EL582/BE620 --- Medical Imaging - I

Physics of Ultrasound Imaging

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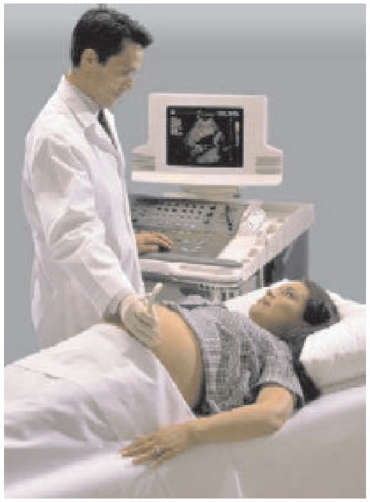
Based on J. L. Prince and J. M. Links, Medical Imaging Signals and Systems, and lecture notes by Prince. Figures are from the textbook except otherwise noted.

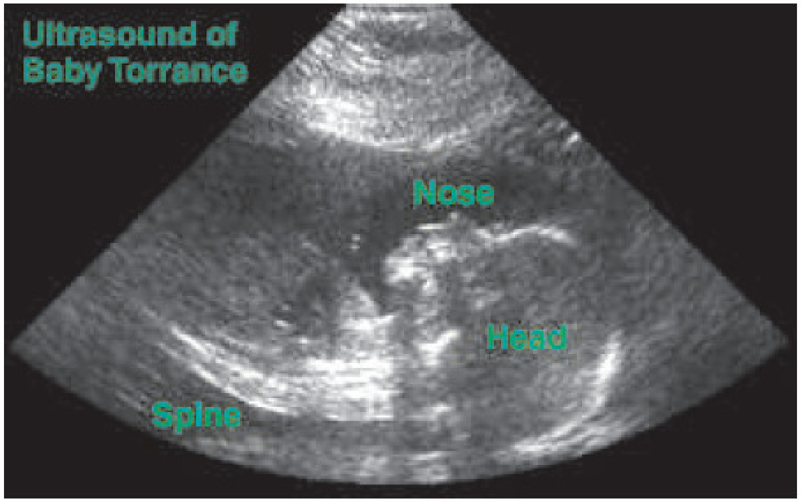
Lecture Outline

- Ultrasound imaging overview
- General characterization of acoustic wave
- Wave equation
  - 3D
  - Plane wave
  - Spherical wave
- Reflection of wave
- Absorption and scattering of wave
- Doppler effect
- Field pattern of a transducer

Ultrasound Imaging

- Measure the reflectivity of tissue to sound waves
- Can also measure velocity of moving objects, e.g. blood flow (Doppler imaging)
- No radiation exposure, completely non-invasive and safe
- Fast
- Inexpensive
- Low resolution
- Medical applications: imaging fetus, heart, and many others





What is Acoustic Wave

- Pressure waves that propagate through matter via compression and expansion o the material
  - Generated by compressing and releasing a small volume of tissue
- Longitudinal wave
  - Particles in the medium move back and force in the same direction that the wave is traveling
- Shear Wave
  - Particles move at right angles to the direction of the wave
  - Not used for medical ultrasound imaging

Longitudinal Wave

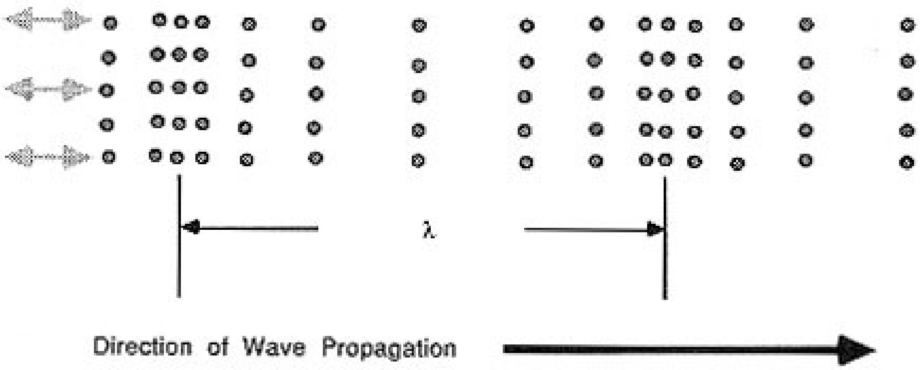
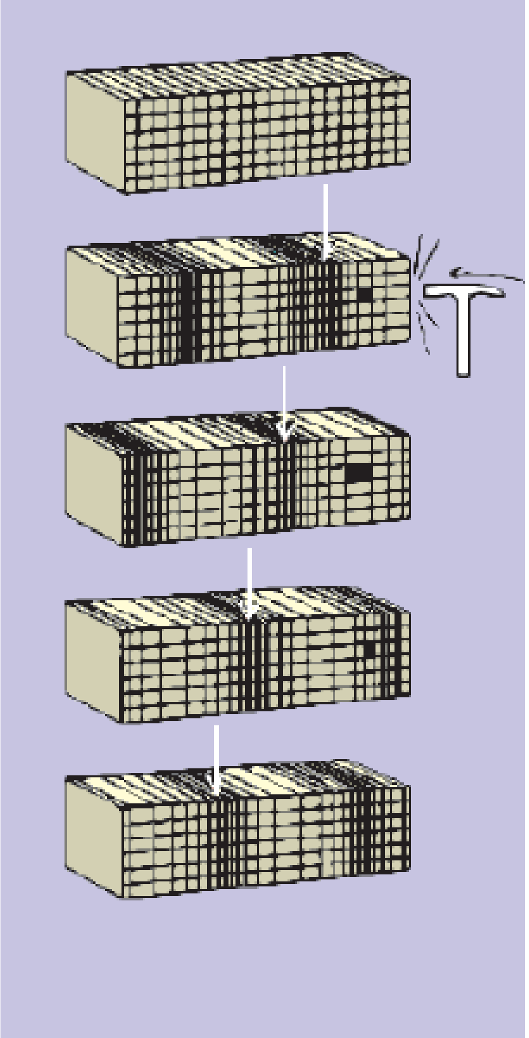


Figure 63 One-dimensional representation of longitudinal acoustic propagation.

From Graber: Lecture note for BMI F05



From Prince, Lecture note

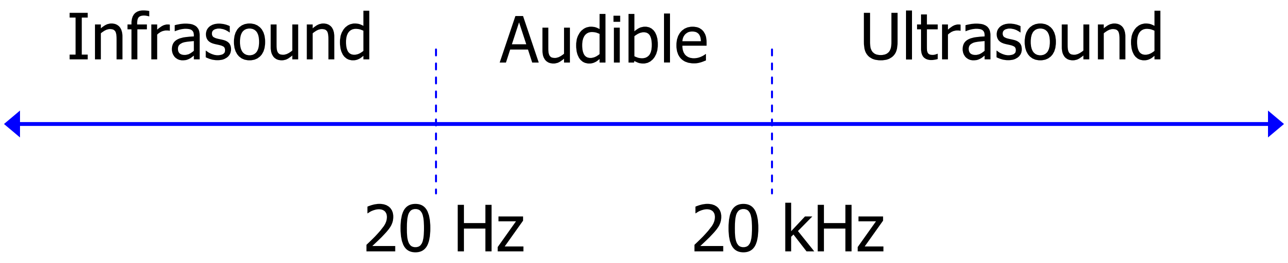
EM vs Acoustic Wave

- Electromagnetic
  - Self propagating, consisting of electric and magnetic components oscillating at right angles to each other, and to the direction of propagation
  - Does not *require* a material medium through which to propagate
  - Classification (increasing in frequency, decreasing in wavelength):
    - radio, microwave, infrared, visible light, ultraviolet, x-ray, gamma ray
- Acoustic
  - Pressure waves that propagate through matter via compression and expansion of the material
  - Requires a material medium through which to propagate
  - *Classification (increasing in frequency):*
    - *Infra sound, audible sound, ultrasound*

Transfer and Transformation of Energy

- Light becomes sound — photoacoustic phenomena
- Sound becomes light — sonoluminescence
- Absorbed electromagnetic (EM) and acoustic energy both become heat
- Nevertheless, EM and acoustic energy are **FUNDAMENTALLY DISTINCT PHENOMENA**!

Acoustic Wave Energy Ranges


Infrasound
Audible
Ultrasound
20 Hz
20 kHz

- Just as there are infrared, visible, and ultraviolet ranges in the EM spectrum, so there are infrasound ("infra" = "below," "beneath"), audible (i.e., sound) and ultrasound ("ultra" = "beyond," "above") ranges of acoustic wave frequencies

- Note that the ratio of the highest to the lowest audible frequencies is 10^3, or almost 10 octaves. On the other hand, the ratio of the highest to the lowest frequencies of *visible light* is a bit less than 2 (i.e., less than one octave).

Characterization of Acoustic Wave

- Speed of sound in a medium depends on the medium property

$$c = \sqrt{1/\kappa\rho}; \quad \kappa: \text{compressibility}; \rho: \text{density}$$

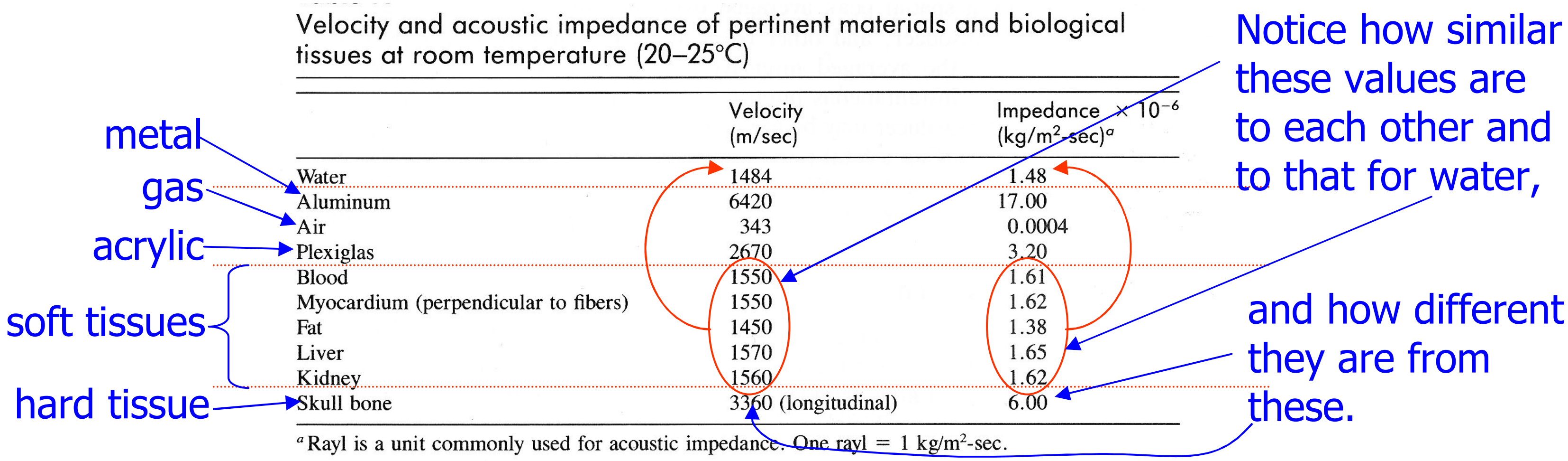
  - Air: 330 m/s; water, c=1480; most tissue: ~1500 m/s; bone: 4080 m/s

- Particle displacement velocity (v)
  - Note that particle speed v is different from sound speed c!
- Acoustic pressure (p): $p = Zv$
  - Analogy: p: voltage, v: current, Z: impedance
- Characteristic impedance of a medium: $Z = \rho c = \sqrt{\rho/\kappa}$
  - Unit: kg/m^2s or rayls (after Lord Rayleigh)

Acoustic energy and intensity

- Particles in motion have kinetic energy; those poised for motion has potential energy
- Kinetic energy density: $w_k = \frac{1}{2}\rho v^2$
- Potential energy density: $w_p = \frac{1}{2}\kappa p^2$
- Acoustic energy density: $w = w_k + w_p$
- Acoustic intensity (acoustic energy flux): $I = pv = \frac{p^2}{Z}$
  - Analogy: I (power), p (voltage), v (current)

Acoustic Properties of Common Material



Velocity and acoustic impedance of pertinent materials and biological tissues at room temperature (20–25°C)

	Velocity (m/sec)	Impedance × 10^{-6} (kg/m^2-sec)[a]
Water	1484	1.48
Aluminum	6420	17.00
Air	343	0.0004
Plexiglas	2670	3.20
Blood	1550	1.61
Myocardium (perpendicular to fibers)	1550	1.62
Fat	1450	1.38
Liver	1570	1.65
Kidney	1560	1.62
Skull bone	3360 (longitudinal)	6.00

[a] Rayl is a unit commonly used for acoustic impedance. One rayl = 1 kg/m^2-sec.

Notice how similar these values are to each other and to that for water,

and how different they are from these.

From [Graber: Lecture note for BMI F95]

Table 10.1 in [Prince] gives more information, including density and absorption coefficient

3D Wave Equation

- Acoustic pressure: $p(x, y, z, t)$
- 3-D wave equation

$$\nabla^2 p(x, y, z, t) = \frac{1}{c^2} p_{tt}(x, y, z, t)$$

where

$$\nabla^2 p = p_{xx} + p_{yy} + p_{zz}$$

and c is the speed of sound

- General solution is very complicated
- We go after plane waves and spherical waves

Plane Wave

- Plane wave in z direction:

$$p(z,t) = p(x,y,z,t)$$

- Plane wave equation:

$$p_{zz}(z,t) = \frac{1}{c^2} p_{tt}(z,t)$$

- General solution:

Forward traveling wave

$$p(z,t) = \phi_f(t - c^{-1}z) + \phi_b(t + c^{-1}z)$$

where $\phi_f(t)$ and $\phi_b(t)$ are arbitrary

Backward traveling wave

One of them may be zero

Example

- An ultrasound transducer is pointing down the +z axis. Starting at time t=0, it generates an acoustic pulse with form

$$\phi(t) = (1 - e^{-t/\tau_1})e^{-t/\tau_2}$$

- Assume the speed of the sound in the tissue is c=1540 m/s. What is the forward traveling wave down the +z axis? At what time does the leading edge of the impulse hit the interface 10 cm away from the transducer?

$$\phi_f(z,t) = \phi(t - z/c)$$

$$= (1 - e^{-(t-z/c)/\tau_1})e^{-(t-z/c/\tau_2}$$

$$T = 0.1m/1540 = 64.9\mu s$$

Harmonic Waves

- Harmonic plane wave:

$$p(z,t) = \cos(k(z - ct))$$

- Viewed at a fixed particle, the pressure changes in time with frequency $f_t=kc/2\pi$ (cycles/s)
- Viewed at a fixed time, the pressure changes in z with frequency $f_z=k/2\pi$
  - k is called wavenumber
- Wavelength is the spacing between peak or valleys of the wave at any time
  - $\lambda=1/f_z=2\pi/k=c/f_t$

- (approximately) Harmonic wave are widely used in ultrasound imaging
- Given ft, the wavelength depends c, which in turn depends on the tissue property!
  - Wavelength determines the resolution of ultra sound imaging
  - Ex: f_t=3.5 MHz, c=1540m/s (most tissue), λ=0.44mm

Spherical Wave

- 3-D spherical wave:

$$p(r,t) = p(x,y,z,t)$$

where $r = \sqrt{x^2 + y^2 + z^2}$.

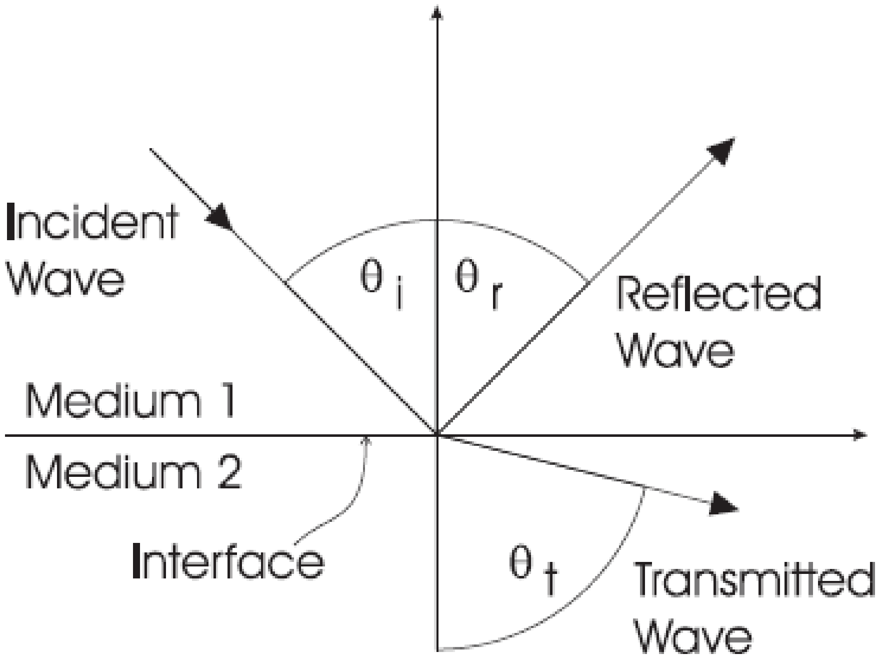
- Spherical wave equation:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rp) = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

- General solution (outward expanding):

$$p(r,t) = \frac{1}{r}\phi_o(t - c^{-1}r)$$

Reflection and Refraction: Geometric Characteristics



- Snell's Laws:

$$\begin{aligned} \theta_r &= \theta_i \\ \frac{\sin\theta_i}{\sin\theta_t} &= \frac{c_1}{c_2} \end{aligned}$$

Reflection and Refraction: Energy Characteristics

- Particle motion conservation
  - Tangential particle motion caused by the incident wave=sum of particle motions of transmitted and reflected waves

$$v_i \cos\theta_i = v_r \cos\theta_r + v_t \cos\theta_t$$

- Pressure conservation

$$p_t - p_r = p_i$$

- Based on above equations, and the relation $p=Zv$, one can derive relation of p_r and p_t with p_i, and consequently pressure reflectivity and pressure transmitivity defined in the next slide

Reflected and Refracted Wave

- Pressure reflectivity:

$$R = \frac{p_r}{p_i} = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}$$

- Pressure transmittivity:

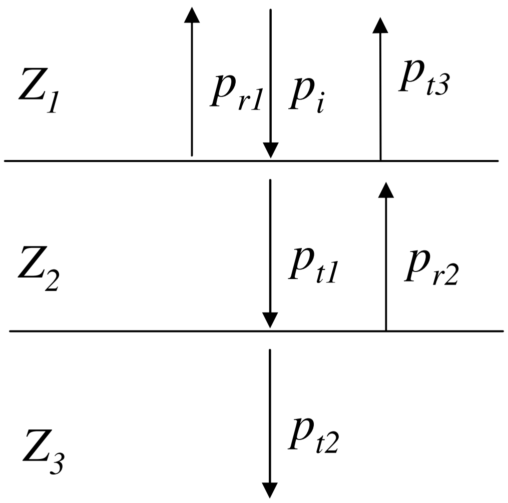
$$T = \frac{p_t}{p_i} = \frac{2 Z_2 \cos\theta_i}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}$$

- At normal incidence:

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

Example

- Layered medium, determine total reflected signal from multiple layers



Total reflected signal = $p_{r1} + p_{t3}$

Energy Attenuation due to Absorption and Scattering

- The pressure of an acoustic wave decreases as the wave propagates due to
  - Absorption
    - The wave energy is converted to thermal energy
    - Two forms
      - Classical: due to frictions between particles as the wave propagates
      - Relaxation: due to particle motion to return (relax) to original position after displacement by the wave pressure
  - Scattering
    - When the sound wave hits an object much larger than its wavelength, reflection occurs.
    - When the object size <= wavelength, scattering occur (reflection in all directions)
    - Note the difference between reflection and scattering!

Overall Attenuation

- Absorption and scattering together causes the pressure and intensity of a sound wave to decrease exponentially in the propagation distance z

$$\text{Suppose } \ p(0,t) = A_0 f(t)$$

$$\text{No attenuation}: p(z,t) = A_0 f(t - c^{-1} z)$$

$$\text{With attenuation}: p(z,t) = A_0 e^{-\mu_a z} f(t - c^{-1} z)$$

$$\mu_a : \ \text{Amplitude attenuation factor [cm}^{-1}]$$

$$\text{Attenuation coefficient in dB}:$$

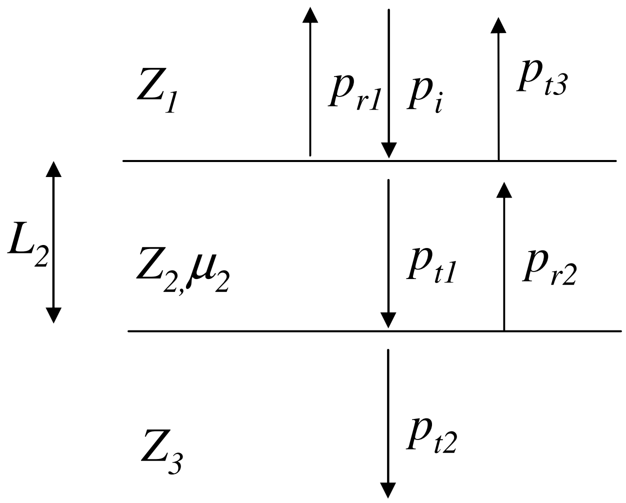
$$\alpha = 20(\log_{10} e)\mu_a \approx 8.7\ \mu_a \ \ \text{[dB/cm]}$$

- The attenuation coefficient depends on the frequency of the wave, generally $\alpha = af^b$
- Rough approximation (1 MHz <=f<=10MHz): b=1, $\alpha = af$

See Table 10.2 in textbook for “a” value for biological tissues
(much larger for bone and lung)

Example

- Same layered medium as before, but considering the attenuation



Total reflected signal = $p_{r1} + p_{t3}$

Scattered Signal

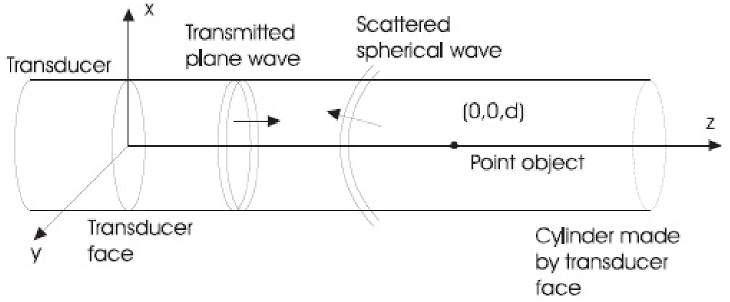
- When the sound wave hits an object much smaller than sound wavelength, scattering occur (reflection in all directions), giving rise to spherical waves
- Consider a plane wave traveling in z, hitting a small target at z=d
    - Source wave

$$p(z,t) = A_0 f(t - c^{-1} z)$$

- Particle at $(0,0,d)$, reflection coefficient R
- Generates spherical wave

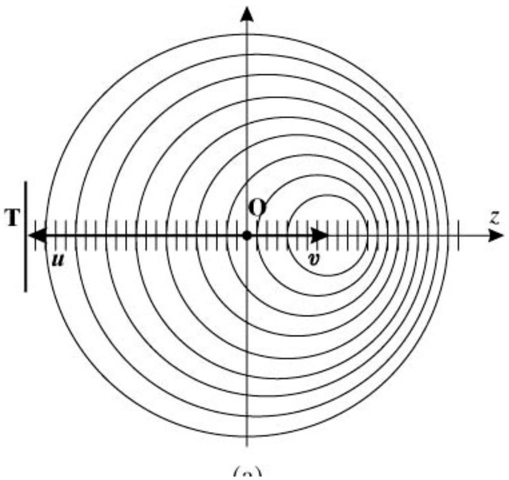
$$p_s(r,t) = \frac{R e^{-\mu_a r} A_0 e^{-\mu_a d}}{r} f(t - c^{-1}d - c^{-1}r)$$

- r is distance from $(0,0,d)$



Doppler Effect: Moving source

- Doppler effect: change in frequency of sound due to the relative motion of the source and receiver
- Case 1: moving source (scatterer), stationary receiver (transducer)
  - Source moving away, wavelength longer, lower freq
  - Source moving closer, wavelength shorter, higher freq.



source (freq $= f_o$)moving with speed v opposit the wave direction :

one period $T = 1/f_o$

crest in wave moves a distance of $cT = c/f_o$ without source motion

source moves a distance of $vT = v/f_o$

With source motion, crest moves a distance (equivalenth wavelength λ_T) of $cT + vT$

Equivalent temporal frequency is

$$f_T = \frac{c}{\lambda_T} = \frac{c}{c+v} f_o$$

General case source moving in a direction θ :

(angle between source- > receiver vector and source motion vector)

$$f_T = \frac{c}{c - v\cos\theta} f_O$$

Doppler frequency :

$$f_D = f_T - f_O = \frac{v\cos\theta}{c - v\cos\theta} f_O \approx \frac{v\cos\theta}{c} f_O$$

$\theta < 0$: source moving away from receiver, $f_D < 0$

$\theta >= 0$: source moving towards receiver, $f_D > 0$

Doppler Effect: Moving Receiver

- Case 2: stationary source (transducer), moving receiver (target)
  - Transducer transmitting a wave at freq fs, wavelength =c/fs
  - Object is a moving receiver with speed v, with angle θ
  - Target moving away (θ<=0), sound moves slower
  - Target moving closer (θ>=0), sound moves faster

$$f_O = \frac{c + v\cos\theta}{c} f_S$$

Doppler frequency:

$$f_D = f_O - f_S = \frac{v\cos\theta}{c} f_S$$

$$\theta < 0 : \text{source moving away from receiver, } f_D < 0$$

$$\theta >= 0 : \text{source moving towards receiver, } f_D > 0$$

Doppler Effect for Transducer

- Transducer:
  - Transmit wave at freq fs to object
  - Object moves with velocity v at angle θ
  - Object receives a wave with freq fo =(c+vcosq)/c fs
  - The object (scatterer) reflects this wave (acting as a moving source)
  - Transducer receives this wave with freq

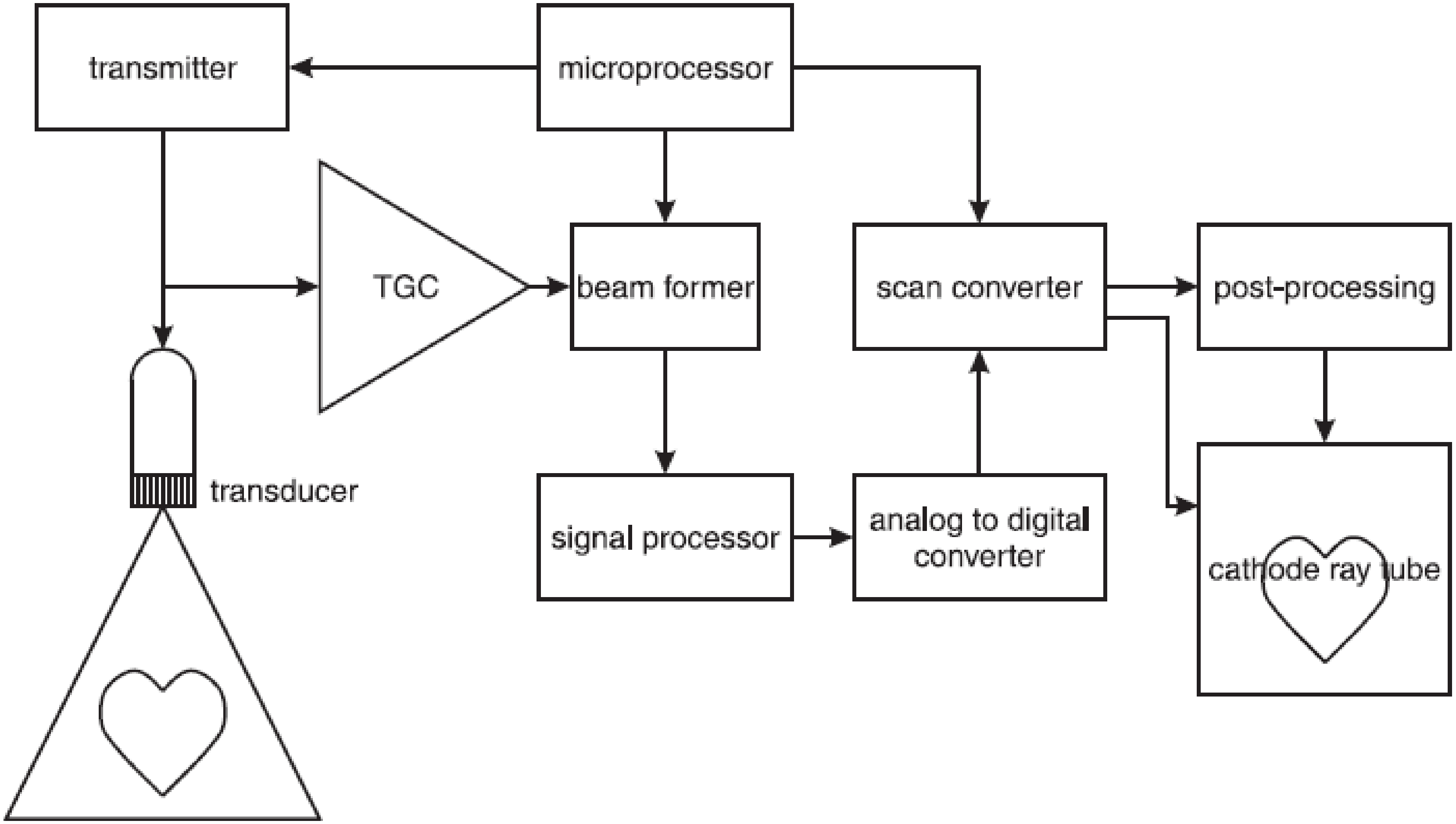
$$f_T = \frac{c}{c - v\cos\theta} f_O = \frac{c + v\cos\theta}{c - v\cos\theta} f_S$$

  - Doppler freq

$$f_D = f_T - f_S = \frac{2v\cos\theta}{c - v\cos\theta} f_S \approx \frac{2v\cos\theta}{c} f_S$$

- Dopper-shift velocimeter: as long as θ not eq 90^0, can recover object speed from doppler freq.
- Doppler imaging: display f_D in space and time
  -

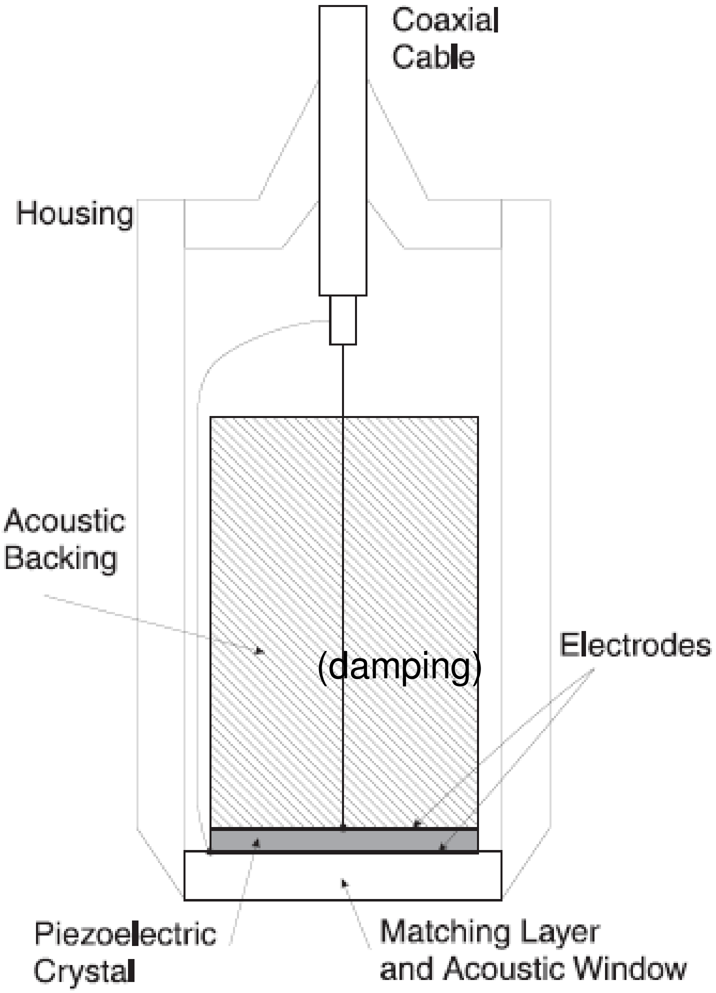
Schematic of an Ultrasound Imaging System

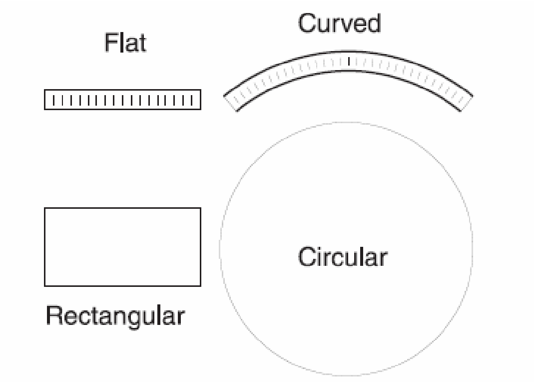


Functions of transducer

- Used both as Transmitter And Receiver
- Transmission mode: converts an oscillating voltage into mechanical vibrations, which causes a series of pressure waves into the body
- Receiving mode: converts backscattered pressure waves into electrical signals

Single Crystal Transducer (Probe)



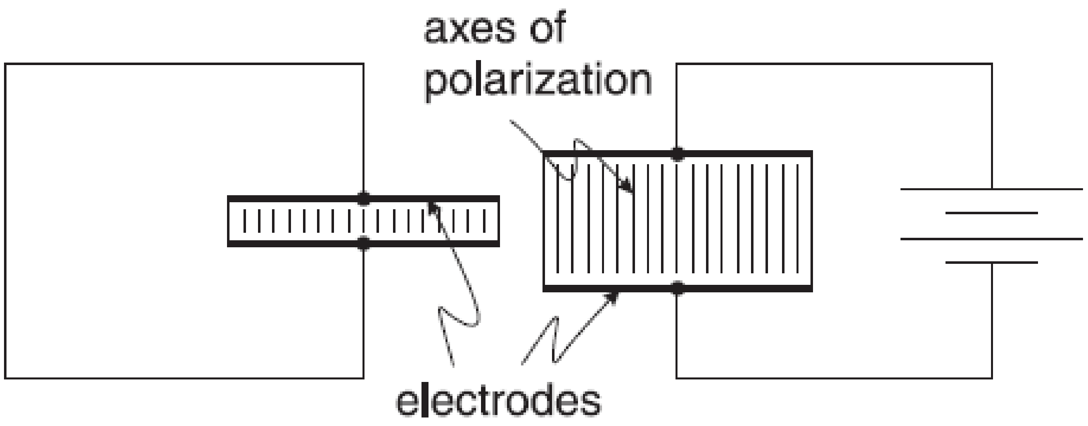


Piezoelectrical Crystal

- Converting electrical voltage to mechanical vibration and vice versa
- The thickness of the crystal varies with the applied voltage
- When an AC voltage is applied across the opposite surface of the crystal, the thickness oscillates at the same frequency of the voltage
- Examples of piezoelectric Materials:
  - Crystalline (quartz), Polycrystalline ceramic (PZT, lead zirconium titanate), Polymers (PVDF)
  - PZT is more efficient in converting between electric signal and pressure wave
- The crystal vibrates sinusoidally after electrical excitation has ended (resonate)
  - Resonant frequency f =c /2d (d=thickness)
  - Practical system: 1-20 Mhz
  - This is the frequency of the pressure wave introduced into the body
  - The damping material damps the vibration after 3-5 cycles
- When the diameter D of the surface is much larger than d, longitudinal waves are transmitted into the body

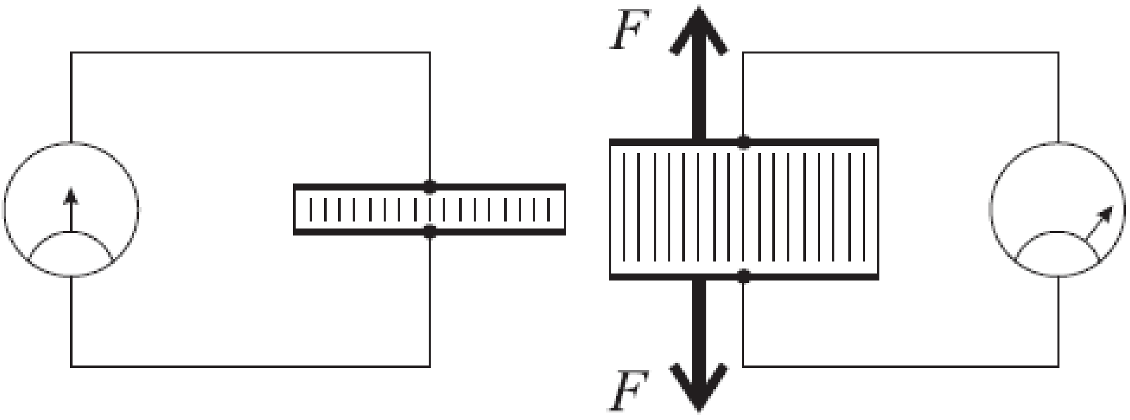
- The crystal is shaped into a disk or rectangle, with either flat or concave surface

Piezoelectrical Effect



(a) electrodes shorted

(b) voltage applied across electrodes



(c) no force applied

(d) force applied

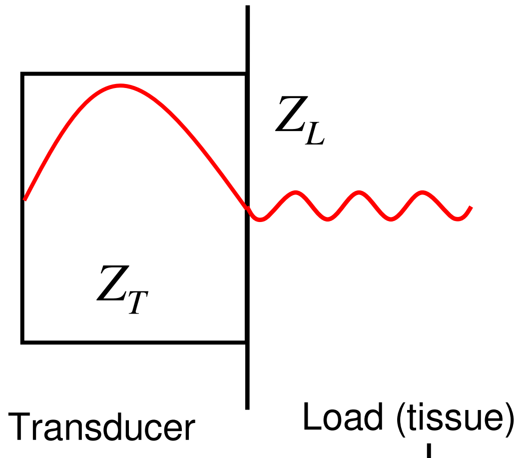
Matching Layer

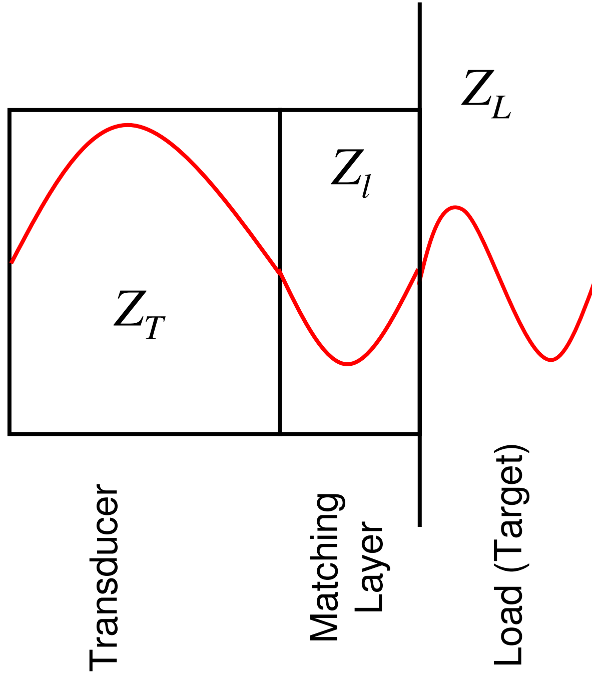
- To provide acoustic coupling between the crystal and patient skin and to protect surface of the crystal
- Z of PZT (ZT) is about 15 times of Z of skin (ZL)
  - Placing crystal directly over skin would result a large amount of energy be reflected back from the boundary
    - R= (ZL-ZT)/(ZL+ZT) ~1
- Matching layer
  - layer thickness = λ/4

$$Z_l = \sqrt{Z_T Z_L}$$

  - Maximize energy transfer into the body
  - Show as a homework
- Problems: Finding material with exact Z_l value
  - Dual-layer:

$$Z_{l,1} = Z_T^{3/4} Z_L^{1/4}; \quad Z_{l,2} = Z_T^{1/4} Z_L^{3/4}$$

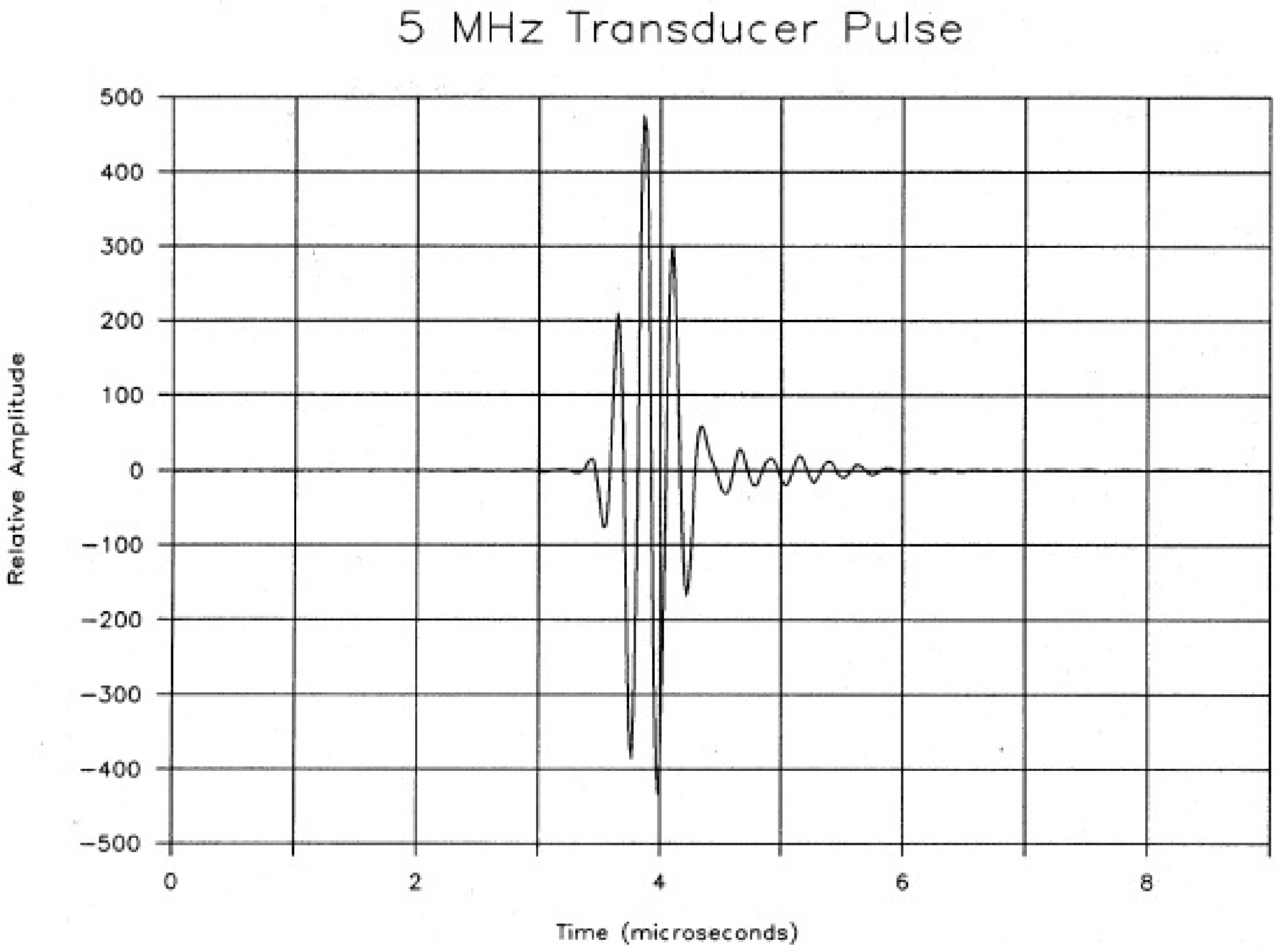




Continuous Wave vs Pulsed

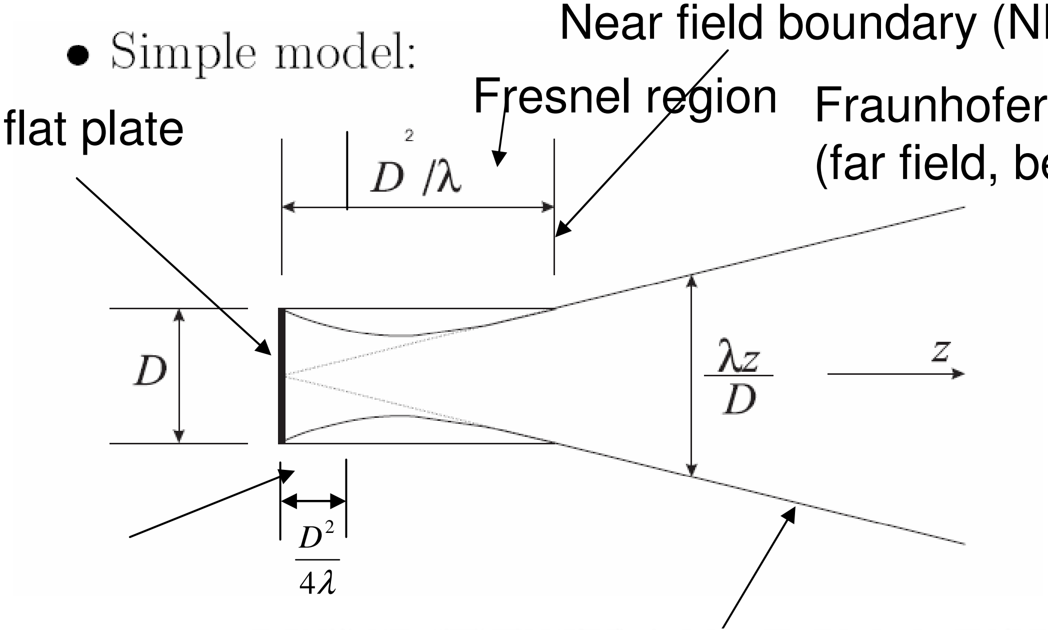
- Continuous wave (CW) Ultrasound
  - The voltage signal applied to the transducer is a continuous sinusoid signal
- Pulsed Ultrasound
  - A pulsed signal is introduced (more common), allowing to separate reflected signal from targets at different distances based on time delay

Typical Transmit Pulse



Flat (Piston) Plate Transducer

- Simple model:



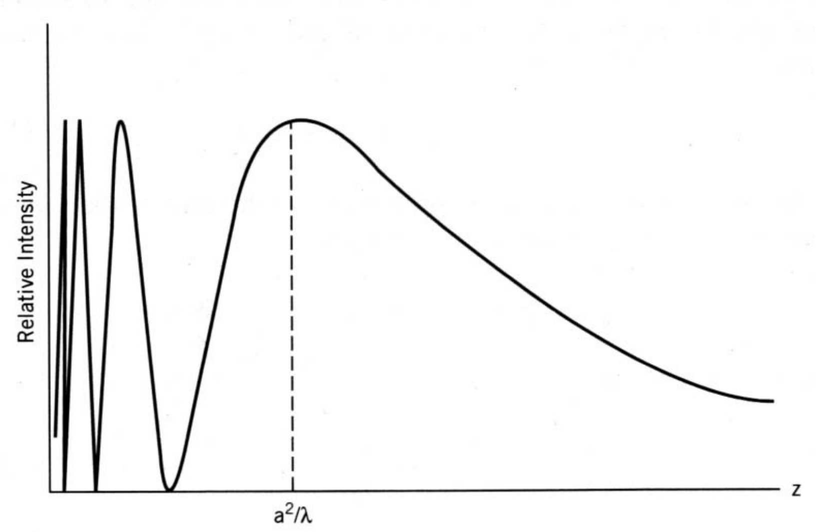
Near field boundary (NFB)

Fresnel region

Vibrating flat plate

Fraunhofer region
(far field, beam spreading)

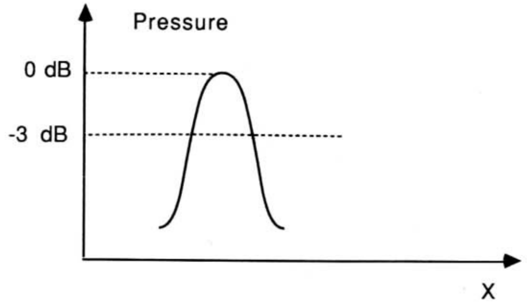
Geometric approximation: Wave is confined in a cylinder



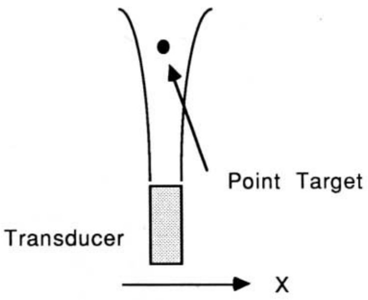
- At border of the beam width, the signal strength drops by a factor of 2, compared to the strength on the z-axis
- Beam width determines the imaging resolution (lateral resolution).
- Smaller D is good only before far field
- D=1~5 cm in practice, very poor lateral resolution
- Focused plate is used to produce narrow beam

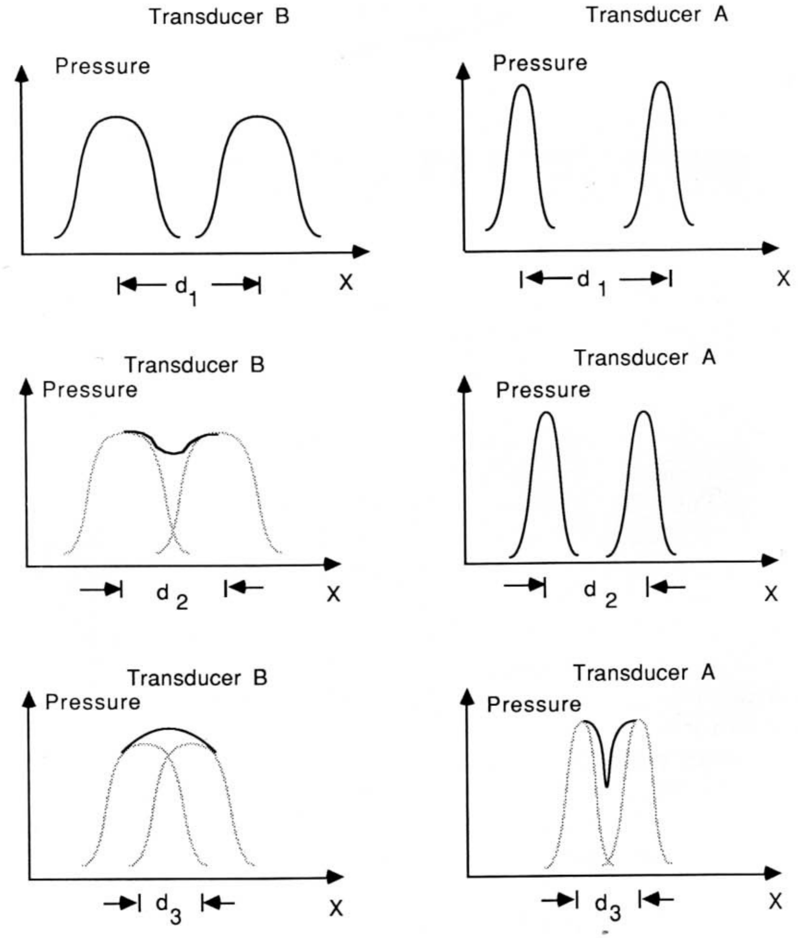
Axial and lateral resolution

- Axial resolution = $0.5\times\tau c$, determined by τ = pulse duration.
  Pulse length determined by location of -3 dB point.



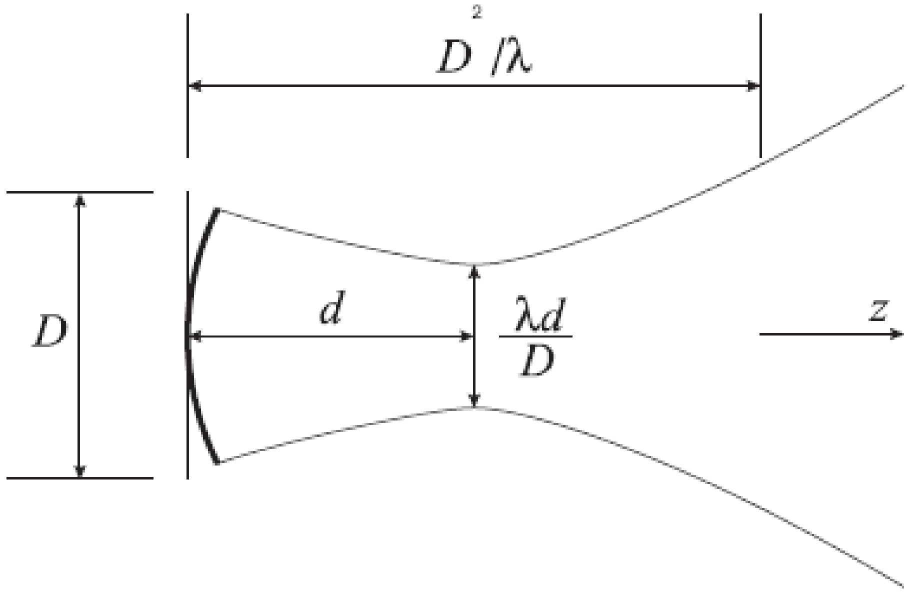
- Lateral resolution determined by beam width (-3 dB beam width or - 6 dB width)





Focused Transducer

- Beam focusing can be accomplished by
  - Using a crystal with a curved surface
  - Placing a concave lens in front of the crystal

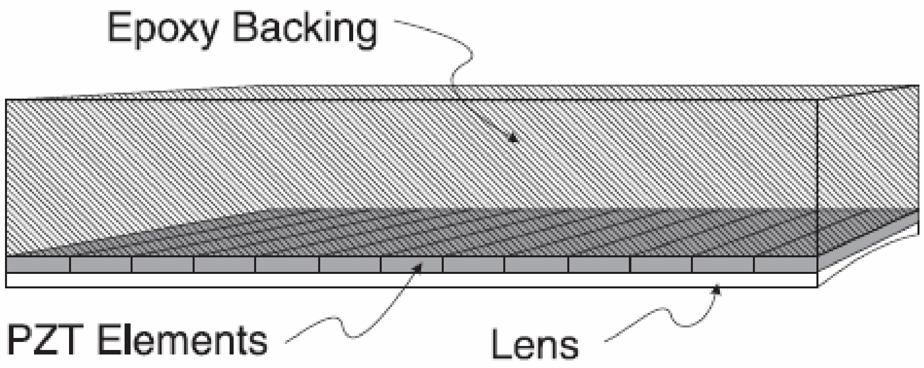


Good resolution at the focal depth, but worse at smaller or larger distances than flat transducer, Diverges more quickly after the focal point

Transducer Array

- With a single crystal, manual or mechanical steering of the beam is needed to produce a two-dimensional image
- Practical systems today use an array of small piezoelectric crystals
  - Allow electronic steering of the beam to optimize the lateral resolution

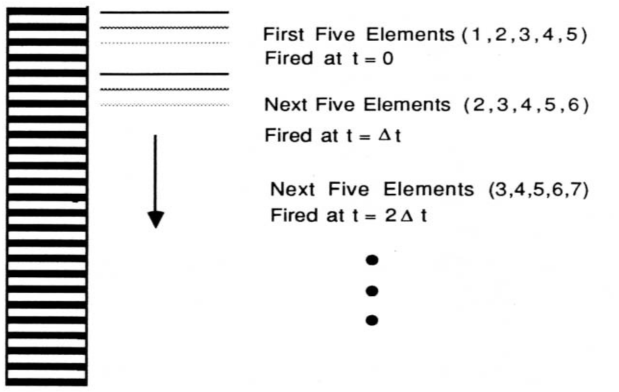
Transducer Array



- Linear arrays:
  - 64–256 elements, fire in groups
  - each element $\approx$ 2 mm by 10 mm
- Phased arrays:
  - 30–128 elements; electronically steered
  - each element $\approx$ 0.2 mm by 8 mm

Transducer arrays

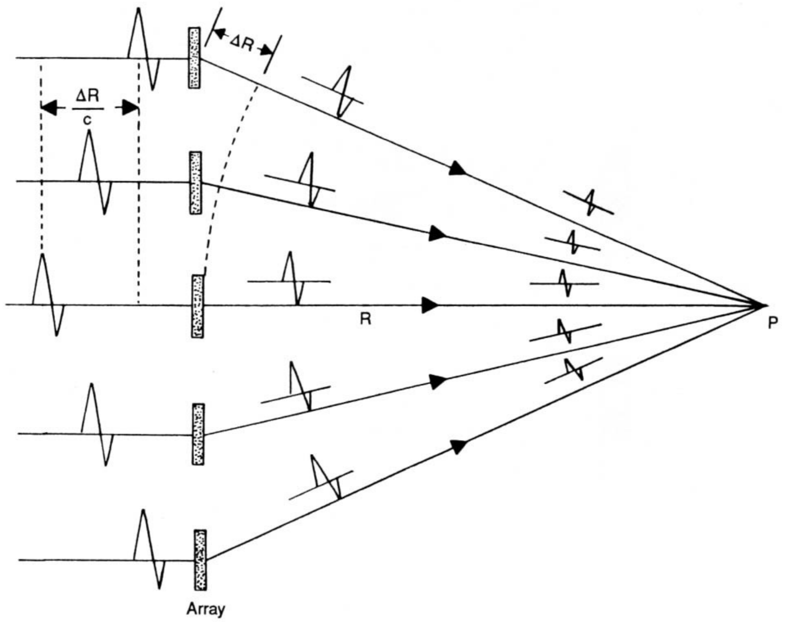
- Linear sequential array → lateral scan

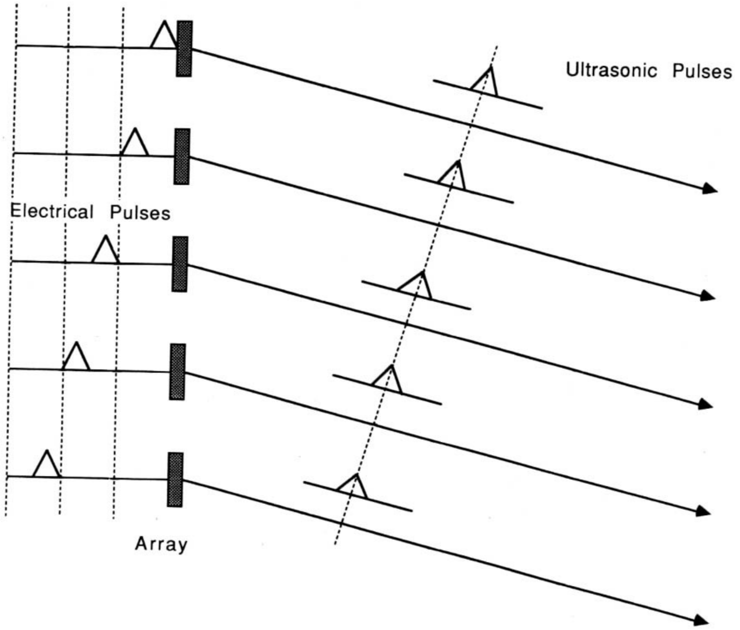

First Five Elements (1,2,3,4,5)
Fired at t = 0

Next Five Elements (2,3,4,5,6)
Fired at t = Δt

Next Five Elements (3,4,5,6,7)
Fired at t = 2Δt

- Linear phased array for beam steering, focusing





Array types

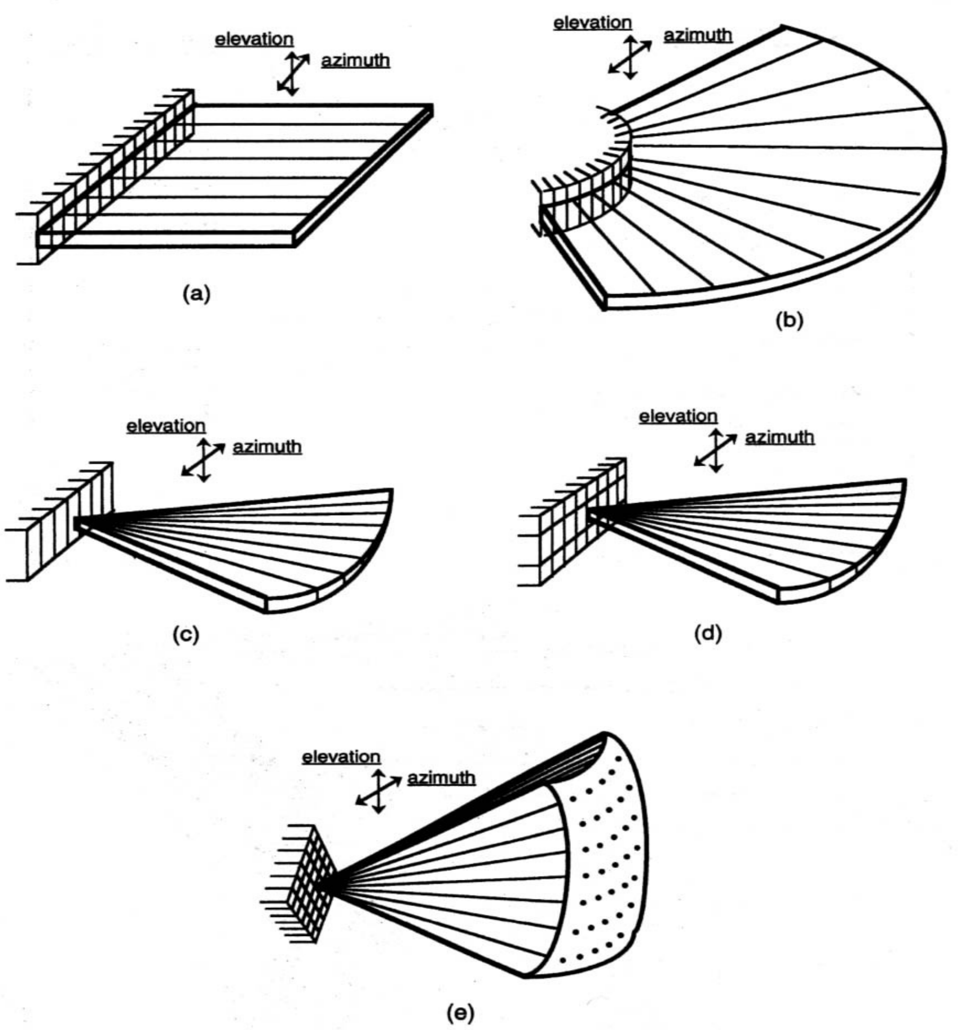
a) Linear Sequential (switched) ~1 cm × 10-15 cm, up to 512 elements

b) Curvilinear
similar to (a), wider field of view

c) Linear Phased
up to 128 elements, small footprint → cardiac imaging

d) 1.5D Array
3-9 elements in elevation allow for focusing

e) 2D Phased
Focusing, steering in both dimensions



Summary

- What is acoustic wave?
- Characterization of wave:
  - Material properties: k, ρ, c, Z
  - Wave properties: particle velocity v, pressure p, intensity I
- Plane wave vs. Spherical wave
- Reflection and refraction of wave at an interface
- Scattering of wave
- Transducer for ultrasound imaging
  - Piezoelectric crystal
  - Resonance frequency (determined by thickness of crystal)
  - Flat surface vs. focused
  - Beam divergence in both types of transducer
  - Beam width determines lateral resolution
  - Pulse period determines axial resolution
  - Array transducer

Reference

- Prince and Links, Medical Imaging Signals and Systems, Chap. 10 (Sec. 10.5 not required),11.2,11.3
- A. Webb, Introduction to Biomedical Imaging, Chap. 3

Homework

- Reading:
  - Prince and Links, Medical Imaging Signals and Systems, Chap. 10 (Sec. 10.5 not required),11.2,11.3
- Note down all the corrections for Ch. 10,11 on your copy of the textbook based on the provided errata (see Course website or book website for update).
- Problems:
  - P10.1
  - P10.3
  - P10.6
  - P10.8
  - P10.10
  - P10.12
  - P10.13
  - Considering the matching layer in a transducer. Show that, when a matching layer of depth λ/4 is used, the transmitted energy into the tissue is maximized with an impedance of $Z_l = \sqrt{Z_T Z_L}$