji MIYASHITA†



$Z_2$  vortex (chirality ordering) transition



Seabra, Momoi, Sindzingre, Shannon 2011



Gvozdkova, Melchy, Zhitomirsky 2010



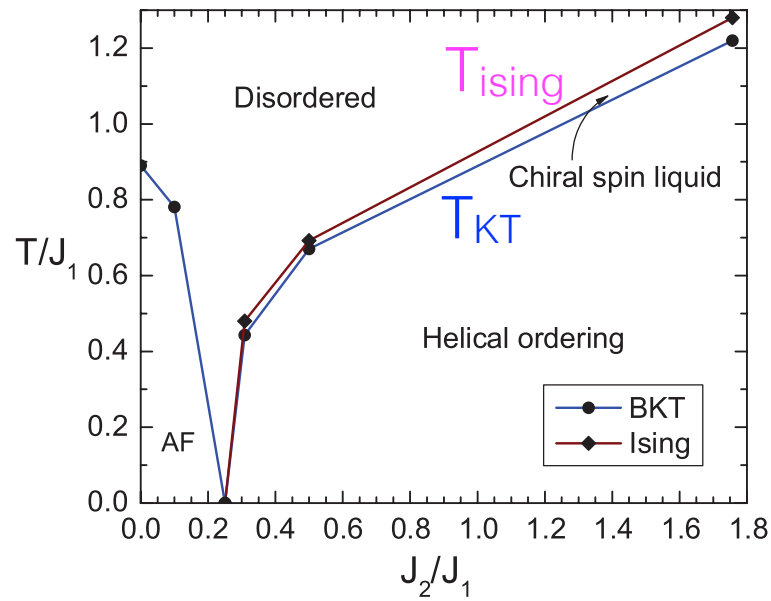
# Emergent Ising order parameters

PHYSICAL REVIEW B **85**, 174404 (2012)

## Chiral spin liquid in two-dimensional XY helimagnets

A. O. Sorokin<sup>1,\*</sup> and A. V. Syromyatnikov<sup>1,2,†</sup>

$$H = \sum_{\mathbf{x}} (J_1 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x}+\mathbf{a}}) + J_2 \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x}+2\mathbf{a}}) - J_b \cos(\varphi_{\mathbf{x}} - \varphi_{\mathbf{x}+\mathbf{b}})),$$



PRL **93**, 257206 (2004)

PHYSICAL REVIEW LETTERS

week ending  
17 DECEMBER 2004

## Low-Temperature Broken-Symmetry Phases of Spiral Antiferromagnets

Luca Capriotti<sup>1,2</sup> and Subir Sachdev<sup>2,3</sup>

$$\hat{H} = J_1 \sum_{\langle i,j \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j + J_3 \sum_{\langle\langle i,j \rangle\rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j,$$

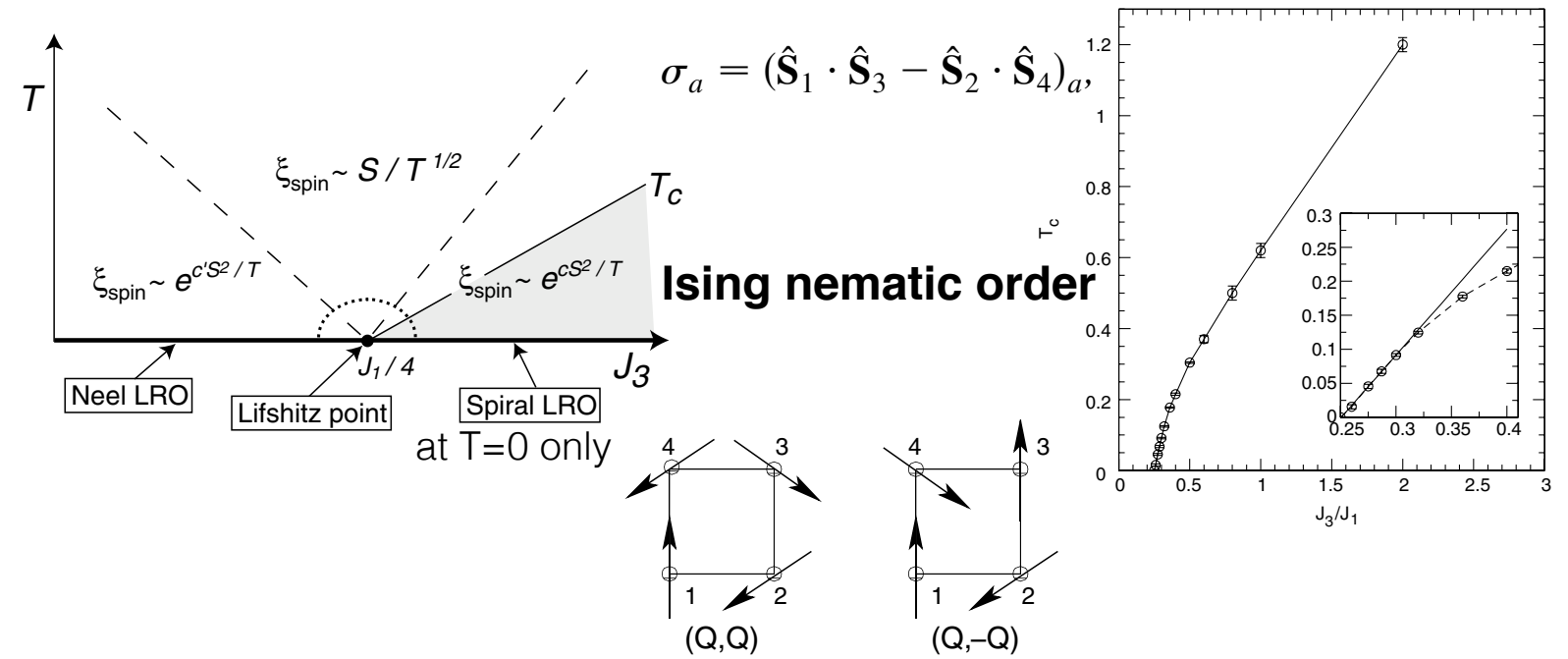


FIG. 2. The two different minimum energy configurations with magnetic wave vectors  $\vec{Q} = (Q, Q)$  and  $\vec{Q}^* = (Q, -Q)$  with  $Q = 2\pi/3$ , corresponding to  $J_3/J_1 = 0.5$ .

# Ising nematic in collinear spin system

VOLUME 64, NUMBER 1

PHYSICAL REVIEW LETTERS

1 JANUARY 1990

## Ising Transition in Frustrated Heisenberg Models

P. Chandra

*Corporate Research Science Laboratories, Exxon Research and Engineering Company, Annandale, New Jersey 08801*

P. Coleman and A. I. Larkin<sup>(a)</sup>

*Serin Physics Laboratory, Rutgers University, P.O. Box 849, Piscataway, New Jersey 08854*

$$\sigma = \vec{N}_1 \cdot \vec{N}_2 = \pm 1$$

VOLUME 91, NUMBER 17

PHYSICAL REVIEW LETTERS

week ending  
24 OCTOBER 2003

## Ising Transition Driven by Frustration in a 2D Classical Model with Continuous Symmetry

Cédric Weber,<sup>1,2</sup> Luca Capriotti,<sup>3</sup> Grégoire Misguich,<sup>4</sup> Federico Becca,<sup>5</sup> Maged Elhajal,<sup>1</sup> and Frédéric Mila<sup>1</sup>

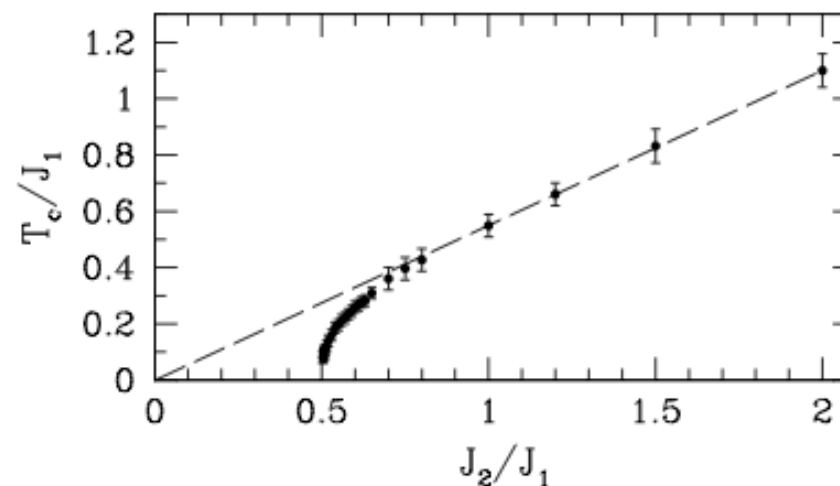
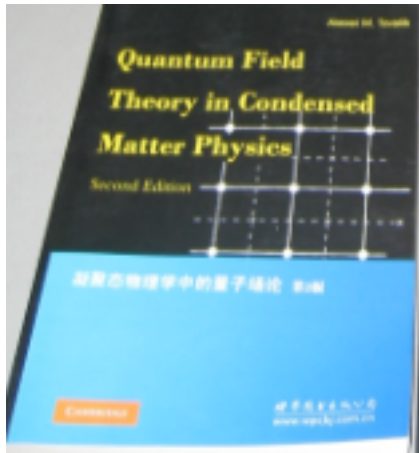


FIG. 4. Monte Carlo results for the critical temperature  $T_c$  as a function of the frustrating ratio  $J_2/J_1$ . The line is an extrapolation of the large  $J_2$  data down to  $J_2 = 0$  (see text).

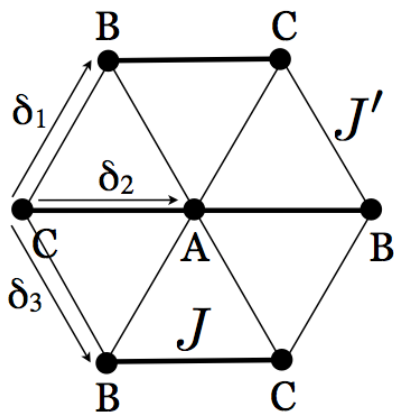




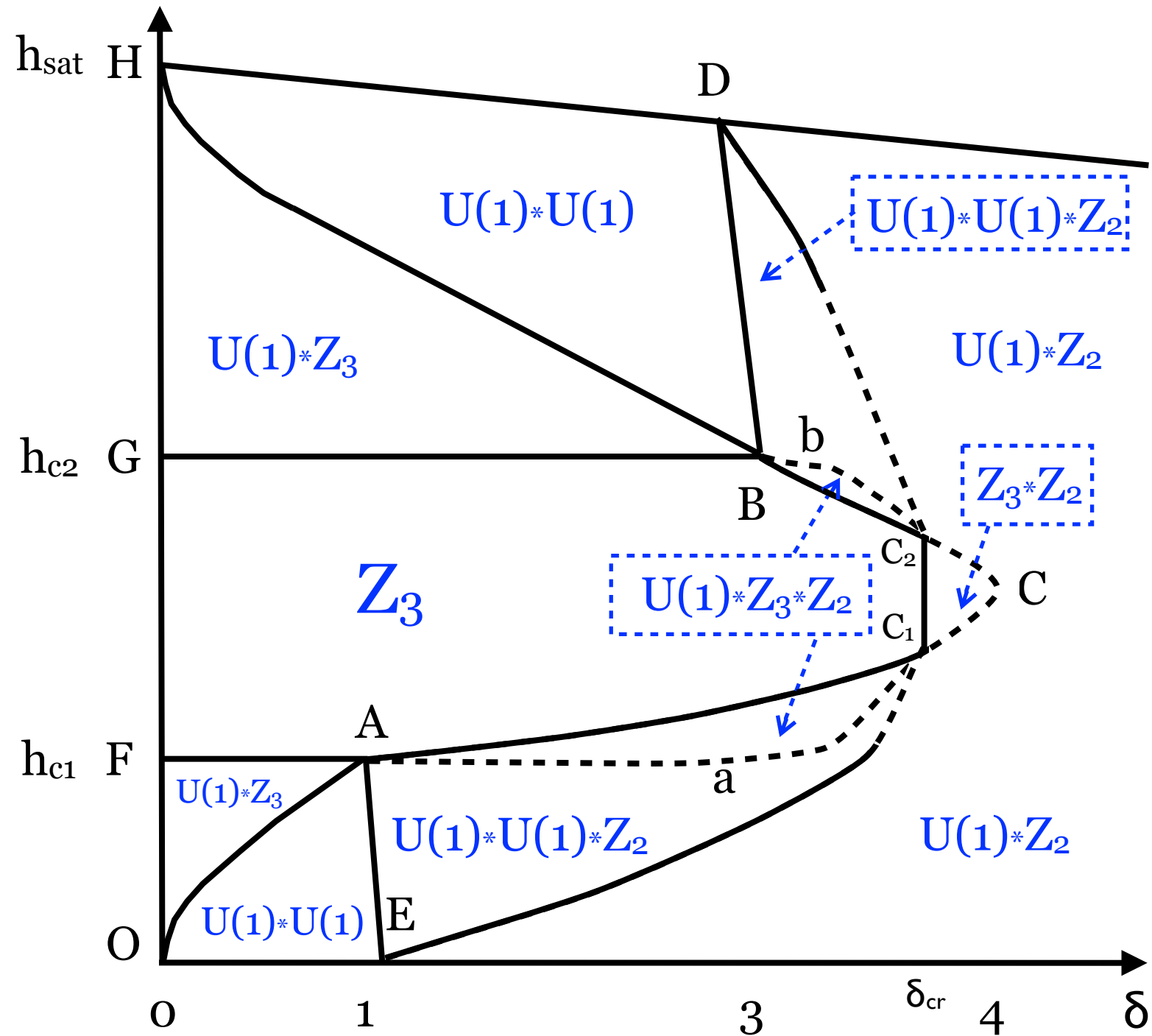
# THIS TALK:

Emergent Ising orders in quantum two-dimensional triangular antiferromagnet at  $T=0$

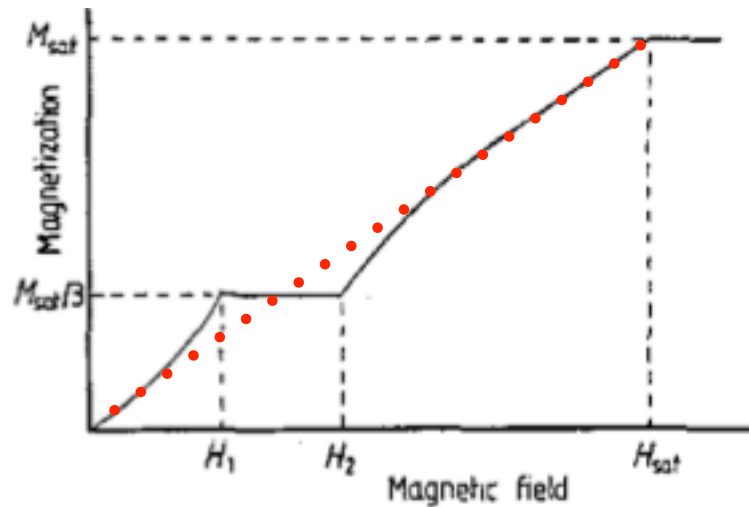
$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



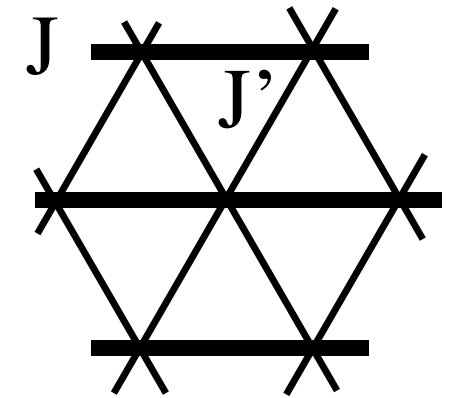
$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$



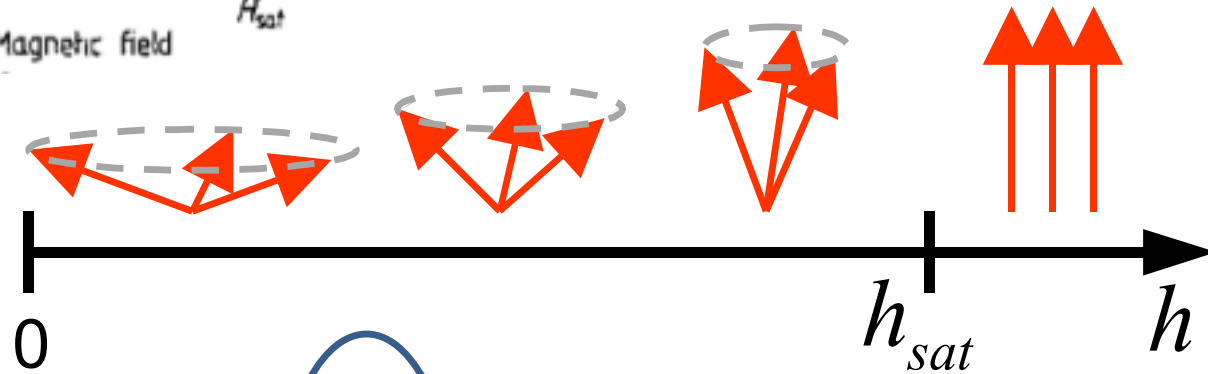
# Spatially anisotropic model: classical vs *quantum*



$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i \mathbf{S}_i^z$$



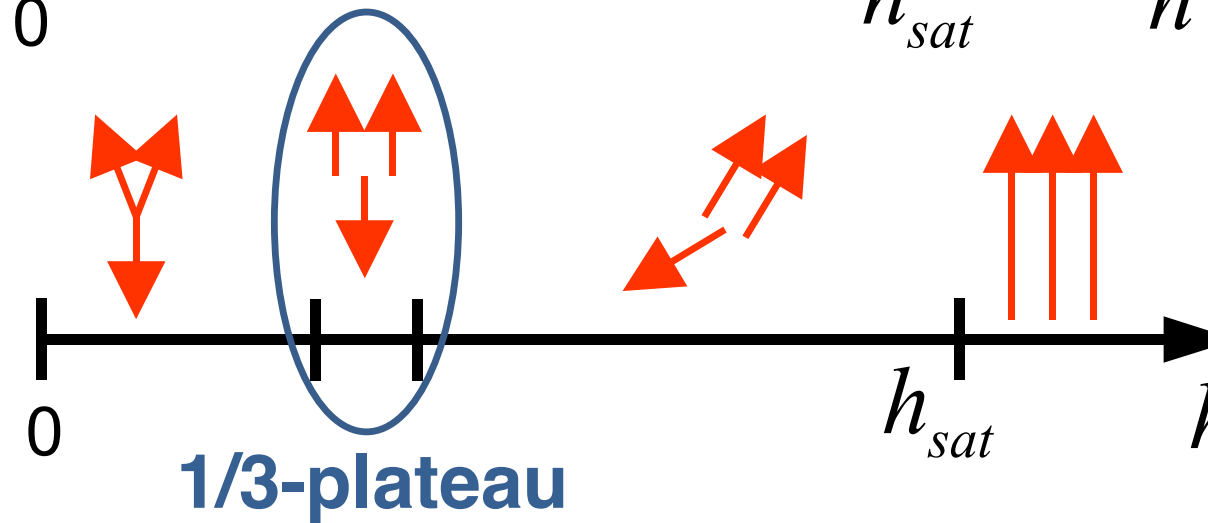
$$S = \infty$$



Umbrella state:  
favored **classically**;  
energy gain  $(J-J')^2/J$

$$J' \neq J$$

$$S = \frac{1}{2}$$

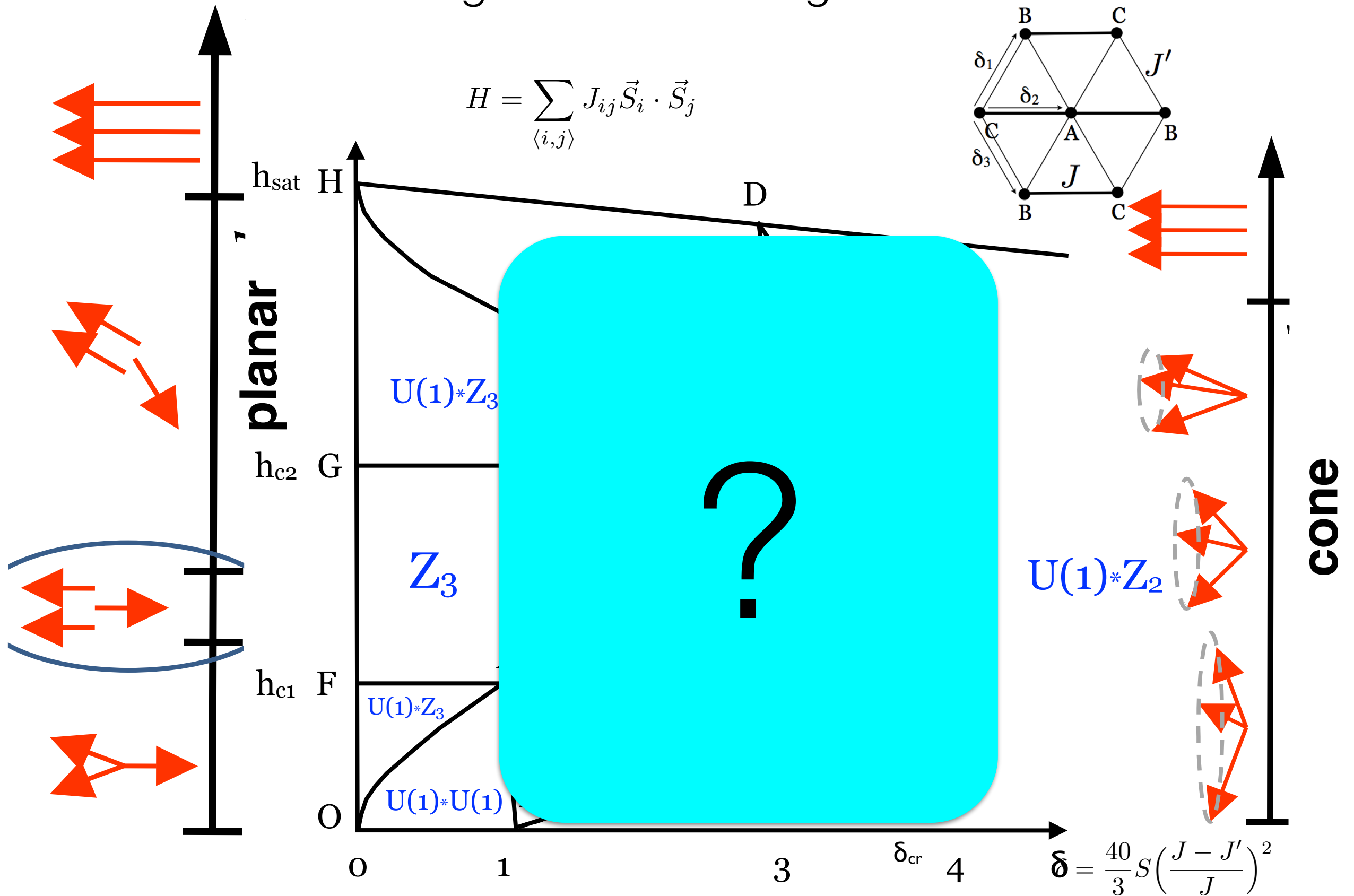


Planar states: favored by  
quantum fluctuations;  
energy gain  $J/S$

The competition is controlled by  
dimensionless parameter

$$\delta = S(J - J')^2 / J^2$$

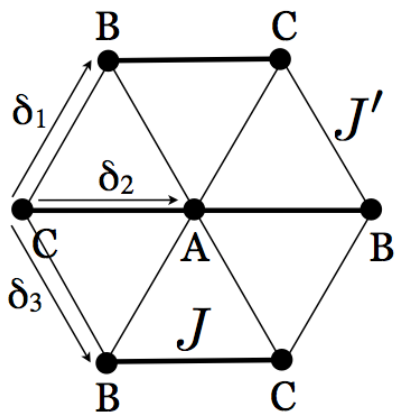
# Emergent Ising orders in quantum two-dimensional triangular antiferromagnet at $T=0$



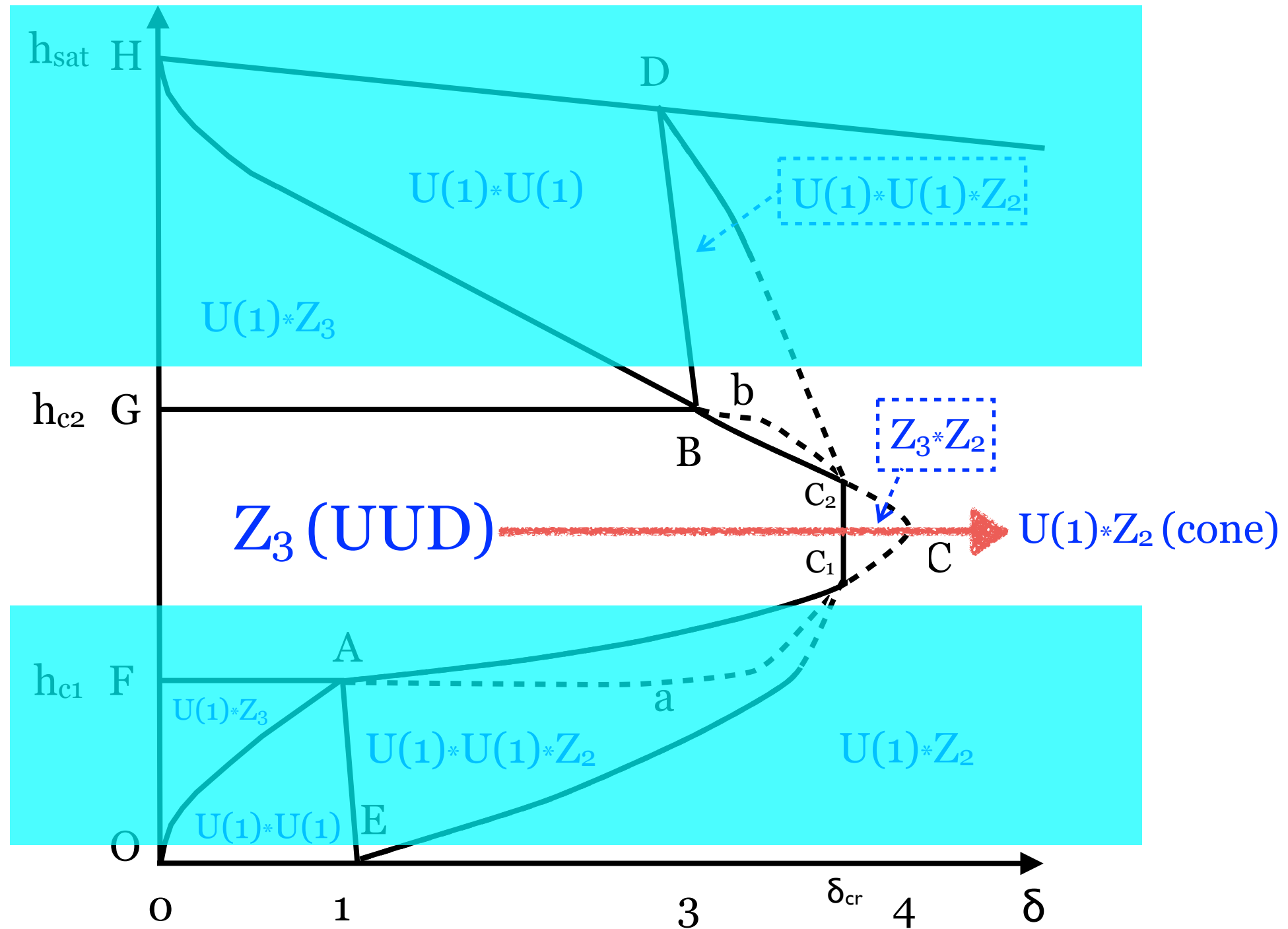
# UUD-to-cone phase transition

$Z_3 \rightarrow U(1) \times Z_2$  or  $Z_3 \rightarrow \text{smth else} \rightarrow U(1) \times Z_2$ ?

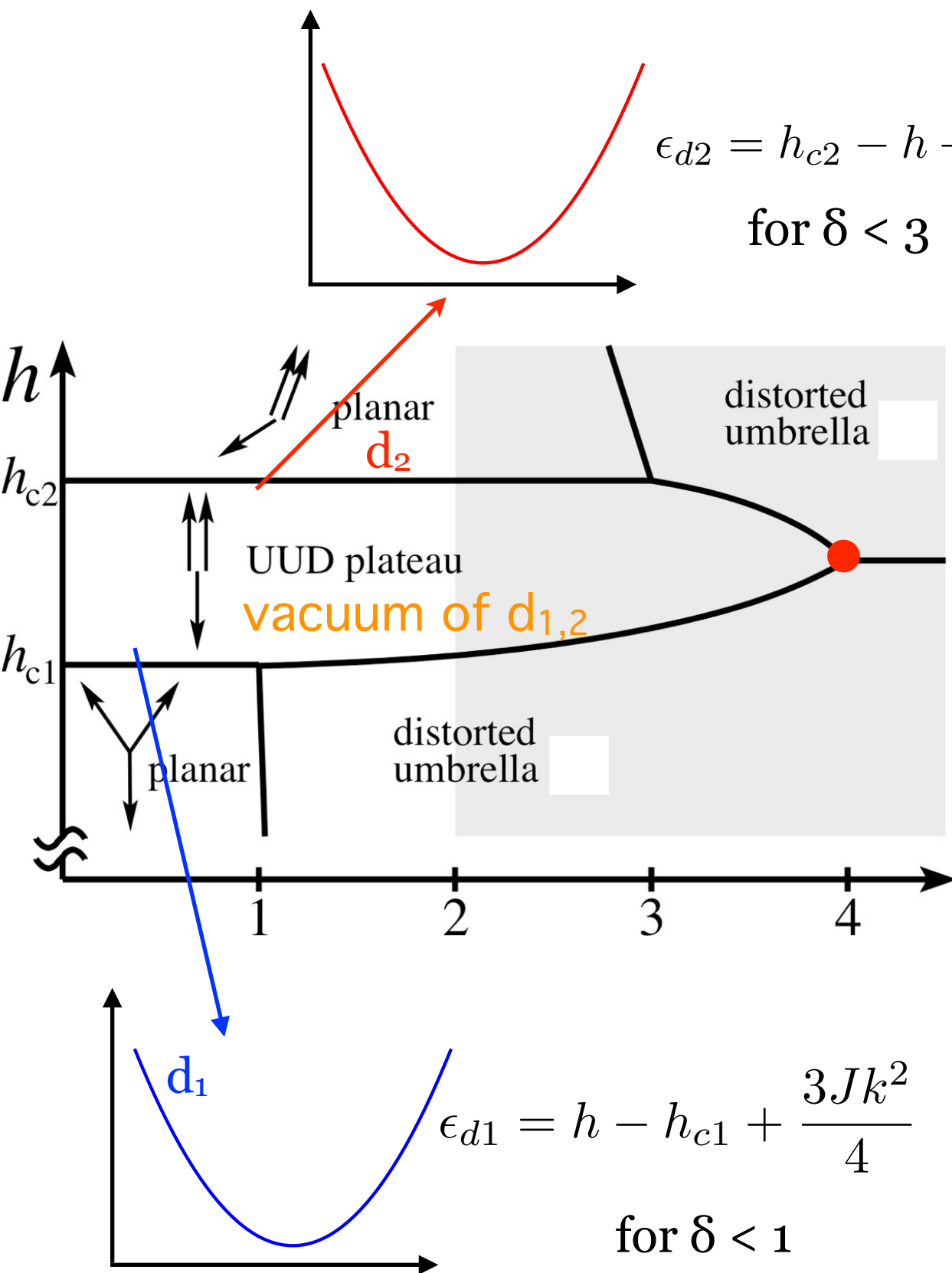
$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$



# Low-energy excitation spectra



Magnetization plateau is **collinear** phase: preserves  $O(2)$  rotations about magnetic field -- no gapless spin waves. Breaks only discrete  $Z_3$ . Hence, **very stable**.

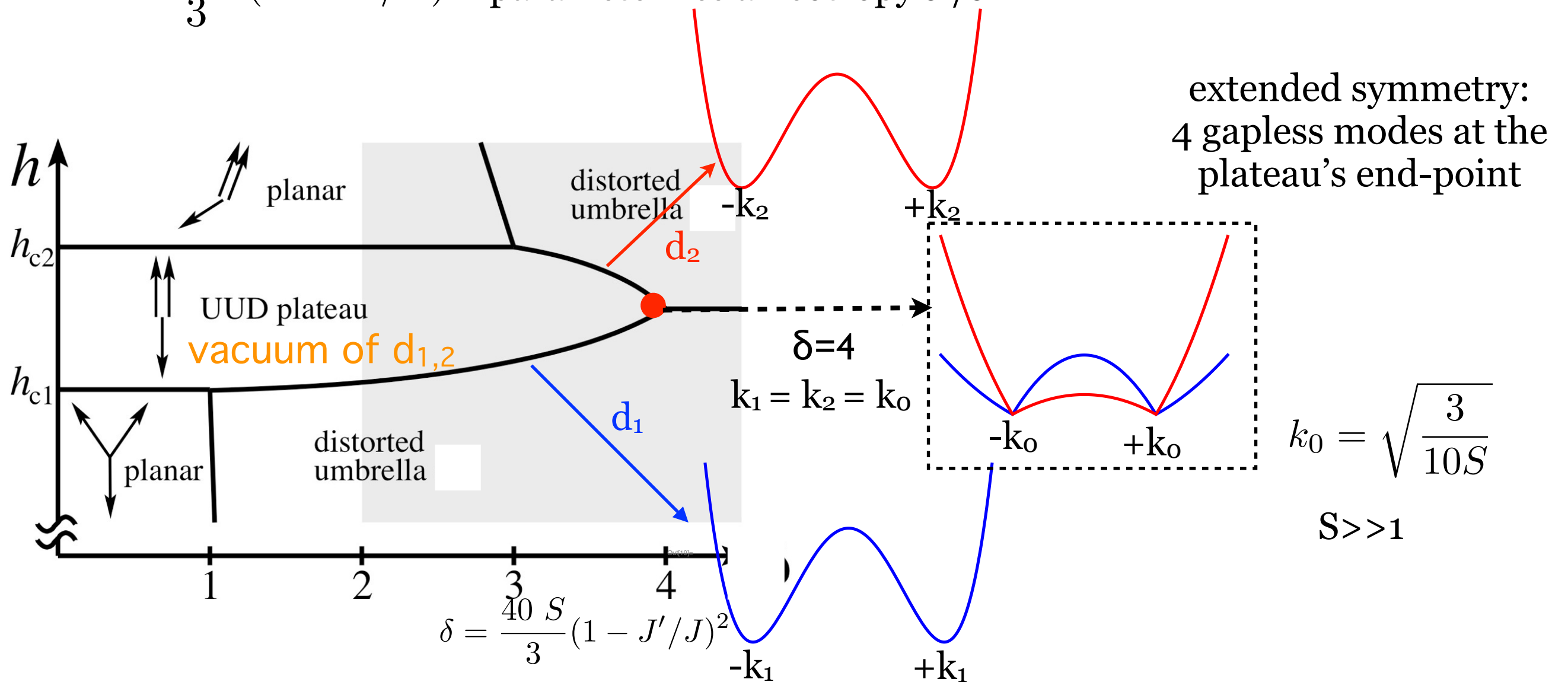
$$h_{c2} - h_{c1} = \frac{0.6}{2S} h_{\text{sat}} = \frac{0.6}{2S} (9JS)$$

$$\delta = \frac{40}{3} \frac{S}{J} (1 - J'/J)^2$$

Bose-Einstein condensation of  $d_1$  ( $d_2$ ) mode at  $k=0$  leads to lower (upper) co-planar phase

# Low-energy excitation spectra near the plateau's end-point

$$\delta = \frac{40 S}{3} (1 - J'/J)^2 \text{ parameterizes anisotropy } J'/J$$



Magnetization plateau is **collinear** phase: preserves  $O(2)$  rotations about magnetic field -- no gapless spin waves. Breaks only discrete  $Z_3$ .



# Bosonization of 2d interacting magnons

$$\mathcal{H}_{d_1 d_2}^{(4)} = \frac{3}{N} \sum_{p,q} \Phi(p,q) \left( \underbrace{d_{1,\mathbf{k}_0+\mathbf{p}}^\dagger d_{2,-\mathbf{k}_0-\mathbf{p}}^\dagger}_{\Psi_{1,p}^\dagger} d_{1,-\mathbf{k}_0+\mathbf{q}} d_{2,\mathbf{k}_0-\mathbf{q}} - \underbrace{d_{1,\mathbf{k}_0+\mathbf{p}}^\dagger d_{2,-\mathbf{k}_0-\mathbf{p}}^\dagger}_{\Psi_{1,p}^\dagger} d_{1,-\mathbf{k}_0+\mathbf{q}} d_{2,\mathbf{k}_0-\mathbf{q}}^\dagger \right) + \text{h.c.}$$

$$\Phi(p,q) \sim \frac{(-3J)k_0^2}{|\mathbf{p}||\mathbf{q}|}$$

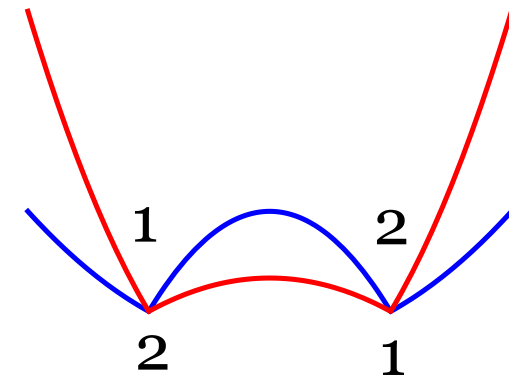
singular magnon interaction

$$\Psi_{1,p}^\dagger \Psi_{2,q}$$

$$\Psi_{1,p}^\dagger \Psi_{2,q}^\dagger$$

magnon pair operators

$$\left\{ \begin{array}{l} \Psi_{1,p} = d_{1,\mathbf{k}_0+\mathbf{p}} d_{2,-\mathbf{k}_0-\mathbf{p}} \\ \Psi_{2,p} = d_{1,-\mathbf{k}_0+\mathbf{p}} d_{2,\mathbf{k}_0-\mathbf{p}} \end{array} \right.$$



Obey canonical Bose commutation relations in the UUD ground state

$$[\Psi_{1,\mathbf{p}}, \Psi_{2,\mathbf{q}}] = \delta_{1,2} \delta_{\mathbf{p},\mathbf{q}} \left( 1 + d_{1,\mathbf{k}_0+\mathbf{p}}^\dagger d_{1,\mathbf{k}_0+\mathbf{p}} + d_{2,\mathbf{k}_0+\mathbf{p}}^\dagger d_{2,\mathbf{k}_0+\mathbf{p}} \right) \rightarrow \delta_{1,2} \delta_{\mathbf{p},\mathbf{q}}$$

In the UUD ground state  $\langle d_1^\dagger d_1 \rangle_{\text{uud}} = \langle d_2^\dagger d_2 \rangle_{\text{uud}} = 0$

★ Interacting magnon Hamiltonian in terms of  $\mathbf{d}_{1,2}$  bosons = non-interacting Hamiltonian in terms of  $\Psi_{1,2}$  magnon pairs

# Two-magnon instability

Magnon pairs  $\Psi_{1,2}$  condense *before* single magnons  $d_{1,2}$

Equations of motion for  $\Psi$  - Hamiltonian

$$\langle \Psi_{1,\mathbf{p}}^\dagger - \Psi_{1,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi_{2,\mathbf{q}}^\dagger - \Psi_{2,\mathbf{q}} \rangle$$

$$\langle \Psi_{2,\mathbf{p}}^\dagger - \Psi_{2,\mathbf{p}} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi_{1,\mathbf{q}}^\dagger - \Psi_{1,\mathbf{q}} \rangle$$

'Superconducting' solution with *imaginary* order parameter

$$\langle \Psi_{1,p} \rangle = \langle \Psi_{2,p} \rangle \sim i \frac{\Upsilon}{\mathbf{p}^2}$$

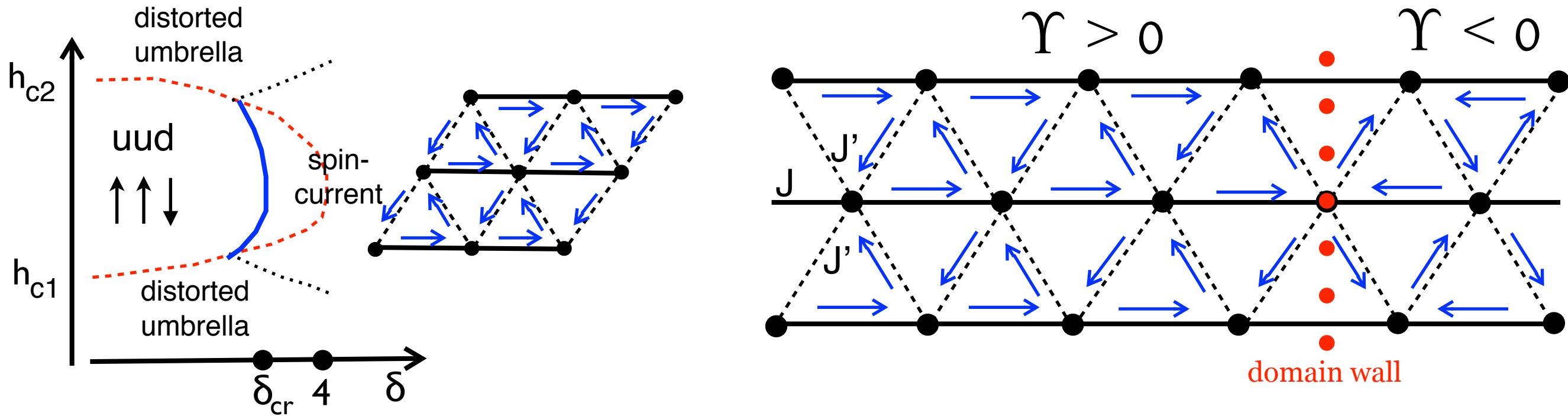
**Instability** = softening of two-magnon mode @  $\delta_{\text{cr}} = 4 - O(1/S^2)$

$$1 = \frac{1}{S} \frac{1}{N} \sum_p \frac{k_0}{\sqrt{|\mathbf{p}|^2 + (1 - \delta/4)k_0^2}}$$

**no** single particle condensate

$$\langle d_1 \rangle = \langle d_2 \rangle = 0$$

# Two-magnon condensate = Spin-current nematic state



no transverse magnetic order  $\langle \mathbf{S}_r^{x,y} \rangle = 0$   $\langle \mathbf{S}_r \cdot \mathbf{S}_{r'} \rangle$  is not affected

Finite scalar (and vector) chiralities. Sign of  $\Upsilon$  determines sense of spin-current circulation

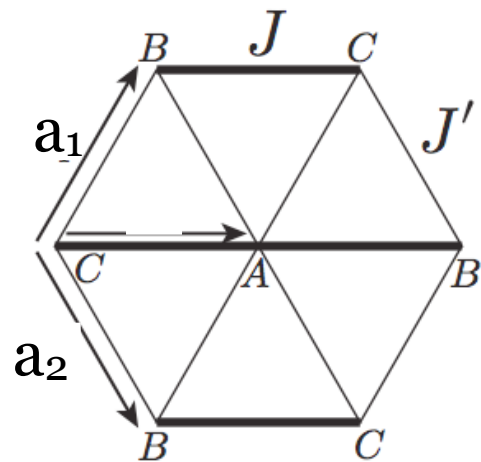
$$\langle \hat{z} \cdot \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \hat{z} \cdot \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \hat{z} \cdot \mathbf{S}_B \times \mathbf{S}_A \rangle \propto \Upsilon$$

**Spontaneously broken  $Z_2$  -- spatial inversion** [in addition to broken  $Z_3$  inherited from the UUD state]

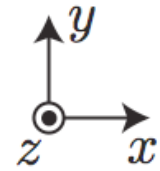
Leads to **spontaneous generation of Dzyaloshinskii-Moriya interaction**

# Spontaneous generation of Dzyaloshinskii-Moriya interaction

$$\mathcal{H}_{d_1 d_2}^{(4)} \propto \frac{1}{N} \sum_{k \in +k_0} \frac{-ik_0}{\sqrt{(k - k_0)^2 + (1 - \delta/4)k_0^2}} (d_{1,\mathbf{k}}^\dagger d_{2,-\mathbf{k}}^\dagger - d_{1,\mathbf{k}} d_{2,-\mathbf{k}}) \sum_{p \in -k_0} \frac{-ik_0}{\sqrt{(p + k_0)^2 + (1 - \delta/4)k_0^2}} (d_{1,\mathbf{p}}^\dagger d_{2,-\mathbf{p}}^\dagger - d_{1,\mathbf{p}} d_{2,-\mathbf{p}})$$



continuum limit of DM in triangular lattice



$$\sum_{\mathbf{r}} \hat{z} \cdot \mathbf{S}_{\mathbf{r}} \times (\mathbf{S}_{\mathbf{r}+\mathbf{a}_1} + \mathbf{S}_{\mathbf{r}+\mathbf{a}_2})$$

$$\mathbf{B} \parallel \mathbf{z}$$

Mean-field approximation:

$$\mathcal{H}_{d_1 d_2}^{(4)} \rightarrow D \sum_{\mathbf{k}} \left( \frac{k_0}{|k - k_0|} + \frac{k_0}{|k + k_0|} \right) (d_{1,\mathbf{k}}^\dagger d_{2,-\mathbf{k}}^\dagger - d_{1,\mathbf{k}} d_{2,-\mathbf{k}})$$

$$D \sim \Upsilon$$

**spin currents** appear due to **spontaneously generated DM**  
 (similar to Lauchli et al (PRL 2005) for Heis.+ring exchange model;  
 also ‘chiral Mott insulator’, Dhar et al, PRB 2013; Zaletel et al, 2013 )

## Incommensurate Spin Correlations in Spin-1/2 Frustrated Two-Leg Heisenberg Ladders

Alexander A. Nersisyan,<sup>1</sup> Alexander O. Gogolin,<sup>2</sup> and Fabian H.L. Eßler<sup>3</sup>

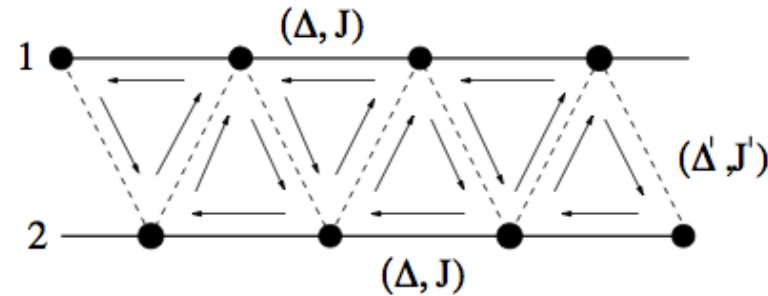


FIG. 3. Structure of the spin currents in the spin nematic phase.

PHYSICAL REVIEW B 87, 174501 (2013)

## Chiral Mott insulator with staggered loop currents in the fully frustrated Bose-Hubbard model

Arya Dhar,<sup>1</sup> Tapan Mishra,<sup>2</sup> Maheswar Maji,<sup>3</sup> R. V. Pai,<sup>4</sup> Subroto Mukerjee,<sup>3,5</sup> and Arun Paramekanti<sup>2,3,6,7</sup>

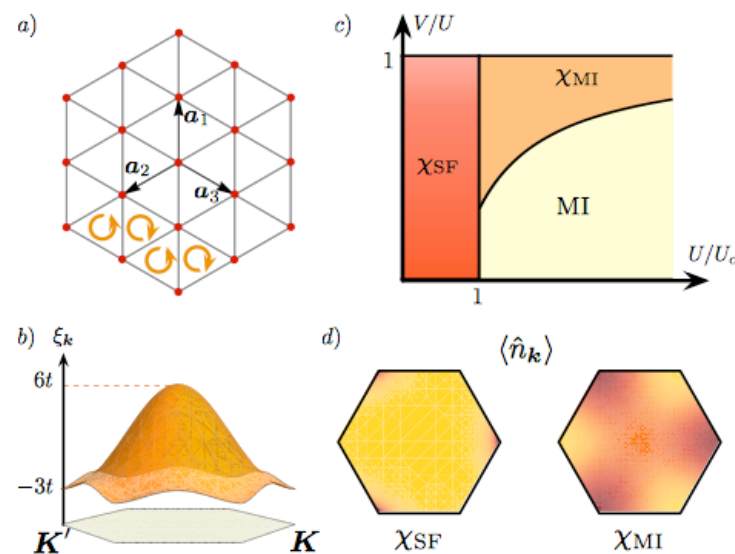


FIG. 1. **Bosons on the Frustrated Triangular Lattice.** (a) Lattice, coordinate system and sample current pattern in the  $\chi$ MI; (b) single-particle dispersion  $\xi_{\mathbf{k}}$ , with minima at the  $\mathbf{K}$ ,  $\mathbf{K}'$  points of the BZ; (c) Variational mean-field phase diagram showing  $\chi$ SF,  $\chi$ MI and MI phases tuned by the on site repulsion  $U$  and nearest neighbor repulsion  $V$ ; (d) Momentum distribution  $\langle \hat{n}_{\mathbf{k}} \rangle$  for the chiral phases.

PHYSICAL REVIEW B 89, 155142 (2014)

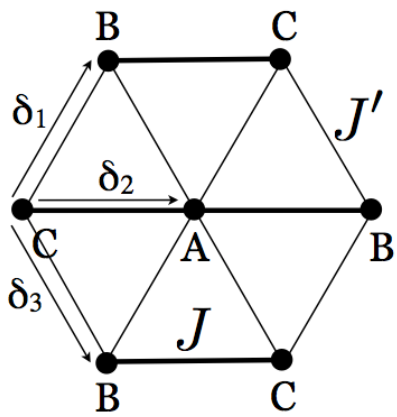
## Chiral bosonic Mott insulator on the frustrated triangular lattice

Michael P. Zaletel,<sup>1</sup> S. A. Parameswaran,<sup>1,2</sup> Andreas Rüegg,<sup>1,3</sup> and Ehud Altman<sup>1,4</sup>

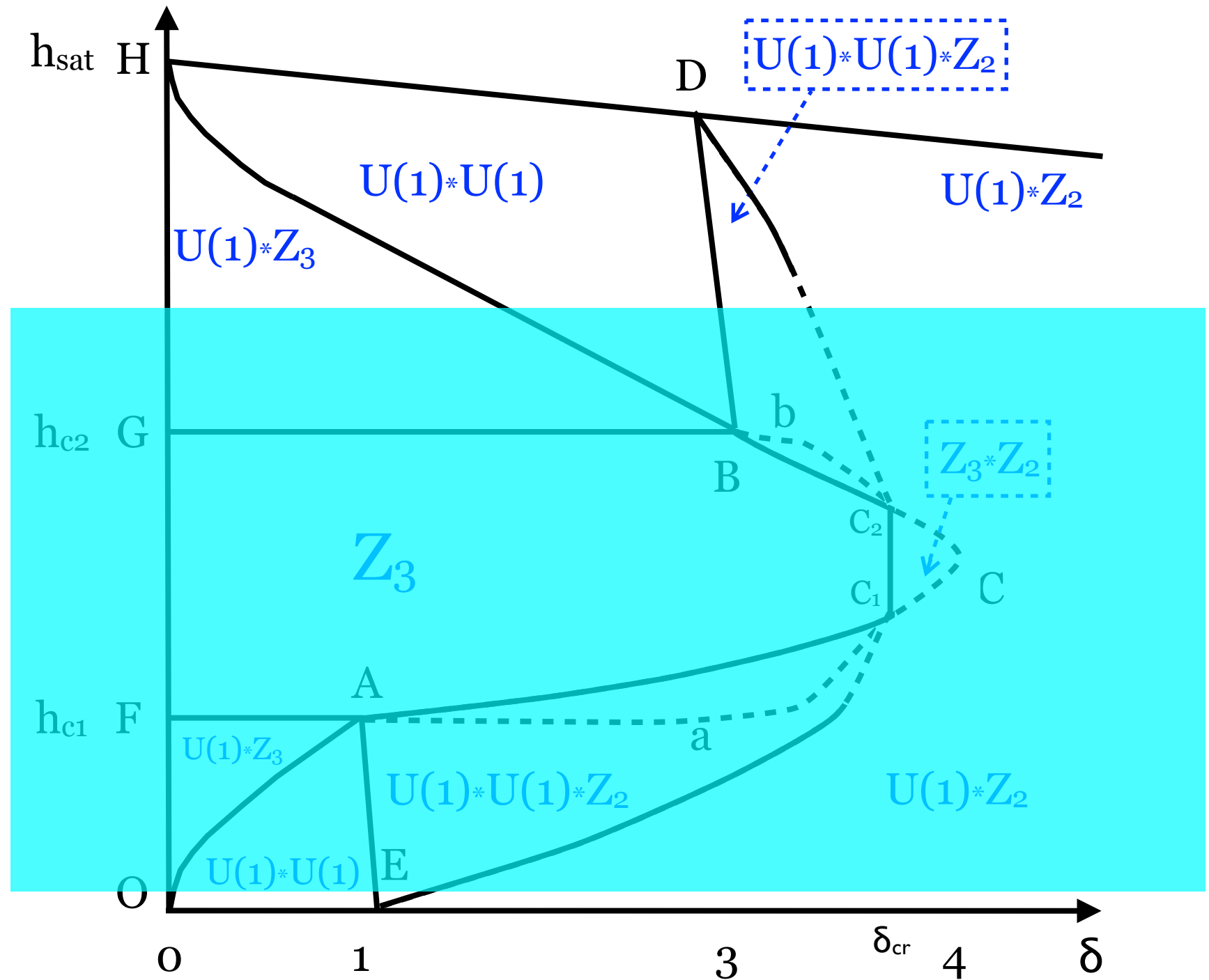
gapped single particles;  
but  
spontaneously broken time-reversal  
= spontaneous circulating  
currents

# Phases of a triangular-lattice antiferromagnet near saturation

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

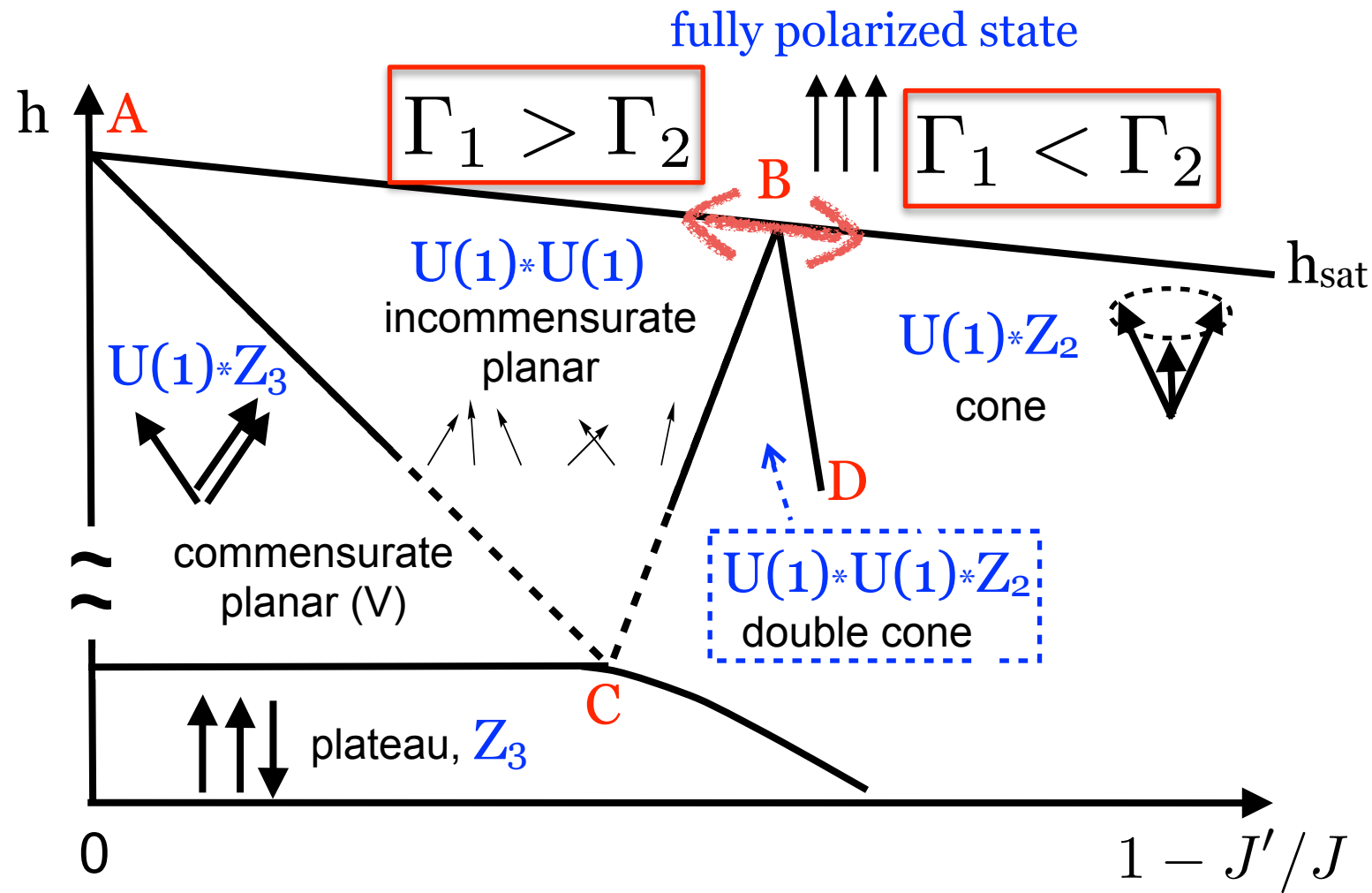


$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$





# High-field phases: from **cone** to **incommensurate planar** at $h = h_{\text{sat}}$



Low-density expansion  $E_0/N = -\mu(|\psi_1|^2 + |\psi_2|^2) + \frac{1}{2}\Gamma_1(|\psi_1|^4 + |\psi_2|^4) + \Gamma_2|\psi_1|^2|\psi_2|^2$

$$\Delta\Gamma = \Gamma_2 - \Gamma_1 = \Delta\Gamma^{(0)} + \Delta\Gamma^{(1)} = \frac{9(\delta J)^2}{J} - \frac{1.6J}{S}$$

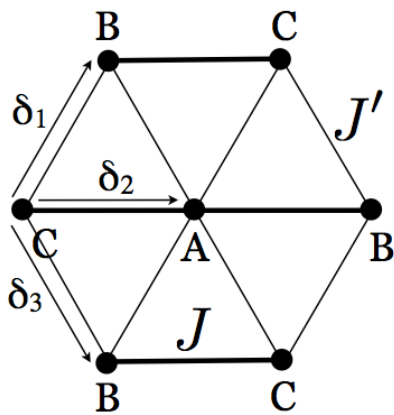
$$\Delta\Gamma^{(1)} = \frac{1}{16S} \sum_{\mathbf{k} \in \text{BZ}} \left( \frac{(J_0 + 5J_{\mathbf{k}})^2}{J_0 - J_{\mathbf{k}}} - \frac{(J_0 - 4J_{\mathbf{Q}+\mathbf{k}})^2}{J_{\mathbf{Q}+\mathbf{k}} - J_{\mathbf{Q}}} \right) + \frac{3J}{8S} \approx -\frac{1.6J}{S}$$

# Phases of a triangular-lattice antiferromagnet near saturation

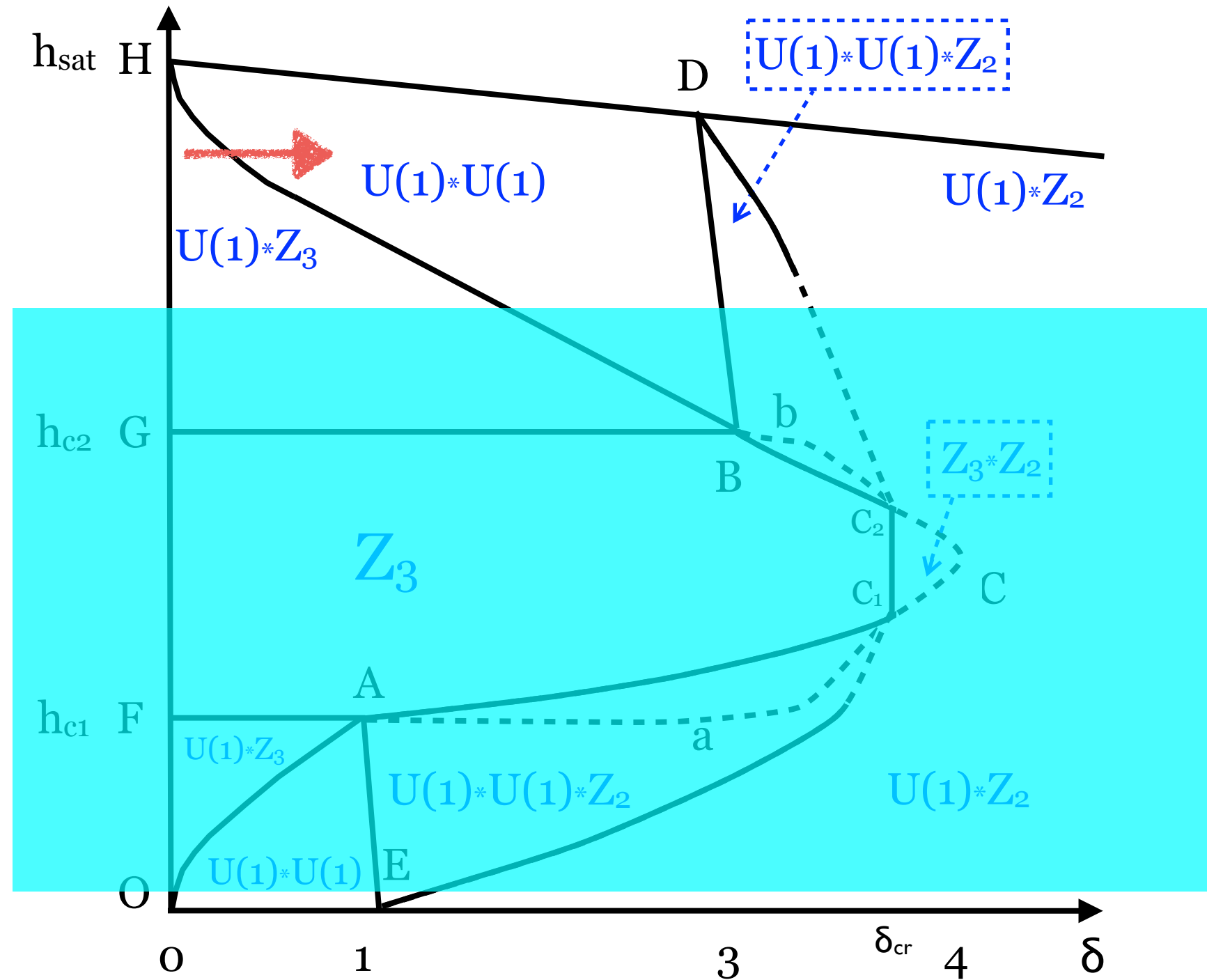
$$U(1) \times Z_3 \rightarrow U(1) \times U(1)$$

$$Q = (4\pi/3, 0) \rightarrow Q_i = \text{incommensurate}$$

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

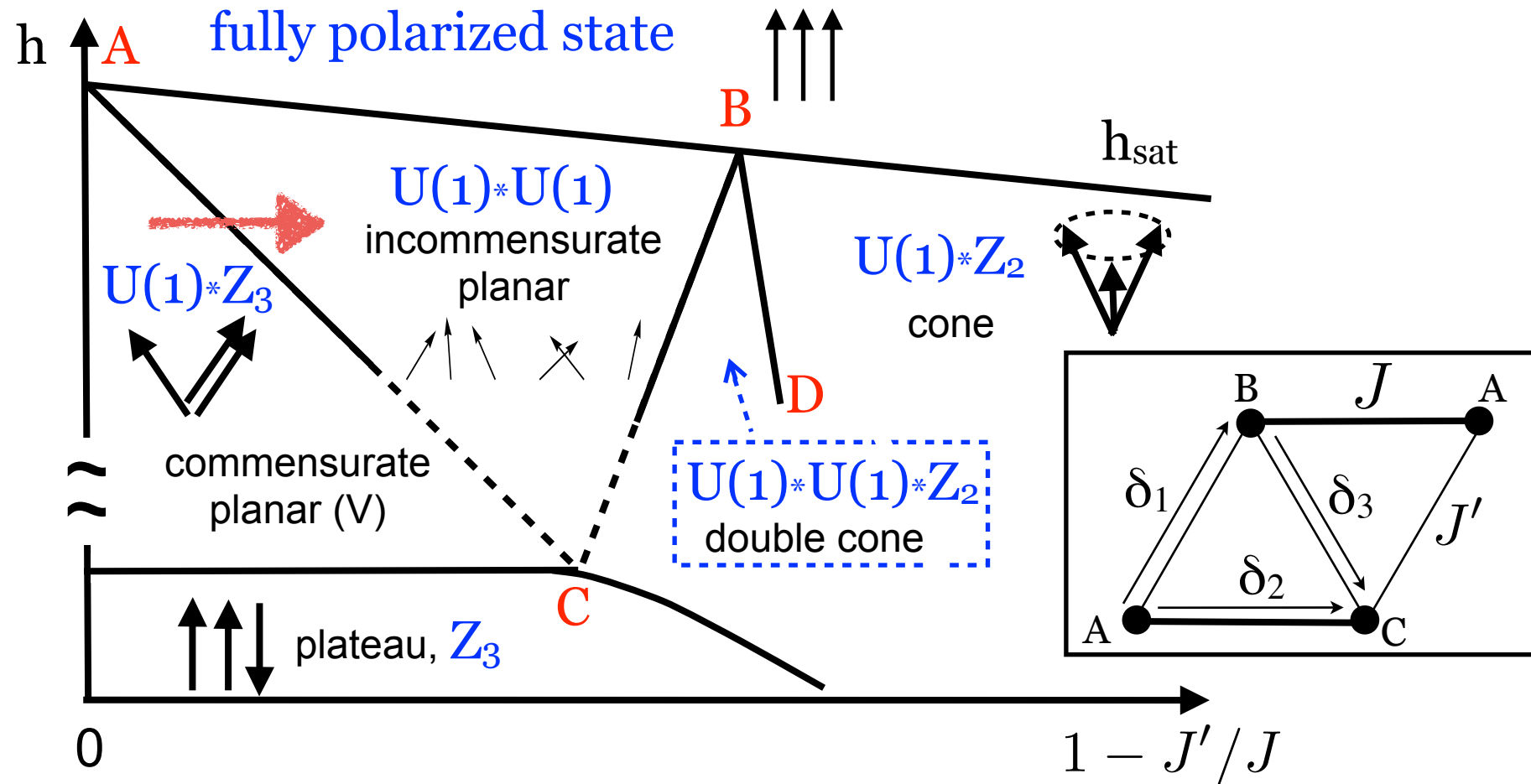


$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$



# High-field phases: commensurate-incommensurate transition

$$Q = (4\pi/3, 0) \rightarrow Q_i = \text{incommensurate}$$



Low-density expansion  $\rightarrow$  classical sine-Gordon model of the (relative) phase fluctuations

$$E_0/N = -\mu(|\psi_1|^2 + |\psi_2|^2) + \frac{1}{2}\Gamma_1(|\psi_1|^4 + |\psi_2|^4) + \Gamma_2|\psi_1|^2|\psi_2|^2 + \Gamma_3((\bar{\psi}_1\psi_2)^3 + \text{h.c.})\dots$$

$$\longrightarrow \mathcal{E}_\theta = \frac{3JS^2\mu}{4h_{\text{sat}}}(\partial_x\theta)^2 + \frac{\sqrt{3}\delta JS^2\mu}{h_{\text{sat}}}\partial_x\theta + S\frac{\Gamma_3 S^2}{4}\frac{\mu^3}{h_{\text{sat}}^3}\cos[6\theta] \dots$$

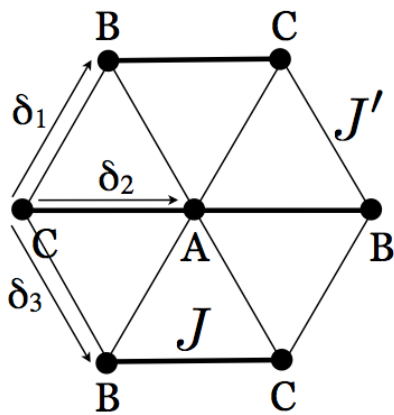
$e^{i3\mathbf{Q}\cdot\mathbf{r}} = 1$

first calculation of  $\Gamma_3 = \frac{3}{32S^2} \sum_{\mathbf{k} \in \text{BZ}} \left( \frac{(5J_{\mathbf{k}} + J_0)(5J_{\mathbf{Q}+\mathbf{k}} + J_0)J_{\mathbf{Q}-\mathbf{k}}}{(J_0 - J_{\mathbf{k}})(J_0 - J_{\mathbf{Q}+\mathbf{k}})} - \frac{(5J_{\mathbf{k}} + J_0)(J_{\mathbf{k}} + J_0)}{2(J_0 - J_{\mathbf{k}})} \right) + \frac{3J_0}{64S^2} \approx -\frac{0.69J}{S^2}$

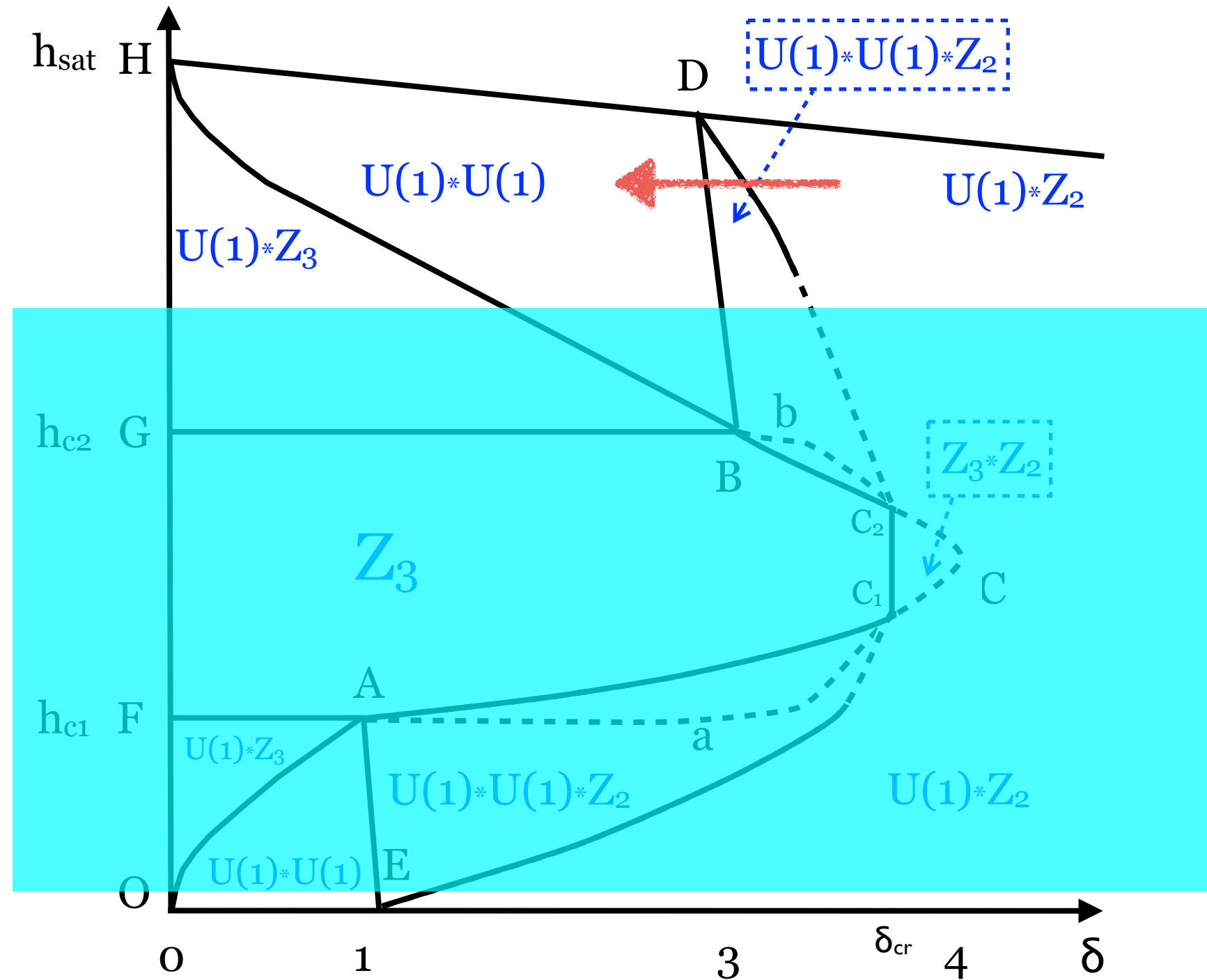
# Phases of a triangular-lattice antiferromagnet near saturation

$$U(1) \times U(1) \rightarrow U(1) \times Z_2 \text{ or } U(1) \times U(1) \rightarrow \text{smth else} \rightarrow U(1) \times Z_2?$$

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



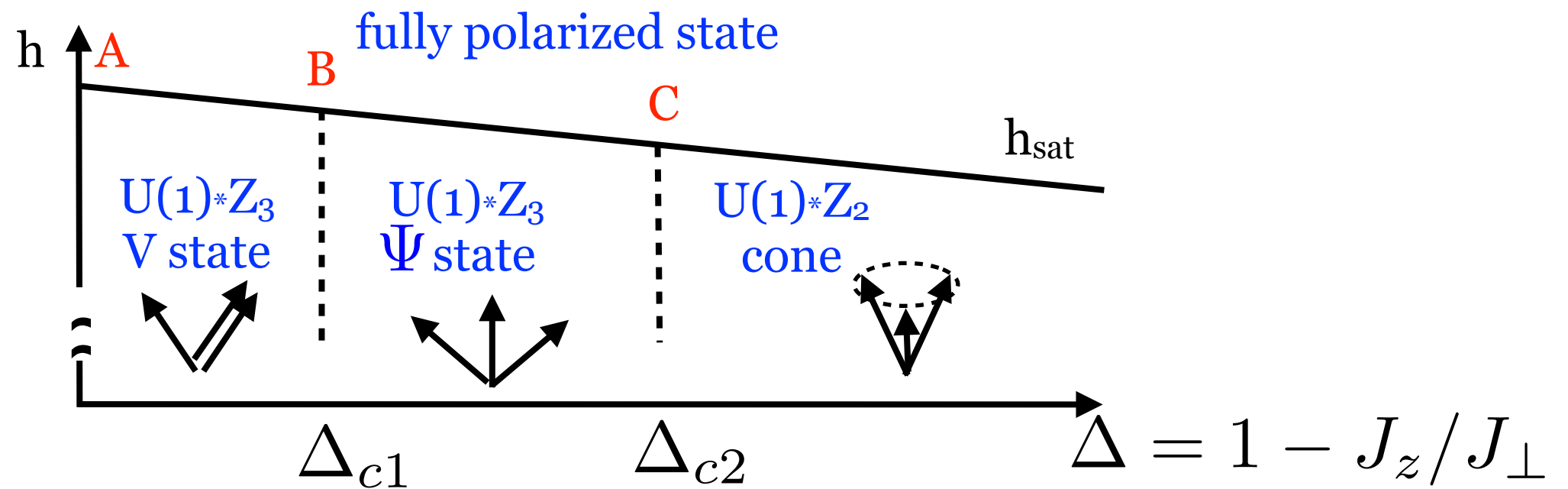
$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$





# High-field phases, XXZ model: commensurate-commensurate transitions

$$H_{\text{XXZ}} = \sum_{\langle i,j \rangle} J(S_i^x S_j^x + S_i^y S_j^y) + J_z S_i^z S_j^z - h \sum_j S_j^z$$



$$\Delta_{c1} = 0.45/S$$

$$\Delta_{c2} = 0.53/S$$

co-planar

commensurate non-coplanar

$$\Gamma_3 = \frac{3J\Delta}{2S} - \frac{0.69J}{S^2}$$

$$\mathcal{E}_\theta = \frac{3JS^2}{4h_{\text{sat}}} (\partial_x \theta)^2 + S\Gamma_3 \frac{\mu^3}{h_{\text{sat}}^3} \cos[6\theta]$$

Solid-Solid transition due to the sign change of  $\Gamma$



# Conclusions

Emergent Ising orders

Two-dimensional chiral spin-current phase  $Z_3^*Z_2$

High-field phases of triangular antiferromagnet  $U(1)^*U(1)^*Z_2$

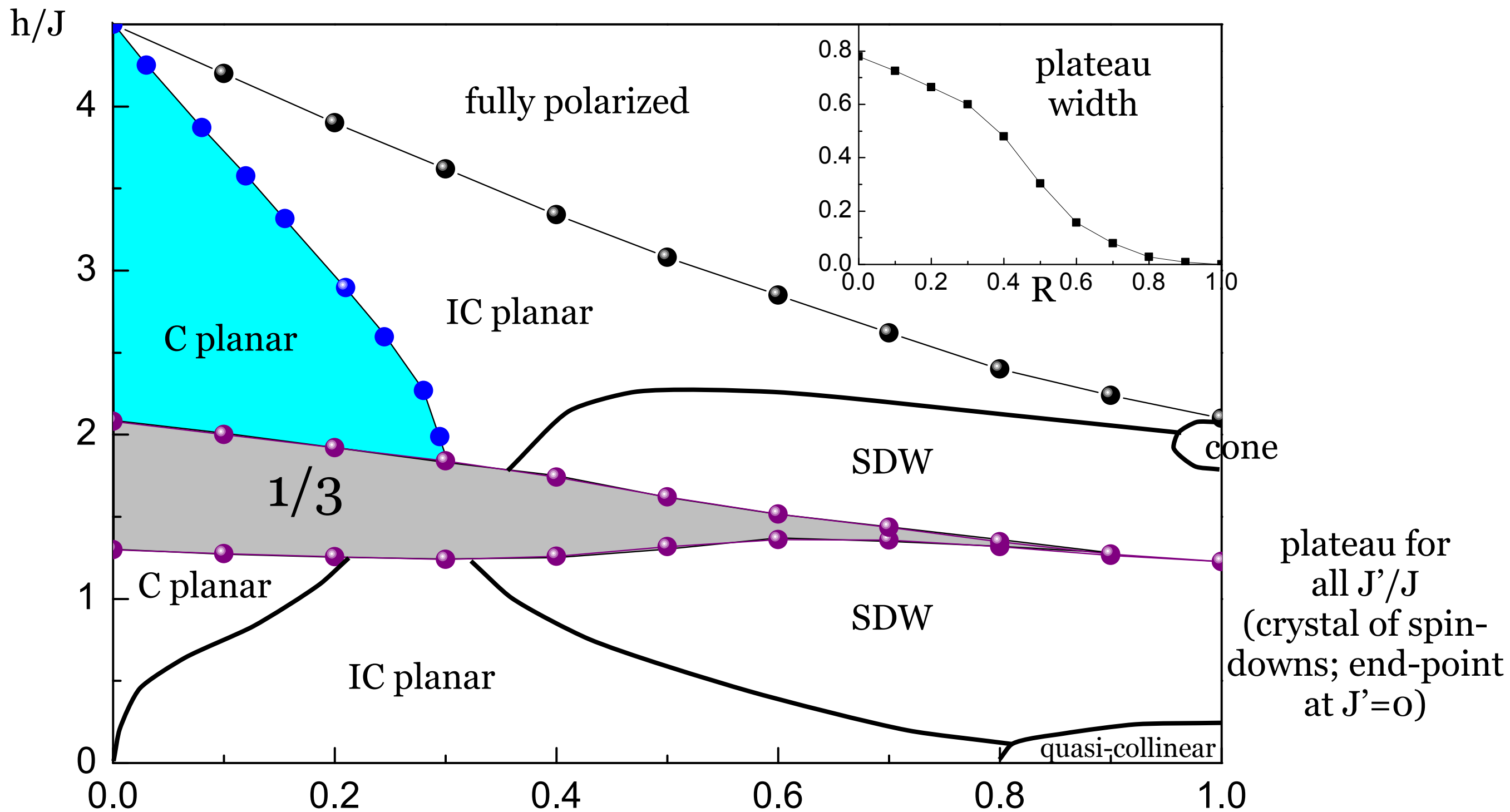
“An exceedingly simple model leads to a surprising richness of phases and critical behavior.

The underlying triangular lattice and the associated degeneracy play a crucial role in this physics.”

HB2U, Alyosha!

*two-dimensional*

# Schematic phase diagram for *spin-1/2* triangular lattice AFM



PHYSICAL REVIEW B **87**, 165123 (2013)

Ground states of  $\text{spin-}\frac{1}{2}$  triangular antiferromagnets in a magnetic field

$R = 1 - J'/J$

*spin-current state does not apply directly to  $s=1/2$  model*

Ru Chen,<sup>1</sup> Hyejin Ju,<sup>1</sup> Hong-Chen Jiang,<sup>2</sup> Oleg A. Starykh,<sup>3</sup> and Leon Balents<sup>2</sup>

<sup>1</sup>Department of Physics, University of California, Santa Barbara, Santa Barbara, California, 93106, USA

<sup>2</sup>Kavli Institute of Theoretical Physics, University of California, Santa Barbara, Santa Barbara, California, 93106, USA

<sup>3</sup>Department of Physics and Astronomy, University of Utah, Salt Lake City, Utah 84112, USA

# Compare with:

PHYSICAL REVIEW B **87**, 060407(R) (2013)

## Quantum stabilization of classically unstable plateau structures

Tommaso Coletta,<sup>1</sup> M. E. Zhitomirsky,<sup>2</sup> and Frédéric Mila<sup>1</sup>

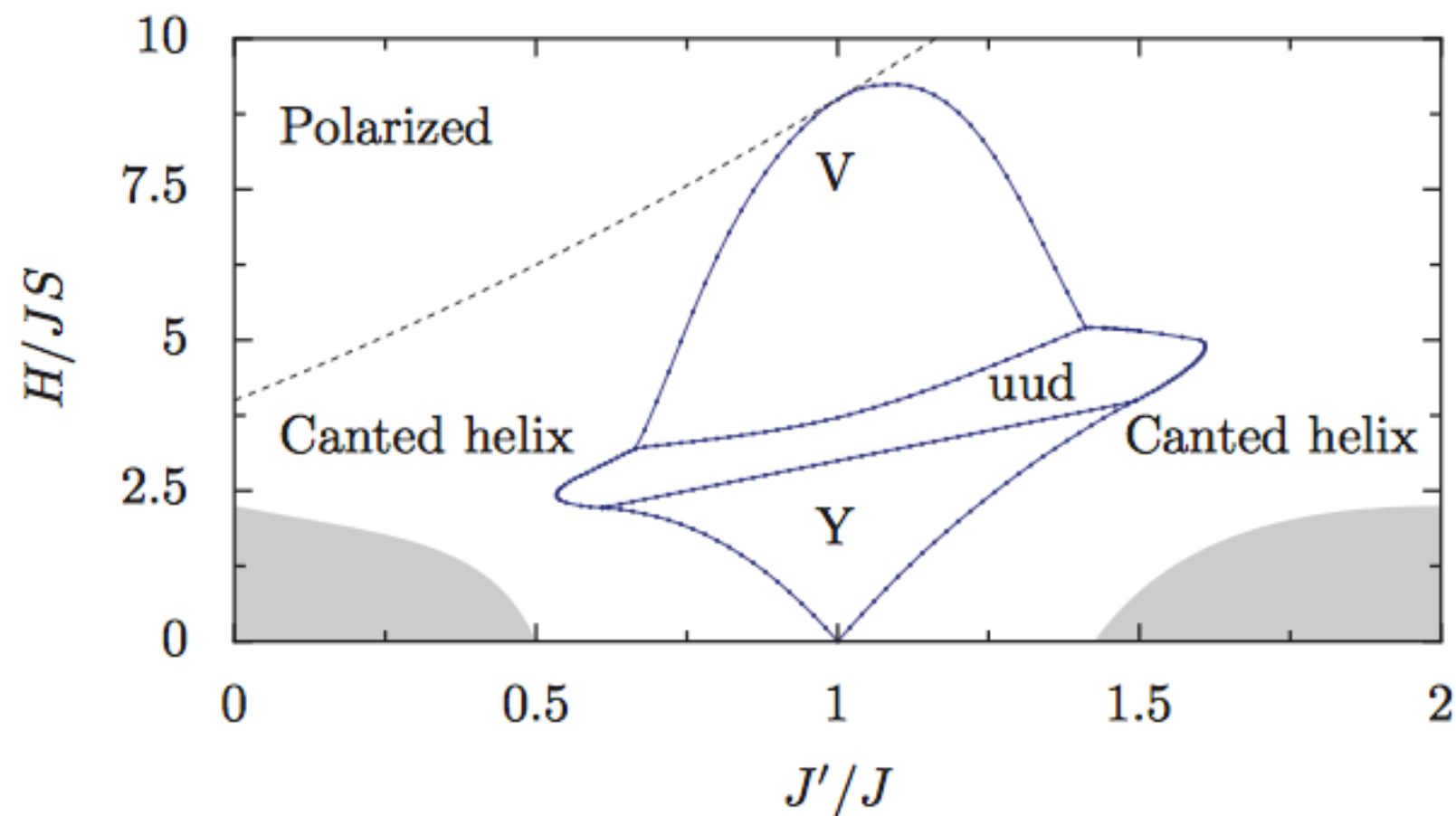
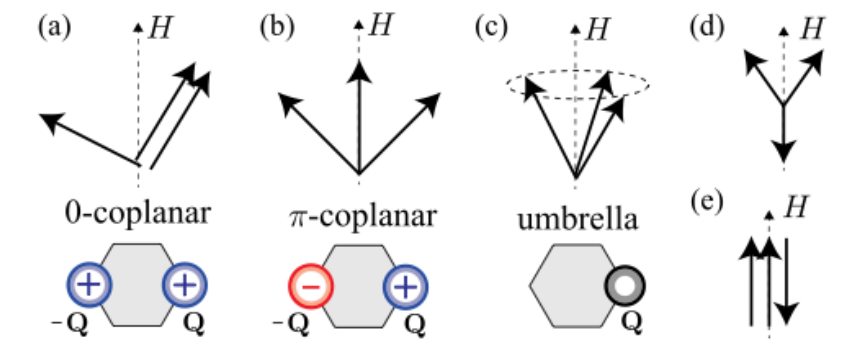
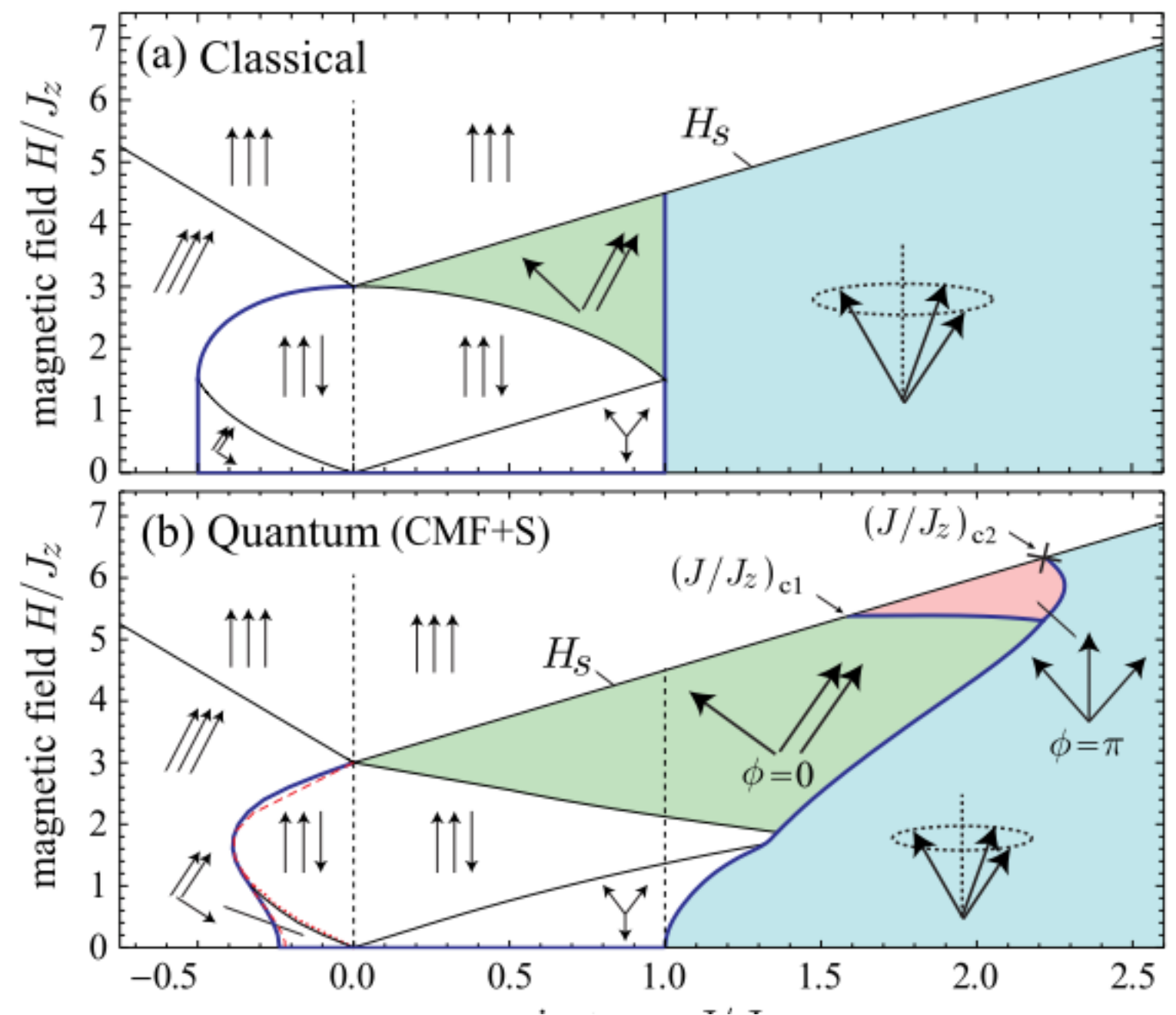


FIG. 4. (Color online) Phase diagram of the spin-1/2 anisotropic triangular lattice in magnetic field. Y and V regions denote three-sublattice planar states. The dashed line is the classical saturation field. The gray shading denotes regions where phases other than the canted helical states may be expected.

### Quantum Phase Diagram of the Triangular-Lattice XXZ Model in a Magnetic Field

Daisuke Yamamoto,<sup>1</sup> Giacomo Marmorini,<sup>1,2</sup> and Ipeei Danshita<sup>3</sup>

PRL 112, 127203 (2014)



Cluster mean-field theory:  
direct 1<sup>st</sup> order transition

between the UUD and the cone phases.

Can there be an intermediate  
spin-current (chiral Mott) phase  
with broken  $Z_3 \times Z_2$  ?

# High-field phases, J-J' model

