Title
Portfolio Performance Evaluation on Various Financial Models

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Publication Date
2015

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Los Angeles

Portfolio Performance Evaluation on Various Financial Models

A thesis submitted in partial satisfaction

of the requirements for the degree Master of Science

in Statistics

by

John Lee Murphy

2015

ABSTRACT OF THE THESIS

Portfolio Performance Evaluation on Various Financial Models

by

John Lee Murphy

Master of Science in Statistics

University of California, Los Angeles, 2015

Professor Yingnian Wu, Chair

Portfolio performance evaluation is a tool used to judge how a portfolio performs during given period. The main evaluation methods include traditional (classical) portfolio performance evaluation and modern portfolio performance evaluation.

This thesis focuses on four typical measures of traditional (classical) portfolio performance evaluation, including Jensen's alpha, Sharpe ratio, generalized Sharpe ratio and Treynor ratio. These four measures will be applied to three financial models: single index model, constant correlation model and multigroup model and be compared to test which measure evaluates more accurately in different situations.

We also apply two market timing ability models to different portfolios based on these three financial models to compare which measure predicts portfolio performance more accurately by collecting the measures from a given period and examining the market timing ability in the following test period.

The thesis of John Lee Murphy is approved.

Nicolas Christou

Qing Zhou

Yingnian Wu, Committee Chair

University of California, Los Angeles

2015

My sincere gratitude to …

the professors in UCLA Statistics Department for huge help,

and my family for support

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Chapter 1

Introduction

During the last few decades, from about the 1980s, there was a huge wave of investment boom as the economic level of public kept increasing during this period. Along with this trend, the demand for the mutual funds and related investments increased greatly. Under such a background, methods of portfolio performance evaluation were studied widely, while more and more methods were invented to measure portfolio performances, which are classified into traditional methods and modern ones generally. During recent years, more and more people do not satisfy with investing through public institutions, such as Mutual Fund Company. People begin to invest by self-operations. In this thesis, we try to analyze how to achieve accurate evaluation of portfolio performance as much as possible on three basic investment models.

Since we try to analyze basic and simple models which can be handled by individual investors, we focus on traditional (classical) methods of portfolio performance evaluation, which reflect the essential principles more clearly. The main idea in most traditional methods is just compare the return of the managed portfolio to the return of a benchmark portfolio. What's more, the benchmark portfolio should be a practically alternative investment, which is comparable to the managed portfolio [1]. Commonly, people choose a market index as the benchmark portfolio [2].

This thesis will analyze four of the most typically classical methods, including Jensen's alpha, Sharpe ratio, generalized Sharpe ratio and Treynor ratio. Jensen's alpha was created by Jensen (1968) based on the capital asset pricing model, which uses the abnormal earn of managed portfolio to evaluate its performance. Treynor ratio was proposed by Treynor (1965) to solve a problem in Jensen's alpha when beta is large than one in some small capitalization stock funds [1]. Sharpe ratio, carried out by Sharpe (1966), is a risk-adjusted performance measure. Compared with the other two measures, Sharpe ratio uses the standard deviation of managed portfolio as the risk instead of using systematic risk. Sharpe also suggested another version of Sharpe ratio, called generalized Sharpe ratio, which uses the standard deviation of the difference between the managed portfolio and the benchmark portfolio instead of the standard deviation of only the managed portfolio [3].

To compare these four kinds of measures, we introduce three financial models to set up portfolios with chosen stocks. The three models are single index model, constant correlation model and multigroup model. All of them can be handled by individual investors. We will arrange the chosen stocks in two different orders: by prices and by betas. Then we build various portfolios based on these two orders on three financial models. Thus we can have a comprehensive comparison of the four classical measures of portfolio performance evaluation in different situations.

Furthermore, we will apply two kinds of market timing ability models on our portfolios. One is Merton-Henriksson market timing measure, while the other is Treynor-Mazuy market timing measure. They both belong to classical market timing measures, which use convexity between managed portfolio and benchmark to indicate the market timing ability [4].

Chapter 2

Data Selection

To set up the portfolios under single index model, constant correlation model and multigroup model, we choose 36 stocks from S&P 500 index, while S&P 500 records 500 leading companies in the United States. We choose S&P 500 as our market index because it includes many representative companies, which makes the risk more dispersed so that it can reflect the changes of the market more comprehensively. To fit multigroup model, these 36 stocks come from five large industries, which are Electric Utilities, Independent Oil &Gas, Money Center Banks, Diversified Utilities and Drug Manufactures. The details are shown in the table below,

Electric Utilities	AEP, CMS, ED, DTE, ETR, FE, PNW, PPL, SCG, TE, AES, WEC
Independent Oil &Gas	APA, APC, OG, CHK, DNR, DVN, MRO, MUR, NBL, NFX, OXY, RRC
Money Center Banks	BAC, WFC, PNC, JPM
Diversified Utilities	CNP, EXC, NRG, PEG
Drug Manufactures-Major	BMY, JNJ, MRK, PFE

Table 2.1: 36 stocks from 5 different industries

For these 36 stocks, we arrange them in orders by two ways, sorted by prices and by betas of stocks. Beta is calculated using regression analysis and can be thought as the measure of the risk arising from exposure to general market movements or the volatility compared with the market.

Chapter 3

Financial Models

3.1 Basic Concepts

First, we introduce some basic concepts about stocks and portfolios, which will serve us in the following analysis of portfolio performance evaluations.

3.1.1 Concepts about Stock & Portfolio

Two of the most basic but important concepts are return and risk. These two concepts apply to both stocks and portfolios.

Return of stock at time t is defined as

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where P_t and P_{t-1} are closing prices of stock at time t and t-1 respectively. When the stock pays dividend, the formula can be modified as

$$R_t = \frac{P_t + D - P_{t-1}}{P_{t-1}}$$

where D is the dividend paid during the period from t-1 to t. In our analysis, we use monthly return of stocks for our calculations and ignore any dividends paid by stocks. Based on this formula, we have the expected return for stock i as

$$\overline{R}_i = \frac{1}{n} \sum_{t=1}^{n} R_{it}$$

Then the return of portfolio can be represented as

$$R_p = X_1\overline{R}_1 + X_2\overline{R}_2 + \ldots\ldots\ldots + X_n\overline{R}_n$$

where X_1,X_2 ... X_n are percentages of each stock in portfolio.

The standard deviation of stock is always referred as the risk of stock, while the variance of stock i is defined as

$$\sigma_i{}^2=\frac{1}{n-1}\sum_{t=1}^{n}(R_{it}-\overline{R}_i)^2$$

And the covariance between the return of stock I and j is defined as

$$\text{cov}(R_i,R_j)=\frac{1}{n-1}\sum_{t=1}^{n}(R_{it}-\overline{R}_i)(R_{jt}-\overline{R}_j)$$

Then the variance of portfolio is defined as

$$\sigma_p{}^2=\text{var}(X_1\overline{R}_1+X_2\overline{R}_2+\ldots\ldots\ldots+X_n\overline{R}_n)$$

$$=\sum_{i=1}^{n}X_i^2\text{var}(R_i)+\sum_{i=1,j\neq i}^{n}cov(R_i,R_j)$$

3.1.2 CAPM (Capital Asset Pricing Model)

Most of the models used in this thesis about stocks and portfolios are based on capital asset pricing model (CAPM). Its formula is

$$\text{E}\ (R_i)-R_f=\beta_i\ (\text{E}\ (R_m)-R_f)$$

where E (R_i) is the expected return of the asset, E (R_m) is the expected return of the market (index), R_f is the risk free rate, and β_i is the beta of stock i.

This model describes the relationship between expected return and risk of assets. And can be used to determine a theoretically appropriate required rate of return of an asset (in our analysis, stocks or portfolios). It shows the asset's sensitivity to non-diversifiable risk (systematic risk), often known as beta of an asset. In other words, the CAPM says that the expected return of an asset equals to the risk free rate plus a risk premium [5].

3.1.3 Risk Free Rate

Risk free rate, often denoted as R_f, is the theoretical rate of return at which investors invest with no possibility of loss. In fact, the ideal risk free rate does not exist since there is no investment which can promise 100% no loss. Therefore, we often use the rate of government Treasury bill as the risk free rate. In our analysis of portfolio performance evaluation, we use the 1-year Treasury bill as the risk free rate.

3.1.4 Beta of Stock

The beta value of asset, as mentioned in the concept of CAPM, measures the volatility of an asset compared with the market index. Based on capital asset pricing model, beta can be calculated by

$$\beta_i = \frac{cov(R_i, R_m)}{var(R_m)}$$

We will use beta value to arrange stocks as mentioned in the data selection part.

3.1.5 Short Sales

Short sales mean that investors can borrow an asset and sell it. In other words, short sales allow the percentages of stocks in a portfolio can be negative. When short sales are allowed, investors have much more opportunities to manage their portfolios better by achieving larger expected return. In our analysis, to make our evaluation more general, we allow short sales in all financial models in this thesis.

3.2 Models for Building Portfolios

3.2.1 Single Index Model

Single index model is based on the capital asset pricing model, which states a linear relationship between the return of stock and market. The basic model of single index model is,

$$R_{it}=\alpha_i + \beta_i R_{mt} + \epsilon_{it}$$

where R_{it} is the return of stock i at time t, while R_{mt} is the return of the market at time t. α_i and β_i are the coefficients of this linear model.

Alternatively, single index model can also be expressed as the linear model between the excess return of stock and the excess return of market as following,

$$r_{it}=\alpha_i + \beta_i r_{mt} + \epsilon_{it}$$

where $r_{it}=R_{it}-R_f$ and $r_{mt}=R_{mt}-R_f$ are the excess returns of stock and market.

3.2.2 Constant Correlation Model

Constant correlation model is another useful model to achieve an optimal portfolio with given stocks, which assumes that the correlations between all pairs of stocks are the same, denoted by ρ.

3.2.3 Multigroup Model

In multigroup model, stocks are divided into different groups by the industries they belong to. Such as auto parts, dairy products, credit services, life insurance, etc. This model assumes that correlations between every pair of stocks within same group are all the same. The new correlation matrix based on this assumption is used to get the optimal portfolio. By grouping stocks into their own industries, the multigroup model makes the results of analysis more reasonable corresponding to the diversification of portfolio.

3.3 Models for Portfolio Performance Evaluation

There are many evaluation methods to test the performances of different portfolios. Among the classical measures, Jensen's alpha, Sharpe ratio, generalized Sharpe ratio and Treynor ratio are typical. The main idea behind the classical measure of investment performance is to compare the return of the portfolio to the return of a benchmark portfolio which is comparable and alternative to the one we evaluate [6]. In our analysis, we choose the S&P 500 market index as our benchmark portfolio.

3.3.1 Jensen's Alpha

The purpose of Jensen's alpha is to compare the excess return of managed portfolio to the benchmark portfolio (market index) in a linear model as,

$$R_p - R_f = \alpha^J + \beta\,(R_m - R_f)$$

where α^J is called Jensen's alpha, which represents the abnormal return an investor earns over the benchmark portfolio. Positive alpha means the investor beats the market, while negative alpha reflects that the investor does worse than market index [7].

3.3.2 Sharpe Ratio

Sharpe ratio is a kind of risk adjusted measure for portfolio performance. It measures the expected excess return of managed portfolio per unit of risk, in other words, the ratio of expected excess return to the risk of portfolio. The formula of Sharpe ratio is

$$SR_P = \frac{E(R_P - R_f)}{\sigma_P}$$

Alternatively, we can use the return of market take the place of risk free rate to get the Sharpe ratio as follows

$$SR_P=\frac{E(R_P-R_m)}{\sigma_P}$$

Large Sharpe ratio means good performance and vice versa.

3.3.3 Generalized Sharpe ratio

Rather than take the ratio of the expected difference between the return of managed portfolio and market to the risk of the portfolio, we can also take the ratio of the expected difference to the standard deviation of the difference. The later one can be expressed as

$$SR_P=\frac{E(R_P-R_m)}{\sigma(R_P-R_m)}$$

which is called generalized Sharpe ratio. Apparently, people prefers large ratio [1].

3.3.4 Treynor Ratio

Treynor ratio is also a risk-adjusted measure, which is similar to Sharpe ratio. Treynor ratio uses beta of the portfolio instead of the risk of the portfolio. In other words, it uses the systematic risk instead of the total risk. The formula of Treynor ratio is

$$T_p=\frac{E(R_P-R_f)}{\beta_P}$$

Still, the larger the Treynor ratio, the better a portfolio performs.

3.4 Models for Market Timing Ability

To evaluate the portfolio performance more completely and comprehensively, we can check the market timing ability of investors on managed portfolio. For classical models we discussed above, the market timing ability are reflected by the convexity in the relationship between managed portfolio and market index. If an investor is good at market timing, he will choose stocks with high beta values before the market goes up and prefers lower betas before market goes down [1]. There are two typically classical market timing ability methods: Merton-Henriksson market timing measure and Treynor-Mazuy market timing measure. In later analysis, we will examine which measure predicts better in various situation.

3.4.1 Merton-Henriksson Market Timing Measure

Merton-Henriksson market timing model uses the convexity in the relationship between the excess return between managed portfolio and market with formula as,

$$r_p = a_p + b_p r_m + \Lambda_p \max(r_m, 0)$$

where $r_p = R_p - R_f$ and $r_m = R_m - R_f$ are the excess returns of managed portfolio and market, while Λ_p measures the market timing ability. When Λ_p is positive, it means that the investor has market timing ability, while negative Λ_p means no market timing ability. What's more, when Λ_p is equal to zero, this formula just becomes the model to calculate Jensen's alpha [1].

3.4.2 Treynor-Mazuy Market Timing Measure

Treynor-Mazuy market timing measure is similar to Merton-Henriksson market timing measure. The difference is that it uses a quadratic model instead of the linear model. The formula is,

$$r_p = a_p + b_p r_m + \Lambda_p r_m^2$$

While each elements in this model represents the same meaning as in Merton-Henriksson measure. Similarly, positive Λ_p means good market timing ability [8].

Chapter 4

Analysis of Portfolio Performance Evaluation & Market Timing Ability

In this part, we begin to examine the applicability of the four measures of portfolio performance evaluation in three financial models with two different ways of grouping stocks. Then we use two market timing ability models to test market timing ability of each portfolio.

4.1 Stocks grouped by Prices

First, we begin with a common way of classifying stocks by individual investors, which is to distinguish stocks by their prices. For the 36 stocks we choose in data selection part, we equally divide them into three groups, each with stocks of different price level. And then we use the method of randomly choosing 4 stocks from each of the three price level to form another three groups with mixed prices, each also with 12 stocks. The final grouping result is shown below as,

Group #1 (High)	Group #2 (Medium)	Group #3 (Low)	Group #4 (Mixed)	Group #5 (Mixed)	Group #6 (Mixed)
DTE	AEP	CMS	AEP	DTE	SCG
ETR	SCG	PPL	FE	ETR	WEC
PNW	WEC	TE	CMS	PNW	AEP
ED	FE	AES	TE	ED	FE
APC	RRC	COG	RRC	RRC	APC
OXY	NBL	CHK	NBL	MUR	OXY
APA	MUR	DNR	PNC	DNR	PNC
DVN	NFX	MRO	JPM	CHK	JPM
PNC	PEG	WFC	WFC	EXC	CNP
JPM	EXC	BAC	BAC	PEG	PEG
JNJ	MRK	CNP	JNJ	NRG	MRK
BMY	PFE	NRG	BMY	CNP	PFE

Table 4.1: Six groups of stocks with different price levels

4.1.1 Single Index Model

For the six groups of stocks with different price levels, we first use single index model to set up portfolios for each group of stocks. Then we apply the four measures of portfolio performance evaluation (Jensen's alpha, Sharpe ratio, generalized Sharpe ratio and Treynor ratio) on these six groups in a five-year period from 1/1/2005 to 12/31/2009. We assume that the average risk free rate during these five years is 0.001.

Then we use the formulas discussed above to calculate these measures. For the Sharpe ratio, we exam both kinds of Sharpe ratio, using both the risk free rate and the market index as benchmark portfolio. The results of each measure can be expressed by either figure or table. For instance, the Jensen's alpha for all six portfolios can be shown in a graph as,

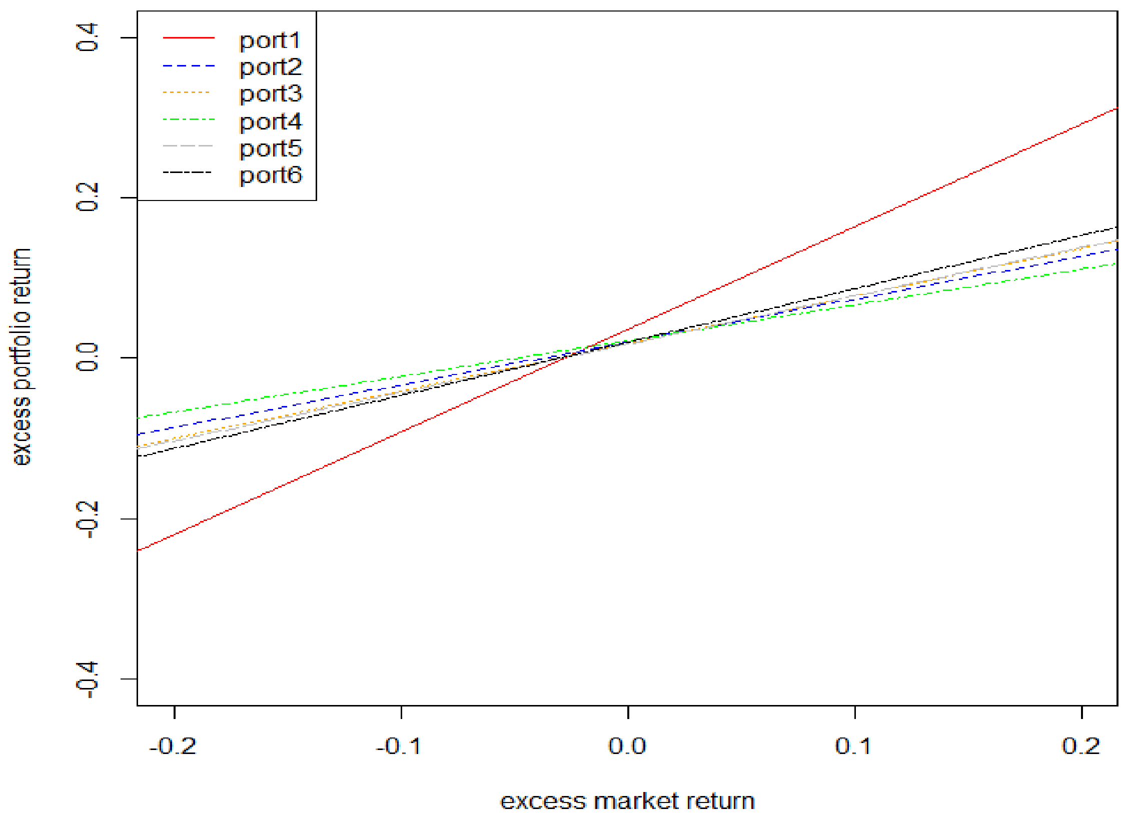

Figure 4.1: Jensen's alpha for each portfolio under SIM (grouped by prices)

In the figure, x axis and y axis present excess return of market and managed portfolios, while the intercepts are Jensen's alpha and the slope of each line can be thought as beta of portfolio.

But as our purpose is to compare all these measures, a table containing exact numbers is more intuitive for our analysis. Thus, we use tables as our main tool to analyzing portfolio performances, while we will plot Jensen's alpha in each situation as this measure can be shown clearly in graph. The results of all measures for the six portfolios are shown in table below,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	0.0351182	0.3117603	0.3197070	0.3779181	0.0265967
Group 2	0.0198646	0.3702323	0.3867762	0.3982576	0.0362003
Group 3	0.0182278	0.3097553	0.3249128	0.3464862	0.0299848
Group 4	0.0211173	0.3529376	0.3676964	0.3557744	0.0464926
Group 5	0.0170757	0.3160501	0.3326026	0.3638157	0.0274198
Group 6	0.0203306	0.3413769	0.3563581	0.4020525	0.0296960

Table 4.2: Portfolio Performances under SIM (grouped by prices)

Based on the table above, we can order the performances of the six portfolios by each measure as followings,

Jensen's alpha	#1 > #4 > #6 > #2 > #3 > #5
Sharpe ratio (R_f)	#2 > #4 > #6 > #5 > #1 > #3
Sharpe ratio (R_m)	#2 > #4 > #6 > #5 > #3 > #1
Generalized Sharpe ratio	#6 > #2 > #1 > #5 > #4 > #3
Treynor ratio	#4 > #2 > #3 > #6 > #5 > #1

Table 4.3: Order of Portfolio Performances under SIM (grouped by prices)

Then we apply the two methods of market timing ability to each of the six portfolio. For convenience, we rewrite the two formulas as,

Merton-Henriksson Model: r_p=a_p+$b_p r_m$+$\Lambda_p$max $(r_m, 0)$

Treynor-Mazuy Model: r_p=a_p+$b_p r_m$+$\Lambda_p r_m^2$.

In both models, we use the coefficient Λ_p to represent the market timing ability. The results are shown below,

	Merton-Henriksson	Treynor-Mazuy
Group 1	-0.1120228	-1.75317228
Group 2	0.2481174	0.60707774
Group 3	-0.61106259	-3.62403370
Group 4	0.01946966	-0.28332126
Group 5	-0.06641002	-0.91213697
Group 6	-0.53892395	-3.44510057

Table 4.4: Results of Market Timing Ability under SIM (grouped by prices)

Finally, we choose a following five-year period from 1/1/2010 to 12/31/2014 to examine how accurately each market timing model predicts the performance of each portfolio. For comparison, during the second period, we still use the same percentages of stocks in each portfolio. In this period, we assume the risk free rate equal to 0.0001.

The measures for all six portfolios are shown below,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	-0.02934857	-0.0330965	-0.1237789	-0.145606	-0.0019838
Group 2	0.001896047	0.1944626	-0.0478029	-0.0524232	0.01463847
Group 3	0.008617106	0.2842762	0.05631809	0.05659317	0.02860281
Group 4	0.001875505	0.1788648	-0.0260718	-0.0288324	0.01431085
Group 5	-0.00352893	0.1098849	-0.1259646	-0.1511134	0.00708307
Group 6	0.0008128599	0.1957836	-0.0308740	-0.0371470	0.01270087

Table 4.5: Portfolio Performances during test period under SIM (grouped by prices)

Finally, we order the performances of the six portfolios during test period as followings,

Jensen's alpha	#3 > #2 > #4 > #6 > #5 > #1
Sharpe ratio (R_f)	#3 > #6 > #2 > #4 > #5 > #1
Sharpe ratio (R_m)	#3 > #4 > #6 > #2 > #1 > #5
Generalized Sharpe ratio	#3 > #4 > #6 > #2 > #1 > #5
Treynor ratio	#3 > #2 > #4 > #6 > #5 > #1

Table 4.6: Order of Portfolio Performances during test period under SIM (grouped by prices)

4.1.2 Constant Correlation Model

Then we use constant correlation model on these six groups to get six new portfolios. Same as what we did above, we first show the figure of Jensen's alpha,

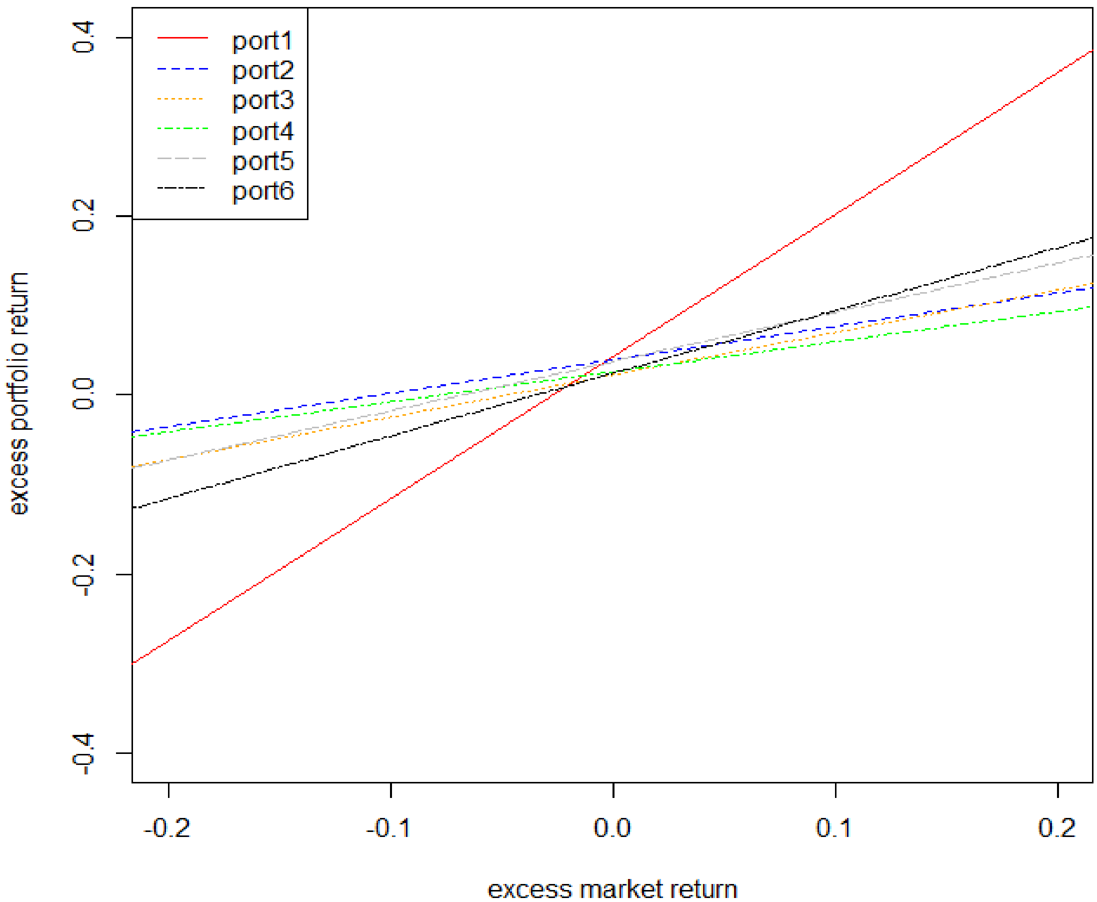

Figure 4.2: Jensen's alpha for each portfolio under CCM (grouped by prices)

The results of all measures are shown below,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	0.0429481	0.3038274	0.3101627	0.358538	0.0262407
Group 2	0.0390057	0.3475441	0.3553328	0.3476717	0.1037135
Group 3	0.0225587	0.2828995	0.2939717	0.2911103	0.0469138
Group 4	0.0267693	0.3240848	0.3346954	0.3181393	0.0788142
Group 5	0.0374920	0.2786288	0.2851538	0.2868246	0.0676507
Group 6	0.0242998	0.3353007	0.3475672	0.381646	0.0340412

Table 4.7: Portfolio Performances under CCM (grouped by prices)

Then the order of performances for each measure is shown in table below,

Jensen's alpha	#1 > #2 > #5 > #4 > #6 > #3
Sharpe ratio (R_f)	#2 > #6 > #4 > #1 > #3 > #5
Sharpe ratio (R_m)	#2 > #6 > #4 > #1 > #3 > #5
Generalized Sharpe ratio	#6 > #1 > #2 > #4 > #3 > #5
Treynor ratio	#2 > #4 > #5 > #3 > #6 > #1

Table 4.8: Order of Portfolio Performances under CCM (grouped by prices)

Then we move on the results of market timing ability as,

	Merton-Henriksson	Treynor-Mazuy
Group 1	-0.18736102	-3.42827863
Group 2	1.35622659	4.91212042
Group 3	-0.72997201	-4.51561322
Group 4	0.17786978	0.22018676
Group 5	0.66574789	0.77909404
Group 6	-0.49022005	-3.44550161

Table 4.9: Results of Market Timing Ability under CCM (grouped by prices)

For the test period, the results of measures are,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	-0.04213124	-0.05564271	-0.124808	-0.141149	-0.00335097
Group 2	0.001290956	0.1125454	-0.0138079	-0.0144867	0.01333206
Group 3	0.01208885	0.2640672	0.08504184	0.083411	0.0391653
Group 4	-0.00059115	0.1169971	-0.0380146	-0.0412361	0.01094335
Group 5	-0.01424263	0.000449136	-0.0980409	-0.105951	0.000043497
Group 6	-0.00060189	0.1708841	-0.0204200	-0.0250043	0.01104029

Table 4.10: Portfolio Performances during test period under CCM (grouped by prices)

The order of performances during test period are as followings,

Jensen's alpha	#3 > #2 > #4 > #6 > #5 > #1
Sharpe ratio (R_f)	#3 > #6 > #4 > #2 > #5 > #1
Sharpe ratio (R_m)	#3 > #2 > #6 > #4 > #5 > #1
Generalized Sharpe ratio	#3 > #2 > #6 > #4 > #5 > #1
Treynor ratio	#3 > #2 > #6 > #4 > #5 > #1

Table 4.11: Order of Portfolio Performances during test period under CCM (grouped by prices)

4.1.3 Multigroup Model

In this part, we use multigroup model on these six portfolios based on the industries these stocks belong to. The procedure is similar as what we did above. First, graph of Jensen's alpha is

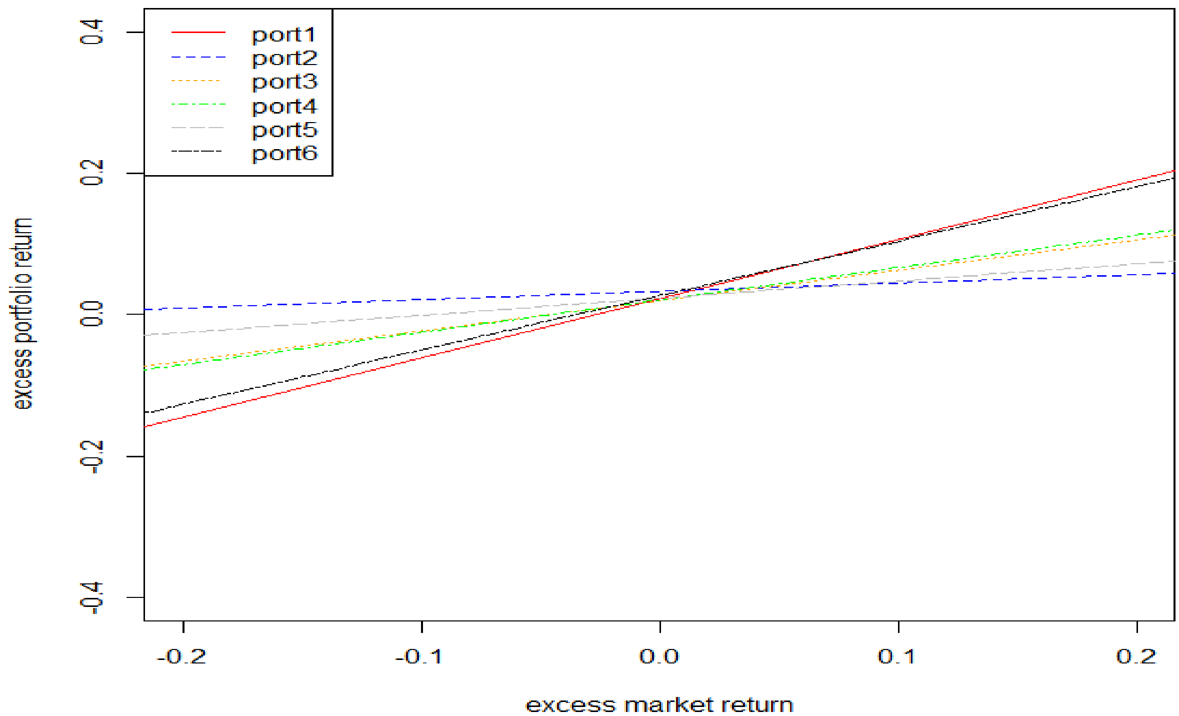

Figure 4.3: Jensen's alpha for each portfolio under MGM (grouped by prices)

Then the results of all measures are,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	0.0229556	0.2903677	0.3016912	0.3483694	0.0265511
Group 2	0.0332018	0.3902512	0.4004721	0.3609783	0.2806807
Group 3	0.0205558	0.2656627	0.2770725	0.2698629	0.0471531
Group 4	0.0204189	0.3189447	0.3327545	0.3253901	0.0436686
Group 5	0.0228810	0.3081946	0.3199802	0.2914186	0.0929185
Group 6	0.0270513	0.3559314	0.3676263	0.4145604	0.0342595

Table 4.12: Portfolio Performances under MGM (grouped by prices)

The order of the performances are as followings,

Jensen's alpha	#2 > #6 > #1 > #5 > #3 > #4
Sharpe ratio (R_f)	#2 > #6 > #4 > #5 > #1 > #3
Sharpe ratio (R_m)	#2 > #6 > #4 > #5 > #1 > #3
Generalized Sharpe ratio	#6 > #2 > #1 > #4 > #5 > #3
Treynor ratio	#2 > #5 > #3 > #4 > #6 > #1

Table 4.13: Order of Portfolio Performances under MGM (grouped by prices)

Then we test the market timing abilities,

	Merton-Henriksson	Treynor-Mazuy
Group 1	-0.2907759	-1.30330125
Group 2	0.72437373	4.68270385
Group 3	0.13049802	0.50638144
Group 4	0.79071158	3.40274719
Group 5	0.50046248	2.30426606
Group 6	0.18350483	0.5323336

Table 4.14: Results of Market Timing Ability under MGM (grouped by prices)

Then we test the performances on test period,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	-0.005845383	0.1137032	-0.0429150	-0.0538399	0.00691169
Group 2	0.01835657	0.2391048	0.08777919	0.07913234	2.281551
Group 3	0.02184734	0.3952841	0.2069947	0.1891963	0.1073197
Group 4	0.001990537	0.2339376	0.02441604	0.03153852	0.01378135
Group 5	0.007530942	0.1477429	-0.0253896	-0.0233847	0.04780863
Group 6	0.0006976039	0.1677049	-0.0060966	-0.0070418	0.01244802

Table 4.15: Portfolio Performances during test period under MGM (grouped by prices)

The order of these performances are as followings,

Jensen's alpha	#3 > #2 > #5 > #4 > #6 > #1
Sharpe ratio (R_f)	#3 > #2 > #4 > #6 > #5 > #1
Sharpe ratio (R_m)	#3 > #2 > #4 > #6 > #5 > #1
Generalized Sharpe ratio	#3 > #2 > #4 > #6 > #5 > #1
Treynor ratio	#2 > #3 > #5 > #4 > #6 > #1

Table 4.16: Order of Portfolio Performances during test period under MGM (grouped by prices)

4.2 Stocks grouped by Betas

After evaluating the portfolio performances based on grouping stocks by prices, we now divide the 36 stocks by their beta values. For the betas, we calculate it according to the linear model between the return of stock and the return of market index. We still divide them into 3 groups from high betas group to low betas group. Then we create another 3 groups with mixed betas stocks by randomly drawing 4 stocks from each beta level. The final result is shown below,

Group #1 (High)	Group #2 (Medium)	Group #3 (Low)	Group #4 (Mixed)	Group #5 (Mixed)	Group #6 (Mixed)
AES	DTE	PNW	FE	DTE	AES
TE	ETR	CMS	WEC	PNW	TE
NFX	APA	AEP	CHK	CHK	DTE
CHK	APC	SCG	DNR	MRO	ETR
COG	NBL	FE	COG	BAC	WEC
MRO	RRC	PPL	DVN	WFC	ED
OXY	JPM	WEC	BAC	JPM	FE
DVN	PNC	ED	WFC	PNC	CMS
DNR	NRG	EXC	BMY	EXC	MUR
MUR	CNP	PEG	JNJ	PEG	DVN
BAC	MRK	JNJ	MRK	BMY	NRG
WFC	PFE	BMY	PFE	JNJ	CNP

Table 4.17: Six groups of stocks with different beta levels

4.2.1 Single Index Model

As what we did in the part 4.1, Jensen's alpha can be shown as

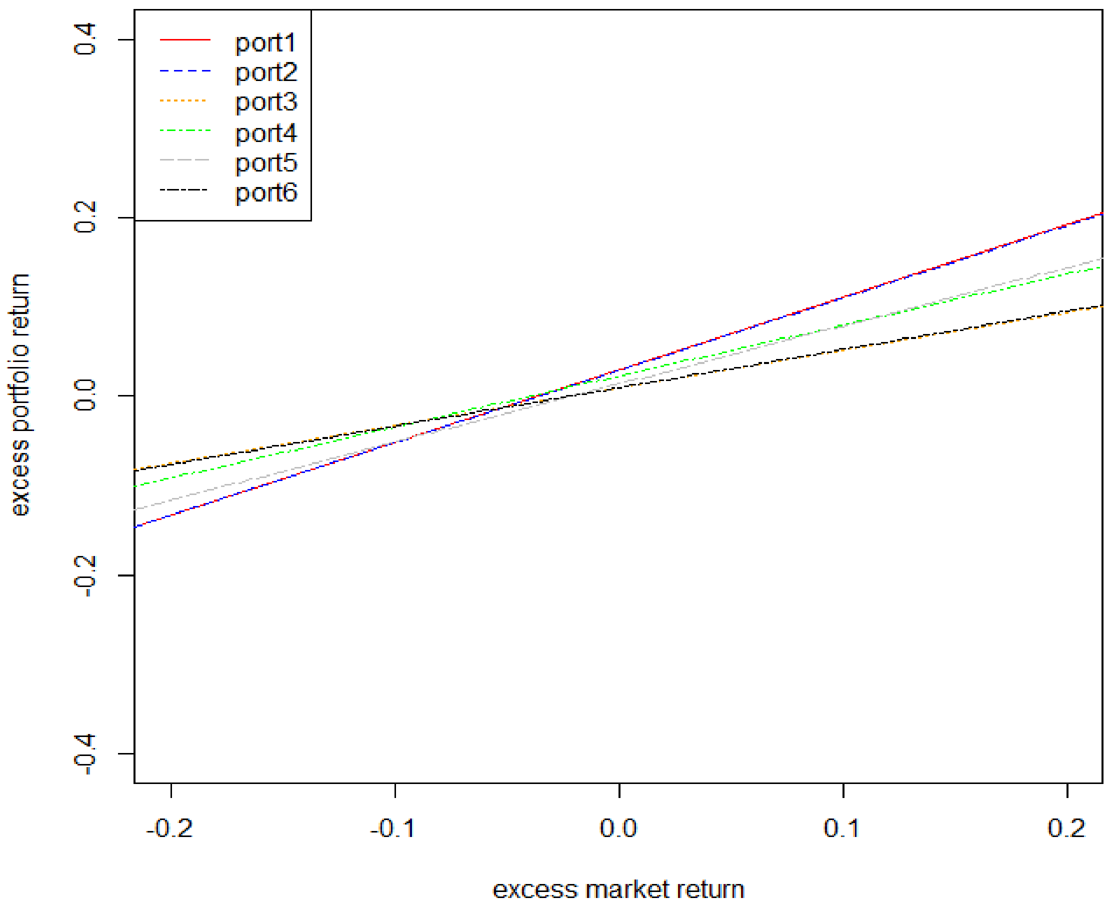

Figure 4.4: Jensen's alpha for each portfolio under SIM (grouped by betas)

Then we check the performances during the first period first as followings,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	0.0297899	0.3592362	0.3699433	0.4154863	0.0357930
Group 2	0.0285752	0.3568399	0.3679378	0.4165905	0.0344617
Group 3	0.0100253	0.2735342	0.2980797	0.2638170	0.0229690
Group 4	0.0223045	0.3449916	0.3587027	0.3726569	0.0384287
Group 5	0.0139831	0.2204529	0.2346963	0.2590215	0.0205599
Group 6	0.0095450	0.2768816	0.3030406	0.2665478	0.0215363

Table 4.18: Portfolio Performances under SIM (grouped by betas)

The order of performances are,

Jensen's alpha	#1 > #2 > #4 > #5 > #3 > #6
Sharpe ratio (R_f)	#1 > #2 > #4 > #6 > #3 > #5
Sharpe ratio (R_m)	#1 > #2 > #4 > #6 > #3 > #5
Generalized Sharpe ratio	#2 > #1 > #4 > #6 > #3 > #5
Treynor ratio	#4 > #1 > #2 > #3 > #6 > #5

Table 4.19: Order of Portfolio Performances under SIM (grouped by betas)

Then we test the market timing abilities of these portfolios,

	Merton-Henriksson	Treynor-Mazuy
Group 1	-0.48351134	-3.01706259
Group 2	0.25392121	-0.3014696
Group 3	-0.33177329	-1.0767096
Group 4	-0.33291359	-1.9963106
Group 5	-0.59874633	-3.01585385
Group 6	-0.33872306	-1.11626424

Table 4.20: Results of Market Timing Ability under SIM (grouped by betas)

Then we evaluate portfolio performances on the test period,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	-0.00789403	0.08633374	-0.0378301	-0.0444247	0.005920881
Group 2	-0.01262812	0.0345764	-0.1009354	-0.1218948	0.002229222
Group 3	0.01164186	0.3142025	0.03756043	0.03001878	0.09554527
Group 4	0.008471994	0.2331307	0.0267308	0.02567807	0.03264748
Group 5	0.003237003	0.2136252	-0.0233986	-0.0252700	0.0168642
Group 6	0.01043703	0.3797537	0.05299148	0.04318111	0.05056528

Table 4.21: Portfolio Performances during test period under SIM (grouped by betas)

The order of performances are as followings,

Jensen's alpha	#3 > #6 > #4 > #5 > #1 > #2
Sharpe ratio (R_f)	#6 > #3 > #4 > #5 > #1 > #2
Sharpe ratio (R_m)	#6 > #3 > #4 > #5 > #1 > #2
Generalized Sharpe ratio	#6 > #3 > #4 > #5 > #1 > #2
Treynor ratio	#3 > #6 > #4 > #5 > #1 > #2

Table 4.22: Order of Portfolio Performances during test period under SIM (grouped by betas)

4.2.2 Constant Correlation Model

Under this model, Jensen's alpha is shown as

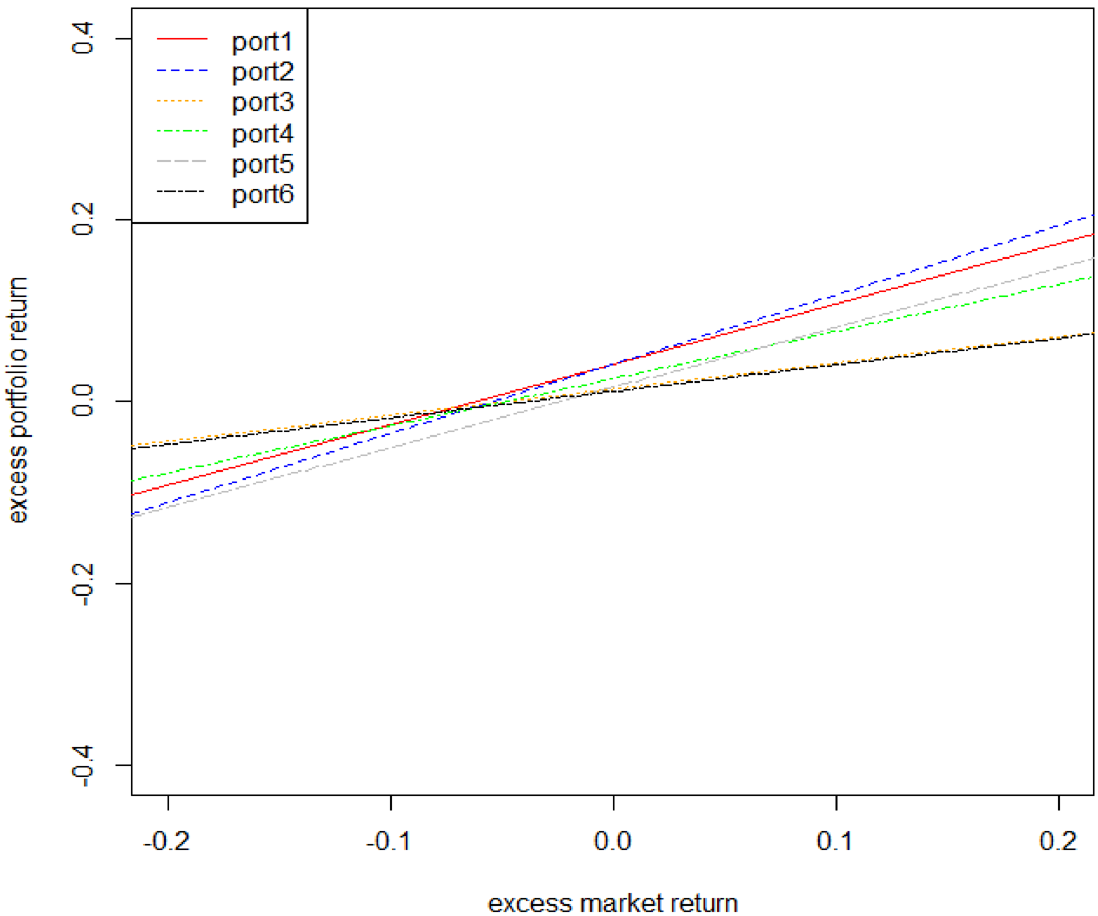

Figure 4.5: Jensen's alpha for each portfolio under CCM (grouped by betas)

The results of all measures during the first period are as followings

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	0.0408694	0.3594452	0.3671786	0.3781027	0.0606194
Group 2	0.0414238	0.3352506	0.3423798	0.3562329	0.0536737
Group 3	0.0143059	0.2417618	0.2566714	0.2274595	0.0489216
Group 4	0.0252683	0.3209696	0.3321808	0.3344325	0.0478696
Group 5	0.0148889	0.2030403	0.2153344	0.2321341	0.0217047
Group 6	0.0109254	0.2484493	0.2686288	0.2199122	0.0366795

Table 4.23: Portfolio Performances under CCM (grouped by betas)

The order of performances are,

Jensen's alpha	#2 > #1 > #4 > #5 > #3 > #6
Sharpe ratio (R_f)	#1 > #2 > #4 > #6 > #3 > #5
Sharpe ratio (R_m)	#1 > #2 > #4 > #6 > #3 > #5
Generalized Sharpe ratio	#1 > #2 > #4 > #6 > #3 > #5
Treynor ratio	#1 > #2 > #3 > #4 > #6 > #5

Table 4.24: Order of Portfolio Performances under CCM (grouped by betas)

Then we test the market timing abilities of these portfolios,

	Merton-Henriksson	Treynor-Mazuy
Group 1	-0.53955216	-4.17730766
Group 2	0.55043694	-0.39970518
Group 3	-0.12828585	-0.17526844
Group 4	-0.32698914	-2.22151917
Group 5	-0.66225692	-3.56827455
Group 6	-0.28147044	-0.45686576

Table 4.25: Results of Market Timing Ability under CCM (grouped by betas)

Then we evaluate the performances during the test period,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	-0.00825615	0.06865367	-0.0243564	-0.0266912	0.005964357
Group 2	-0.0202935	-0.0057537	-0.0965374	-0.1078011	-0.00044198
Group 3	0.01661424	0.3054886	0.08702459	0.07077004	-0.6684545
Group 4	0.01016829	0.2195975	0.03957511	0.03773192	0.04080757
Group 5	0.002980777	0.20146	-0.0233471	-0.0252594	0.01632838
Group 6	0.01567175	0.4162473	0.1308444	0.1021931	0.1463248

Table 4.26: Portfolio Performances during test period under CCM (grouped by betas)

And the orders of performances in this period are,

Jensen's alpha	#3 > #6 > #4 > #5 > #1 > #2
Sharpe ratio (R_f)	#6 > #3 > #4 > #5 > #1 > #2
Sharpe ratio (R_m)	#6 > #3 > #4 > #5 > #1 > #2
Generalized Sharpe ratio	#6 > #3 > #4 > #5 > #1 > #2
Treynor ratio	#6 > #4 > #5 > #1 > #2 > #3

Table 4.27: Order of Portfolio Performances during test period under CCM (grouped by betas)

4.2.3 Multigroup Model

Finally, Jensen's alpha in this condition is shown below,

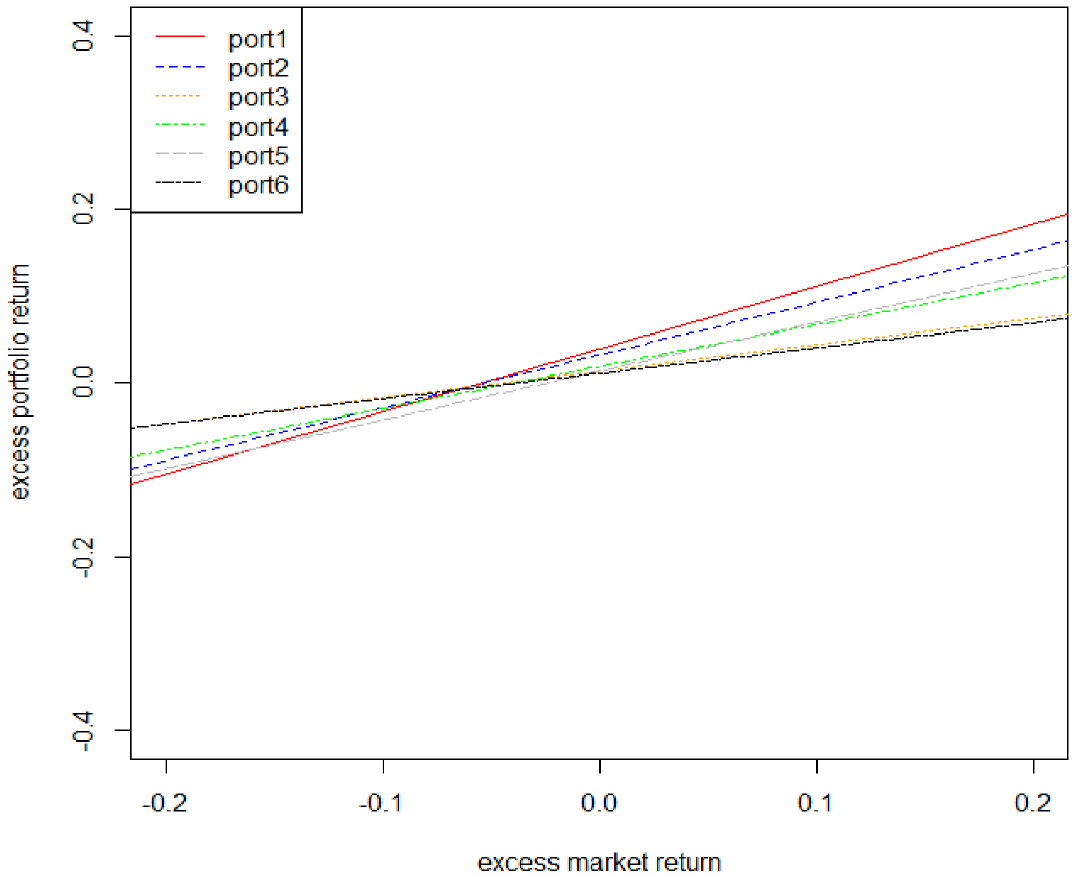

Figure 4.6: Jensen's alpha for each portfolio under MGM (grouped by betas)

We then evaluate all other measures as followings,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	0.0390471	0.3385687	0.3462079	0.3599935	0.0531677
Group 2	0.0320747	0.3131857	0.3217911	0.3293103	0.0521110
Group 3	0.0143331	0.246753	0.2619566	0.2331882	0.0462816
Group 4	0.0195911	0.3074966	0.3213992	0.3180773	0.0398450
Group 5	0.0143837	0.2283262	0.2425709	0.2521206	0.0246657
Group 6	0.0114202	0.2613773	0.2816667	0.2302134	0.0383489

Table 4.28: Portfolio Performances under MGM (grouped by betas)

The orders of these performances are,

Jensen's alpha	#1 > #2 > #4 > #5 > #3 > #6
Sharpe ratio (R_f)	#1 > #2 > #4 > #6 > #3 > #5
Sharpe ratio (R_m)	#1 > #2 > #4 > #6 > #3 > #5
Generalized Sharpe ratio	#1 > #2 > #4 > #5 > #3 > #6
Treynor ratio	#1 > #2 > #3 > #4 > #6 > #5

Table 4.29: Order of Portfolio Performances under MGM (grouped by betas)

Then we test the market timing abilities,

	Merton-Henriksson	Treynor-Mazuy
Group 1	-0.33860355	-0.87140618
Group 2	0.81220810	3.37047352
Group 3	0.16218138	0.99163686
Group 4	0.35739679	2.63425933
Group 5	0.286899498	0.99760418
Group 6	-0.21406254	-0.05681618

Table 4.30: Results of Market Timing Ability under MGM (grouped by betas)

Next, we move on the test period. The measures during this period are,

	Jensen's Alpha	Sharpe Ratio(R_f)	Sharpe Ratio(R_m)	Generalized Sharpe	Treynor Ratio
Group 1	0.008606426	0.1838556	0.07396084	0.07829881	0.02087234
Group 2	-0.00207771	0.09623291	-0.0457019	-0.0494751	0.009249926
Group 3	0.018534	0.4170317	0.1583477	0.1232575	0.7544628
Group 4	0.02273887	0.4552324	0.2360767	0.2021623	0.1866646
Group 5	0.005868224	0.3282882	0.08860096	0.1175139	0.01844482
Group 6	0.0153584	0.404771	0.1226087	0.09632582	0.1404124

Table 4.31: Portfolio Performances during test period under MGM (grouped by betas)

And the orders of performances are,

Jensen's alpha	#4 > #3 > #6 > #1 > #5 > #2
Sharpe ratio (R_f)	#4 > #3 > #6 > #5 > #1 > #2
Sharpe ratio (R_m)	#4 > #3 > #6 > #5 > #1 > #2
Generalized Sharpe ratio	#4 > #3 > #5 > #6 > #1 > #2
Treynor ratio	#3 > #4 > #6 > #1 > #5 > #2

Table 4.32: Order of Portfolio Performances during test period under MGM (grouped by betas)

Chapter 5

Conclusion

In the conclusion part, we will compare the measures of portfolio performance evaluation and the market timing ability for each portfolio first and then take an overview of all these measures.

For each financial model, we check the orders of portfolios given by all measures and decide the best order by the principle in which we decide each position in the order with the portfolio matches majority measures in this position. Then we compare the accuracy of the two market timing ability measures by the orders of prediction combined with the portfolio performances during the test period. We need to mention that sometimes Sharpe ratio with risk free rate as benchmark and Sharpe ratio with market index as benchmark have the same result, in this case, we just treat them together as Sharpe ratio.

First, for portfolios under single index model with stocks grouped by prices, we decide the proper order of portfolio performances as #2 > #4 > #6 > #5 > #3 > #1. Based on this order, we can see that Sharpe ratio with market index as benchmark is the most suitable measure, while Sharpe ratio with the risk free rate as benchmark also performs not bad. Besides Sharpe ratio, Treynor ratio does well for portfolio of stocks with high price level and Jensen's Alpha does well for portfolio with low price stocks. Then, both market timing measures give the same result as #2 > #4 > #5 > #1 > #6 > #3, while the best order of performances during test period should be #3 > #2 > #4 > #6 > #5 > #1. Combine these two information, we can see that both market timing measures work well for all portfolios except portfolio with low price stocks, in which both measures predict totally wrong.

Then we move on portfolios under constant correlation model with stocks grouped by prices. Using the same logic, we can conclude that Sharpe ratio works best. Besides, Treynor ratio works well for portfolio of medium price stocks and generalized Sharpe ratio does well in portfolio of low price stocks. For the market timing ability, the two measures still give the same result, which is still good for all portfolios but the one with low price stocks.

For portfolios based on multigroup model with stocks grouped by prices, we find that Sharpe ratio still does best in evaluation. While both Jensen's Alpha and Treynor ratio work well for portfolio with medium price stocks, and generalized Sharpe ratio does well in portfolio with low price stocks. For market timing ability, the two measures still both predict badly in portfolio with low price stocks, but work really well in others portfolios.

For portfolios under single index model with stocks grouped by betas, we can see that Sharpe ratio keeps working best, while Jensen's Alpha and generalized Sharpe ratio are also not bad. In details, Jensen's Alpha just has a little bit mistake in one portfolio with mixed beta value stocks and generalized Sharpe ratio does well in portfolio with low beta value stocks. For market timing ability, we can see that both measures predict well for portfolios with high and low beta value stocks. And for portfolios with mixed beta value stocks, Treynor-Mazuy does better. But both of them have bad performances in portfolio with medium beta value stocks.

Then for portfolios of constant correlation model with stocks grouped by betas, both Sharpe ratio and generalized Sharpe ratio work well. Jensen's Alpha is good at portfolio with low beta stocks, while Treynor ratio does well at portfolios with high and medium beta stocks. For market timing ability, we can see that Treynor-Mazuy measure predicts better, and both of them still have bad performances in prediction of portfolio of medium beta stocks.

Finally, for portfolios based on multigroup model with stocks grouped by beats. Sharpe ratio still performs best among all measures. Under this model, other measures are also not bad for most portfolios. Both market timing ability measures predict are really good for portfolio of high beta stocks and not bad for portfolios with low and mixed beta stocks. But they still both bad at predicting portfolio of medium beta value stocks.

Based on our analysis above, we can see that for portfolios with stocks grouped by their prices, Sharpe ratio beats other measures in all of the three financial models, while Sharpe ratio with market index as benchmark is even more reliable. The other three measures work well in some portfolios, but there is no obvious patterns to show which measures among these three kept being good at any kind of portfolio. For the market timing ability, both measures predict totally wrong in portfolio with low price stocks in all three financial models. While they both predict well in other portfolios. For portfolios with stocks grouped by betas, Sharpe ratio still the best one, while other measures do not have stable pattern of their performances. Both market timing ability measures are bad at predicting performance of portfolio with medium beta value stocks in all three financial models. For all other portfolios, both of them predict well, while Treynor-Mazuy market timing measure performs better in some cases.

Overall, based on our analysis, no matter which way is used to group stocks for building portfolio and which financial model we use, Sharpe ratio is the most reliable portfolio performance evaluation measure among these four typically classical measures, while using market index as benchmark is even more stable. And Treynor-Mazuy market timing ability measure is more accurate in predicting portfolio performance. The only two big problems about market timing ability are predicting portfolios with all low price stocks or all medium beta value stocks.

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