

Chapter 15: Game Theory: The Mathematics of Competition

Lesson Plan

- Two-Person Total-Conflict Games: Pure Strategies
- Two-Person Total-Conflict Games: Mixed Strategies
- Partial-Conflict Games
- Larger Games
- Using Game Theory

For All Practical Purposes



Mathematical Literacy in Today's World, 9th ed.

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Game Theory

■ Game Theory

- Uses mathematical tools to study situations, called games, involving both conflict and cooperation.



- Players – The people, organizations, or even countries, in a game that choose from a list of options available to them.
- Strategies – The courses of action they may take to lead to outcomes, which describe the consequences of their choices.
- Rational choice – Strategies are analyzed in game theory and players select strategies to obtain preferred outcomes.

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Two-Person Total-Conflict Games: Pure Strategies

■ Two-Person Games of Total Conflict

- Games in which what one player wins the other player loses, so cooperation never benefits the players.

■ Pure Strategy

- A course of action a player can choose in a game that does not involve randomized choices.

■ Payoff Matrix

- A rectangular array of numbers. In a two-person game, the rows and columns correspond to the strategies of the two players, and the numerical entities give the payoffs to the players when these strategies are selected.

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Two-Person Total-Conflict Games: Pure Strategies

■ Maximin Strategy

- In a two-person zero-sum game, the pure strategy of the row player corresponding to the maximin in a payoff matrix.
- Maximin – The largest of the minimum payoff in each row of a payoff matrix.

■ Minimax Strategy

- In a two-person zero-sum game, the pure strategy of the column player corresponding to the minimax in a payoff matrix.
- Minimax – The smallest of the maximum payoff in each column of a payoff matrix.

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Two-Person Total-Conflict Games: Pure Strategies

■ Saddlepoint

- The payoff that results when the maximin and the minimax are the same, which is the value of the game.
- The saddlepoint has the shape of a saddle-shaped surface.
 - The middle point on a horse saddle is simultaneously the lowest point along the spine of the horse and the highest point between the rider's legs.

■ Value

- The best outcome that both players can guarantee in a two-person zero-sum game.
- If there is a saddlepoint, this is the value.
- Otherwise, it is the expected payoff resulting when the players choose their optimal mixed strategies.

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Two-Person Total-Conflict Games: Mixed Strategies

■ Mixed Strategy

- ❑ A strategy that involves the random choice of pure strategies, according to particular probabilities.
- ❑ A mixed strategy of a player is optimal if it guarantees the value of the game.
- ❑ Players must take care to conceal the strategy they will select until the encounter actually takes place (example in baseball).
- ❑ Players may also vary their strategies in order to surprise the opponent.
- ❑ Players may also use the tactic of bluffing, which involves secrecy and deception (example: bluffing when playing poker).



In baseball, a pitcher may not signal the type of pitch he/she intends to throw in advance.

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Two-Person Total-Conflict Games: Mixed Strategies

■ Pure and Mixed Strategy

- A pure strategy is each of the definite courses of action that a player can choose.
- A mixed strategy is a strategy in which the course of action is randomly chosen from one of the pure strategies in the following way:
 - Each pure strategy is assigned some probability, indicating the relative frequency with which that pure strategy will be played.
 - The specific strategy used in any given play of the game can be selected using some appropriate random device.

■ Expected Value E

- If each of the n payoffs, s_1, s_2, \dots, s_n will occur with the probabilities p_1, p_2, \dots, p_n , respectively, then the average, or expected value E , is given by:

$$E = p_1 s_1 + p_2 s_2 + \dots + p_n s_n$$

Assume all probability rules hold.

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Two-Person Total-Conflict Games: Mixed Strategies

■ Fair Game

- A game is fair if its value is 0 and, consequently, it favors neither player when at least one player uses an optimal (mixed) strategy—one that guarantees that the resulting payoff is the best that this player can obtain against all possible strategy choices (pure or mixed) by an opponent.

■ Nonsymmetrical Game

- A two-person constant-sum game in which the row player's gains are different from the column player's gains, except when there is a tie.

■ Zero-Sum Game

- A constant-sum game in which the payoff to one player is the negative of the payoff to the other player, so the sum of the payoffs to the players at each outcome is zero.

Minimax Theorem

Guarantees that there is a unique game value and an optimal strategy for each player, so that either player alone can realize at least this value by playing this strategy, which may be pure or mixed.

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Partial-Conflict Games

■ Partial-Conflict Games

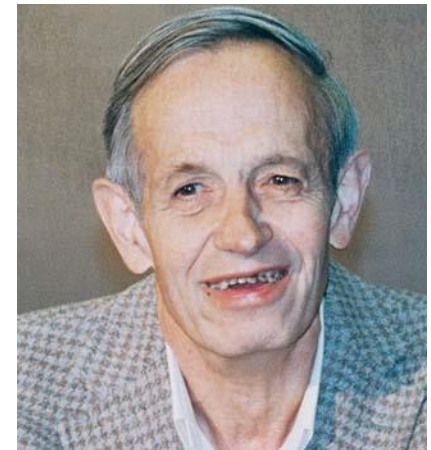
- A variable-sum game in which both players can benefit by cooperation but may have strong incentives not to cooperate.

■ Variable-Sum Games

- Games of partial conflict are variable-sum games in which the sum of payoff to the players at the different outcomes varies.

■ Nash Equilibrium

- Strategies associated with an outcome such that no player can benefit by choosing a different strategy, given that the other players do not depart from their strategies.
- Technically, while it is the set of strategies that define the equilibrium, the choice of these strategies leads to an outcome that we shall also refer to as an equilibrium.



John F. Nash, Jr.

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Partial-Conflict Games

■ Prisoners' Dilemma

- A two-person variable-sum symmetric game in which each player has two strategies—cooperate or defect (not cooperate).
- Cooperate dominates defect for both players, even though the mutual-defection outcome, which is the unique Nash equilibrium in the game, is worse for both players than mutual cooperation.

■ Chicken

- A two-person variable-sum symmetric game in which each player has two strategies:
 1. To swerve to avoid a collision
 2. Not to swerve and possibly cause a collision
- Neither player has a dominant strategy.
- The compromise outcome (both players swerve), and the disaster outcome (both players do not), are both not Nash equilibria.
- The other two outcomes, in which one player swerves and the other does not, are Nash equilibria.

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Larger Games

■ Sincere Voting

- Under sincere voting, every voter votes for his/her most-preferred alternative, based on his/her true preferences, without taking into account what the other voters might do.

■ Sophisticated Voting

- The successive elimination of dominated strategies by voters (insofar as this is possible).

■ Status-Quo Paradox

- Status quo is defeated by another alternative, even if there is no Condorcet winner, when the voters are sophisticated.
- The situation, in which supporting the apparently favored outcome hurts, is called the status-quo paradox.

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Using Game Theory

■ Theory of Moves

- The theory of moves (TOM) is a dynamic theory that describes optimal choices in strategic-form games in which the players think ahead and can make moves and countermoves.

■ Practical Applications

- Mixed strategies, such as the element of surprise, is essential in many random applications, such as making inspections or auditing choices that are random and unpredictable, to help deter potential cheaters.
- Optimal detection and optimal concealment strategies can be modeled as a game to help military or intelligence services trying to intercept a weapon hidden among many decoys.
- Bluffing occurs not only in poker but is common in many bargaining situations, such as labor threatening a strike in labor-management negotiations, even if one may have to back down.