IV. Cumulative Damage Leading to Fatigue and Creep Failure for General Materials

Of the various headings under mechanics of materials, failure is usually considered to be the most difficult one. Then proceeding further to the sub-fields under failure, probably fatigue is considered to be its most difficult one. The concept of damage comes into play with fatigue, but deducing useful and reasonably general damage forms has been a taxing and uncertain exercise. The organized attempts to quantify damage usually fall under the subject of cumulative damage with the understanding that the damage leads to and terminates with materials failure.

Quantifying damage relative to failure allows the optimization of stress loading programs and allows the means for including overload type damage events. Although this website is not aimed as a literature survey, this section will be used to examine several different approaches for cumulative damage. The possible applications are to isotropic or anisotropic materials (including composites) with the proviso that in either case the conditions of proportional loading apply.

Creep rupture also comes under the heading of cumulative damage. Creep rupture is the time dependent damage growth leading to failure in polymers and in metals at high temperature. As developed here, fatigue conditions and creep rupture conditions admit the same general formulation. Both will be treated and with suitable notational changes either form can be found from the other.

There are considered to be two main approaches for cumulative damage. One is that of the direct postulation of lifetime damage forms such as Miner's rule and the other is that of residual strength. Residual strength is the reduced (instantaneous) static strength that the material can still deliver after being subjected to loads causing damage. Of course both are relevant to the general problem and a well founded cumulative damage theory must contain both descriptions in a compatible form. The overall objective of both and of all approaches is to secure a life prediction methodology.

In the area of the fatigue of metals, a common approach is to assume a pre-existing crack that grows according to a power law form with regard to the stress level. This is often called the Paris law. When the crack reaches a certain pre-specified size, the service life is considered to be completed. While this is certainly a useful approach it cannot be considered to be a life prediction methodology based upon the approach to failure. Accordingly it will not be followed here.

The general topics of fatigue or of damage are covered in many self contained, inclusive books such as Suresh [1] and Krajcinovic [2]. As part of a general damage approach, constitutive relations are often taken for damage involving scalar or tensor valued damage variables and a whole framework of behavior is built up. In a considerably different direction to be followed here, cumulative damage leading to failure is more akin to failure criteria, but with the capability to characterize damage as the prelude to failure. The first credible damage form was that of Miner's rule [3]. Broutman and Sahu [4] and Hashin and Rotem [5] much later produced other damage forms that with the passage of time have gained credibility. Reifsnider [6] initiated a different damage formalism which has been further developed by him and others in other papers. Other also notable efforts have been given by Adam et al [7] and by many other workers. Christensen [8] recently developed a new and different approach. Post, Case, and Lesko [9] have recently given a survey of many different cumulative damage models and applied them to situations of spectrum type loadings for composites.

Due to the complexity of the topic many approaches and models contain adjustable parameters, sometimes many parameters. In the coverage undertaken here only models without any adjustable parameters will be considered. That is to say, the damage formulations to be considered will be based solely upon the properties contained within a database of constant amplitude fatigue or creep testing and the static strength. Then the damage forms will be used to predict life under variable amplitude programs of load application. This circumstance is analogous to viscoelastic behavior where mechanical properties of creep or relaxation function type are inserted into convolution integrals to predict general behavior. Materials with memory could encompass viscoelastic effects at one extreme, and damage memory effects at the other extreme. The four damage models to be examined here are those of Miner's rule, Hashin-Rotem, Broutman-Sahu, and then the recently developed one by Christensen. The Broutman-Sahu form is representative of a large class of models, as will be explained later.

In terms of evaluating damage models, there does not appear to be any definitive set or sets of experimental data. The problem is the lack of repeatability and in a more general sense the great variability in the data. This places even more emphasis on the need for a careful theoretical evaluation, looking for physical consistency in the predictions and conversely examining aspects of inconsistency in important special cases.

Three main problems will be used to evaluate the four models. These include examples of prescribed major damage followed by the predicted life at a lower, safer stress level. Another problem is that of the ability to predict residual instantaneous strength after a long time load application that nevertheless is shorter than that time which would cause fatigue or creep failure. Thirdly is the problem of residual life. In this situation a load is applied right up to the time of, but just an instant before, fatigue or creep failure. At that time the load is removed and replaced by one of a lower, less stringent level. The additional life that results at the lower stress level is designated as the residual life.

All of these models and methods are taken from the peer reviewed literature. The new dimension which is added here is that of a more inclusive theoretical evaluation than appears to have been previously given.

Four Alternative Approaches

The general concept of damage that would or could lead to failure was rather vague and nebulous until Miner gave it a specific form and it became recognized. With background from Palmgren, Miner [3] postulated the damage form for fatigue as

$$\sum_i \frac{n_i}{N_i} = 1 \tag{1}$$

where n_i is the number of applied cycles at nominal stress σ_i and N_i is the limit number of cycles to failure at the same stress and for the same cycle type. Thus each value n_i/N_i is viewed as a quanta of damage, the sum of which specifies failure. As with all cumulative damage forms, when the left hand side of (1) is less than one, it still quantifies the damage level but does not imply failure. The spectrum of values of N( ) versus constant stress, σ, is as shown in Fig. 1. Relation (1) then allows the life prediction for a combination of different load levels. All of the fatigue conditions considered here will be taken to be of the same frequency and cycle type.

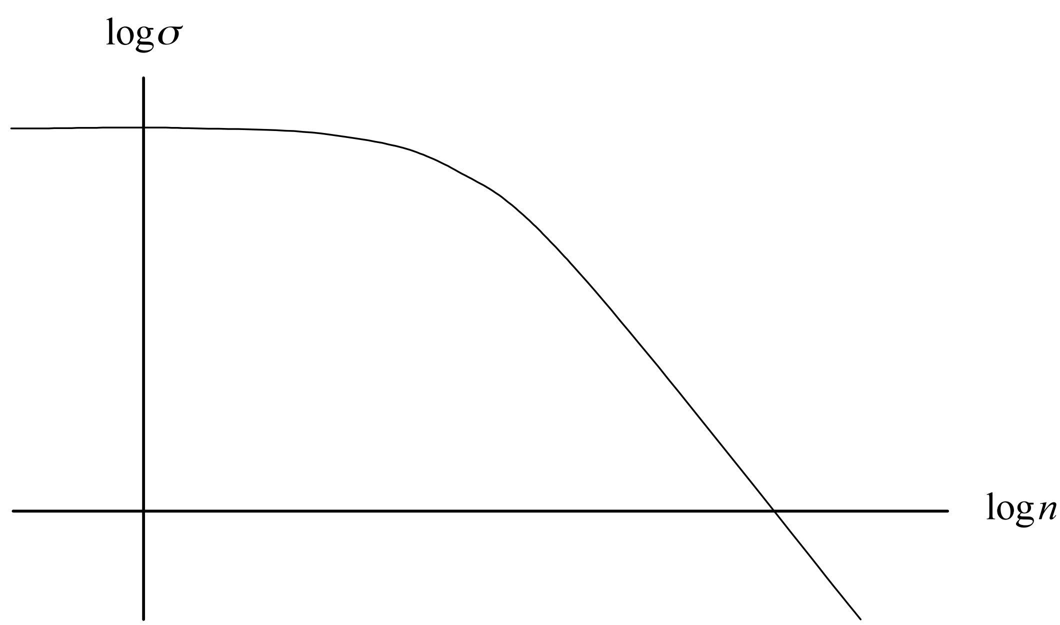

Fig. 1 Fatigue Life at Constant Stress

Relation (1) is a completely empirical form but it was a reasonable conjecture at the time. It is usually called Miner's rule. It also sometimes goes by the name Palmgren-Miner rule or law and also by the term Linear Cumulative Damage.

The first evidence of possible inadequacy of Miner's rule is that it predicts the independence of the order of application of the loads leading to failure, so long as the duration of each sequence of cycles is preserved. Despite the shortcomings which will be shown in the following evaluation, Miner's rule has always been by far the most widely used cumulative damage form.

Writing Miner's rule in its two step form gives

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \tag{2}$$

Hashin and Rotem [5] recognized that there could be a problem with the calibration of the first term relative to the second. Specifically, for a given proportion of the life being expended in step 1 at stress level σ_1, this proportion of the expended life would be expected to be different relative to the following stress level of step 2. With a set of conditions and assumptions they modified (2) to the form

$$\left(\frac{n_1}{N_1}\right)^{\frac{\log N_2}{\log N_1}} + \frac{n_2}{N_2} = 1 \tag{3}$$

For more than two steps, the form (3) can be applied iteratively. The form (3) can also be modified to accommodate a continuous variation of n/N, but the result is quite complex and not needed here.

In a completely different approach the problem of residual strength can be approached. Residual strength is the static strength that would exist if a fatigue test were interrupted and tested to failure in the usual static strength manner. One of the simplest possible residual strength forms was given by Broutman and Sahu [4]. It states the residual strength σ_R as

$$\sigma_R = \sigma_s - \sum_i (\sigma_s - \sigma_i) \frac{n_i}{N_i} \tag{4}$$

where σ_s is the instantaneous tensile static strength as measured on the virgin material before any fatigue damage is induced. At $\sigma_i = \sigma_s$ (with σ_i being the maximum stress in the cycle) this residual strength form properly reduces to the static strength. The form (4) is emblematic of a much larger class of models whereby (σ_s - σ_i) and (n_i /N_i) are each raised to some different power, thus introducing adjustable parameters to be determined into the process.

The life of the material under a prescribed stress program is implicit in (4) and determined by the total number of cycles at which $\sigma_R = \sigma_i$, the nominal (maximum) applied stress and the residual strength become identical. Before this number of cycles, the residual strength is greater than the applied maximum stress but less than the static strength.

At this point it is convenient to recast these fatigue forms into analogous creep rupture forms. Creep rupture occurs in polymers and in metals at high temperature. Let $t_c(\sigma)$ be the time to failure under constant stress σ. Miner's rule (1) for creep conditions then takes the form

$$\int_0^{\tilde{t}} \frac{d\tau}{\tilde{t}_c(\tilde{\sigma})} = 1 \tag{5}$$

where stress is nondimensionalized by the static strength

$$\tilde{\sigma} = \frac{\sigma}{\sigma_s}$$

and time is nondimensionalized by a time constant, t_0 which calibrates the time scale,

$$\tilde{t} = \frac{t}{t_0}$$

The basic property $\tilde{t}_c\left(\tilde{\sigma}\right)$ gives the failure times versus the constant stress levels, as shown in Fig. 1 but for creep rupture rather than fatigue.

The Hashin-Rotem fatigue form (3) then for a two step creep condition becomes

$$\left(\frac{\tilde{t}_1}{\tilde{t}_{c1}}\right)^{\frac{\log \tilde{t}_{c2}}{\log \tilde{t}_{c1}}} + \frac{\left(\tilde{t} - \tilde{t}_1\right)}{\tilde{t}_{c2}} = 1 \tag{6}$$

where

$$\tilde{t}_{c1} = \tilde{t}_c\left(\tilde{\sigma}_1\right)$$

$$\tilde{t}_{c2} = \tilde{t}_c\left(\tilde{\sigma}_2\right)$$

The Broutman-Sahu residual strength fatigue form (4) becomes the creep condition residual strength as

$$\tilde{\sigma}_R = 1 - \int_0^{\tilde{t}} \frac{(1-\tilde{\sigma})\,d\tau}{\tilde{t}_c(\tilde{\sigma})} \tag{7}$$

The corresponding lifetime form for Broutman-Sahu is found by taking $\tilde{\sigma}_R(t) = \tilde{\sigma}(t)$ in (7) to get

$$\frac{1}{1-\tilde{\sigma}(t)} \int_0^{\tilde{t}} \frac{(1-\tilde{\sigma})\,d\tau}{\tilde{t}_c(\tilde{\sigma})} = 1 \tag{8}$$

Compare (8) with the Miner's rule form (5). Considerable differences must be expected.

Now a recently derived fourth model will be introduced. In a program to develop a physically based flaw growth model, Christensen [8] has obtained a new formalism. Since a complete flaw growth method cannot simply be postulated, its derivation will be outlined here in a brief manner. The manuscript for this work can be downloaded from the homepage.

Take an existing microscale crack and specify its rate of growth according to a power law as

$$\dot{a} = \lambda\left(\sigma\sqrt{a}\right)^r \tag{9}$$

where the crack size a(t) and the stress σ(t) are functions of time, r is the power law exponent, and λ is a constant.

Integrate (9) to get

$$\left(\frac{a}{a_0}\right)^{1-\frac{r}{2}} - 1 = \lambda\left(1-\frac{r}{2}\right)a_0^{\frac{r}{2}-1}\int_0^t \sigma^r(\tau)d\tau \tag{10}$$

where a_0 is the initial crack size.

Following classical fracture mechanics as an initial approach, take the crack as growing to a size that becomes unstable when it reaches the same stress intensity factor as that which gives the static strength, σ_s , thus

$$\sigma(t)\sqrt{a(t)} = \sigma_s\sqrt{a_0} \tag{11}$$

Combining (10) and (11) then gives a life prediction form to be solved for the lifetime *t* under a prescribed stress history *σ(t)*. When this form is specialized to the constant stress case, the creep rupture time to failure is found to be given by

$$\tilde{t}_c = \frac{1}{\tilde{\sigma}^r} - \frac{1}{\tilde{\sigma}^2} \tag{12}$$

where nondimensional stress and nondimensional time will be used from this point on. Relation (12) has a behavior as shown in the first case of Fig. 2 below on log-log scales. At long time there is a power law behavior controlled by the exponent r and at short time it approaches the static strength asymptote. That much is perfectly acceptable. However, the form (12) exhibits a rather sharp transition from one asymptote to the other. Most data show a more gradual transition. Thus a more general approach is required , although the form (12) could remain as a special case. The single, isolated, ideal crack is of limited value in understanding the explicit failure of homogeneous materials. It gives useful, qualitative guidelines but not the concrete, quantitative results that can be used in applications.

Going back to relation (11) which gives the criterion for unstable growth of the crack, it must be considered as being inadequate. Instead, replace (11) by the more general form

$$\tilde{a} = F(\tilde{\sigma}) \tag{13}$$

where

$$\tilde{a} = a(t)/a_0$$

and F( ) is a function of stress yet to be determined. The necessity for this generalization beyond the behavior of a single ideal crack relates to more complex matters such as the possible interaction between cracks, crack coalescence and many other non-ideal types of damage and damage growth. The basic flaw growth relation (9) will be retained, but now a(t) represents some more general measure of flaw size as it grows.

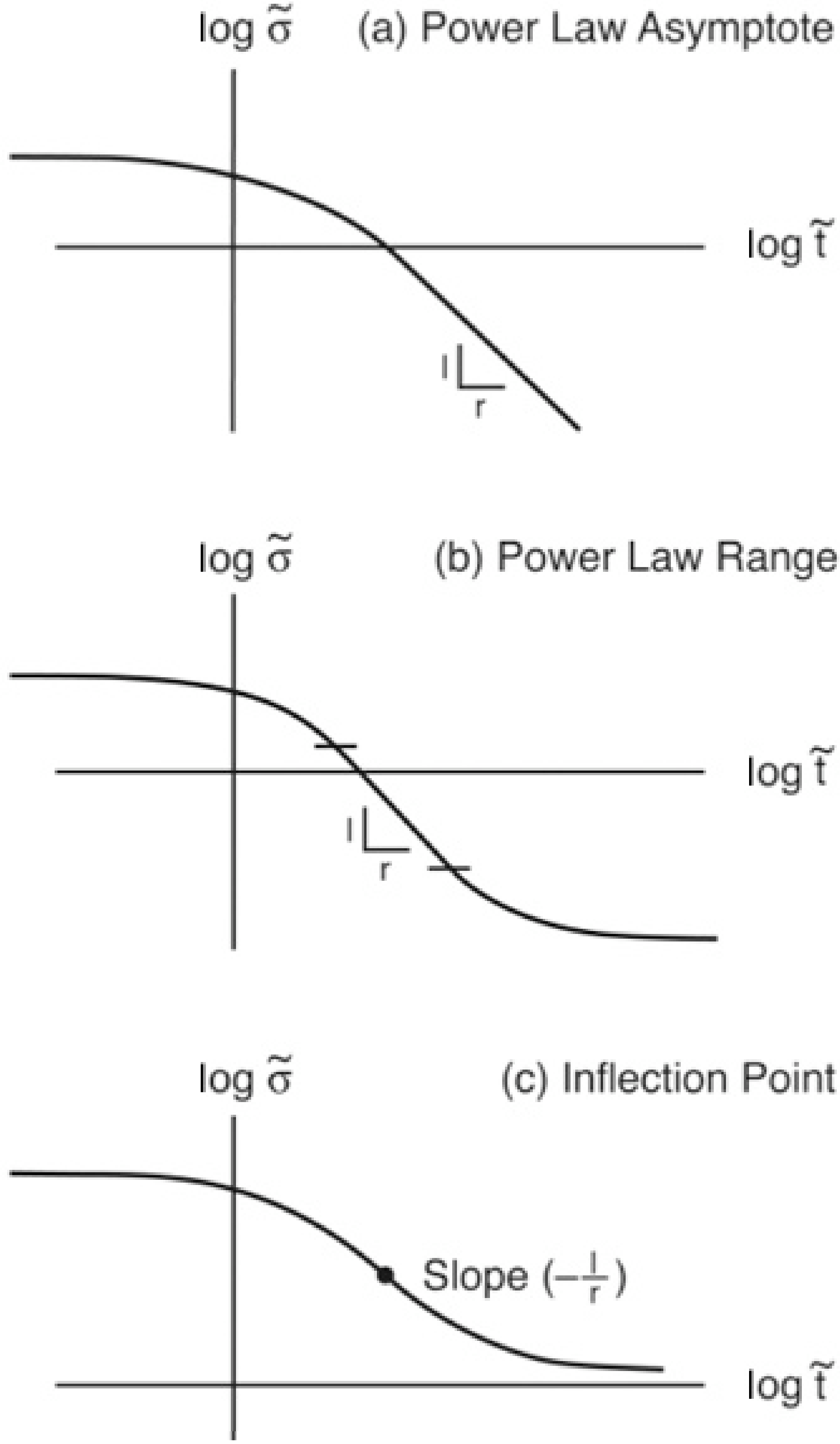

Fig. 2 Determination of Exponent "r" from Creep Rupture at Constant Stress

Combining (10) and (13) gives

$$f\left(\tilde{\sigma}\left(\tilde{t}\right)\right) = \int_{0}^{\tilde{t}} \tilde{\sigma}^{r}\left(\tau\right)d\tau \tag{14}$$

where

$$f\left(\tilde{\sigma}\right) = 1 - F^{1-\frac{r}{2}}\left(\tilde{\sigma}\right) \tag{15}$$

and where the various properties combine to form the calibrating time constant.

For constant stress, relation (14) becomes

$$f\left(\tilde{\sigma}\right) = \tilde{\sigma}^{r}\tilde{t}_{c}\left(\tilde{\sigma}\right) \tag{16}$$

where $\tilde{t}_{c}\left(\tilde{\sigma}\right)$ is the spectrum of creep rupture properties at stress levels $\tilde{\sigma}$, taken to be known from tests.

Finally, substituting (16) into (14) gives the flaw growth lifetime criterion as

$$\frac{1}{\tilde{\sigma}^{r}\left(\tilde{t}\right)\tilde{t}_{c}\left(\tilde{\sigma}\right)}\int_{0}^{\tilde{t}} \tilde{\sigma}^{r}\left(\tau\right)d\tau = 1 \tag{17}$$

For a given stress history $\tilde{\sigma}(\tau)$, relation (17) determines the lifetime, $\tilde{t}$, of the material. The exponent r is determined from the basic creep rupture forms as shown in Fig. 2. The creep rupture properties and its specific property r calibrate the theory behind the life prediction form (17). It is not surprising that the property $\tilde{t}_c(\tilde{\sigma})$ in (17) is at current time, $\tilde{t}$, rather than being inside the integral as in the other models. This relates to the method whereby the flaw grows until it reaches a critical size at the then existing current stress. This present approach also admits a full statistical generalization, see Christensen [8]. The manuscript of this published work can be downloaded from the www.FailureCriteria.com homepage.

The four basic forms under consideration here, Miner's rule, Hashin-Rotem, Broutman-Sahu, and the present form are respectively given by (5), (6), (8), and (17), It is important to observe that all of these are completely calibrated by and determined by only the basic experimentally determined creep rupture property $\tilde{t}_c(\tilde{\sigma})$ and the static strength. There are no additional parameters to be adjusted or fine tuned. Apparently these four are the only forms that have been proposed or derived that do not involve additional parameters beyond the mechanical properties.

It is interesting to note that there is a special case in which two of these basic forms become identical. Specifically for a creep rupture property of the power law type, as in

$$\tilde{t}_c = \frac{A}{\tilde{\sigma}^r} \tag{18}$$

then Miner's rule (5) and the present form (17) reduce to the same form. This is shown directly by substituting (18) into each of these. Otherwise these two forms make completely different predictions, sometimes extremely different predictions. In this power law special case, (18), the creep rupture conforms to the limiting (degenerate) case show in Fig. 3.

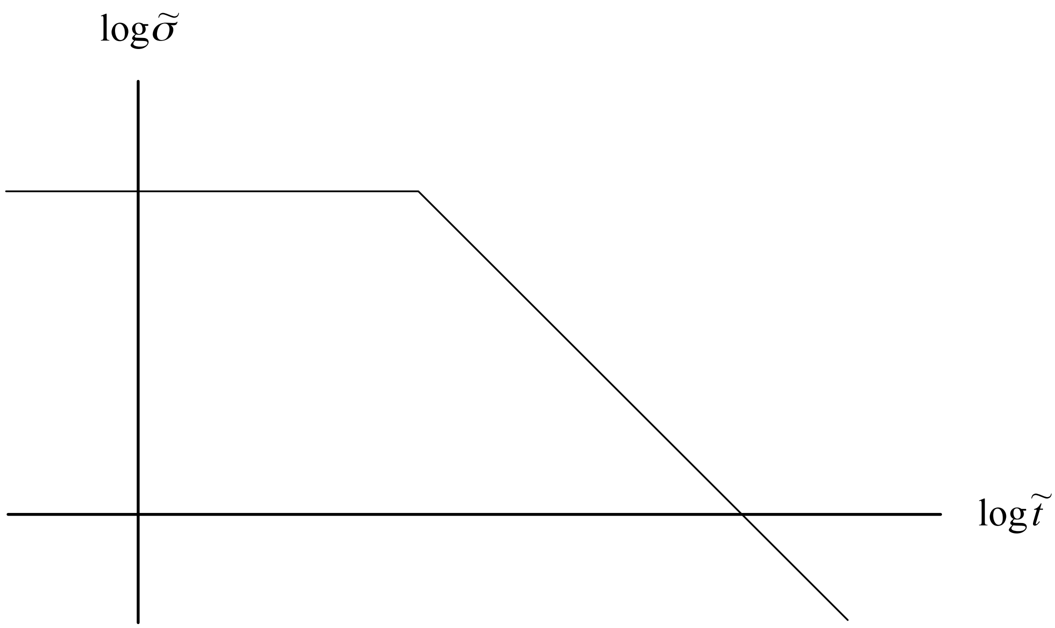

Fig. 3 Creep Rupture Power Law Behavior

In physical reality there probably is no such thing as true power law behavior. It is however a convenient mathematical approximation in some cases. However, care must be taken using power law idealizations to represent real data because slight deviations from the data on log scales can cause large differences in actual life predictions.

The residual strength form corresponding to the damage/lifetime form (17) is given by

$$\left(\tilde{\sigma}_R\right)^r \tilde{t}_c\left(\tilde{\sigma}_R\right) = \int_0^{\tilde{t}} \tilde{\sigma}^r(\tau)\,d\tau \tag{19}$$

The residual strength form of the Broutman-Sahu type is given by (7). The other two models don't directly give a residual strength determination.

The fatigue life forms for the first two models are given by Miner's rule (1) and Hashin-Rotem (3). The Broutman-Sahu form for fatigue life is given from (4) by setting $\sigma_R = \sigma_i$. The fatigue life form from the present derivation is given by

$$\frac{1}{(\sigma_K)^r N(\sigma_K)} \sum_{i=1}^{K} (\sigma_i)^r n_i = 1 \tag{20}$$

where exponent r is given by the basic fatigue property envelope at constant amplitude, the same as in the creep rupture curves of Fig. 2.

This completes the background and specification of the four basic forms under consideration. Now a comparative evaluation of these forms will be given using the creep failure notation.

Residual Strength

When a general program of stress history is interrupted and suddenly tested for static strength, Fig. 4, the residual strength, σ_R , is determined.

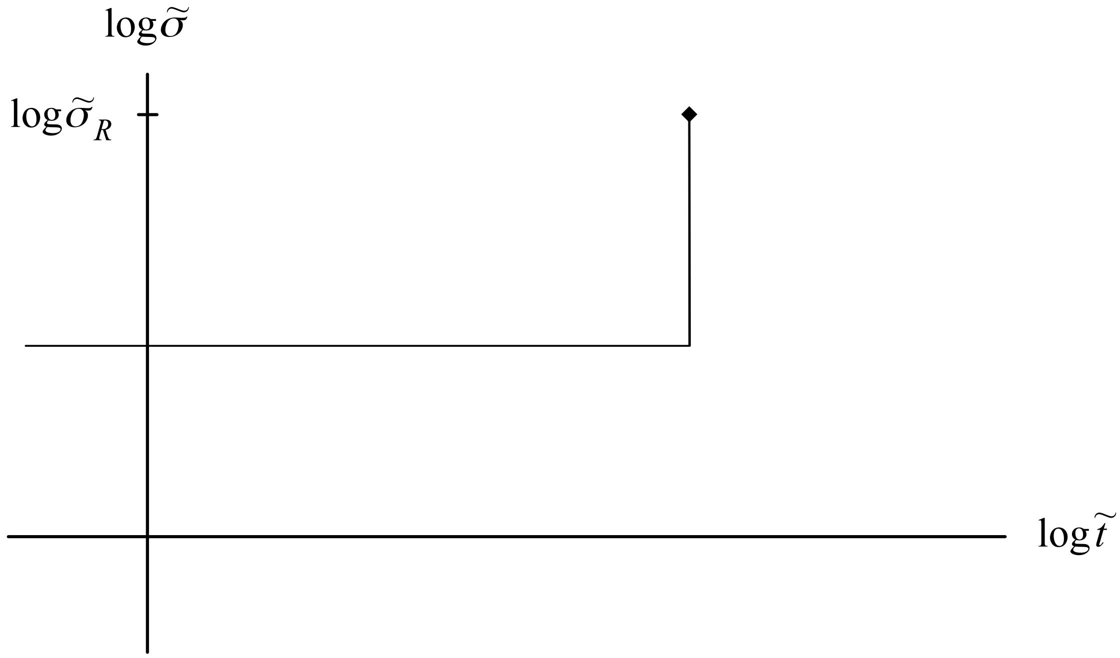

Fig. 4 Residual Strength

Under constant stress $\tilde{\sigma}$, before testing for $\tilde{\sigma}_R$ at time $\tilde{t}_1$ the Broutman-Sahu form (7) becomes

$$\tilde{\sigma}_R = 1 - (1 - \tilde{\sigma}) \frac{\tilde{t}_1}{\tilde{t}_c(\tilde{\sigma})} \tag{21}$$

Under constant stress until time $\tilde{t}_1$, the present form (19) becomes

$$(\tilde{\sigma}_R)^r \tilde{t}_c(\tilde{\sigma}_R) = \tilde{\sigma}^r t_1 \tag{22}$$

To determine $\tilde{\sigma}_R$ from (22) requires knowledge of the range of values for $\tilde{t}_c(\tilde{\sigma}_R)$. In contrast (21) only requires knowledge of the value of $\tilde{t}_c$ at one value of stress. This difference has important implications as will be seen next.

Consider two different creep rupture envelopes 1 and 2 in Fig. 5.

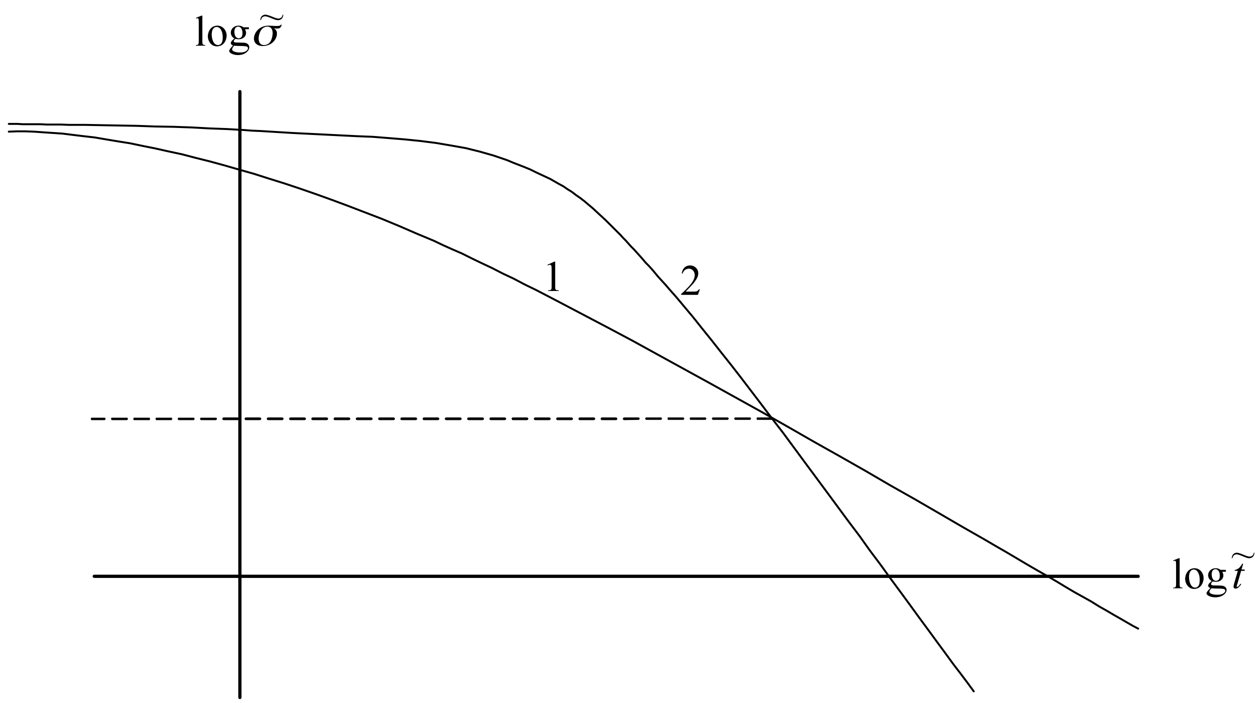

Fig. 5 Residual Strength for Two Cases

A time line through the intersection point between envelopes 1 and 2 will be used to establish the stress level for residual strength determination. At any time point on the (dashed) time line in Fig. 5 the Broutman-Sahu prediction (21) will give the same σ_R for Case 1 as for Case 2. It cannot distinguish creep rupture curve 1 from 2 in these cases, and for this reason its behavior is inconsistent and probably unacceptable. The present method (22) does have the capability to distinguish Cases 1 and 2.

The Miner's rule and the Hashin-Rotem methods do not have the direct capability to make a prediction for $\tilde{\sigma}_R$.

Damage/Life Examples

Another useful way to compare the models is to consider the remaining life after a major damage event. To simulate this, a two step loading program will be taken with the first step, of specified duration, being at a high stress overload, and the second step consisting of the service level stress up to failure.

The four damage models from (5), (6), (8), and (17) in this two step form are given by

Miner's Rule

$$\frac{\tilde{t}_1}{\tilde{t}_{c1}} + \frac{(\tilde{t} - \tilde{t}_1)}{\tilde{t}_{c2}} = 1 \tag{23}$$

Hashin-Rotem

$$\left(\frac{\tilde{t}_1}{\tilde{t}_{c1}}\right)^{\frac{\log \tilde{t}_{c2}}{\log \tilde{t}_{c1}}} + \frac{(\tilde{t} - \tilde{t}_1)}{\tilde{t}_{c2}} = 1 \tag{24}$$

Broutman-Sahu

$$\frac{\tilde{t}_1}{\left(\dfrac{1-\tilde{\sigma}_2}{1-\tilde{\sigma}_1}\right)\tilde{t}_{c1}} + \frac{\left(\tilde{t}-\tilde{t}_1\right)}{\tilde{t}_{c2}} = 1 \tag{25}$$

Present

$$\frac{\tilde{t}_1}{\left(\dfrac{\tilde{\sigma}_2}{\tilde{\sigma}_1}\right)^r \tilde{t}_{c2}} + \frac{\left(\tilde{t}-\tilde{t}_1\right)}{\tilde{t}_{c2}} = 1 \tag{26}$$

where index 1 refers to the first step and index 2 the second, and as before with

$$\tilde{t}_{c1} = \tilde{t}_c\left(\tilde{\sigma}_1\right)$$

$$\tilde{t}_{c2} = \tilde{t}_c\left(\tilde{\sigma}_2\right)$$

Before going to the examples, a significant difference in the models can be seen from (23)-(26). In each case the first term represents the damage due to the stress overload and the second term gives the normal accrual of damage at the service stress level, up to failure. The second terms in (23)-(26) are identical. Only the first terms, the major damage terms in the following examples, are fundamentally different. The first three forms have the first terms in them as mainly controlled by $\tilde{t}_{c1}$ but the last form, (26), has its first term as influenced by $\tilde{t}_{c2}$. The effective creep rupture time controlling the damage in the first step of (26) is as shown in Fig. 6.

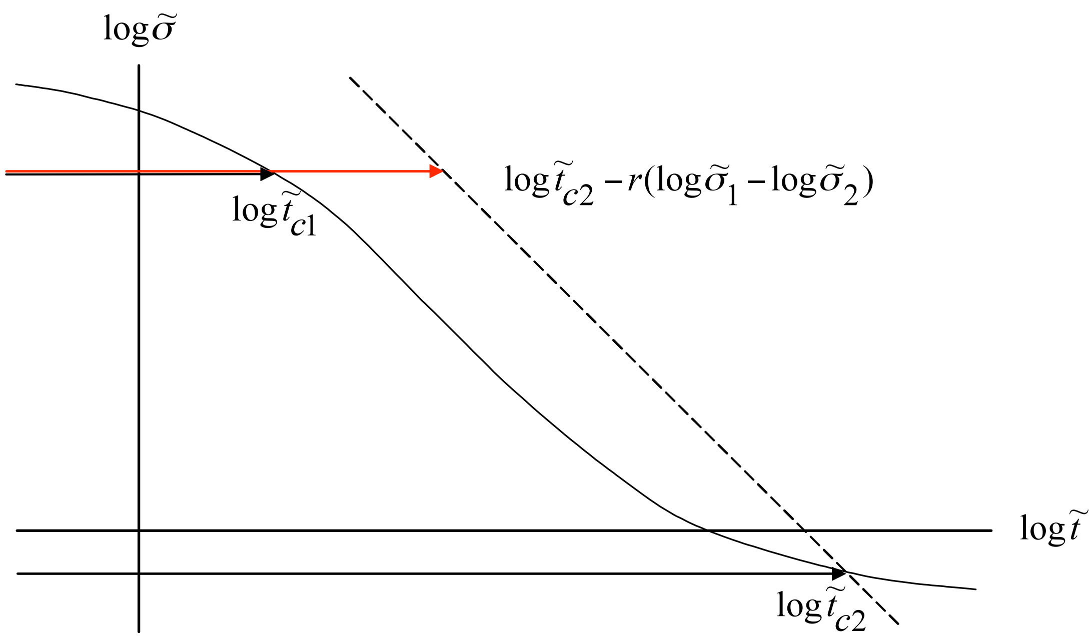

Fig. 6 Effective Creep Rupture Time, First Step of (26)

From a damage growth point of view, for step 1 the damage grows up until time $\tilde{t}_1$ and that amount of damage is independent of whatever $\tilde{t}_{c1}$ may be. The present model, (26), calibrates the first step damage relative to that which ultimately causes failure, $\tilde{t}_{c2}$, not to the hypothetical level given by $\tilde{t}_{c1}$.

The two damage/life examples to be given are specified by

$$\tilde{\sigma}_1 = 0.9$$
$$\tilde{\sigma}_2 = 0.5$$

$$\tilde{t}_1 = 9, \qquad \tilde{t}_{c1} = 10, \qquad \textit{Case A}$$
$$\tilde{t}_1 = 252.1, \quad \tilde{t}_{c1} = 280.1, \quad \textit{Case B}$$

$$\tilde{t}_{c2} = 10^5$$

$$r = 10$$

Only $\tilde{t}_1$ and $\tilde{t}_{c1}$ are varied between the two examples. Both cases have $\tilde{t}_1/\tilde{t}_{c1}=$ 0.9. The governing creep rupture curves are as shown schematically in Fig. 7. Case B represents that of a power law behavior as specified by the power 10, which is in the range of polymer behavior.

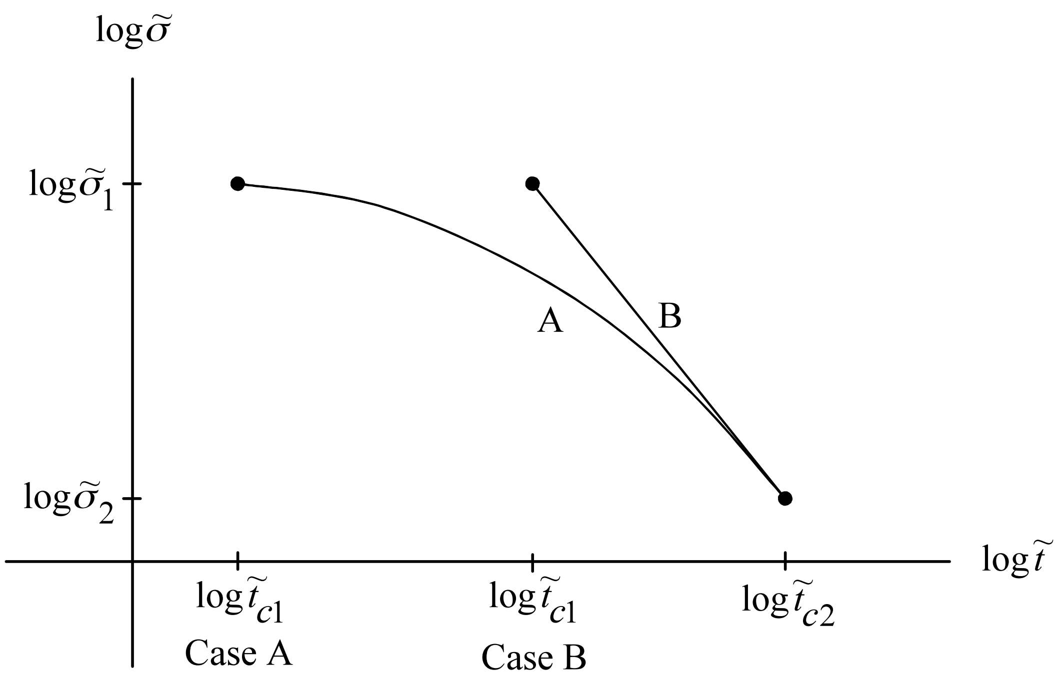

Fig. 7 Creep Rupture Curves for Cases A and B

Let the lifetime $\tilde{t}$ be specified by

$$\tilde{t} = \tilde{t}_1 + \Delta\tilde{t}$$

with $\Delta\tilde{t}$ designating the remaining life after the inflicted damage state of the first step. The results predicted by (23)-(26) are given below

Remaining Life, $\Delta\tilde{t}$

	Present	Broutman-Sahu	Hashin-Rotem	Miner's Rule
Case A	96,787.	82,000.	40,951.	10,000.
Case B	10,000.	82,000.	19,366.	10,000.

If there were negligible damage from the first step, the remaining life would be 10^5. Comparing the four models for Case A shows the present model to give the least damage in the first step while Miner's rule gives by far the most damage, resulting in the shortest remaining life. There is no consistency between any of the models, they all make completely distinct predictions in this case.

Comparing the four models for the power law form, Case B, Broutman-Sahu gives a unrealistically small amount of damage (large remaining life) while the other three models are about the same at small values of remaining life.

As seen from the above table Miner's rule and Broutman-Sahu cannot distinguish the amount of stress overload damage between Cases A and B even though the duration of the stress overload is 28 times greater in Case B than it is in Case A. It is only the present model that properly shows the strong effect in going from Case A to Case B.

Residual Life

As the final evaluation condition, the problem of residual life will be posed. For a given program of stress history, if the load were terminated just an instant before failure, the question is what would be the additional life at a reduced stress level. This is called the residual life problem Of course it would be extremely difficult to conduct this as an actual physical experiment because of the usual scatter in the data. However, it is very useful to use this as a conceptual test with various deterministic models to see which ones predict reasonable results and which ones predict unrealistic and unacceptable results.

The problem will now be posed in its simplest form as a two step process. The duration of the first step is taken as $\tilde{t}_1 \rightarrow \tilde{t}_{c1}$,but understood to be terminated just before failure. The second step is then for $\tilde{\sigma}_2 < \tilde{\sigma}_1$ reduced by a specified amount. Since both $\tilde{t}_{c1}$and $\tilde{t}_{c2}$ enter the problem formulation, it will be revealing to see which models involve both of these physical properties.

Let the residual life beyond step 1 be notated as

$$\tilde{t}_R = \tilde{t} - \tilde{t}_1$$

From (8) the Broutman-Sahu prediction for the residual life is given by

$$\tilde{t}_R = \left(\frac{\tilde{\sigma}_1 - \tilde{\sigma}_2}{1 - \tilde{\sigma}_2}\right)\tilde{t}_{c2} \qquad (27)$$

From (17) the present prediction for the residual life is given by

$$\tilde{t}_R = \tilde{t}_{c2} - \left(\frac{\tilde{\sigma}_1}{\tilde{\sigma}_2}\right)^r \tilde{t}_{c1} \tag{28}$$

Miner's rule, (5), and Hashin-Rotem, (6), both predict that there is no remaining residual life even though the stress level is reduced. It is seen that only the present model brings both $\tilde{t}_{c1}$ and $\tilde{t}_{c2}$ into the residual life prediction.

As an example of residual life take

$$\tilde{\sigma}_1 = 0.5$$
$$\tilde{\sigma}_2 = 0.4$$

$$\tilde{t}_{c1} = 10^5$$
$$\tilde{t}_{c2} = 9.5 \times 10^5$$

$$r = 10$$

The residual life predictions are

$$\tilde{t}_R = 18{,}677. \qquad \text{Present}$$
$$\tilde{t}_R = 158{,}333. \qquad \text{Broutman-Sahu}$$

Conclusion

The three model testing conditions: damage/life, residual strength, and residual life, show that the present model is the only one of the four that satisfies these consistency tests. More specifically, the damage/life examples reveal the importance of the maximum slope of the creep rupture-life envelope on log-log scales, Fig. 2. Only the present model includes this characteristic, as property r in (17). This property is important because it calibrates the kinetics of the flaw growth process. Without this defining property, two points on a stress versus life envelope cannot distinguish power law behavior from anything else. It is analogous to trying to define curvature by only two points. With this property included, it can be shown from (17) that in general for two step programs of loading, the high stress to low stress sequence produces longer lifetimes than does the low to high sequence. General observations usually confirm this specific sequence effect, Found and Quaresimin [10]. Miner's rule says there is no sequence effect. Interested readers could examine other models and deduce other conditions of evaluation in addition to those considered here.

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