UK Junior Mathematical Challenges

2007 to 2011

Organised by the

United Kingdom Mathematics Trust

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UK JUNIOR MATHEMATICAL CHALLENGE

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RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 8 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 9 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK JMC is about solving interesting problems, not about lucky guessing.

2007

1. What is the value of $0.1 + 0.2 + 0.3 \times 0.4$?

A 0.24 B 0.312 C 0.42 D 1.0 E 1.5

2. My train was scheduled to leave at 17:40 and to arrive at 18:20. However, it started five minutes late and the journey then took 42 minutes. At what time did I arrive?

A 18:21 B 18:23 C 18:25 D 18:27 E 18:29

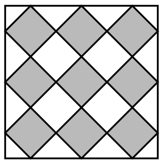
3. What is the remainder when 354972 is divided by 7 ?

A 1 B 2 C 3 D 4 E 5

4. Which of the following numbers is three less than a multiple of 5 and three more than a multiple of 6?

A 12 B 17 C 21 D 22 E 27

5. In the diagram, the small squares are all the same size. What fraction of the large square is shaded?

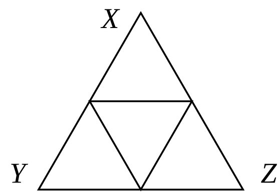


A $\frac{9}{20}$ B $\frac{9}{16}$ C $\frac{3}{7}$ D $\frac{3}{5}$ E $\frac{1}{2}$

6. When the following fractions are put in their correct places on the number line, which fraction is in the middle?

A $-\frac{1}{7}$ B $\frac{1}{6}$ C $-\frac{1}{5}$ D $\frac{1}{4}$ E $-\frac{1}{3}$

7. The equilateral triangle XYZ is fixed in position. Two of the four small triangles are to be painted black and the other two are to be painted white. In how many different ways can this be done?

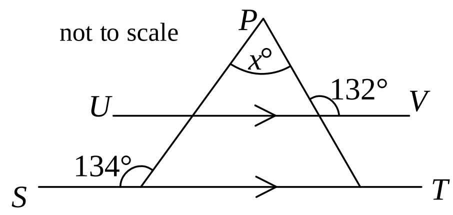


A 3 B 4 C 5 D 6 E more than 6

8. Amy, Ben and Chris are standing in a row. If Amy is to the left of Ben and Chris is to the right of Amy, which of these statements must be true?

A Ben is furthest to the left B Chris is furthest to the right C Amy is in the middle D Amy is furthest to the left E None of statements A, B, C, D is true

9. In the diagram on the right, ST is parallel to UV. What is the value of x ?

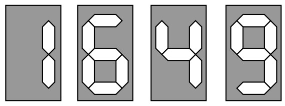


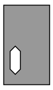
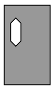
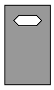
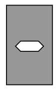
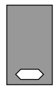
A 46 B 48 C 86 D 92 E 94

10. Which of the following has the largest value?

A $\frac{1}{2} + \frac{1}{4}$ B $\frac{1}{2} - \frac{1}{4}$ C $\frac{1}{2} \times \frac{1}{4}$ D $\frac{1}{2} \div \frac{1}{4}$ E $\frac{1}{4} \div \frac{1}{2}$

11. A station clock shows each digit by illuminating up to seven bars in a display. For example, the displays for 1, 6, 4 and 9 are shown. When all the digits from 0 to 9 are shown in turn, which bar is used least?



A  B  C  D  E 

12. The six-member squad for the Ladybirds five-a-side team consists of a 2-spot ladybird, a 10-spot, a 14-spot, an 18-spot, a 24-spot and a pine ladybird (on the bench). The average number of spots for members of the squad is 12. How many spots has the pine ladybird?

A 4 B 5 C 6 D 7 E 8

13. Points P and Q have coordinates (1, 4) and (1, −2) respectively. For which of the following possible coordinates of point R would triangle PQR **not** be isosceles?

A (−5, 4) B (7, 1) C (−6, 1) D (−6, −2) E (7, −2)

14. If the line on the right were 0.2 mm thick, how many metres long would the line need to be to cover an area of one square metre?

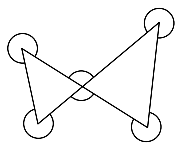
A 0.5 B 5 C 50 D 500 E 5000

15. I choose three numbers from this number square, including one number from each row and one number from each column. I then multiply the three numbers together. What is the largest possible product?

1	2	3
4	5	6
7	8	9

A 72 B 96 C 105 D 162 E 504

16. What is the sum of the six marked angles?



A 1080° B 1440° C 1620° D 1800°
E more information needed

17. Just William's cousin, Sweet William, has a rectangular block of fudge measuring 2 inches by 3 inches by 6 inches. He wants to cut the block up into cubes whose side lengths are whole numbers of inches. What is the smallest number of cubes he can obtain?

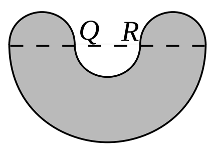
A 3 B 8 C 15 D 29 E 36

18. The letters J, M, C represent three different non-zero digits. What is the value of $J + M + C$?

$$\begin{array}{ccc} & J & J \\ & M & M \\ & C & C \\ \hline J & M & C \\ \hline \end{array}$$

A 19 B 18 C 17 D 16 E 15

19. The points P, Q, R, S lie in order along a straight line, with $PQ = QR = RS = 2$ cm. Semicircles with diameters PQ, QR, RS and SP join to make the shape shown on the right. What, in cm², is the area of the shape?



A 5π B $9\pi/2$ C 4π D $7\pi/2$ E 3π

20. At halftime, Boarwarts Academy had scored all of the points so far in their annual match against Range Hill School. In the second half, each side scored three points. At the end of the match, Boarwarts Academy had scored 90% of the points. What fraction of the points in the match was scored in the second half?

A $\frac{3}{100}$ B $\frac{3}{50}$ C $\frac{1}{10}$ D $\frac{9}{50}$ E $\frac{1}{5}$

21. A list of ten numbers contains two of each of the numbers 0, 1, 2, 3, 4. The two 0s are next to each other, the two 1s are separated by one number, the two 2s by two numbers, the two 3s by three numbers and the two 4s by four numbers. The list starts 3, 4, ... What is the last number?

A 0 B 1 C 2 D 3 E 4

22. Only one choice of the digit *d* gives a prime number for each of the three-digit numbers read across and downwards in the diagram on the right. Which digit is *d* ?

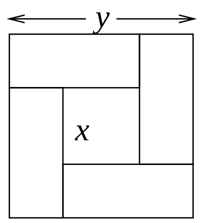
	5	
1	*d*	3
	7	

A 4 B 5 C 6 D 7 E 8

23. The diagram shows a square with sides of length *y* divided into a square with sides of length *x* and four congruent rectangles.

What is the length of the longer side of each rectangle?

A $\frac{y-x}{2}$ B $\frac{y+2x}{3}$ C $y-x$ D $\frac{2y}{3}$ E $\frac{y+x}{2}$



24. The pages of a book are numbered 1, 2, 3, … . In total, it takes 852 digits to number all the pages of the book. What is the number of the last page?

A 215 B 314 C 320 D 329 E 422

25. A piece of paper in the shape of a polygon is folded in half along a line of symmetry. The resulting shape is also folded in half, again along a line of symmetry. The final shape is a triangle. How many possibilities are there for the number of sides of the original polygon?

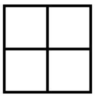
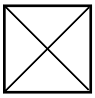
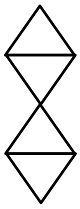
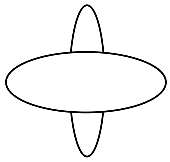
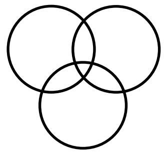
A 3 B 4 C 5 D 6 E 7

2008

1. Which of these calculations produces a multiple of 5?

A $1 \times 2 + 3 + 4$ B $1 + 2 \times 3 + 4$ C $1 \times 2 + 3 \times 4$ D $1 + 2 \times 3 \times 4$ E $1 \times 2 \times 3 \times 4$

2. Which of these diagrams could be drawn without taking the pen off the page and without drawing along a line already drawn?

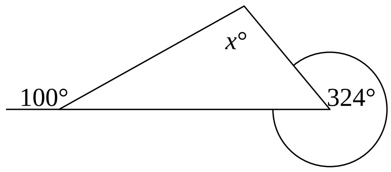
A  B  C  D  E 

3. All of the Forty Thieves were light-fingered, but only two of them were caught red-handed. What percentage is that?

A 2 B 5 C 10 D 20 E 50

4. In this diagram, what is the value of x?

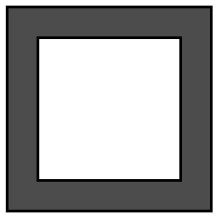
A 16 B 36 C 64 D 100 E 144



5. At Spuds-R-Us, a 2.5kg bag of potatoes costs £1.25. How much would one tonne of potatoes cost?

A £5 B £20 C £50 D £200 E £500

6. The diagram shows a single floor tile in which the outer square has side 8cm and the inner square has side 6cm. If Adam Ant walks once around the perimeter of the inner square and Annabel Ant walks once around the perimeter of the outer square, how much further does Annabel walk than Adam?

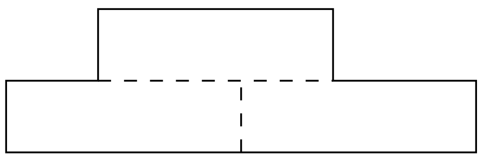


A 2 cm B 4 cm C 6 cm D 8 cm E 16 cm

7. King Harry's arm is twice as long as his forearm, which is twice as long as his hand, which is twice as long as his middle finger, which is twice as long as his thumb. His new bed is as long as four arms. How many thumbs length is that?

A 16 B 32 C 64 D 128 E 256

8. The shape on the right is made up of three rectangles, each measuring 3cm by 1cm. What is the perimeter of the shape?



A 16 cm B 18 cm C 20 cm D 24 cm E More information needed

9. Which of the following has the smallest value?

A $\frac{1}{2} - \frac{1}{3}$ B $\frac{1}{3} - \frac{1}{4}$ C $\frac{1}{4} - \frac{1}{5}$ D $\frac{1}{5} - \frac{1}{6}$ E $\frac{1}{6} - \frac{1}{7}$

10. The faces of a cube are painted so that any two faces which have an edge in common are painted different colours. What is the smallest number of colours required?

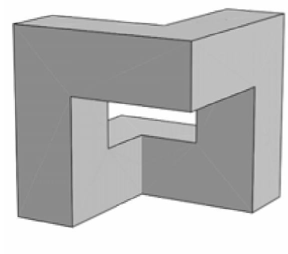
A 2 B 3 C 4 D 5 E 6

11. In 1833 a ship arrived in Calcutta with 120 tons remaining of its cargo of ice. One third of the original cargo was lost because it had melted on the voyage. How many tons of ice was the ship carrying when it set sail?

A 40 B 80 C 120 D 150 E 180

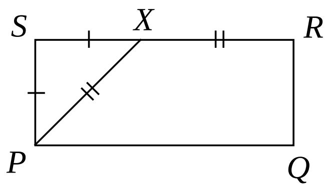
12. The sculpture 'Cubo Vazado' [Emptied Cube] by the Brazilian artist Franz Weissmann is formed by removing cubical blocks from a solid cube to leave the symmetrical shape shown.

If all the edges have length 1, 2 or 3, what is the volume of the sculpture?



A 9 B 11 C 12 D 14 E 18

13. A rectangle $PQRS$ is cut into two pieces along PX, where $PX = XR$ and $PS = SX$ as shown. The two pieces are reassembled without turning either piece over, by matching two edges of equal length. Not counting the original rectangle, how many different shapes are possible?



A 1 B 2 C 3 D 4 E 5

14. A solid wooden cube is painted blue on the outside. The cube is then cut into eight smaller cubes of equal size. What fraction of the total surface area of these new cubes is blue?

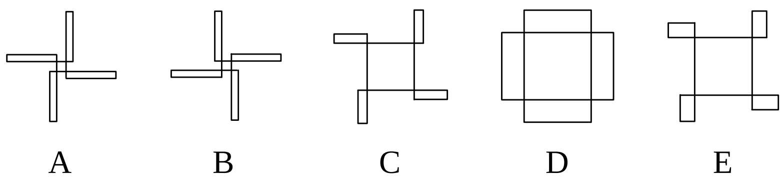
A $\frac{1}{8}$ B $\frac{1}{3}$ C $\frac{3}{8}$ D $\frac{1}{2}$ E $\frac{3}{4}$

15. An active sphagnum bog deposits a depth of about 1 metre of peat per 1000 years. Roughly how many millimetres is that per day?

A 0.0003 B 0.003 C 0.03 D 0.3 E 3

16. The figures below are all drawn to scale. Which figure would result from repeatedly following the instructions in the box on the right?

Move forward 2 units.
Turn right.
Move forward 15 units.
Turn right.
Move forward 20 units.
Turn right.



17. In this *Multiplication Magic Square*, the **product** of the three numbers in each row, each column and each of the diagonals is 1. What is the value of $r + s$?

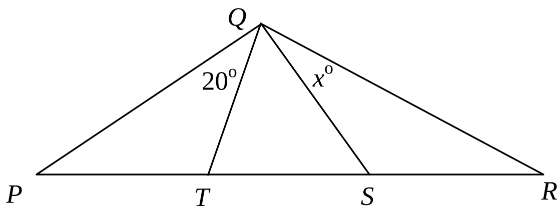
p	q	r
s	1	t
u	4	$\frac{1}{8}$

A $\frac{1}{2}$ B $\frac{9}{16}$ C $\frac{5}{4}$ D $\frac{33}{16}$ E 24

18. Granny swears that she is getting younger. She has calculated that she is four times as old as I am now, but remembers that 5 years ago she was five times as old as I was at that time. What is the sum of our ages now?

A 95 B 100 C 105 D 110 E 115

19. In the diagram on the right, $PT = QT = TS$, $QS = SR$, $\angle PQT = 20°$. What is the value of x?



A 20 B 25 C 30
D 35 E 40

20. If all the whole numbers from 1 to 1000 inclusive are written down, which digit appears the smallest number of times?

A 0 B 2 C 5 D 9 E none: no single digit appears fewer times than all the others

21. What is the value of ♥ if each row and each column has the total given?

				Total
	♥	☼	♬	12
	♬	♥	♥	11
	☼	☼	♬	13
Total	12	11	13	

A 3 B 4 C 5 D 6 E more information needed

22. On a digital clock displaying hours, minutes and seconds, how many times in each 24-hour period do all six digits change simultaneously?

A 0 B 1 C 2 D 3 E 24

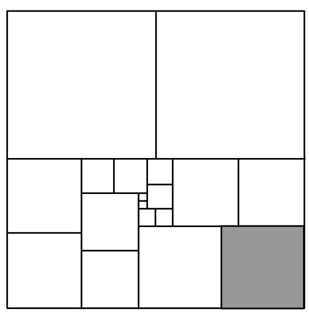
23. In a 7-digit numerical code each group of four adjacent digits adds to 16 and each group of five adjacent digits adds to 19. What is the sum of all seven digits?

A 21 B 25 C 28 D 32 E 35

24. The list 2, 1; 3, 2; 2, 3; 1, 4; describes itself, since there are two 1s, three 2s, two 3s and one 4. There is exactly one other list of eight numbers containing only the numbers 1, 2, 3, and 4 that, in the same way, describes the numbers of 1s, 2s, 3s and 4s in that order. What is the total number of 1s and 3s in this other list?

A 2 B 3 C 4 D 5 E 6

25. A large square is divided into adjacent pairs of smaller squares with integer sides, as shown in the diagram (which is not drawn to scale). Each size of smaller square occurs only twice. The shaded square has sides of length 10. What is the area of the large square?



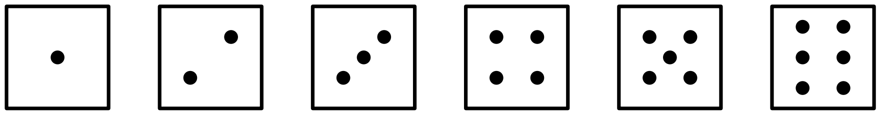
A 1024 B 1089 C 1156 D 1296 E 1444

2009

1. What is the value of $9002 - 2009$?

A 9336 B 6993 C 6339 D 3996 E 3669

2. How many of the six faces of a die (shown below) have fewer than three lines of symmetry?



A 2 B 3 C 4 D 5 E 6

3. Which of the following is correct?

A $0 \times 9 + 9 \times 0 = 9$ B $1 \times 8 + 8 \times 1 = 18$ C $2 \times 7 + 7 \times 2 = 27$
D $3 \times 6 + 6 \times 3 = 36$ E $4 \times 5 + 5 \times 4 = 45$

4. Which of the following points is *not* at a distance of 1 unit from the origin?

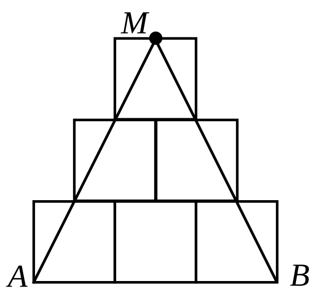
A $(0, 1)$ B $(1, 0)$ C $(0, -1)$ D $(-1, 0)$ E $(1, 1)$

5. Which of the following numbers is divisible by 7?

A 111 B 1111 C 11 111 D 111 111 E 1 111 111

6. Each square in the figure is 1 unit by 1 unit. What is the area of triangle *ABM* (in square units)?

A 4 B 4.5 C 5 D 5.5 E 6

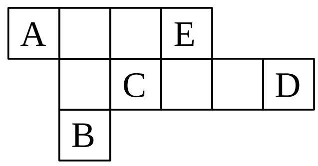


7. How many minutes are there from 11:11 until 23:23 on the same day?

A 12 B 720 C 732 D 1212 E 7212

8. The figure on the right shows an arrangement of ten square tiles. Which labelled tile could be removed, but still leave the length of the perimeter unchanged?

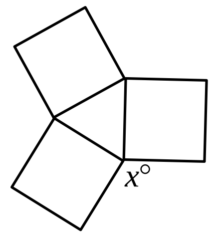
A B C D E



9. How many different digits appear when $\frac{20}{11}$ is written as a recurring decimal?

A 2 B 3 C 4 D 5 E 6

10. The diagram shows three squares of the same size. What is the value of x?



A 105 B 120 C 135 D 150 E 165

11. In a sequence of numbers, each term after the first three terms is the sum of the previous three terms. The first three terms are $-3, 0, 2$. Which is the first term to exceed 100?

A 11th term B 12th term C 13th term D 14th term E 15th term

12. Gill is 21 this year. At the famous visit to the clinic in 1988, her weight was calculated to be 5kg, but she now weighs 50kg. What has been the percentage increase in Gill's weight from 1988 to 2009?

A 900% B 1000% C 5000% D 9000% E 10 000%

13. The sum of ten consecutive integers is 5. What is the largest of these integers?

A 2 B 3 C 4 D 5 E more information needed

14. Karen was given a mark of 72 for Mayhematics. Her average mark for Mayhematics and Mathemagics was 78. What was her mark for Mathemagics?

A 66 B 75 C 78 D 82 E 84

15. In Matt's pocket there are 8 watermelon jellybeans, 4 vanilla jellybeans and 4 butter popcorn jellybeans. What is the smallest number of jellybeans he must take out of his pocket to be certain that he takes at least one of each flavour?

A 3 B 4 C 8 D 9 E 13

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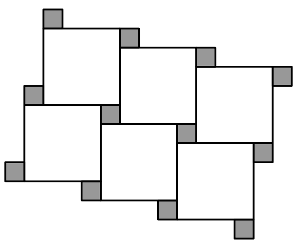
16. The kettle in Keith's kitchen is 80% full. After 20% of the water in it has been poured out, there are 1152 ml of water left. What volume of water does Keith's kitchen kettle hold when it is full?

A 1400 ml B 1600 ml C 1700 ml D 1800 ml E 2000 ml

17. The tiling pattern shown uses two sizes of square, with sides of length 1 and 4. A very large number of these squares is used to tile an enormous floor in this pattern. Which of the following is closest to the ratio of the number of grey tiles on the floor to the number of white tiles?



A 1:1 B 4:3 C 3:2 D 2:1 E 4:1

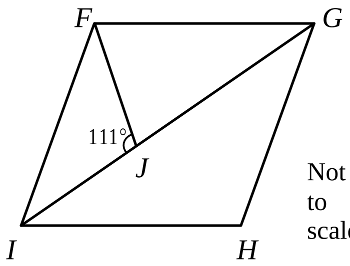
18. Six friends are having dinner together in their local restaurant. The first eats there every day, the second eats there every other day, the third eats there every third day, the fourth eats there every fourth day, the fifth eats there every fifth day and the sixth eats there every sixth day. They agree to have a party the next time they all eat together there. In how many days' time is the party?

A 30 days B 60 days C 90 days D 120 days E 360 days

19. The diagram on the right shows a rhombus *FGHI* and an isosceles triangle *FGJ* in which $GF = GJ$. Angle $FJI = 111°$.

What is the size of angle *JFI* ?

A 27°   B 29°   C 31°   D 33°   E $34\frac{1}{2}°$



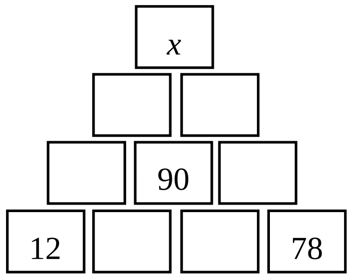
Not to scale

20. In the diagram on the right, the number in each box is obtained by adding the numbers in the two boxes immediately underneath.

What is the value of *x*?

A 300   B 320   C 340

D 360   E more information needed



21. A rectangular sheet of paper is divided into two pieces by a single straight cut. One of the pieces is then further divided into two, also by a single straight cut.

Which of the following could *not* be the total number of edges of the resulting three pieces?

A 9   B 10   C 11   D 12   E 13

22. Starting at the square containing the 2, you are allowed to move from one square to the next either across a common edge, or diagonally through a common corner. How many different routes are there passing through exactly two squares containing a 0 and ending in one of the squares containing a 9?

2	0	0	9
0	0	0	9
0	0	0	9
9	9	9	9

A 7   B 13   C 15   D 25   E 32

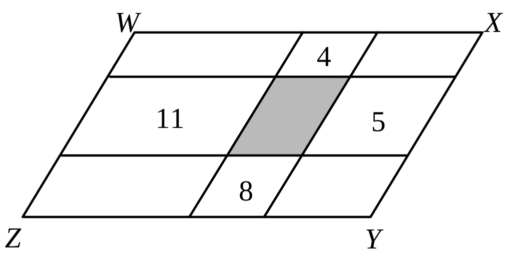
23. The currency used on the planet Zog consists of bank notes of a fixed size differing only in colour. Three green notes and eight blue notes are worth 46 zogs; eight green notes and three blue notes are worth 31 zogs. How many zogs are two green notes and three blue notes worth?

A 13 zogs   B 16 zogs   C 19 zogs   D 25 zogs   E 27 zogs

24. The parallelogram *WXYZ* shown in the diagram on the right has been divided into nine smaller parallelograms. The perimeters, in centimetres, of four of the smaller parallelograms are shown.

The perimeter of *WXYZ* is 21 cm.

What is the perimeter of the shaded parallelogram?



A 5 cm   B 6 cm   C 7 cm   D 8 cm   E 9 cm

25. In Miss Quaffley's class, one third of the pupils bring a teddy bear to school. Last term, each boy took 12 books out of the library, each girl took 17 books and each teddy bear took 9 books. In total, 305 books were taken out. How many girls are there in Miss Quaffley's class?

A 4   B 7   C 10   D 13   E 16

2010

1. What is 2010 + (+2010) + (–2010) – (+2010) – (–2010) ?

A 0 B 2010 C 4020 D 6030 E 8040

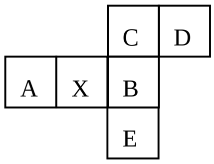
2. Each letter in the abbreviation shown is rotated through 90° clockwise. Which of the following could be the result?

U K M T

3. Which of the following could have a length of 2010 mm?

A a table B an oil tanker C a teaspoon D a school hall E a hen's egg

4. If the net shown is folded to make a cube, which letter is opposite X ?

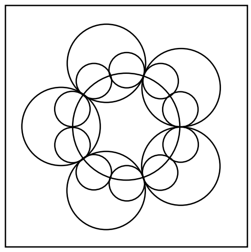


A B C D E

5. The diagram shows a pattern of 16 circles inside a square.

The central circle passes through the points where the other circles touch.

The circles divide the square into regions. How many regions are there?



A 17 B 26 C 30 D 32 E 38

6. Which of the following has the largest value?

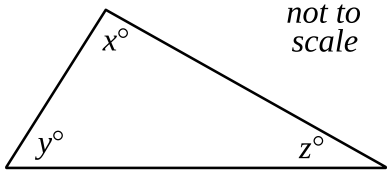
A $6 \div \frac{1}{2}$ B $5 \div \frac{1}{3}$ C $4 \div \frac{1}{4}$ D $3 \div \frac{1}{5}$ E $2 \div \frac{1}{6}$

7. Mr Owens wants to keep the students quiet during a Mathematics lesson. He asks them to multiply all the numbers from 1 to 99 together and then tell him the last-but-one digit of the result. What is the correct answer?

A 0 B 1 C 2 D 8 E 9

8. In a triangle with angles $x°$, $y°$, $z°$ the mean of y and z is x.

What is the value of x?



not to scale

A 90 B 80 C 70 D 60 E 50

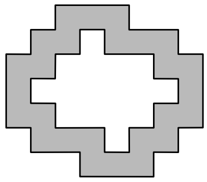
9. Which of the following is the longest period of time?

A 3002 hours B 125 days C $17\frac{1}{2}$ weeks D 4 months E $\frac{1}{3}$ of a year

10. At the Marldon Apple-Pie-Fayre bake-off, prize money is awarded for 1st, 2nd and 3rd places in the ratio 3 : 2 : 1. Last year Mrs Keat and Mr Jewell shared third prize equally. What fraction of the total prize money did Mrs Keat receive?

A $\frac{1}{4}$ B $\frac{1}{5}$ C $\frac{1}{6}$ D $\frac{1}{10}$ E $\frac{1}{12}$

11. In the diagram shown, all the angles are right angles and all the sides are of length 1 unit, 2 units or 3 units.
What, in square units, is the area of the shaded region?

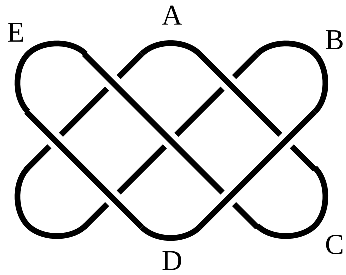


A 22 B 24 C 26 D 28 E 30

12. Sir Lance has a lot of tables and chairs in his house. Each rectangular table seats eight people and each round table seats five people. What is the smallest number of tables he will need to use to seat 35 guests and himself, without any of the seating around these tables remaining unoccupied?

A 4 B 5 C 6 D 7 E 8

13. The diagram shows a Lusona, a sand picture of the Tshokwe people from the West Central Bantu area of Africa. To draw a Lusona the artist uses a stick to draw a single line in the sand, starting and ending in the same place without lifting the stick in between. At which point could this Lusona have started?

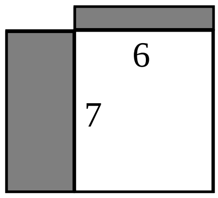


A B C D E

14. The Severn Bridge has carried just over 300 million vehicles since it was opened in 1966. On average, roughly how many vehicles is this per day?

A 600 B 2 000 C 6 000 D 20 000 E 60 000

15. A 6 by 8 and a 7 by 9 rectangle overlap with one corner coinciding as shown.
What is the area (in square units) of the region *outside* the overlap?

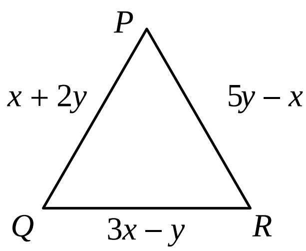


A 6 B 21 C 27 D 42 E 69

16. One of the examination papers for Amy's Advanced Arithmetic Award was worth 18% of the final total. The maximum possible mark on this paper was 108 marks. How many marks were available overall?

A 420 B 480 C 540 D 560 E 600

17. The lengths, in cm, of the sides of the equilateral triangle *PQR* are as shown.
Which of the following could *not* be the values of x and y?

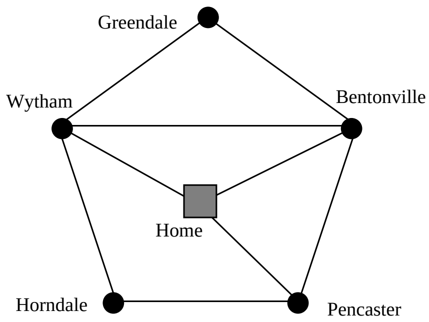


A (18, 12) B (15, 10) C (12, 8) D (10, 6) E (3, 2)

18. Sam's 101st birthday is tomorrow. So Sam's age in years changes from a square number (100) to a prime number (101). How many times has this happened before in Sam's lifetime?

A 1 B 2 C 3 D 4 E 5

19. Pat needs to travel down every one of the roads shown at least once, starting and finishing at home. What is the smallest number of the five villages that Pat will have to visit more than once?



A 1 B 2 C 3 D 4 E 5

20. Nicky has to choose 7 different positive whole numbers whose mean is 7. What is the largest possible such number she could choose?

A 7 B 28 C 34 D 43 E 49

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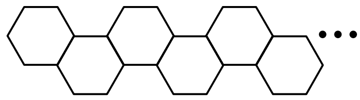
21. A shape consisting of a number of regular hexagons is made by continuing to the right the pattern shown in the diagram, with each extra hexagon sharing one side with the preceding one. Each hexagon has a side length of 1 cm. How many hexagons are required for the perimeter of the whole shape to have length 2010 cm?

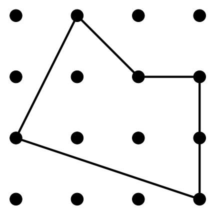


A 335 B 402 C 502 D 670 E 1005

22. Kiran writes down six different prime numbers, p, q, r, s, t, u, all less than 20, such that $p + q = r + s = t + u$. What is the value of $p + q$?

A 16 B 18 C 20 D 22 E 24

23. A single polygon is made by joining dots in the 4×4 grid with straight lines, which meet only at dots at their end points. No dot is at more than one corner. The diagram shows a five-sided polygon formed in this way. What is the greatest possible number of sides of a polygon formed by joining the dots using these same rules?



A 12 B 13 C 14 D 15 E 16

24. The year 2010 belongs to a special sequence of twenty-five consecutive years: each number from 1988 to 2012 contains a repeated digit.

Each of the following belongs to a sequence of consecutive years, where each number in the sequence contains at least one repeated digit.

Which of them belongs to the next such sequence of at least twenty years?

A 2099 B 2120 C 2199 D 2989 E 3299

25. What is the value of $P + Q + R$ in the multiplication on the right?

$$\begin{array}{cccccc} & & P & Q & P & Q \\ & \times & & R & R & R \\ \hline 6 & 3 & 9 & 0 & 2 & 7 \\ \hline \end{array}$$

A 13 B 12 C 11 D 10 E 9

2011

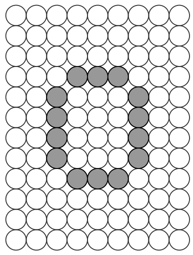
1. What is the value of $2 \times 0 \times 1 + 1$?

A 0 B 1 C 2 D 3 E 4

2. How many of the integers 123, 234, 345, 456, 567 are multiples of 3?

A 1 B 2 C 3 D 4 E 5

3. A train display shows letters by lighting cells in a grid, such as the letter 'o' shown. A letter is made **bold** by also lighting any unlit cell immediately to the right of one in the normal letter. How many cells are lit in a **bold** 'o'?



A 22 B 24 C 26 D 28 E 30

4. The world's largest coin, made by the Royal Mint of Canada, was auctioned in June 2010.
The coin has mass 100 kg, whereas a standard British £1 coin has mass 10 g.
What sum of money in £1 coins has the same mass as the record-breaking coin?

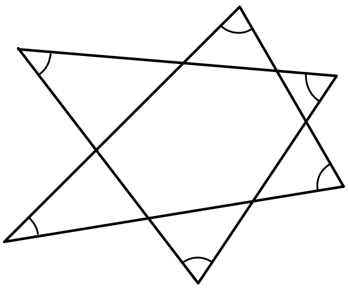
A £100 B £1000 C £10 000 D £100 000 E £1 000 000

5. All old Mother Hubbard had in her cupboard was a Giant Bear chocolate bar. She gave each of her children one-twelfth of the chocolate bar. One third of the bar was left. How many children did she have?

A 6 B 8 C 12 D 15 E 18

6. What is the sum of the marked angles in the diagram?

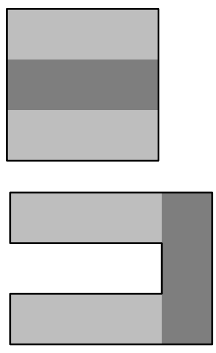
A 90° B 180° C 240° D 300° E 360°



7. Peter Piper picked a peck of pickled peppers. 1 peck = $\frac{1}{4}$ bushel and 1 bushel = $\frac{1}{9}$ barrel.
How many **more** pecks must Peter Piper pick to fill a barrel?

A 12 B 13 C 34 D 35 E 36

8. A square is divided into three congruent rectangles.
The middle rectangle is removed and replaced on the side of the original square to form an octagon as shown.
What is the ratio of the length of the perimeter of the square to the length of the perimeter of the octagon?

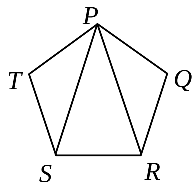


A 3:5 B 2:3 C 5:8 D 1:2 E 1:1

9. What is the smallest possible difference between two different nine-digit integers, each of which includes all of the digits 1 to 9?

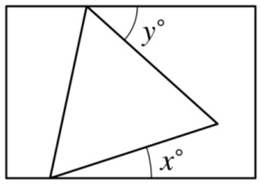
A 9 B 18 C 27 D 36 E 45

10. You want to draw the shape on the right without taking your pen off the paper and without going over any line more than once. Where should you start?



A only at T or Q B only at P C only at S or R D at any point E the task is impossible

11. The diagram shows an equilateral triangle inside a rectangle. What is the value of $x + y$?



A 30 B 45 C 60 D 75 E 90

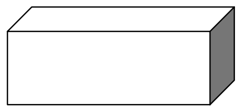
12. If ▲ + ▲ = ■ and ■ + ▲ = ● and ♦ = ● + ■ + ▲, how many ▲s are equal to ♦ ?

A 2 B 3 C 4 D 5 E 6

13. What is the mean of $\frac{2}{3}$ and $\frac{4}{9}$?

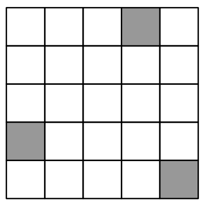
A $\frac{1}{2}$ B $\frac{2}{9}$ C $\frac{7}{9}$ D $\frac{3}{4}$ E $\frac{5}{9}$

14. The diagram shows a cuboid in which the area of the shaded face is one-quarter of the area of each of the two visible unshaded faces. The total surface area of the cuboid is 72 cm^2. What, in cm^2, is the area of one of the visible unshaded faces of the cuboid?



A 16 B 28.8 C 32 D 36 E 48

15. What is the smallest number of *additional* squares which must be shaded so that this figure has at least one line of symmetry *and* rotational symmetry of order 2 ?



A 3 B 5 C 7 D 9 E more than 9

16. The pupils in Year 8 are holding a mock election. A candidate receiving more votes than any other wins. The four candidates receive 83 votes between them. What is the smallest number of votes the winner could receive?

A 21 B 22 C 23 D 41 E 42

17. Last year's match at Wimbledon between John Isner and Nicolas Mahut, which lasted 11 hours and 5 minutes, set a record for the longest match in tennis history. The fifth set of the match lasted 8 hours and 11 minutes.

Approximately what fraction of the whole match was taken up by the fifth set?

A $\frac{1}{5}$ B $\frac{2}{5}$ C $\frac{3}{5}$ D $\frac{3}{4}$ E $\frac{9}{10}$

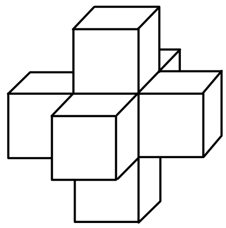
18. Peri the winkle leaves on Monday to go and visit Granny, 90m away. Except for rest days, Peri travels 1m each day (24-hour period) at a constant rate and without pause. However, Peri stops for a 24-hour rest every tenth day, that is, after every nine days' travelling. On which day of the week does Peri arrive at Granny's?

A Sunday B Monday C Tuesday D Wednesday E Thursday

19. A list is made of every digit that is the units digit of at least one prime number. How many of the following numbers appear in the list?

A 1 B 2 C 3 D 4 E 5

20. One cube has each of its faces covered by one face of an identical cube, making a solid as shown. The volume of the solid is 875 cm^3. What, in cm^2, is the surface area of the solid?



A 750 B 800 C 875 D 900 E 1050

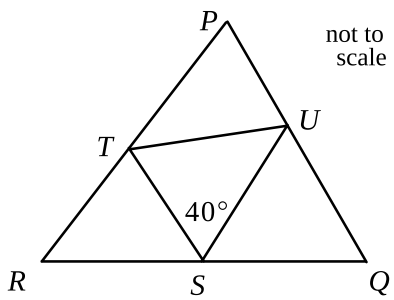
21. Gill leaves Lille by train at 09:00. The train travels the first 27 km at 96 km/h. It then stops at Lens for 3 minutes before travelling the final 29 km to Lillers at 96 km/h. At what time does Gill arrive at Lillers?

A 09:35 B 09:38 C 09:40 D 09:41 E 09:43

22. Last week Evariste and Sophie both bought some stamps for their collections. Each stamp Evariste bought cost him £1.10, whilst Sophie paid 70p for each of her stamps. Between them they spent exactly £10. How many stamps did they buy in total?

A 9 B 10 C 11 D 12 E 13

23. The points S, T, U lie on the sides of the triangle PQR, as shown, so that $QS = QU$ and $RS = RT$.
$\angle TSU = 40°$. What is the size of $\angle TPU$?

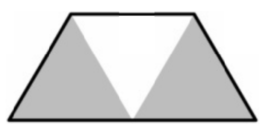


A 60° B 70° C 80° D 90° E 100°

24. Two adults and two children wish to cross a river. They make a raft but it will carry only the weight of one adult or two children. What is the minimum number of times the raft must cross the river to get all four people to the other side? (N.B. The raft may not cross the river without at least one person on board.)

A 3 B 5 C 7 D 9 E 11

25. The diagram shows a trapezium made from three equilateral triangles.
Three copies of the trapezium are placed together, without gaps or overlaps and so that only complete edges coincide, to form a polygon with N sides.
How many different values of N are possible?



A 4 B 5 C 6 D 7 E 8

2007 solutions

1. **C** $0.1 + 0.2 + 0.3 \times 0.4 = 0.3 + 0.12 = 0.42$.

2. **D** The train arrived $5 + 42 = 47$ minutes after 17:40, that is at 18:27.

3. **B** Note that 7 divides 35, 49 and 7, so it divides 354970. So the remainder is 2.

4. **E** Of the options given, only 27, which is three less than a multiple of 5, namely 30, and three more than a multiple of 6, namely 24, has both of the properties in the question.

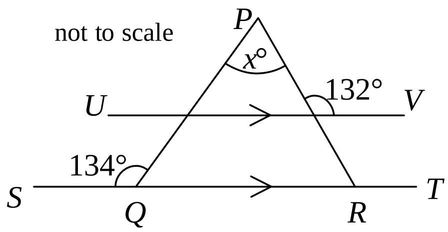
5. **E** The area of the large square may be considered to consist of thirteen equal squares (nine of which are shaded) plus eight 'half squares' and four 'quarter squares' (all of which are unshaded).
So the total unshaded area is $\left(4 + 8 \times \frac{1}{2} + 4 \times \frac{1}{4}\right)$ squares = 9 squares.
Hence half of the large square is shaded.

6. **A** When put in their correct places on the number line, the order of the fractions is:
$-\frac{1}{3}, \ -\frac{1}{5}, \ -\frac{1}{7}, \ \frac{1}{6}, \ \frac{1}{4}$.

7. **D** If the top triangle is painted black, then any one of the three remaining triangles may also be painted black. Similarly, if the top triangle is painted white, then any one of the three remaining triangles may also be painted white. So there are six different ways.

8. **D** From the information, we see that Amy is to the left of both Ben and Chris. So the three are in the order Amy, Ben, Chris or the order Amy, Chris, Ben. So D is certainly true and the others are all false either in one case or in both.

9. **C** As ST is parallel to UV, $\angle PRT = 132°$ (corresponding angles).
So $\angle PRQ = 48°$ (angles on a straight line).
From the exterior angle of a triangle theorem, $\angle SQP = \angle QPR + \angle PRQ$, so $x = 134 - 48 = 86$.



10. **D** The values of the five expressions are: A $\frac{3}{4}$; B $\frac{1}{4}$; C $\frac{1}{8}$; D 2; E $\frac{1}{2}$.

11. **A** The number of times each bar is used is: A 4; B 6; C 8; D 7; E 7.

12. **A** The total number of spots which the six ladybirds have is $6 \times 12 = 72$. So the number of spots which the pine ladybird has is
$72 - (2 + 10 + 14 + 18 + 24) = 4$.

13. **D** If R is $(-5,\ 4)$ then $PQ = PR = 6$. If R is $(7, 1)$ or if R is $(-6,\ 1)$ then R lies on the perpendicular bisector of PQ (the line $y = 1$), so in both cases $PR = QR$. If R is $(7, -2)$, then $QP = QR = 6$. However if R is $(-6, -2)$, then $PQ = 6$, $QR = 7$ and $PR > 7$, so triangle PQR is scalene.

14. **E** The thickness of the line is 0.2 mm, that is 0.0002 m. So, in order to cover an area of one square metre, the length of the line would need to be $\frac{1}{0.0002}$ m, that is 5000 m.

15. **C** We consider the different possible choices from the top row. If 1 is chosen, then the options are 1, 5, 9 and 1, 6, 8 giving products 45 and 48 respectively. If 2 is chosen, the options are 2, 4, 9 and 2, 6, 7 giving products 72 and 84 respectively. Finally, if 3 is chosen, the options are 3, 4, 8 and 3, 5, 7 giving products 96 and 105. So 105 is the maximum.

16. **B** The six marked angles, together with the six interior angles of the two triangles, comprise all of the angles around five separate points. So the required sum is $(5 \times 360 - 2 \times 180)^\circ = 1440^\circ$.

17. **C** The only possible cubes have edge size 1 or 2. It takes 8 of the former to replace one of the latter, so William needs to cut as many cubes of edge size 2 as possible, namely 3. The number of one inch cubes, therefore, is $2 \times 3 \times 6 - 3 \times 8$, that is 12. So the smallest number of cubes is $3 + 12 = 15$.

18. **B** The hundreds column shows us that $J = 1$ or 2. [We can't carry more than 2 from the units to the tens; and 2 plus the biggest feasible values 7, 8, 9 for the three letters is only 26.] The units column shows that $J + M$ is a multiple of 10 and it can't be 0 (or else $J + M = 0$); so $J + M = 10$ and $M = 9$ or 8 respectively. Also, the sum of the units column is $10 + C$, so there is exactly 1 to carry to the tens column. The tens column now tells us that $J + C + 1 = 10J$. So $J = 2$ is not possible and therefore $J = 1$, $C = 8$ and $M = 9$.

19. **A** If the semicircle with diameter PQ is rotated through 180° about Q, the new shape formed has the same area as the original shape. It consists of a semicircle of diameter 6 cm and a semicircle of diameter 2 cm. So its area is $\left(\frac{1}{2} \times \pi \times 3^2 + \frac{1}{2} \times \pi \times 1^2\right)$ cm^2, that is 5π cm^2.

20. **E** Range Hill scored only three points in the match and these were scored in the second half. They represent 10% of the total points scored. As Boarwarts Academy also scored three points in the second half, the proportion of points scored after halftime was 20%, that is $\frac{1}{5}$.

21. **B** Let the list be $3,\ 4, a, b, c, d, e, f, g, h$. We can see that $c = 3$ and $e = 4$. So the list now reads $3,\ 4, a, b, 3, d, 4, f, g, h$. Now, the only pairs of letters two apart from each other are a, d and d, g. Therefore $d = 2$ and the list is $3,\ 4, a$, b, $3,\ 2,\ 4,\ f, g, h$. The only pair now one apart are f, h. The list is 3, 4, a, b, 3, 2, 4, 1, g, 1. Now a, b are the only pair zero apart. So $a = b = 0$ and $g = 2$.

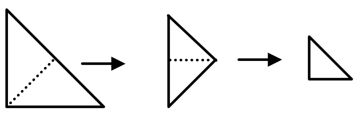
22. **D** Four of the given values for d may be rejected since $143 = 11 \times 13$; $153 = 3 \times 51$; $567 = 3 \times 189$; $183 = 3 \times 61$. However, 173 and 577 are both prime, so $d = 7$.

23. **E** Let the length of the longer side of each rectangle be l. Then the length of each shorter side is $l - x$. So $y = l + l - x$ and hence $l = \frac{1}{2}(y + x)$.

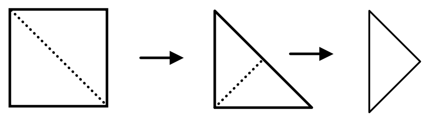
24. **C** Pages 1 to 9 inclusive require 9 digits; pages 10 to 99 inclusive require 180 digits. So, in total, 189 digits are required to number all of the pages before the three-digit page numbers commence with page number 100. This leaves 663 digits, so the last page in the book is the 221st page which has a three-digit number, namely page 320.

25 **B** Imagine unfolding the final triangle once. Then one edge of the final triangle is inside the new shape obtained; and the other two triangle edges have 'mirror image' copies. So the new shape has at most 4 edges. After unfolding once more, one of these edges is now on the inside; and the remaining edges get mirror images again. So the shape obtained (the original shape) has no more than 6 edges. The diagrams below show that 3, 4, 5 and 6 sides are all possible.

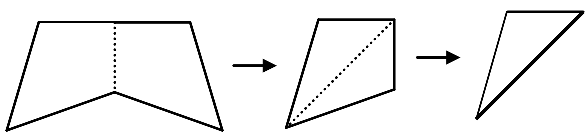
Triangle:



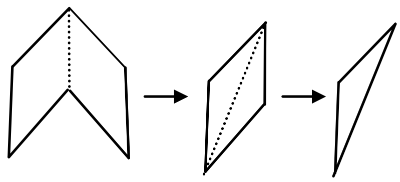
Square:



Pentagon:



Hexagon:



2008 solutions

1. **D** The results of the five calculations are 9, 11, 14, 25, 24 respectively.

2. **E** For it to be possible to draw a figure without taking the pen off the paper and without drawing along an existing line, there must be at most two points in the figure at which an odd number of lines meet. Only E satisfies this condition.

3. **B** $\frac{2}{40} = \frac{1}{20} = \frac{5}{100} = 5\%$.

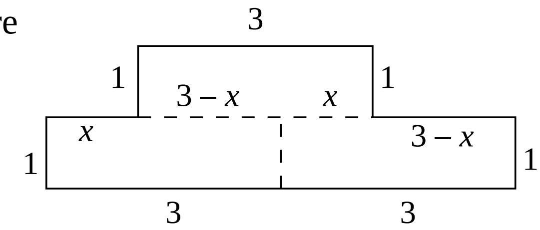
4. **C** The unmarked interior angle on the right of the triangle $= (360 - 324)° = 36°$. So, by the exterior angle theorem, $x = 100 - 36 = 64$.

5. **E** The cost of 1 kg of potatoes is £$1.25 \div 2.5 = 50$ p. So the cost of 1 tonne, that is 1000 kg, is 1000×50 p $=$ £500.

6. **D** Adam Ant walks 24 cm, whilst Annabel Ant walks 32 cm.

7. **C** In terms of length, 1 arm = 2 forearms = 4 hands = 8 middle fingers = 16 thumbs. So 4 arms have the same total length as 64 thumbs.

8. **A** From the diagram, in which all lengths are in cm, it can be seen that the perimeter $= [4 \times 1 + 3 \times 3 + x + (3 - x)]$ cm $= 16$ cm.



9. **E** The values of the five expressions are $\frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \frac{1}{42}$ respectively.

10. **B** Consider one corner of the cube. There are three faces which meet there, and each pair of them has an edge in common. So three different colours are needed. No other colours will be needed provided that opposite faces are painted in the same colour since opposite faces have no edges in common.

11. **E** The 120 tons of ice which remain represent two-thirds of the original cargo. So one-third of the original cargo was 60 tons.

12. **C** Consider the sculpture to consist of three layers, each of height 1. Then the volumes of the bottom, middle and top layers are 5, 2, 5 respectively. So the volume of the sculpture is 12.

(*Alternatively: the sculpture consists of a* $3 \times 3 \times 3$ *cube from which two* $2 \times 2 \times 2$ *cubes have been removed. The* $2 \times 2 \times 2$ *cubes have exactly one* $1 \times 1 \times 1$ *cube* (*the cube at the centre of the* $3 \times 3 \times 3$ *cube*) *in common. So the volume of the sculpture* $= 27 - (2 \times 8 - 1) = 12$.)

13. **C** New shapes may be formed by joining *PX* to *XR* (quadrilateral) or *SP* to *RQ* (parallelogram) or *XS* to *RQ* (trapezium). Triangle *SPX* shows that *PX* and *SX* have different lengths; and *PX* and *PQ* have different lengths because *XR* is shorter than *SR*. So there are no other places to position the triangle.

14. D As the original cube was divided into eight cubes of equal size, these smaller cubes have side equal to half the side of the original cube. So each of the new cubes originally occupied one corner of the large cube and hence has three faces painted blue and three faces unpainted. So the fraction of the total surface area of the new cubes which is blue equals one half.

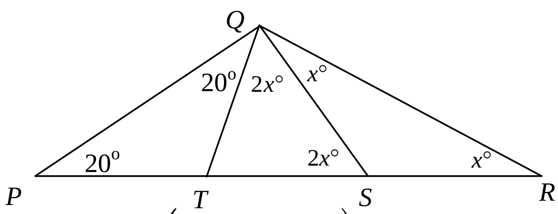
15. B A rate of 1 metre per 1000 years is equivalent to 1 mm per year, that is just under three thousandths of 1 mm per day.

16. A Of the five alternatives, only A and B have straight lines in the ratio 2:15:20. However, B would be formed by repeatedly moving forward 2 units, turning right, moving forward 20 units, turning right, moving forward 15 units, turning right.

17. B Consider the leading diagonal: $p \times 1 \times \frac{1}{8} = 1$ so $p = 8$.
Consider the bottom row: $u \times 4 \times \frac{1}{8} = 1$ so $u = 2$.
Consider the left-hand column: $p \times s \times u = 8 \times s \times 2 = 1$ so $s = \frac{1}{16}$.
Consider the non-leading diagonal: $r \times 1 \times u = r \times 1 \times 2 = 1$ so $r = \frac{1}{2}$.
Therefore $r + s = \frac{1}{2} + \frac{1}{16} = \frac{9}{16}$.

18. B Let my age now be x. So Granny's age is $4x$. Considering five years ago: $4x - 5 = 5(x - 5)$, giving $x = 20$. So Granny is 80 and I am 20.

19. D As $QS = SR$, $\angle SRQ = \angle SQR = x°$.
So $\angle QST = 2x°$ (exterior angle theorem). Also $\angle TQS = 2x°$ since $QT = TS$.
As $PT = QT$, $\angle TPQ = \angle TQP = 20°$.



Consider the interior angles of triangle PQR: $20 + (20 + 2x + x) + x = 180$.
So $4x + 40 = 180$, hence $x = 35$.

20. A Consider the nine numbers from 1 to 9 inclusive: each digit appears once, with the exception of zero. Now consider the 90 two-digit numbers from 10 to 99 inclusive: each of the 10 digits makes the same number of appearances (9) as the second digit of a number and the digits from 1 to 9 make an equal number of appearances (10) as the first digit of a number, but zero never appears as a first digit. There is a similar pattern in the 900 three-digit numbers from 100 to 999 inclusive with zero never appearing as a first digit, but making the same number of appearances as second or third digit as the other nine digits. This leaves only the number 1000 in which there are more zeros than any other digit, but not enough to make up for the fact that zero appears far fewer times than the other nine digits in the numbers less than 1000. (*It is left to the reader to check that* 0 *appears* 192 *times*, 1 *appears* 301 *times and each of* 2 *to* 9 *appears* 300 *times*.)

21. A Consider the third column: 2♫ + ♥ = 13 [1]

Consider the second row: ♫ + 2♥ = 11 [2]

2 × [2] − [1] 3♥ = 9, so ♥ = 3.

(*Although their values are not requested, it is now straightforward to show that* ♫ = 5, ☼ = 4.)

22. D The only such occasions occur when the clock changes from 09 59 59 to 10 00 00; from 19 59 59 to 20 00 00 and from 23 59 59 to 00 00 00.

23. B Let the 7-digit code be $abcdefg$. It may be deduced that $a = 3$ since $b + c + d + e = 16$ and $a + b + c + d + e = 19$. By using similar reasoning, it may be deduced that $b = c = e = f = g = 3$.

As $a + b + c + d = 16$, $d = 7$; so the code is 3337333.

24. E Let the other such list of numbers be a, 1; b, 2; c, 3; d, 4 and note that $a + b + c + d = 8$ since there are 8 numbers in the list.

If $d = 4$, then exactly two of a, b, c equal 4, but this would make $a + b + c + d > 8$, so $d \neq 4$.

Similar reasoning shows that $d \neq 3$, so $d = 1$ or $d = 2$.

If $d = 2$, then exactly one of a, b, c equals 4 and the remaining two both equal 1 since $a + b + c + d = 8$. So we have a, 1; b, 2; c, 3; 2, 4 and it is b which must equal 4 since we already have more than one 2. However, as a and c are now both equal to 1, we have 1, 1; 4, 2; 1, 3; 2, 4 and this is not correct.

So $d = 1$ and we have $a + b + c = 7$ and a, b, $c \neq 4$. Clearly $a \neq 1$, since that would give at least two 1s so $a = 2$ or $a = 3$.

If $a = 2$, then we have 2, 1; b, 2; c, 3; 1, 4 with $b + c = 5$ and b, $c \neq 4$. So $b = 2$, $c = 3$ or vice versa. This gives either 2, 1; 2, 2; 3, 3; 1, 4 (incorrect), or 2, 1; 3, 2; 2, 3; 1, 4 (the example given in the question).

Finally, if $a = 3$, then we have 3, 1; b, 2; c, 3; 1, 4 with $b + c = 4$. The possibilities are 3, 1; 1, 2; 3, 3; 1, 4 or 3, 1; 2, 2; 2, 3; 1, 4 or 3,1; 3, 2; 1, 3; 1, 4 but only the first of these describes itself correctly. So the total number of 1s and 3s is 6.

25. D Let the lengths of the sides of the squares, in increasing order, be $a, b, c, d, e, f, g, h, i$ respectively. So $h = 10$.

Note that $c = 2b - a$ and $d = 2c - 2a = 4b - 4a$. Also, $e = 2d - a = 8b - 9a$.

As $h = 2e - 2a - b = 15b - 20a$, we may deduce that $15b - 20a = 10$, that is $3b - 4a = 2$.

Since a and b are positive integers less than 10, the only possibilities are $a = 1$, $b = 2$ or $a = 4$, $b = 6$. However, $h = 10$ therefore b cannot be greater than 4. So $a = 1$ and $b = 2$. It may now be deduced that $c = 4 - 1 = 3$; $d = 8 - 4 = 4$; $e = 16 - 9 = 7$. Also $2g = 2e + d$, so $g = 9$.

Now the length of the side of the larger square is $2h + e + g = 20 + 7 + 9 = 36$, so its area is $36^2 = 1296$.

(*Note that it was not necessary to find the values of f and i, but it is now quite simple to deduce that $f = 8$ and $i = 18$.*)

2009 solutions

1. **B** $9002 - 2002 = 7000$ so $9002 - 2009 = 7000 - 7 = 6993$.

2. **B** Each of faces 1, 4 and 5 has four axes of symmetry, whilst each of faces 2, 3 and 6 has two axes of symmetry only.

3. **D** The values of the left-hand sides of the expressions are 0, 16, 28, 36 and 40 respectively.

4. **E** Each of points A, B, C and D is 1 unit from the origin, but the point (1, 1) is at a distance $\sqrt{2}$ units from the origin.

5. **D** The problem may be solved by dividing each of the alternatives in turn by 7, but the prime factorisation of 1001, i.e. $1001 = 7 \times 11 \times 13$, leads to the conclusion that 111 111, which is 111×1001, is a multiple of 7.

6. **B** Triangle ABM has base 3 units and height 3 units, so its area is $\frac{1}{2} \times 3 \times 3$ units², that is $4\frac{1}{2}$ units².

7. **C** The time difference is 12 hours and 12 minutes, that is 732 minutes.

8. **E** Removing tile A or tile B or tile D has the effect of reducing the perimeter by a distance equal to twice the side of one tile, whilst removing tile C increases the perimeter by that same distance. Removing tile E, however, leaves the length of the perimeter unchanged.

9. **A** $\frac{20}{11} = 1\frac{9}{11} = 1.818181\ldots$, so only two different digits appear.

10. **B** The triangle in the centre of the diagram is equilateral since each of its sides is equal in length to the side of one of the squares. The sum of the angles at a point is 360°, so $x = 360 - (90 + 90 + 60) = 120$.

11. **C** The first thirteen terms of the sequence are −3, 0, 2, −1, 1, 2, 2, 5, 9, 16, 30, 55, 101, ….

12. **A** The increase in Gill's weight is 45 kg, which is 9 times her weight in 1988. So the percentage increase in weight is 900%.

(*The problem refers to* Q14 *in the very first Schools Mathematical Challenge − the forerunner of the current Junior and Intermediate Mathematical Challenges – in* 1988. *This was* '*Weighing the baby at the clinic was a problem. The baby would not keep still and caused the scales to wobble. So I held the baby and stood on the scales while the nurse read off* 78 kg. *Then the nurse held the baby while I read off* 69 kg. *Finally I held the nurse while the baby read off* 137 kg. *What is the combined weight of all three* (*in* kg)?

A 142 B 147 C 206 D 215 E 284.')

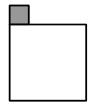
13. **D** Let the ten consecutive integers be $x - 4$, $x - 3$, $x - 2$, $x - 1$, x, $x + 1$, $x + 2$, $x + 3$, $x + 4$ and $x + 5$ respectively. The sum of these is $10x + 5$ so $10x + 5 = 5$, that is $x = 0$. Hence the largest of the integers is 5.

14. **E** The sum of Karen's two marks was 78×2, that is 156. So her mark for Mathemagics was $156 - 72$, that is 84.

15. **E** If Matt takes 12 jellybeans then he will have taken at least one of each flavour unless he takes all 8 watermelon jellybeans and either all 4 vanilla jellybeans or all 4 butter popcorn jellybeans. In this case the 4 remaining jellybeans will all be of the flavour he has yet to take, so taking one more jellybean ensures that he will have taken at least one of each flavour.

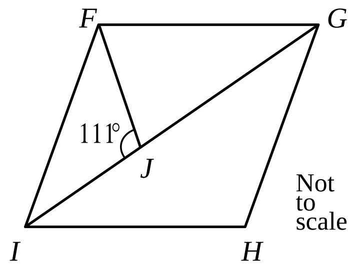
16. **D** 20% of the 80% is 16% of the kettle's capacity. Therefore the volume of water left in the kettle after Keith has poured out 20% of the original amount is 64% of the kettle's capacity. So when full, the kettle holds $\frac{1152}{64} \times 100$ ml, that is 1800 ml.

17. **A** The tiling pattern may be considered to be a tessellation by the shape shown, so the required ratio is 1:1.

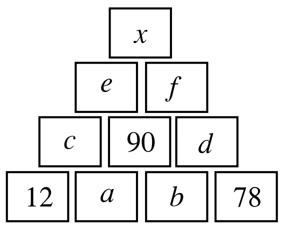


18. **B** The lowest common multiple of 2, 3, 4, 5 and 6 is required. Of these numbers, 2, 3 and 5 are prime whilst $4 = 2^2$ and $6 = 2 \times 3$. So their lowest common multiple is $2^2 \times 3 \times 5$, that is 60.

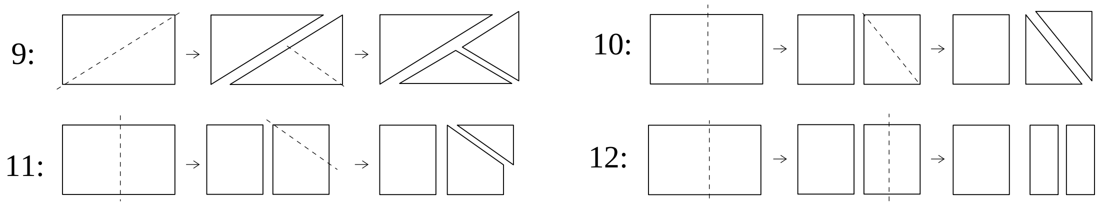
19. **A** Adjacent angles on a straight line add up to 180°, so $\angle GJF = 180° - 111° = 69°$. In triangle FGJ, $GJ = GF$ so $\angle GFJ = \angle GJF$. Therefore $\angle FGJ = (180 - 2 \times 69)° = 42°$. As $FGHI$ is a rhombus, $FG = FI$ and therefore $\angle GIF = \angle FGI = 42°$. Finally, from triangle FJI, $\angle JFI = (180 - 111 - 42)° = 27°$.



20. **D** Let the numbers in the boxes be as shown in the diagram.
Then $b = 90 - a$; $c = 12 + a$; $d = b + 78 = 168 - a$.
Also, $e = 90 + c = 102 + a$; $f = 90 + d = 258 - a$.
So $x = e + f = 102 + a + 258 - a = 360$.



21. **E** The diagrams below show how the total number of edges of the resulting three pieces may be 9, 10, 11 or 12. However, 12 is the maximum value of the total number of edges since the original number of edges is four and any subsequent cut adds a maximum of four edges (by dividing two existing edges and adding the new 'cuts').



22. D In order to reach a 9 in three steps, the first zero must be one of the three adjacent to the 2 and the second zero must be one of the five adjacent to a 9. The table shows the number of such routes to that point.

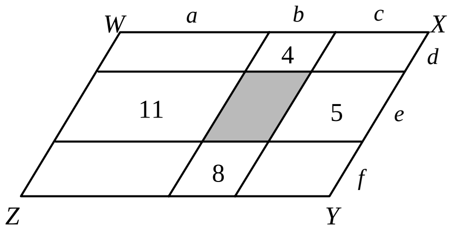
So the total number of different routes is 25.

	1	2	**4**
1	1	2	**5**
2	2	1	**3**
4	**5**	**3**	**1**

23. C Let the value of a green note and the value of a blue note be g zogs and b zogs respectively. Then $3g + 8b = 46$ and $8g + 3b = 31$. Adding these two equations gives $11g + 11b = 77$, so $b + g = 7$.

Therefore $3g + 3b = 21$. Subtracting this equation from the original equations in turn gives $5b = 25$ and $5g = 10$ respectively. So $b = 5$, $g = 2$ and $2g + 3b = 19$.

24. C Let the lengths a, b, c, d, e, f be as shown in the diagram. Then the sum of the perimeters of the four labelled parallelograms is



$2(a + e) + 2(b + d) + 2(b + f) + 2(c + e)$

$= 2(a + b + c + d + e + f) + 2(b + e)$

= perimeter of $WXYZ$ + perimeter of shaded parallelogram.

So the perimeter of the shaded parallelogram is $\big((11 + 8 + 4 + 5) - 21\big)\text{cm} = 7\ \text{cm}$.

25. B Let the number of boys in Miss Quaffley's class be b and the number of girls be g. Then the number of teddy bears is $\frac{1}{3}(b + g)$. Also, in total, the boys took out $12b$ library books last term and the girls took out $17g$ books. The total number of books taken out by the bears was $9 \times \frac{1}{3}(b + g)$ that is $3(b + g)$.

So $12b + 17g + 3(b + g) = 305$, that is $15b + 20g = 305$, that is $3b + 4g = 61$.

Clearly, b and g are positive integers. The positive integer solutions of the equation $3b + 4g = 61$ are $b = 3, g = 13$; $b = 7, g = 10$; $b = 11, g = 7$; $b = 15, g = 4$; $b = 19, g = 1$.

However, there is one further condition: the number of teddy bears, that is $\frac{1}{3}(b + g)$, is also a positive integer and of the five pairs of solutions above, this condition is satisfied only by $b = 11, g = 7$.

Check: the 11 boys take out 132 books, the 7 girls take out 119 books and the 6 teddy bears take out 54 books, giving a total of 305 books.

(*The equation* $3b + 4g = 61$ *in which b and g both represent positive integers is an example of a Diophantine equation*.)

2010 solutions

1. **B** The expression $= 2010 + 2010 - 2010 - 2010 + 2010$
$= (2010 - 2010) + (2010 - 2010) + 2010 = 2010.$

2. **E** In A, the letter T is incorrect; in B it is U which is incorrect; in C and D the incorrect letters are M and K respectively.

3. **A** 2010 mm = 2.01 m so, of the alternatives given, only a table could be expected to have a length of 2010 mm.

4. **D** Let X be on the top face of the cube. If the base is placed on a horizontal surface, then A, B, C, E will all be on vertical faces of the cube and D will be on the base, opposite X.

5. **D** Each of the five outer circles is divided into six regions, giving 30 regions in total. In addition, there is one region in the centre of the diagram and one region between the circles and the sides of the square. So, in all, there are 32 regions.

6. **C** The values of the expressions are A 12; B 15; C 16; D 15 and E 12.

7. **A** As 2, 5 and 10 are all factors of the correct product, this product is a multiple of 100. So the last digit and the last-but-one digit are both zero.

8. **D** If the mean of y and z is x, then $y + z = 2x$. So the sum of the interior angles of the triangle is $(x + y + z)^\circ = 3x^\circ$. So $3x = 180$, that is $x = 60$.

9. **A** One year is, at most, 366 days, so one-third of a year is less than 125 days. No month is longer than 31 days, so 4 months is also less than 125 days, as is 17.5 weeks which equals 122.5 days. However 3002 hours equals 125 days 2 hours, so this is the longest of the five periods of time.

10. **E** Third prize is worth one-sixth of the total prize money, so Mrs Keat received half of that amount, that is one-twelfth of the total.

11. **C** Divide the whole figure into horizontal strips of height 1 unit: its area is $(3 + 6 + 8 + 8 + 8 + 6 + 3)$ units^2 $= 42$ units^2. Similarly, the unshaded area is $(1 + 4 + 6 + 4 + 1)$ units^2 $= 16$ units^2. So the shaded area is 26 units^2.

Alternative solution: notice that if the inner polygon is moved a little, the answer remains the same – because it is just the difference between the areas of the two polygons. So, although we are not told it, we may assume that the inner one is so positioned that the outer shaded area can be split neatly into 1 by 1 squares – and there are 26 of these.

12. **C** There are 36 people to be seated so at least five tables will be required. The number of circular tables must be even. However, five rectangular tables will seat 40 people and three rectangular and two circular will seat 34. So at least six tables are needed. Two rectangular and four circular tables do seat 36 people: so six is the minimum number of tables.

13. **B** It is necessary to find a route for which the line is broken the first time it passes through any intersection and solid when it passes through that intersection for the second time. Only the route which starts at B and heads away from D satisfies this condition.

14. **D** The average number of vehicles per day $\approx \dfrac{300\,000\,000}{44 \times 365} \approx \dfrac{300\,000\,000}{40 \times 400}$ $= \dfrac{300\,000\,000}{16\,000} \approx \dfrac{300\,000\,000}{15\,000} = 20\,000.$

15. **C** The two shaded regions measure 3 by 7 and 1 by 6, so the total area outside the overlap is 27 units^2.

16. **E** As 108 marks represented 18% of the final total, 6 marks represented 1% of the final total. So this total was 600.

17. **D** As triangle *PQR* is equilateral, $x + 2y = 3x - y = 5y - x$. Equating any two of these expressions gives $2x = 3y$.

The only pair of given values which does not satisfy this equation is $x = 10$, $y = 6$.

18. **D** The other times that this has happened previously are when Sam's age in years went from 1 to 2; from 4 to 5; from 16 to 17 and from 36 to 37.

Note that since primes other than 2 are odd, the only squares which need to be checked, other than 1, are of even numbers.

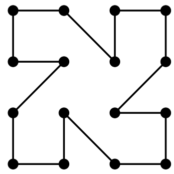
19. **C** Villages which have more than two roads leading to them (or from them) must all be visited more than once as a single visit will involve at most two roads. So Bentonville, Pencaster and Wytham must all be visited more than once. The route Home, Bentonville, Greendale, Wytham, Bentonville, Pencaster, Home, Wytham, Horndale, Pencaster, Home starts and finishes at Home and visits both Greendale and Horndale exactly once so the minimum number of villages is three.

20. **B** The seven numbers must total 49 if their mean is to be 7. The largest possible number will occur when the other six numbers are as small as possible, that is 1, 2, 3, 4, 5, 6. So the required number is $49 - 21 = 28$.

21. **C** The first and last hexagons both contribute 5 cm to the perimeter of the pattern. Every other hexagon in the pattern contributes 4 cm to the perimeter. The first and last thus contribute 10 cm, so we need another $2000 \div 4 = 500$ hexagons. Therefore the total number of hexagons required is 502.

22. **E** The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19. It is not possible for 2 to be one of the six numbers Kiran wrote down, since that would give one of the pairs an odd sum, whereas both of the other pairs would add up to an even number. The sum of the remaining 7 primes is 75 which is a multiple of 3. The sum of the six primes making up the three pairs must also be a multiple of 3 since each pair has the same total. So the odd prime not used in the six pairs must be a multiple of 3 too. Therefore 3 is the odd prime not used. So each pair totals $72 \div 3$, that is 24, and the pairs are $5 + 19$, $7 + 17$, $11 + 13$.

23. **E** The number of sides of the polygon is equal to the number of corners it has. As no dot is at more than one corner, the maximum number of corners is 16. So the maximum possible number of sides is 16, provided that a 16-sided figure may be drawn. The figure on the right shows one of several ways in which this can be achieved.



24. **B** In the 21st Century, to obtain a sequence of two years or more then either a 2 or a 0 must be repeated in each year, or the sequence include years such as 2011, 2033, 2044 etc. So the only sequence after that mentioned in the question will be from 2020 to 2030, but this is too short.

In the 22nd Century, either a 2 or a 1 must be repeated. The first such sequence is 2110 to 2129 which does include 20 years, one of which is 2120.

25. **A** The three-digit number *RRR* is equal to 111 multiplied by the single digit *R*. So $PQPQ \times R = 639027 \div 111 = 5757$. Now *PQPQ* equals the two-digit number *PQ* multiplied by 101. So $PQ \times R = 5757 \div 101 = 57$. The only ways in which 57 may be expressed as the product of a two-digit number and a single digit are 57×1 and 19×3. So $P = 5$, $Q = 7$, $R = 1$ or $P = 1$, $Q = 9$, $R = 3$. In both cases, $P + Q + R = 13$.

2011 solutions

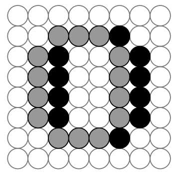
1. **B** $2 \times 0 \times 1 + 1 = 0 \times 1 + 1 = 0 + 1 = 1$.

2. **E** If the sum of the digits is a multiple of 3 then the number is a multiple of 3.

The sums of the digits of the given numbers are 6, 9, 12, 15, 18, so they are all multiples of 3.

(*Can you prove that all numbers consisting of three consecutive digits are multiples of 3? Hint: let the second digit be n.*)

3. **B** In the diagram, the extra cells which need to be lit are shown in black.



So, in total, 24 cells are lit in a bold 'o'.

4. **C** 100 kg = 100 000 g. So the sum of money in £1 coins which would have the same mass as the world's largest coin is £(100 000 ÷ 10) = £10 000.

(*The coin was sold for* \$4m (£2.6m) *at an auction in Vienna in June 2010*.)

5. **B** One third is equal to four twelfths. Hence the children ate eight twelfths of the bar between them. Each child ate one twelfth of the bar, so old Mother Hubbard had eight children.

6. **E** The six marked angles are the interior angles of the two large triangles which make up the star shape in the diagram, so their sum is $2 \times 180° = 360°$.

7. **D** There are 9 bushels in a barrel. Each bushel is 4 pecks, so there are 36 pecks in a barrel. Therefore 35 more pecks are needed.

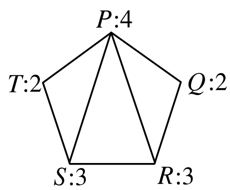
8. **A** Let the original square have side $3x$. Then its perimeter is $12x$.

The perimeter of the octagon is $2 \times 4x + 3 \times 3x + 3 \times x = 8x + 9x + 3x = 20x$.

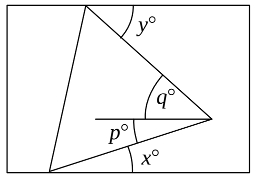
So the required ratio is 12:20 = 3:5.

9. **A** $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$ is the sum of the digits of each such number. As 45 is a multiple of 9, each such number is a multiple of 9 and so too is the difference between two of them. Thus the smallest feasible difference is 9. The two numbers 123 456 798 and 123 456 789 show that this can occur.

10. **C** The diagram shows the number of lines which meet at the vertices P, Q, R, S, T. When the path around the diagram passes through a vertex, it uses up two of the edges. So, apart from the first and last vertex used, each vertex must have an even number of edges meeting at it. So we are obliged to use R or S as the first vertex, and the other as the last. The path *RQPTSRPS*, together with its reverse, shows that either is a possible start. (It is a fact that such a path can be drawn through a connected graph precisely when either all, or all but 2, vertices have an even number of edges meeting there.)



11. C A line segment which is parallel to two sides of the rectangle has been added to the diagram, as shown. The angle marked $p°$ is equal to the angle marked $x°$ as these are alternate angles between parallel lines. So $x = p$. Similarly $y = q$. The angles marked $p°$ and $q°$ together form one interior angle of an equilateral triangle. Therefore $x + y = p + q = 60$.

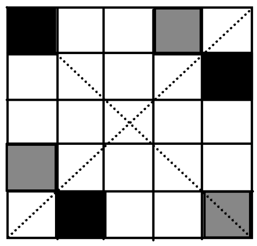


12. E ● = ■ + ▲ = ▲ + ▲ + ▲ = 3▲. Therefore ♦ = ● + ■ + ▲ = 3▲ + 2▲ + ▲ = 6▲.

13. E The mean of $\frac{2}{3}$ and $\frac{4}{9}$ is $\left(\frac{2}{3} + \frac{4}{9}\right) \div 2 = \left(\frac{6}{9} + \frac{4}{9}\right) \div 2 = \frac{10}{9} \div 2 = \frac{5}{9}$.
(*Note that the mean of two numbers lies midway between those two numbers.*)

14. A Let the area of the shaded face be x cm^2. Then the cuboid has two faces of area x cm^2 and four faces of area $4x$ cm^2. So its total surface area is $18x$ cm^2. Therefore $18x = 72$, that is $x = 4$.
So the area of one of the visible unshaded faces is 4×4 cm^2 $= 16$ cm^2.

15. A In order that the figure has rotational symmetry of order 2, the three squares which appear in black must be shaded. When this has been done, we note that the broken lines shown are both lines of symmetry. So the minimum number of squares which must be shaded is 3.



16. B The smallest possible number of votes the winner could receive corresponds to the situation in which the numbers of votes received by each of the candidates are as close together as possible.
As $83 \div 4 = 20.75$, at least one of the candidates receives 21 votes or more. However, it is not possible for the winner to receive 21 votes, since there are still 62 votes to be allocated which makes it impossible for each of the other three candidates to receive fewer than 21 votes. So the winner must receive more than 21 votes. If the numbers of votes received by the candidates are 22, 21, 20, 20 then there is a winner and, therefore, 22 is the smallest number of votes the winner could receive.

17. D The lengths in minutes of the fifth set and the whole match are 491 and 665 respectively.
So the required fraction is $\frac{491}{665} = \frac{491 \times 3}{665 \times 3} \approx \frac{1500}{2000} = \frac{3}{4}$.

18. C Until Peri reaches Granny's, he travels 9m in every 10 days. So he takes 90 days to travel the first 81m of his journey. There remains a distance of 9m to be covered and so, after a further 9 days, Peri is at Granny's. Therefore the length of Peri's journey is 99 days, that is 14 weeks 1 day. So Peri arrives at Granny's on Tuesday.

19. D Of the given numbers, 2, 3 and 5 are all prime and therefore appear in the list. In addition, 1 appears in the list as it is the units digit of 11 and also of many other primes. However, all numbers with units digit 4 are even and therefore not prime, because the only even prime is 2. So only 1, 2, 3, 5 appear in the list.

20. A Let the length of the side of each cube be x cm. Then the volume of the solid is $7x^3$ cm^3. Therefore $7x^3 = 875$, that is $x^3 = 125$. So $x = 5$. The surface area of the solid comprises five of the faces of each of six cubes. Each face has area 25 cm^2 so the required area is $5 \times 6 \times 25$ cm^2 $= 750$ cm^2.

21. B In total the train travels 27 km + 29 km = 56 km.

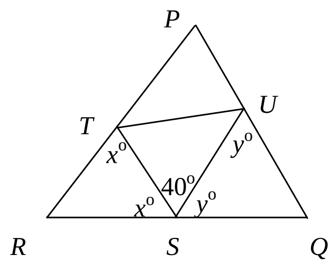
So the combined time for these two parts of the journey is $\frac{56}{96}$ hours = $\frac{7}{12}$ hours = 35 minutes.

The total journey time, therefore, is 38 minutes. So Gill arrives at 09:38.

22. D Let the numbers of stamps bought by Evariste and Sophie be x and y respectively.

Then $1.1x + 0.7y = 10$, that is $11x + 7y = 100$. As 100 has remainder 2 when divided by 7, we need a multiple of 11 which is two more than a multiple of 7. The multiples of 11 less than 100 are 11, 22, 33, 44, 55, 66, 77, 88, 99. Of these only 44 is two more than a multiple of 7. So the only positive integer solutions of the Diophantine equation $11x + 7y = 100$ are $x = 4, y = 8$. Therefore Evariste buys 4 stamps, costing £4.40, and Sophie buys 8 stamps, costing £5.60.

23. E Let $\angle RTS = x°$. Then $\angle RST = x°$ as $RS = RT$.
Let $\angle QUS = y°$. Then $\angle QSU = y°$ as $QS = QU$.



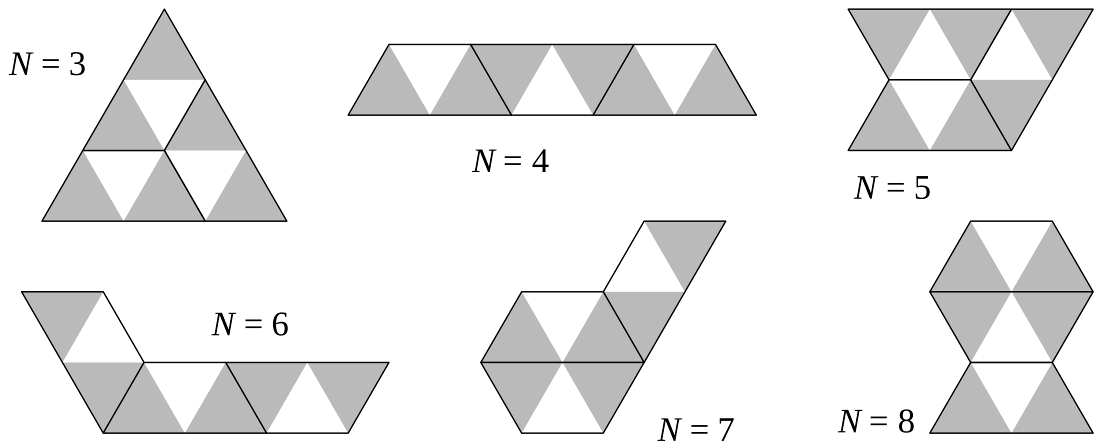
As RSQ is a straight line, $x + y + 40 = 180$; so $x + y = 140$.

$$
\begin{aligned}
\text{Now} \quad \angle TPU &= 180° - \angle TRS - \angle SQU \\
&= 180° - (180 - 2x)° - (180 - 2y)° \\
&= 180° - 180° + 2x° - 180° + 2y° \\
&= 2(x + y)° - 180° \\
&= 2 \times 140° - 180° \\
&= 100°.
\end{aligned}
$$

24. **D** (We may assume that the party is initially on the near bank and wishes to cross to the far bank.)

If an adult crosses to the far bank then there has to be a child waiting there to bring the raft back (unless an adult immediately brings the raft back − but this represents a wasted journey). This is possible only if the first two crossings involve both children crossing to the far bank and one of them staying there whilst the other brings the raft back. The third crossing involves the first adult crossing to the far bank and on the fourth crossing the child waiting on the far bank brings the raft back to the near bank. So after four crossings, one of the adults is on the far bank and the remainder of the party is on the near bank. This procedure is repeated so that after eight crossings, both adults are on the far bank and both children are on the near bank. A ninth and final crossing then takes both children to the far bank.

25. **C** The three trapezia have 12 edges in total. Whenever two trapezia are joined together the total number of edges is reduced by at least 2. Therefore the maximum possible value of N is $12 - 2 \times 2 = 8$. As the shapes form a polygon, N cannot be less than 3. The diagrams below show that all values of N from 3 to 8 are indeed possible, so there are 6 different values of N.



Summary of answers

	2007	2008	2009	2010	2011
1.	C	D	B	B	B
2.	D	E	B	E	E
3.	B	B	D	A	B
4.	E	C	E	D	C
5.	E	E	D	D	B
6.	A	D	B	C	E
7.	D	C	C	A	D
8.	D	A	E	D	A
9.	C	E	A	A	A
10.	D	B	B	E	C
11.	A	E	C	C	C
12.	A	C	A	C	E
13.	D	C	D	B	E
14.	E	D	E	D	A
15.	C	B	E	C	A
16.	B	A	D	E	B
17.	C	B	A	D	D
18.	B	B	B	D	C
19.	A	D	A	C	D
20.	E	A	D	B	A
21.	B	A	E	C	B
22.	D	D	D	E	D
23.	E	B	C	E	E
24.	C	E	C	B	D
25.	B	D	B	A	C