# Understanding Celestial Navigation

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I grew up on the Jersey Shore very near the entrance to New York harbor and was fascinated by the comings and goings of the ships, passing the Ambrose and Scotland light ships that I would watch from my window at night. I wondered how these mariners could navigate these great ships from ports hundreds or thousands of miles distant and find the narrow entrance to New York harbor.

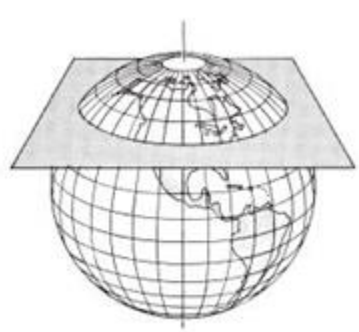
Celestial navigation was always shrouded in mystery that so intrigued me that I eventually began a journey of discovery. One of the most difficult tasks for me, after delving into the arcane knowledge presented in most reference books on the subject, was trying to formulate the “big picture” of how celestial navigation worked. Most texts were full of detailed "cookbook" instructions and mathematical formulas teaching the mechanics of sight reduction and how to use the almanac or sight reduction tables, but frustratingly sparse on the overview of the critical scientific principles of WHY and HOW celestial works. My end result was that I could reduce a sight and obtain a Line of Position but I was unsatisfied not knowing “why” it worked.

This article represents my efforts at learning and teaching myself 'celestial' and is by no means comprehensive. As a matter of fact, I have purposely ignored significant detail in order to present the big picture of how celestial principles work so as not to clutter the mind with arcane details and too many magical formulas. The USPS JN & N courses will provide all the details necessary to ensure your competency as a celestial navigator.

I have borrowed extensively from texts I've studied over the years including: **Primer of Navigation**, by George W. Mixter, **The American Practical Navigator** by Nathaniel Bowditch, **Dutton's Navigation & Piloting** by Elbert S. Maloney, **Marine Navigation Celestial and Electronic** by Richard R. Hobbs, **Celestial Navigation in the GPS Age** by John Karl, and, of course, the USPS Junior Navigation and Navigation manuals past (pre 2006) and present editions, et al.

My hope is that this treatise serves as a supplement to the USPS Junior Navigation and Navigation courses to help our instructors and students see the big picture of celestial navigation and not be left to ferret out the details on their own or have some expectation of an “Ah-Ha!” moment of revelation when it all suddenly makes sense.

A great circle.



A small circle.

# The Essence of Celestial Navigation

Imagine you are standing somewhere on the Earth's surface and you're not exactly sure where that is latitude and longitude-wise, but you have some idea.  We call that your deduced reckoning (DR) position.

Now imagine that the sun's light were focused like a laser pointer shining directly down onto the Earth's surface and where it hits the surface it is marked with an X.  We call that spot the sun's geographic position (GP).

We now have the Earth with two spots on its surface: your DR position and an X marking the GP of the sun.  In celestial navigation, we measure and plot the distance between these two spots.  If we know the distance to the sun's GP at a particular moment, then we could draw a circle on the Earth's surface with a radius equal to that distance (a Circle of Position (COP)), and we would surmise we were somewhere on that circle of position.

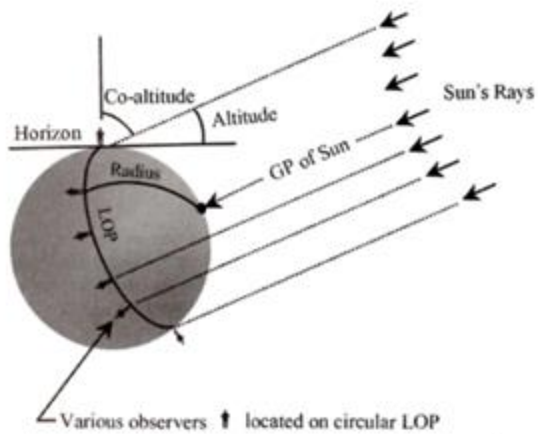



**Figure 1.3** All the equal-altitude points lie on a circular LOP with radius equal to the co-altitude, centered on the geographical position of the sun.

Using a sextant it is easy to measure the distance between the two locations.  The distance between your DR position and the sun's GP is directly related to the altitude of the sun as measured by the sextant.  The higher the altitude, the closer you are to the GP.  If the sun were directly overhead, you would be at the GP and your circle of position would be quite small.  If the sun were near the horizon, you may be thousands of miles from the GP and the circle of position would be very large.  Either way, your DR position would be somewhere on the circle of position.

To determine the distance between the DR and the GP, we subtract the measured sextant altitude from 90° to determine Co-Altitude and then multiply the Co-Altitude by 60.  The result is your distance from the GP in nautical miles.  For example, if the measured sextant altitude were 61°, as might be measured near midday in summer from Puget Sound, you would be 90° - 61° = 29° X 60 = 1,740 miles from the GP.

If we sighted a second celestial body, say the Moon, using the Moon's GP we would have two GPs.  And if we measured the altitude of the Moon with our sextant, we would have two circles of position on which we were located.  Basic navigation theory tells us that if we are located on two different circles of position, we must be located at one of the two places where the circles intersect; the one that is closest to our DR position.

We have now determined our position on the Earth's surface.

That **IS** the essence of celestial navigation. There are many more details that we need to take into account however. We must apply corrections to our sextant reading necessary to account for the fact that our eyes are not at sea level and for the refraction (bending) of light by the atmosphere we experience when viewing celestial bodies. We also need to learn about the Navigational Triangle that allows us to associate measured altitude to distance to the GP. And lastly, our Circles of Position are very large so, how do we plot them? Some of these details are covered below.

Now that you have the basic premise of celestial navigation read on to learn more.

## The Overview

In preparation to taking a sextant sighting we first determine, record, and plot our deduced reckoning (DR) position. We then use our sextant to measure the altitude of our selected celestial body above the visible horizon and record the altitude measured (Hs) along with the exact time (second, minute, and hour) of our sighting. Once that is completed we next apply some corrections (covered later) to our measurement to arrive at our Observed Altitude (Ho). The altitude measured tells us (indirectly (explained below)) our location's distance from the GP of the selected celestial body.

We now must ask: Were we actually located at our DR position when we took the sighting? We have no way of confirming do we? How can we verify our location? Here's how: The nature of the data contained in the Nautical Almanac is detailed such that we can use the latitude and longitude of our DR position to calculate what the altitude of the sighted celestial body would be if measured from that latitude and longitude at the time we took our sighting! Once the altitude calculation (Hc) is completed we can then compare the altitude we calculated (Hc) to the altitude we actually measured (Ho).

If the two altitudes are identical then our location is confirmed to be at our DR position. If the two altitudes differ then our location is not at our DR. Then where are we located relative to the GP? The answer is simple: What is the difference between our two altitudes Hc & Ho? This difference is called the *intercept* We learned earlier that one minute of angle is equal to one nautical mile. So, for example, if our Hc were say 31° 41.8' and our Ho was 31° 38.9 the difference between Hc & Ho is 2.9' or 2.9 nautical miles. This tells us that we are located 2.9 nautical miles from our DR position, but in which direction? Are we closer to the celestial GP or farther away? Once again the answer is pretty simple. If Hc is greater than Ho we must be farther **away** from the GP because the altitude we measured (Ho) is smaller than calculated (Hc). If Ho were greater than Hc we must be closer to (**toward**) the GP because the altitude we measured (Ho) is greater than calculated (Hc).

In order to plot our position accurately, we also need an accurate bearing (azimuth). Where can we find one? Once again we can use the data from the Nautical Almanac to calculate the azimuth from our location to the Geographic Position (GP) of the selected celestial body that we must have been on at the moment we took our sextant measurement.

Once we have calculated the azimuth, we lightly plot the azimuth through our DR position and mark our intercept (2.9 nautical miles in this example) on that azimuth in the direction opposite the GP (AWAY). Again, it is plotted away because Hc is greater than Ho in this example. Had Ho been greater than Hc we would plot the intercept TOWARD the GP.

**Rule: If Ho > Hc - Plot the intercept in the direction (TOWARD) of the GP; If Ho < Hc - Plot the intercept farther AWAY from the GP.**

The point plotted is our estimated position (EP). It is an EP because it is based upon a single sextant sighting. If we solved a sighting on a second celestial body (within 20 minutes of time) we could then plot both points for a "fix" of our position.

## Understanding Celestial Navigation

When we think of celestial navigation, for many, our thoughts wander to the age of exploration and names like Magellan, da Gama, Vespucci, Columbus, Drake, Hudson, Cook, (circa 1454 -1779) however, the principles used in celestial navigation were discovered well before that time. Think of the celestial knowledge necessary for placing the stones of Stonehenge which began in 2600 BC.

The Greek astronomer and mathematician Eratosthenes (276 - 194 BC) made some practical observations that lead to the discovery of the principles used today in celestial navigation. Eratosthenes observed that at noon, around the time of the summer solstice, vertical posts at Alexandria cast a shadow on the ground, whereas at Syene (present day Aswan) it was reported that posts there cast no shadow at that time and the sun illuminated the entire bottom of a well at noontime.

This observation led Eratosthenes to believe the earth must be spherical and the sun's rays are essentially parallel to each other. This inference enabled him to make some calculations that were truly elegant in their simplicity and that proved the earth is spherical and moreover, allowed him to calculate the earth's circumference at 25,000 miles (today's measurement is 24,901 sm or 21,653 nm miles). He determined the sun's rays were vertical at Syene and $7^{1/4°}$ from vertical at Alexandria or $1/50^{th}$ of a circle. He then used the distance between Alexandria and Syene, 500 miles, to calculate 50 * 500 = 25,000.

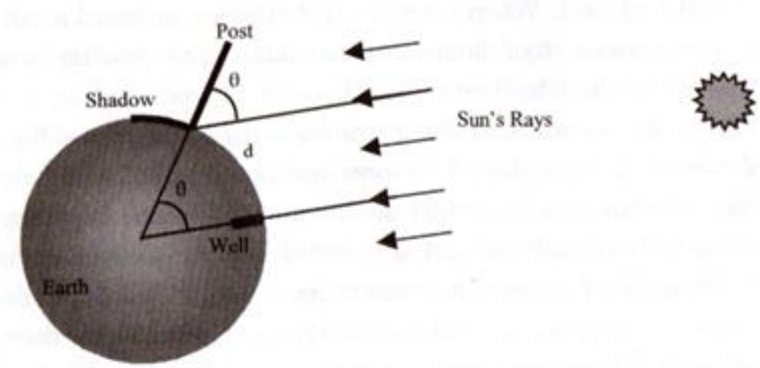



**Figure 1.1** The sun casts a shadow of the post at Alexandria, but the sun is directly overhead the well at Syene. The distance d, between Alexandria and Syene, is proportional to the angle θ, the angle between the sun's rays and the post.

Figure 1.1 Eratosthenes' Angle θ

Although Eratosthenes made some assumptions that affected the accuracy of his measurements, many of today's experts are astonished at the accuracy of his calculations. So, how are Eratosthenes' observations related to celestial navigation? They provided a method to calculate the distance (see d in Figure 1.1) between two places on the earth using Eratosthenes' angle θ! From his observations comes the formula **distance = 60 X θ**. This formula has become the guts of celestial navigation.

## The Mariner's Angle

A ship's motion at sea makes measuring Eratosthenes' angle from vertical too difficult to measure. Instead, navigators use a sextant to measure the angle from the horizontal, as seen in Figure 1.2 below. The sextant measures the sun's altitude from the horizon and we find that the altitude is 90° minus the angle of Eratosthenes! In celestial navigation we call the Eratosthenes' angle the Co-Altitude. The two angles are complements of each other, meaning their sum is 90°.

In celestial navigation, the Co-Altitude is used to calculate the distance of the observer from a point on the earth directly beneath the sun (or other celestial body), called the geographical position or GP.

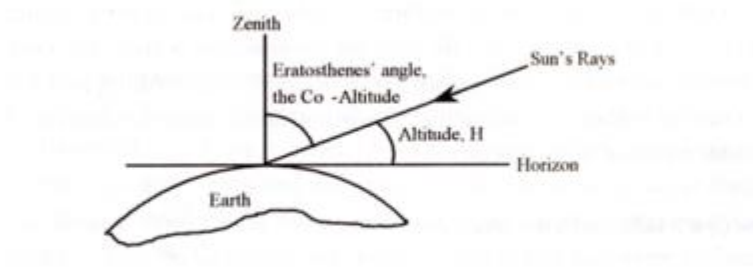



**Figure 1.2** The sextant measures the sun's altitude H, which is the angle the sun's rays make with the horizon at the observer's location.

Figure 1.2 Co-Altitude

## The Equal-Altitude Line of Position (Circle of Position)

Figure 1.3 below shows us a picture of the sun's rays in relationship to the spherical earth. The altitude and co-altitude of the sun's rays at one observer's position are shown at the top of the figure. All the other observers shown in the figure are located where they see the identical altitude. We can see that these equal-altitude locations must lie on a circle centered on the sun's geographical position (GP) with a radius equal to the observer-to-GP distance. This radius is the same distance d as in the example of Eratosthenes' as shown in Figure 1.1, the radius length is just 60 nm (60 nm per degree) times the co-altitude. So **distance = 60 nm x co-altitude**.

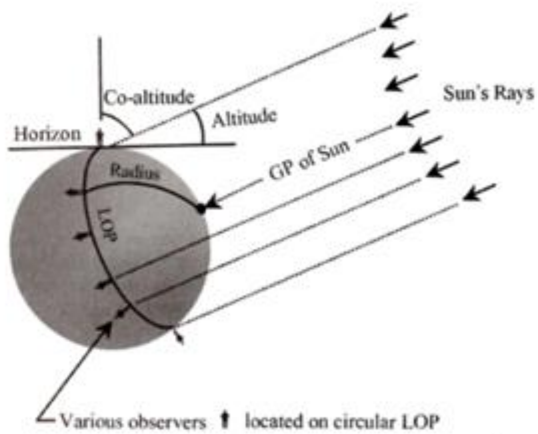



**Figure 1.3** All the equal-altitude points lie on a circular LOP with radius equal to the co-altitude, centered on the geographical position of the sun.

Figure 1.3 Equal Altitude Circle

So, by measuring the sun's altitude and subtracting that altitude from 90°, we learn that our position lies somewhere on this circle of equal altitude. For example, if the altitude we measured was 21° 23.7' then 90 - 21° 23.7' = 68° 36.3' (68.605°) now using Eratosthenes' formula d = 60 * 68.605 = 4116.3 nm is the radius of the circle of equal altitude. We are located somewhere on that circle. Our job becomes one of narrowing the possibilities to find a plausible location.

The altitude measured by each observer depends on his/her distance from the sun's GP. The closer the observer is to the sun's GP, the greater the observed altitude, and conversely, the farther away the observer is, the less the altitude. If you were located at the sun's GP, the sun would be directly overhead, its altitude would be 90°, and your co-altitude would be zero thus your distance from the GP would be zero nm (90° - 90° = 0). To see what I mean, find a room in your house with a ceiling light. Position yourself near a wall and point at the light. Now walk toward the light while continuing to point at the light. See how you have to raise your arm as you move closer? The altitude increases as you get closer.

Since one sextant observation just tells us we're somewhere on this large circle of position, we need more information to fix our location on this circle. In celestial navigation, this is usually done by making an observation of a second celestial body to obtain a second circle of position. With just one observation and using celestial mathematics we'll be able to identify our Estimated Position (EP) with just a few miles of error. However, with two observations, we'd be able to develop a "fix" of our position as shown in figure 1507 below.

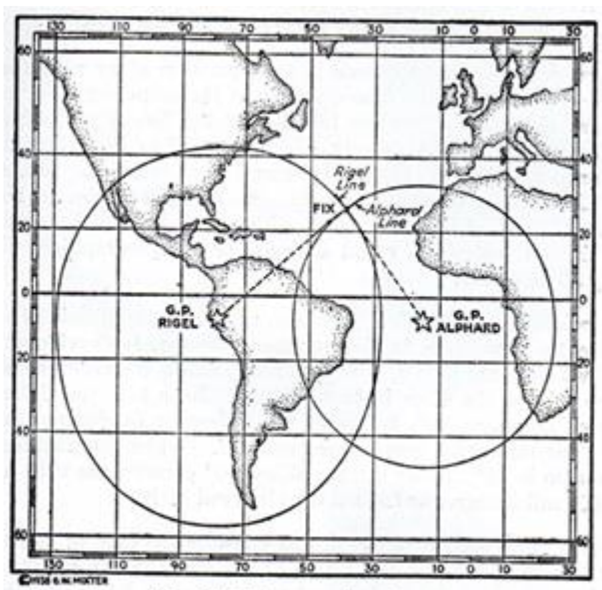



FIG. 1507. FIX FROM TWO STARS.

Figure 1507 Fix from two stars

As shown in the Figure 1507 above, these two circles of position would intersect in two places, leaving an ambiguity between the two possible locations. However, as we can see in the figure, these circles of position are quite large making the elimination of one of the two intersections quite easy; we must be located at the intersection closest to our DR position.

## Using the Nautical Almanac

The British first published the Nautical Almanac and Astronomical Ephemeris in 1766, with data for 1767. The Nautical Almanac is published yearly and jointly by Her Majesty's Nautical Almanac Office and the Nautical Almanac Office of the U. S. Naval Observatory. Today's Nautical Almanac contains data that we can use to determine the precise Geographic Position (GP) of the celestial bodies used in navigation (Sun, Moon, Venus, Mars, Jupiter, Saturn, and 57 selected stars) at any second of time throughout the year of the almanac. By knowing this GP location and our observed altitude taken with the sextant, we learn the radius and location of our circle of position. Remember the GP is at the center.

## Time

The positions of celestial bodies are listed in the almanac by date and time. It is extremely important that we record the exact date and time that we took our sextant sighting, the Day, Hour, Minute, and Second. I'll leave a detailed discussion of time to your JN and N course instructors.

To view a real-time interactive graphic of how the Sun's altitude and azimuth changes over time, visit TimeandDate.com

## Interesting Note About Almanac Data

If our Latitude and Longitude are known, we can use the almanac data for a particular day and a little arithmetic to determine any listed celestial body's altitude, azimuth, or geographic position (GP) at any second of any minute of any hour of that day from our position.

Conversely, in Celestial Navigation, we use the almanac data and a little arithmetic to determine a celestial body's altitude, azimuth, and GP to find our Latitude and Longitude at the hour, minute, and second of our sextant sighting.

## The Limitations of Mechanical Methods

Plotting such huge circles of position on our charts however, is impractical for two reasons; 1) a chart covering an area that large would have such a small scale that accurate plotting of our position would not be possible and conversely, 2) a chart with a large enough scale to allow accurate plotting would be physically too large and impractical for use on the vessel.

Since mechanical methods will not work, we'll have to use a mathematical solution. We will not delve into how the mathematics, known for over one thousand years, were developed we'll just use it.

## The Captain Marq de St Hilaire Method (Azimuth & Intercept)

The purpose of sight reduction is to determine the latitude and longitude of some point on the all-important circular equal-altitude COP and to do it in a relatively simple way. After all, mariners should not have to be mathematicians in order to navigate. Captain St Hilaire published his method in 1875 and it meets those requirements.

Captain St Hilaire discovered a method of reducing a celestial observation for finding position using the circle of equal altitudes that does NOT require attempting to plot these huge circles on our charts.

He learned that the Nautical Almanac data could be used to locate the GP of a celestial body as normal and, after locating the GP, also was sufficient to allow him to choose virtually any position (latitude & longitude) and then be able to calculate the altitude an observer would measure of that particular celestial body, if the observer were actually located at that position. He could then compare the two positions, the calculated position and his position based on his sextant measurement to find the difference in the two altitudes, one measured and one calculated. This difference tells him that his measured observation was taken at a position a distance equal to the difference from the chosen position he used for the calculated solution.

Of course, he would not choose just any position, he would choose a reference position close to where he believed he was located, such as his DR position. He would then calculate what the altitude of the celestial body would be from that location and then compare that calculated altitude to his actually observed altitude of his sextant reading to learn if there was a difference. If the two altitudes, observed and calculated, were exactly the same then he could conclude that the ship was indeed located at the reference position when the sight was taken. If the altitudes differed then he was not located at the reference position and his position was actually "off" by the difference in altitudes.

Here's an example using the data used earlier:

An observed altitude (Ho) of 21° 23.7' resulted in a COP with a radius of 4116.3 nm. Suppose we calculated an altitude (Hc) from a reference position that resulted in an Hc of 21° 21.6' with a resulting radius of 4118.4 nm. If we compare the radii of the circles we have a difference of 2.1 nm. Now here's the magical part that Captain St Hilaire discovered: Instead of determining Co-Altitudes (90 - Ho and 90 - Hc) and calculating and comparing the radii, just compare Ho to Hc. In our example Ho is 21° 23.7', and Hc is 21° 21.6' what is the difference between them? 2.1 nm the same as when we compared the radii!! So, we learn the difference without having to calculate Co-Altitude and therefore, eliminate the need for huge charts!

But, what does the 2.1 nm difference mean? It means that at the time of our observation of the celestial body with the sextant we were actually a distance of 2.1 nm offset from the chosen reference position! So, at this point, we know a bit more but we'll have to determine the bearing to use to plot a point 2.1 nm different. We also need to determine if the 2.1 nm is in a direction closer to the GP or farther away.

By comparing Ho & Hc we can see that if Ho is greater than Hc we must have been 2.1 nm closer to the GP or if Hc is greater than Ho we must have been 2.1 nm farther away. (See the earlier narrative about altitude as we approach the GP).

Now, having two known locations on the globe, 1) the GP of the body and 2) the reference position, Captain St Hilaire learned he could mathematically calculate the azimuth from the reference position to the GP. This is the azimuth the ship must have been on at the moment he took the sextant reading!

Once the azimuth is calculated, we can now plot the reference position (L, Lo) on our active chart, and then measure and plot a position 2.1 nm TOWARD the GP (if Ho > Hc as in our example) or AWAY from the GP (if Ho < Hc) along the azimuth calculated as the azimuth from the reference position to the GP and thus we arrive at a plotted latitude & longitude as our Estimated Position (EP).

**Note:** Look closely at Figure 1507 above. Hover your mouse on the image to zoom and note, in the FIX location, that, because the COP is so large, we plot only a small segment of each COP in our vicinity, as a straight line (Line of Position) perpendicular to the azimuth to the GP. If we have only one COP, we have an Estimated Position (EP), with two COPs we would have a FIX. This plotting strategy eliminates the need to plot huge COPs. We can instead use a plotting sheet scaled to our area of interest, plot our DR position, the segment(s) of our COP, then plot our position along the calculated azimuth.

It is an Estimated Position because we have only a single sight. To be able to plot a fix we'd need to take a second sighting (see Figure 1507 above) of another celestial body (within 20 minutes of time) or if the celestial body is the sun, we can wait and sight the sun a second time 2 - 6 hours later and plot that LOP and advance the earlier LOP for an R-Fix as we learn in the JN course.

Does all this mean we can just forget about Co-Altitude? No! Our Law of Cosines formulas that we'll use to calculate Hc and the azimuth to the GP will use Co-Altitude, et al to arrive at the solutions we seek. But we need not bother with calculating and comparing the radii of the COPs, we'll just compare Ho and Hc to find the difference, which is called the **intercept**, and determine if the intercept is TOWARD or AWAY from the GP and plot that point along the calculated **azimuth**, hence the name "Azimuth - Intercept" method.

It's appropriate to call our preselected point a reference point, because it refers to the geographical area where we want to plot our LOP. This reference point can be anywhere but is usually a position chosen near the ship and, in the USPS JN & N courses, is typically our DR position but, does not have to be. Traditionally, this point has been called the Assumed Position (AP) but that is misleading, we are not assuming we are located there. The AP simply says we want to locate a portion of the COP nearest this location. Don't get confused, no assumptions are being made, we are just choosing a L, Lo in the vicinity of our location and using it to calculate what the altitude of the celestial body would actually be from that position.

Our reference position, the GP of the celestial body, and the earth's pole nearest to our reference position become the three vertices of a very important triangle called the **Navigational Triangle**. Using the Law of Cosines formulas and the data we extract from the Nautical Almanac, we will solve the Navigational Triangle, compare the Ho we've obtained from taking our sight to Hc, to obtain a Line of Position and subsequent EP. We'll learn the details about the Navigational Triangle and how to use the Law of Cosines formulas during our JN course.

Here's a look at the Law of Cosines formulas that we will learn to use in the JN course to find Hc and the azimuth to the GP. They may look intimidating at the moment but we're only concerned with how to use them not how they were derived so it'll be rather straight-forward.

$$\sin H_C = (\cos LHA * \cos Lat * \cos Dec) + (\sin Lat * \sin Dec)$$

$$\cos Z = (\sin Dec - (\sin Lat * \sin H_C)) \div (\cos Lat * \cos H_C)$$

Note that the formulas do not use Ho! We measure Ho from our *actual* sextant sighting. We then solve a *calculated* altitude (Hc) to compare with our actual altitude (Ho) to learn the **intercept**. The formulas solve the Navigational Triangle to find the calculated altitude (Hc) and the azimuth (Z) from our reference position to the GP of the body. After finding Hc, we compare our observed altitude (Ho) to the calculated altitude (Hc) to find the intercept difference that we then plot along the azimuth (Zn) either TOWARD or AWAY from the GP. This plotted position becomes our EP.

## Summary - The Navigational Triangle

The objective of celestial navigation is to locate our position by solving a spherical triangle comprised of three vertices:

1. A location chosen near our position (DR) - Provides Co-L (90 - Latitude) an easily determined value

2. The Geographic Position (GP) of the observed celestial body - Provides Co-Dec (90 - Declination) an easily determined value
3. The pole of the earth nearest our location. - A known location

   The side labelled Co-H (our distance from the GP) is the **unknown** side for which we are solving.

**Note:** Many students get confused by using the DR as a vertex to find position, believing the DR position as uncertain, it is **not**. The DR has an unambiguous latitude and longitude that can be precisely plotted. What is uncertain is our location. We may be at the DR or not; that is what we'll learn by solving the Navigational Triangle.

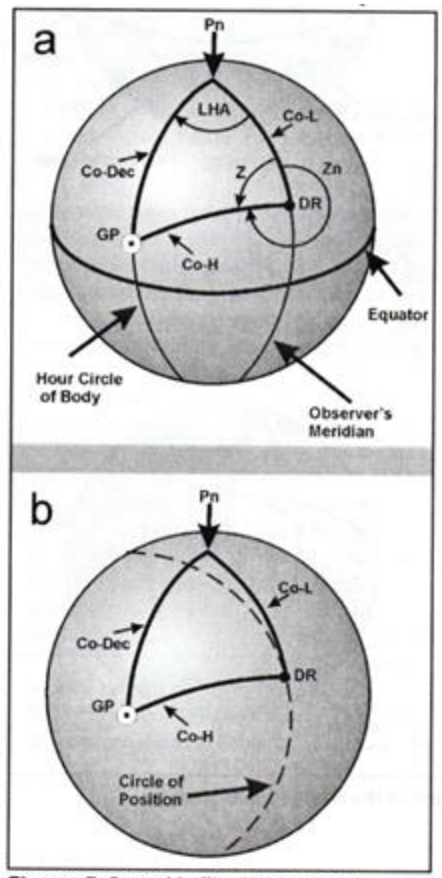
See Figure 7-5a below. We'll also be able to determine the Local Hour Angle labelled LHA in the figure as it is the difference in longitude between our chosen reference position and the longitude of the GP of the celestial body. We now know two sides of the triangle (Co-L & Co-Dec) and the LHA angle. This gives us enough information to solve the triangle for Co-H (complement of Hc, the distance to the GP).

We now solve to determine Co-H (Co-Altitude), the unknown side of the triangle and the angle Z. While it may appear complicated, extracting data from the Nautical Almanac along with our Law of Cosines formulas, used to solve the triangle, are pretty straight forward and does not require complicated work on our part, just some simple arithmetic.

The solution of the triangle provides the mariner with:

1. The true azimuth (Zn) from the reference position to the GP of the celestial body at the time of the sight, 360° - Z in the figure.
2. The calculated altitude (Hc) of the celestial body from the reference position at the precise time the sextant sight was taken.

Looking at Figure 7-5b you can see that the solution results in a COP (dashed line) with the GP of the body at the center. That is the COP of the reference position we chose. Comparing our observed altitude (Ho) to our calculated altitude (Hc) tells us that, at the time of our sight, we were "off" that COP (and hence the DR) along azimuth Zn by the difference (the Intercept).




**Figures 7–5a and b The Navigational Triangle**

When we compare the calculated altitude (Hc) to our sextant sighting observed altitude (Ho), we plot the difference either TOWARD (Ho > Hc) or AWAY (Ho < Hc) from the GP along the true azimuth (Zn).

At this point, I hope I have reached my objective that you have a big picture view of the principles of celestial navigation in action. I have purposely omitted many of the details and minutia of reducing a sextant sight to a position, for clarity. These details will be amply covered in your JN and N courses. Happy navigating!