

Chapter 422

Two-Sample T-Tests Assuming Equal Variance

Introduction

This procedure provides sample size and power calculations for one- or two-sided two-sample t-tests when the variances of the two groups (populations) are assumed to be equal. This is the traditional two-sample t-test (Fisher, 1925). The assumed difference between means can be specified by entering the means for the two groups and letting the software calculate the difference or by entering the difference directly.

The design corresponding to this test procedure is sometimes referred to as a *parallel-groups* design. This design is used in situations such as the comparison of the income level of two regions, the nitrogen content of two lakes, or the effectiveness of two drugs.

There are several statistical tests available for the comparison of the center of two populations. This procedure is specific to the two-sample t-test assuming equal variance. You can examine the sections below to identify whether the assumptions and test statistic you intend to use in your study match those of this procedure, or if one of the other **PASS** procedures may be more suited to your situation.

Other PASS Procedures for Comparing Two Means or Medians

Procedures in **PASS** are primarily built upon the testing methods, test statistic, and test assumptions that will be used when the analysis of the data is performed. You should check to identify that the test procedure described below in the Test Procedure section matches your intended procedure. If your assumptions or testing method are different, you may wish to use one of the other two-sample procedures available in **PASS**. These procedures are Two-Sample T-Tests Allowing Unequal Variance, Two-Sample Z-Tests Assuming Equal Variance, Two-Sample Z-Tests Allowing Unequal Variance, and the nonparametric Mann-Whitney-Wilcoxon (also known as the Mann-Whitney U or Wilcoxon rank-sum test) procedure. The methods, statistics, and assumptions for those procedures are described in the associated chapters.

If you wish to show that the mean of one population is larger (or smaller) than the mean of another population by a specified amount, you should use one of the clinical superiority procedures for comparing means. Non-inferiority, equivalence, and confidence interval procedures are also available.

Test Assumptions

When running a two-sample equal-variance t-test, the basic assumptions are that the distributions of the two populations are normal, and that the variances of the two distributions are the same. If those assumptions are not likely to be met, another testing procedure could be used, and the corresponding procedure in **PASS** should be used for sample size or power calculations.

Test Procedure

If we assume that μ_1 and μ_2 represent the means of the two populations of interest, and that $\delta = \mu_1 - \mu_2$, the null hypothesis for comparing the two means is $H_0: \mu_1 = \mu_2$ (or $H_0: \delta = 0$). The alternative hypothesis can be any one of

Two-Sided: $H_1: \mu_1 \neq \mu_2$ (or $H_1: \delta \neq 0$)

Upper One-Sided: $H_1: \mu_1 > \mu_2$ (or $H_1: \delta > 0$)

Lower One-Sided: $H_1: \mu_1 < \mu_2$ (or $H_1: \delta < 0$)

depending upon the desire of the researcher or the protocol instructions. A suitable Type I error probability (α) is chosen for the test, the data is collected, and a t -statistic is generated using the formula:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

This t -statistic follows a t distribution with $n_1 + n_2 - 2$ degrees of freedom. The null hypothesis is rejected in favor of the alternative if,

for $H_1: \mu_1 \neq \mu_2$ (or $H_1: \delta \neq 0$),

$$t < t_{\alpha/2} \text{ or } t > t_{1-\alpha/2},$$

for $H_1: \mu_1 > \mu_2$ (or $H_1: \delta > 0$),

$$t > t_{1-\alpha},$$

or, for $H_1: \mu_1 < \mu_2$ (or $H_1: \delta < 0$),

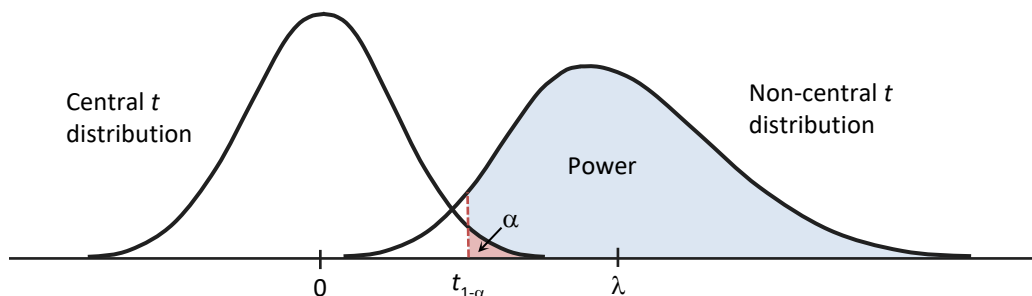
$$t < t_{\alpha}.$$

Comparing the t -statistic to the cut-off t -value (as shown here) is equivalent to comparing the p -value to α .

Power Calculation

This section describes the procedure for computing the power from n_1 and n_2 , α , the assumed μ_1 and μ_2 , and the assumed common standard deviation, $\sigma_1 = \sigma_2 = \sigma$. Two good references for these methods are Julious (2010) and Chow, Shao, Wang, and Lohkhygina (2018).

The figure below gives a visual representation for the calculation of power for a one-sided test.



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If we call the assumed difference between the means, $\delta = \mu_1 - \mu_2$, the steps for calculating the power are as follows:

1. Find $t_{1-\alpha}$ based on the central- t distribution with degrees of freedom,

$$df = n_1 + n_2 - 2.$$

2. Calculate the non-centrality parameter:

$$\lambda = \frac{\delta}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}.$$

3. Calculate the power as the probability that the test statistic t is greater than $t_{1-\alpha}$ under the non-central- t distribution with non-centrality parameter λ :

$$Power = \Pr_{Non-central-t}(t > t_{1-\alpha} | df = n_1 + n_2 - 2, \lambda).$$

The algorithms for calculating power for the opposite direction and the two-sided hypotheses are analogous to this method.

When solving for something other than power, **PASS** uses this same power calculation formulation, but performs a search to determine that parameter.

A Note on Specifying the Means or Difference in Means

When means are specified in this procedure, they are used to determine the assumed difference in means for power or sample size calculations. When the difference in means is specified in this procedure, it is the assumed difference in means for power or sample size calculations. It does not mean that the study will be powered to show that the mean difference is this amount, but rather that the design is powered to reject the null hypothesis of equal means if this were the true difference in means. If your purpose is to show that one mean is greater than another by a specific amount, you should use one of the clinical superiority procedures for comparing means.

A Note on Specifying the Standard Deviation

The sample size calculation for most statistical procedures is based on the choice of alpha, power, and an assumed difference in the primary parameters of interest – the difference in means in this procedure. An additional parameter that must be specified for means tests is the standard deviation. Here, we will briefly discuss some considerations for the choice of the standard deviation to enter.

If a number of previous studies of a similar nature are available, you can estimate the variance based on a weighted average of the variances, and then take the square root to give the projected standard deviation.

Perhaps more commonly, only a single pilot study is available, or it may be that no previous study is available. For both of these cases, the conservative approach is typically recommended. In PASS, there is a standard deviation estimator tool. This tool can be used to help select an appropriate value or range of values for the standard deviation.

If the standard deviation is not given directly from the previous study, it may be obtained from the standard error, percentiles, or the coefficient of variation. Once a standard deviation estimate is obtained, it may be useful to then use the confidence limits tab to obtain a confidence interval for the standard deviation estimate. With regard to power and sample size, the upper confidence limit will then be a conservative

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estimate of the standard deviation. Or a range of values from the lower confidence limit to the upper confidence limit may be used to determine the effect of the standard deviation on the power or sample size requirement.

If there is no previous study available, a couple of rough estimation options can be considered. You may use the data tab of the standard deviation estimator to enter some values that represent typical values you expect to encounter. This tool will allow you to see the corresponding population or sample standard deviation. A second rough estimation technique is to base the estimate of the standard deviation on your estimate of the range of the population or the range of a data sample. A conservative divisor for the population range is 4. For example, if you are confident your population values range from 45 to 105, you would enter 60 for the Population Range, and, say, 4, for 'C'. The resulting standard deviation estimate would be 15.

If you are unsure about the value you should enter for the standard deviation, we recommend that you additionally examine a range of standard deviation values to see the effect that your choice has on power or sample size.

Example 1 – Finding the Sample Size

Researchers wish to compare two types of local anesthesia to determine whether there is a difference in time to loss of pain. Subjects will be randomized to treatment, the treatment will be administered, and the time to loss of pain measured. The anticipated time to loss of pain for one of the types of anesthesia is 9 minutes. The researchers would like to generate a sample size for the study with 90% power to reject the null hypothesis of equal loss-of-pain time if the true difference is at least 2 minutes. How many participants are needed to achieve 90% power at significance levels of 0.01 and 0.05?

Past experiments of this type have had standard deviations in the range of 1 to 5 minutes. It is anticipated that the standard deviation of the two groups will be equal. It is unknown which treatment has lower time to loss of pain, so a two-sided test will be used.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

| | |
|------------------------------|------------------------|
| Solve For | Sample Size |
| Alternative Hypothesis | Two-Sided |
| Power..... | 0.90 |
| Alpha..... | 0.01 0.05 |
| Group Allocation | Equal (N1 = N2) |
| Input Type..... | Means |
| μ_1 | 11 |
| μ_2 | 9 |
| σ | 1 to 5 by 1 |

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Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results for an Equal-Variance T-Test

Solve For: **Sample Size**

Difference: $\delta = \mu_1 - \mu_2$

Hypotheses: $H_0: \delta = 0$ vs. $H_1: \delta \neq 0$

| Target Power | Actual Power | N1 | N2 | N | μ_1 | μ_2 | δ | σ | Alpha |
|--------------|--------------|-----|-----|-----|---------|---------|----------|----------|-------|
| 0.9 | 0.92949 | 10 | 10 | 20 | 11 | 9 | 2 | 1 | 0.01 |
| 0.9 | 0.92907 | 7 | 7 | 14 | 11 | 9 | 2 | 1 | 0.05 |
| 0.9 | 0.90596 | 32 | 32 | 64 | 11 | 9 | 2 | 2 | 0.01 |
| 0.9 | 0.91250 | 23 | 23 | 46 | 11 | 9 | 2 | 2 | 0.05 |
| 0.9 | 0.90182 | 69 | 69 | 138 | 11 | 9 | 2 | 3 | 0.01 |
| 0.9 | 0.90434 | 49 | 49 | 98 | 11 | 9 | 2 | 3 | 0.05 |
| 0.9 | 0.90083 | 121 | 121 | 242 | 11 | 9 | 2 | 4 | 0.01 |
| 0.9 | 0.90323 | 86 | 86 | 172 | 11 | 9 | 2 | 4 | 0.05 |
| 0.9 | 0.90062 | 188 | 188 | 376 | 11 | 9 | 2 | 5 | 0.01 |
| 0.9 | 0.90148 | 133 | 133 | 266 | 11 | 9 | 2 | 5 | 0.05 |

Target Power The desired power value (or values) entered in the procedure. Power is the probability of rejecting a false null hypothesis.

Actual Power The power obtained in this scenario. Because N1 and N2 are discrete, this value is often (slightly) larger than the target power.

N1 and N2 The number of items sampled from each population.

N The total sample size. $N = N_1 + N_2$.

μ_1 and μ_2 The assumed population means.

δ The difference between population means at which power and sample size calculations are made. $\delta = \mu_1 - \mu_2$.

σ The assumed population standard deviation for each of the two groups.

Alpha The probability of rejecting a true null hypothesis.

Summary Statements

Group sample sizes of 10 and 10 achieve 92.949% power to reject the null hypothesis of equal means when the population mean difference is $\mu_1 - \mu_2 = 11 - 9 = 2$ with a standard deviation for both groups of 1 and with a significance level (alpha) of 0.01 using a two-sided two-sample equal-variance t-test.

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Dropout-Inflated Sample Size

| Dropout Rate | Sample Size | | | Dropout-Inflated Enrollment Sample Size | | | Expected Number of Dropouts | | |
|--------------|-------------|-----|-----|---|-----|-----|-----------------------------|----|----|
| | N1 | N2 | N | N1' | N2' | N' | D1 | D2 | D |
| 20% | 10 | 10 | 20 | 13 | 13 | 26 | 3 | 3 | 6 |
| 20% | 7 | 7 | 14 | 9 | 9 | 18 | 2 | 2 | 4 |
| 20% | 32 | 32 | 64 | 40 | 40 | 80 | 8 | 8 | 16 |
| 20% | 23 | 23 | 46 | 29 | 29 | 58 | 6 | 6 | 12 |
| 20% | 69 | 69 | 138 | 87 | 87 | 174 | 18 | 18 | 36 |
| 20% | 49 | 49 | 98 | 62 | 62 | 124 | 13 | 13 | 26 |
| 20% | 121 | 121 | 242 | 152 | 152 | 304 | 31 | 31 | 62 |
| 20% | 86 | 86 | 172 | 108 | 108 | 216 | 22 | 22 | 44 |
| 20% | 188 | 188 | 376 | 235 | 235 | 470 | 47 | 47 | 94 |
| 20% | 133 | 133 | 266 | 167 | 167 | 334 | 34 | 34 | 68 |

| | |
|------------------|---|
| Dropout Rate | The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR. |
| N1, N2, and N | The evaluable sample sizes at which power is computed. If N1 and N2 subjects are evaluated out of the N1' and N2' subjects that are enrolled in the study, the design will achieve the stated power. |
| N1', N2', and N' | The number of subjects that should be enrolled in the study in order to obtain N1, N2, and N evaluable subjects, based on the assumed dropout rate. After solving for N1 and N2, N1' and N2' are calculated by inflating N1 and N2 using the formulas $N1' = N1 / (1 - DR)$ and $N2' = N2 / (1 - DR)$, with N1' and N2' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lohknygina, Y. (2018) pages 32-33.) |
| D1, D2, and D | The expected number of dropouts. $D1 = N1' - N1$, $D2 = N2' - N2$, and $D = D1 + D2$. |

Dropout Summary Statements

Anticipating a 20% dropout rate, 13 subjects should be enrolled in Group 1, and 13 in Group 2, to obtain final group sample sizes of 10 and 10, respectively.

References

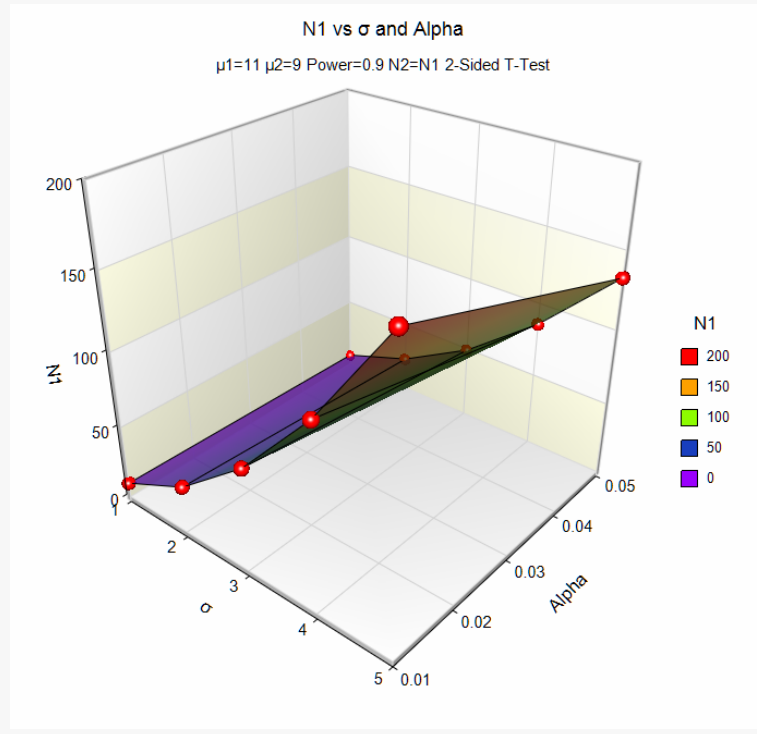
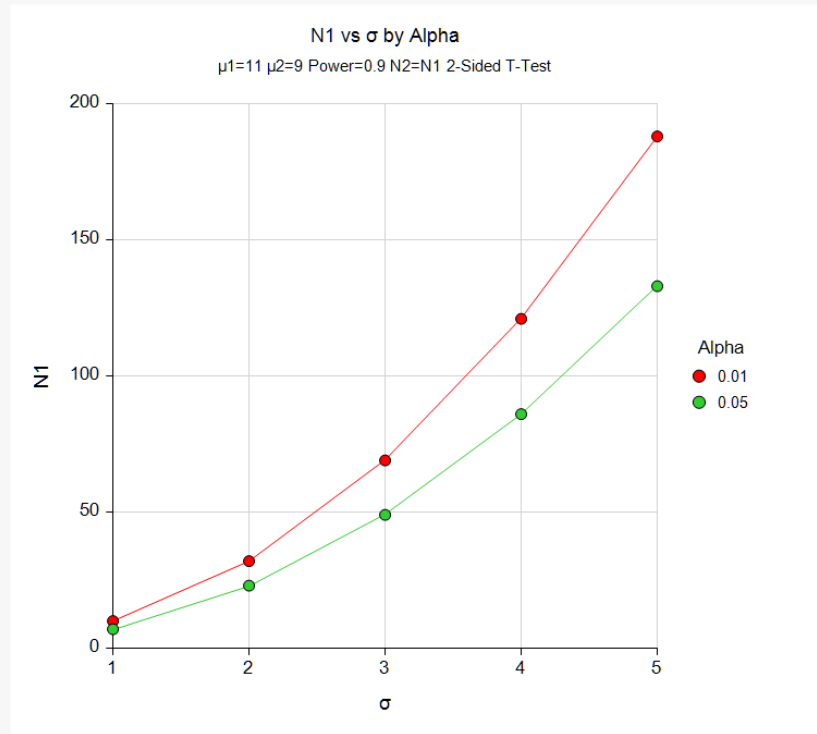
- Chow, S.C., Shao, J., Wang, H., and Lohknygina, Y. 2018. Sample Size Calculations in Clinical Research, Third Edition. Taylor & Francis/CRC. Boca Raton, Florida.
- Julious, S. A. 2010. Sample Sizes for Clinical Trials. Chapman & Hall/CRC. Boca Raton, FL.
- Machin, D., Campbell, M., Fayers, P., and Pinol, A. 1997. Sample Size Tables for Clinical Studies, 2nd Edition. Blackwell Science. Malden, MA.
- Zar, Jerrold H. 1984. Biostatistical Analysis (Second Edition). Prentice-Hall. Englewood Cliffs, New Jersey.

These reports show the values of each of the parameters, one scenario per row.

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Plots Section

Plots



These plots show the relationship between the standard deviation and sample size for the two alpha levels.

Two-Sample T-Tests Assuming Equal Variance

To specify the difference directly to get the same results, change **Input Type** to **Difference** and enter **2** for **δ** , leaving everything else the same. You may then make the appropriate entries as listed below, or open **Example 1b** by going to the **File** menu and choosing **Open Example Template**.

Design Tab

Input Type.....**Difference**
 δ**2**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for an Equal-Variance T-Test

Solve For: [Sample Size](#)
 Difference: $\delta = \mu_1 - \mu_2$
 Hypotheses: $H_0: \delta = 0$ vs. $H_1: \delta \neq 0$

| Target Power | Actual Power | N1 | N2 | N | δ | σ | Alpha |
|--------------|--------------|-----|-----|-----|----------|----------|-------|
| 0.9 | 0.92949 | 10 | 10 | 20 | 2 | 1 | 0.01 |
| 0.9 | 0.92907 | 7 | 7 | 14 | 2 | 1 | 0.05 |
| 0.9 | 0.90596 | 32 | 32 | 64 | 2 | 2 | 0.01 |
| 0.9 | 0.91250 | 23 | 23 | 46 | 2 | 2 | 0.05 |
| 0.9 | 0.90182 | 69 | 69 | 138 | 2 | 3 | 0.01 |
| 0.9 | 0.90434 | 49 | 49 | 98 | 2 | 3 | 0.05 |
| 0.9 | 0.90083 | 121 | 121 | 242 | 2 | 4 | 0.01 |
| 0.9 | 0.90323 | 86 | 86 | 172 | 2 | 4 | 0.05 |
| 0.9 | 0.90062 | 188 | 188 | 376 | 2 | 5 | 0.01 |
| 0.9 | 0.90148 | 133 | 133 | 266 | 2 | 5 | 0.05 |

The results are the same as those obtained when the means are entered.

Example 2 – Finding the Power

Suppose a new corn fertilizer is to be compared to a current fertilizer. The current fertilizer produces an average of about 74 lbs. per plot. The researchers need only show that there is difference in yield with the new fertilizer. They would like to consider the effect of the number of plots used on the power of the test if the improvement in yield is at least 10 lbs.

Researchers plan to use a one-sided two-sample t-test with alpha equal to 0.05. Previous studies indicate the standard deviation for plot yield to be between 20 and 30 lbs. The plot group sizes of interest are 10 to 100 plots per group.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

| Design Tab | |
|------------------------------|------------------------|
| Solve For | Power |
| Alternative Hypothesis | One-Sided |
| Alpha..... | 0.05 |
| Group Allocation | Equal (N1 = N2) |
| Sample Size Per Group | 10 to 100 by 10 |
| Input Type..... | Means |
| μ_1 | 84 |
| μ_2 | 74 |
| σ | 20 25 30 |

Two-Sample T-Tests Assuming Equal Variance

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Reports

Numeric Results for an Equal-Variance T-Test

Solve For: Power

Difference: $\delta = \mu_1 - \mu_2$

Hypotheses: $H_0: \delta \leq 0$ vs. $H_1: \delta > 0$

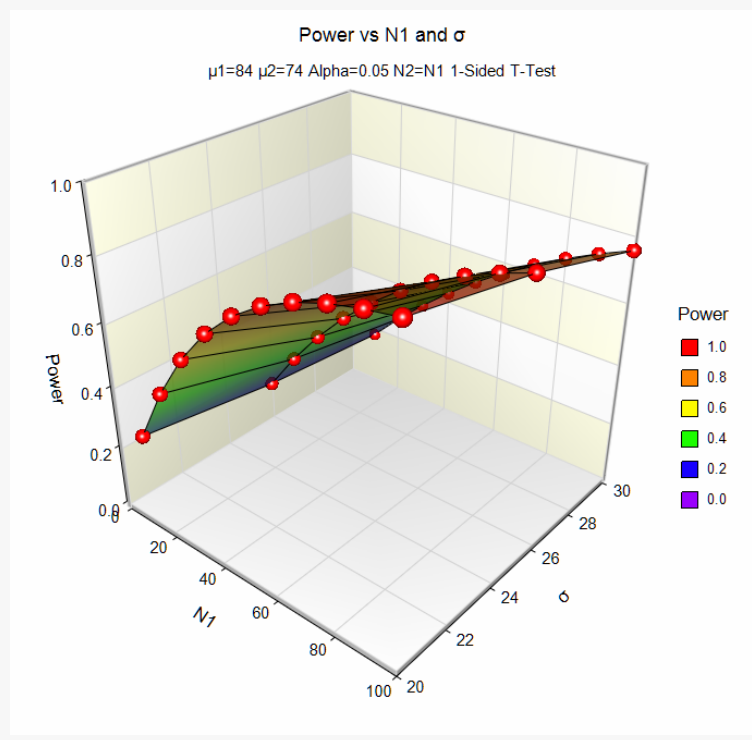
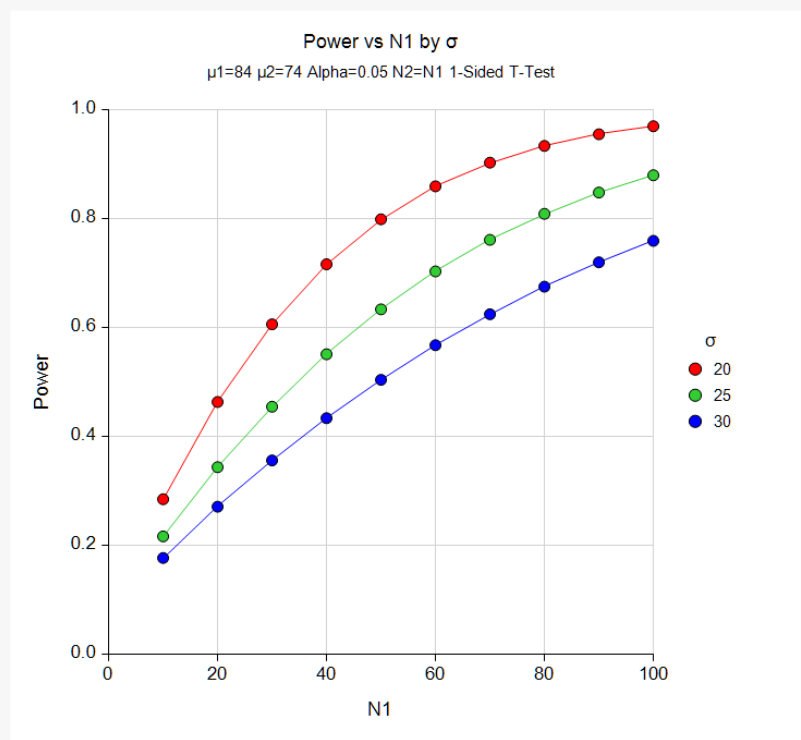
| Power | N1 | N2 | N | μ_1 | μ_2 | δ | σ | Alpha |
|---------|-----|-----|-----|---------|---------|----------|----------|-------|
| 0.28476 | 10 | 10 | 20 | 84 | 74 | 10 | 20 | 0.05 |
| 0.46337 | 20 | 20 | 40 | 84 | 74 | 10 | 20 | 0.05 |
| 0.60603 | 30 | 30 | 60 | 84 | 74 | 10 | 20 | 0.05 |
| 0.71625 | 40 | 40 | 80 | 84 | 74 | 10 | 20 | 0.05 |
| 0.79894 | 50 | 50 | 100 | 84 | 74 | 10 | 20 | 0.05 |
| 0.85948 | 60 | 60 | 120 | 84 | 74 | 10 | 20 | 0.05 |
| 0.90297 | 70 | 70 | 140 | 84 | 74 | 10 | 20 | 0.05 |
| 0.93369 | 80 | 80 | 160 | 84 | 74 | 10 | 20 | 0.05 |
| 0.95510 | 90 | 90 | 180 | 84 | 74 | 10 | 20 | 0.05 |
| 0.96985 | 100 | 100 | 200 | 84 | 74 | 10 | 20 | 0.05 |
| 0.21656 | 10 | 10 | 20 | 84 | 74 | 10 | 25 | 0.05 |
| 0.34367 | 20 | 20 | 40 | 84 | 74 | 10 | 25 | 0.05 |
| 0.45471 | 30 | 30 | 60 | 84 | 74 | 10 | 25 | 0.05 |
| 0.55111 | 40 | 40 | 80 | 84 | 74 | 10 | 25 | 0.05 |
| 0.63357 | 50 | 50 | 100 | 84 | 74 | 10 | 25 | 0.05 |
| 0.70314 | 60 | 60 | 120 | 84 | 74 | 10 | 25 | 0.05 |
| 0.76113 | 70 | 70 | 140 | 84 | 74 | 10 | 25 | 0.05 |
| 0.80897 | 80 | 80 | 160 | 84 | 74 | 10 | 25 | 0.05 |
| 0.84807 | 90 | 90 | 180 | 84 | 74 | 10 | 25 | 0.05 |
| 0.87978 | 100 | 100 | 200 | 84 | 74 | 10 | 25 | 0.05 |
| 0.17689 | 10 | 10 | 20 | 84 | 74 | 10 | 30 | 0.05 |
| 0.27109 | 20 | 20 | 40 | 84 | 74 | 10 | 30 | 0.05 |
| 0.35609 | 30 | 30 | 60 | 84 | 74 | 10 | 30 | 0.05 |
| 0.43365 | 40 | 40 | 80 | 84 | 74 | 10 | 30 | 0.05 |
| 0.50411 | 50 | 50 | 100 | 84 | 74 | 10 | 30 | 0.05 |
| 0.56765 | 60 | 60 | 120 | 84 | 74 | 10 | 30 | 0.05 |
| 0.62456 | 70 | 70 | 140 | 84 | 74 | 10 | 30 | 0.05 |
| 0.67519 | 80 | 80 | 160 | 84 | 74 | 10 | 30 | 0.05 |
| 0.71995 | 90 | 90 | 180 | 84 | 74 | 10 | 30 | 0.05 |
| 0.75932 | 100 | 100 | 200 | 84 | 74 | 10 | 30 | 0.05 |

These reports show the values of each of the parameters, one scenario per row.

Two-Sample T-Tests Assuming Equal Variance

Plots Section

Plots



These plots show the relationship between the power and sample size for the three values of σ .

Two-Sample T-Tests Assuming Equal Variance

Again, to specify the difference directly to get the same results, change **Input Type** to **Difference** and enter **10** for δ , leaving everything else the same. You'll get the same results as those above.

Design Tab

Input Type.....**Difference**

δ**10**

Example 3 – Finding the Difference

In some cases, it may be useful to determine how different the means of two populations would need to be to achieve the desired power with a specific constrained sample size.

Suppose, for example, that 80 subjects are available for a study to compare weight loss regimens. Researchers would like to determine which of the two regimens is better. The anticipated weight loss after two months is anticipated to be about 20 lbs. The researchers would like to know how different the mean weight loss must be to have a study with 90% power when alpha is 0.05 and the standard deviation is assumed to be 7 lbs. in both groups.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 3** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

| | |
|---------------------------------|--|
| Design Tab | |
| Solve For | Effect Size (Means or Difference) |
| Alternative Hypothesis | Two-Sided |
| Power..... | 0.90 |
| Alpha..... | 0.05 |
| Group Allocation | Equal (N1 = N2) |
| Sample Size Per Group | 40 |
| Input Type..... | Difference |
| δ | Search > 0 |
| σ | 7 |
| Reports Tab | |
| Means, Difference Decimals..... | 3 |

Output

Click the Calculate button to perform the calculations and generate the following output.

| | | | | | | |
|--|-----------|-----------|----------|----------------------------|----------------------------|--------------|
| Numeric Results for an Equal-Variance T-Test | | | | | | |
| Solve For: Effect Size (Means or Difference) | | | | | | |
| Difference: $\delta = \mu_1 - \mu_2$ | | | | | | |
| Hypotheses: $H_0: \delta = 0$ vs. $H_1: \delta \neq 0$ | | | | | | |
| Power | N1 | N2 | N | δ | σ | Alpha |
| 0.9 | 40 | 40 | 80 | 5.137 | 7 | 0.05 |

If the true population mean weight loss for one group is 5.137 lbs. more or less than the other, the researchers would have 90% power to show a difference between the groups.

Example 4 – Validation of Sample Size using Machin et al. (1997)

Machin *et al.* (1997) page 35 present an example for a two-sided test in which the mean difference is 5, the common standard deviation is 10, the power is 90%, and the significance level is 0.05. They calculate the per group sample size as 86.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 4** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

| Design Tab | |
|------------------------------|------------------------|
| Solve For | Sample Size |
| Alternative Hypothesis | Two-Sided |
| Power..... | 0.90 |
| Alpha..... | 0.05 |
| Group Allocation | Equal (N1 = N2) |
| Input Type..... | Difference |
| δ | 5 |
| σ | 10 |

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for an Equal-Variance T-Test

| | |
|-------------|--|
| Solve For: | Sample Size |
| Difference: | $\delta = \mu_1 - \mu_2$ |
| Hypotheses: | $H_0: \delta = 0$ vs. $H_1: \delta \neq 0$ |

| Target Power | Actual Power | N1 | N2 | N | δ | σ | Alpha |
|--------------|--------------|----|----|-----|----------|----------|-------|
| 0.9 | 0.90323 | 86 | 86 | 172 | 5 | 10 | 0.05 |

The sample size of 86 per group matches Machin's result exactly.

Example 5 – Validation of Power using Zar (1984)

Zar (1984) page 136 give an example in which the mean difference is 1, the common standard deviation is 0.7206, the sample sizes are 15 in each group, and the significance level is 0.05. They calculate the power as 0.96.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 5** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

| | |
|----------------------------------|------------------------|
| Design Tab | |
| Solve For | Power |
| Alternative Hypothesis | Two-Sided |
| Alpha..... | 0.05 |
| Group Allocation | Equal (N1 = N2) |
| Sample Size Per Group | 15 |
| Input Type..... | Means |
| μ_1 | 1 |
| μ_2 | 0 |
| σ | 0.7206 |
| Reports Tab | |
| Standard Deviation Decimals..... | 4 |

Output

Click the Calculate button to perform the calculations and generate the following output.

| | | | | | | | | |
|--|-----------|-----------|----------|---------------------------|---------------------------|----------------------------|----------------------------|--------------|
| Numeric Results for an Equal-Variance T-Test | | | | | | | | |
| Solve For: Power | | | | | | | | |
| Difference: $\delta = \mu_1 - \mu_2$ | | | | | | | | |
| Hypotheses: $H_0: \delta = 0$ vs. $H_1: \delta \neq 0$ | | | | | | | | |
| Power | N1 | N2 | N | μ_1 | μ_2 | δ | σ | Alpha |
| 0.95611 | 15 | 15 | 30 | 1 | 0 | 1 | 0.7206 | 0.05 |

The power of 0.95611 matches Zar’s result of 0.96 to the two decimal places given.

Example 6 – Validation using Julious (2010)

Julious (2010) page 49, as part of a table, gives an example in which the mean difference is 0.05, the common standard deviation is 1, the power is 90%, and the significance level is 0.05. He calculates the sample size for each group to be 8,407.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 6** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

| | |
|----------------------------------|------------------------|
| Design Tab | |
| Solve For | Sample Size |
| Alternative Hypothesis | Two-Sided |
| Power..... | 0.90 |
| Alpha..... | 0.05 |
| Group Allocation | Equal (N1 = N2) |
| Input Type..... | Difference |
| δ | 0.05 |
| σ | 1 |
| Reports Tab | |
| Means, Difference Decimals | 2 |

Output

Click the Calculate button to perform the calculations and generate the following output.

| | | | | | | | |
|---|--|-----------|-----------|----------|----------------------------|----------------------------|--------------|
| Numeric Results for an Equal-Variance T-Test | | | | | | | |
| <hr/> | | | | | | | |
| Solve For: | Sample Size | | | | | | |
| Difference: | $\delta = \mu_1 - \mu_2$ | | | | | | |
| Hypotheses: | H0: $\delta = 0$ vs. H1: $\delta \neq 0$ | | | | | | |
| <hr/> | | | | | | | |
| Target Power | Actual Power | N1 | N2 | N | δ | σ | Alpha |
| 0.9 | 0.90003 | 8407 | 8407 | 16814 | 0.05 | 1 | 0.05 |
| <hr/> | | | | | | | |

The sample size matches Julious' result of 8,407 exactly.

Example 7 – Validation using Chow, Shao, Wang, and Lokhnygina (2018)

Chow, Shao, Wang, and Lokhnygina (2018) presents an example on page 53 of a two-sided two-sample *t*-test sample size calculation for equal group sizes in which $\delta = 0.05$, $\sigma = 0.1$, $\alpha = 0.05$, and power = 0.80. They obtain a sample size of 64 for each group.

Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 7** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Design Tab

Solve For **Sample Size**
 Alternative Hypothesis **Two-Sided**
 Power..... **0.80**
 Alpha..... **0.05**
 Group Allocation **Equal (N1 = N2)**
 Input Type..... **Difference**
 δ **0.05**
 σ **0.1**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results for an Equal-Variance T-Test

Solve For: [Sample Size](#)
 Difference: $\delta = \mu_1 - \mu_2$
 Hypotheses: $H_0: \delta = 0$ vs. $H_1: \delta \neq 0$

| Target Power | Actual Power | N1 | N2 | N | δ | σ | Alpha |
|--------------|--------------|----|----|-----|----------|----------|-------|
| 0.8 | 0.80146 | 64 | 64 | 128 | 0.05 | 0.1 | 0.05 |

The sample size of 64 in each group matches the expected result.