Unemployment Rates Forecasts – Unobserved Component Models versus SARIMA models

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Abstract:

In this paper we focus on the comparison of unemployment rates forecasting accuracy using time-varying parameter models and SARIMA models. We are particularly interested in the forecasts of the unemployment rate of eight Central and Eastern European first-wave accession countries: Estonia, Latvia, Lithuania, Czech Republic, Poland, Slovakia, Hungary and Slovenia within 1999-2015 years. We use a rolling short-term forecast experiment in order to obtain out-of-sample test of forecast accuracy. Moreover, we examine also the dynamic asymmetries in unemployment rates and the forecasting performance of different models. We find that the forecasting ability of the models depends not only on the forecasting horizon, but also on the direction of the movement in unemployment rates. The empirical evidence derived from our investigation suggests that there is no best single model, however SARIMA models although not including a "cyclical" component tend to perform better than others for a longer forecasts horizons.

JEL classification codes: C22, C53, E27;

Keywords: unemployment rate, unobserved component, SARIMA models, forecasting accuracy

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Introduction

An important question in forecasting of time series is which model is the best one. For more or less forty years ARMA-type models have been used for modelling and forecasting economic time series. This approach has a certain feature: all shocks, coming either from the cycle or from other sources, are included in these model's innovations. Simultaneously in the last years the unobserved component models seem to become very promising tool in forecasting different economic series as it allows to separate time series components. In this paper we compare the forecasting accuracy of few unobserved component models and few specifications of seasonal autoregressive integrated moving average (SARIMA) models. We are interested in comparison of different models of unemployment rates series in several Central and Eastern European (CEE) countries..

Neftci [1984] indicate that some macroeconomic series display asymmetric behaviour. In case of the unemployment rates they have a tendency to rise suddenly, but fall gradually (Koop and Potter 1999). In this paper we are interested if there are the differences of the unemployment rates' forecasts accuracy at the time of increase and decrease of these rates. For the purpose of forecasting we use linear models: in case of structural time series modelling level, trend, seasonality and cyclical components are included, allowing for the coefficients on each predictor to be either time variable or constant over time. In the case of SARIMA models we consider two different specifications. The forecasting performance of different models is compared in different horizons and different times in order to indicate the best model.

A number of research papers have used time series models for forecasting unemployment rates. These works are devoted either to single unemployment rate, where clearly the most popular is the US unemployment rate (e.g. Montgomery et al. 1998, Altissimo and Violante 2001, Caner and Hansen 2001, Proietti 2003, Koop and Potter 1999) or a comparison of models used in forecasting unemployment rates from different economies, eg. OECD countries (Skalin and Teräsvirta 2002, U.S., U.K., Canada, and Japan (Milas and Rothman 2005), G7 countries (Teräsvirta et al. 2005) and the Baltic States (Będowska-Sójka 2015).

Many works are devoted to comparison of different models. Montgomery, Zarnowitz, Tsay and Tiao [1998] in a rolling forecasts experiment for the US quarterly unemployment rates show that non-linear models performed better than the linear ARMA model in terms of forecasting errors when the unemployment increased rapidly but not elsewhere. Stock and Watson [1999] used a large data set of U.S. macroeconomic time series, including the monthly unemployment rate, and showed that linear models have better forecasting accuracy than nonlinear ones. Oppositely, Teräsvirta et al. [2005] find that the nonlinear LSTAR model turns out to be better than the linear or neural network models when modelling unemployment rates in G7 countries.

Marcellino (2002) generated forecasts of three key economic variables: the growth rate of industrial production, the unemployment rate and the inflation. He showed that best forecast for industrial production was obtained within linear models, whereas for the unemployment rate the non-linear models generate better forecasts. Proietti [2003] investigated the out-of-sample performance of linear and nonlinear structural time series models of the seasonally adjusted US unemployment rate. Generally linear models are said to perform significantly better than nonlinear models, but a nonlinear specification outperforms the selected linear model at short lead times in periods of slowly decreasing unemployment rate.

The main purpose of this paper is to compare an accuracy of unemployment rate forecast's obtained from different linear models, namely structural time series models and SARIMA models. Our approach is much in the same spirit of Proietti (2003) and Będowska-Sójka (2015) as it concentrates on the comparison of forecasting models on the basis of the short-term forecasts. With respect to Proietti (2003) paper we focus on seasonally unadjusted data from several countries and use linear structural time series models only. In Będowska-Sójka (2015) few unobserved component models are compared from the perspective of forecasts generated for unemployment rates of the three Baltic states: Lithuania, Latvia and Estonia. In that paper it is shown that models which contain cyclical components perform better than other unobserved component models (Będowska-Sójka 2015).

Our sample data consists of seasonally unadjusted monthly unemployment rates of the eight CEE countries that joined European Union in 2004 in the first-wave accession. These countries are: Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia, and Slovenia. The sample starts in January 1999 and ends in March 2015 (with some exceptions described below). The forecasts of unemployment rates are generated from the rolling forecasts experiment where seasonality effects are built directly into the forecasting procedure. In this paper we consider all forecasts origin starting from January 2008 and ending in March 2014. The forecasts are set to horizons from one month to one year. As the rolling window generally consists of 108 observations we obtain 75 forecasts for each series and each models. In order to compare forecasts from different models, we use common forecasting error measures. To the best of our knowledge this is the first study that compares unemployment rate forecasts within these eight CEE countries.

Our contribution is as follows: first in six out of eight cases seasonal ARIMA models offered better forecasting accuracy than the unobserved component models. Second, when comparing models across all countries in the sample, there are substantial differences between their forecasting abilities; the lowest mean percentage forecasting error for 12-month horizon is 1.82% in case of Slovakian unemployment rate and the highest is 8.67% for the Estonian one. In case of Estonian, Latvian and Slovenian unemployment rates shocks that increase unemployment rates tend to have greater negative impact on the model's forecasting ability than shocks that lower unemployment rate. Finally, the differences in forecasting errors obtained from different methods are generally not serious.

The plan of the paper is as follows. Next section describes the methodology used in the empirical study. Then data are presented and empirical results of the comparison of forecasts are shown. In the last section the conclusions are presented.

Methodology

Our paper aims to compare forecasts from two alternative specifications that are used to represent the dynamic properties of series, namely unobserved component models (UC) and seasonal ARIMA models. When the disturbances are independent, identically distributed and Gaussian, an ARIMA model with restrictions in the parameters is the reduced form of an unobserved component model [Harvey 1989]. There is one aggregated disturbance within the specification of ARIMA models, whereas unobserved component models include component disturbances. Thus, the latter may allow to discover the characteristics, that are not observed in the reduced form of ARIMA model. In this paper we try examine which of these two classes of the models is more appropriate when forecasting the unemployment rates.

The theory of structural time series models and ARIMA models is presented by Harvey [1989] – below a short presentation of the models used in the study is given.

Within ARIMA models we use two specifications:

I. Seasonal ARIMA(0,1,1)(0,1,1) – henceforth SARIMA1

II. Seasonal ARIMA(2,1,0)(0,1,1) – henceforth SARIMA2.

When unobserved component models are taken into account, the general structural model is written as [Harvey 1989]:

$$y_t = \mu_t + \gamma_t + \psi_t + \varepsilon_t \qquad \varepsilon_t \sim \mathrm{NID}(0,\ \sigma_\varepsilon^2) \qquad t = 1,...,T \tag{1}$$

where y_t represents the time series to be modelled and forecast, μ_t is the trend component, γ_t is the seasonal component, ψ_t is the cyclical component, ε_t represents the irregular component and NID denotes Normally and Independently Distributed. All of these components are assumed to be unobserved. We use three specifications of UC models:

III. Basic Structural Model (BSM)

$$y_t = \mu_t + \gamma_t + \varepsilon_t$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \qquad \eta_t \sim \mathrm{NID}(0, \sigma_\eta^2)$$

$$\beta_t = \beta_{t-1} + \zeta_t \qquad \zeta_t \sim \mathrm{NID}(0,\ \sigma_\zeta^2) \tag{2}$$

where μ_t represents the stochastic level of the trend and β_t represents the stochastic slope of the trend. It is also assumed that ε_t, η_t and ζ_t are independent variables. Additionally, γ_t is trigonometric seasonal component described as:

$$\gamma_t = \sum_{j=1}^{s/2} \gamma_{j,t} \tag{3}$$

with *s* standing for the number of seasons, *s* = 12 in our case.

Each $\gamma_{jt,t}$ is generated by:

$$\begin{bmatrix} \gamma_{j,t} \\ \gamma^*_{j,t} \end{bmatrix} = \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{j,t-1} \\ \gamma^*_{j,t-1} \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega^*_{j,t} \end{bmatrix}, \quad j = 1,.....,[s/2],\ t = 1,....,T \tag{4}$$

where $\lambda_j = 2\pi j / s$ is the frequency and $\omega_{j,t}$, $\omega^*_{j,t}$, the seasonal disturbances, are mutually uncorrelated ($\omega_{j,t} \sim \mathrm{NID}(0, \sigma^2_{\omega_j})$, $\omega^*_{j,t} \sim \mathrm{NID}(0, \sigma^2_{\omega^*_j})$) and uncorrelated with ε_t.

As the unemployment rate tends to move in a countercyclical way [Montgomery et al. 1998], we expect that a cyclical component might improve unemployment rates forecasts. Therefore the next model is:

IV. Structural Model Plus Cycle (SMC)

$$y_t = \mu_t + \gamma_t + \psi_t + \varepsilon_t, \qquad \mu_t = \mu_{t-1} + \eta_t$$

In the model the statistical specification of a cycle, ψ_t, is defined by:

$$\begin{bmatrix}\psi_t \\ \psi_t^*\end{bmatrix} = \rho \begin{bmatrix}\cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c\end{bmatrix}\begin{bmatrix}\psi_{t-1} \\ \psi_{t-1}^*\end{bmatrix} + \begin{bmatrix}\kappa_t \\ \kappa_t^*\end{bmatrix}, \qquad t = 1, \ldots T \tag{5}$$

where: λ_c is the frequency (in radians), $0 < \lambda_c < \pi$, ρ is a damping factor, $0 \le \rho \le 1$ and κ_t, κ_t^* are mutually uncorrelated white noise disturbances with zero means and common variance σ_κ^2.

The last model included in the study is:

V. Autoregressive Structural Model (ARSM)

$$y_t = \mu_t + y_{t-1} + \gamma_t + \varepsilon_t, \qquad \mu_t = \mu_{t-1} + \eta_t \qquad \eta_t \sim \mathrm{NID}(0,\ \sigma_\eta^2) \tag{6}$$

where μ_t represents the stochastic level of the trend , γ_t is trigonometric seasonal component described in equations (3) and (4), ε_t represents the irregular component (as in equation 1).

As the primary use of time series models is forecasting, it seems that mean square error MSE would be adequate criterion in judging the models performance [Montgomery et al. 1998]. We use out-of-sample forecasts to assess which model gives the better accuracy. These forecasts are generated in a rolling forecasts window: for the given origin the model is estimated and forecasts are generated. Next, this step is repeated for each model and each series – hence we obtain 75 forecasts from one-step ahead till twelve-step ahead for each series. The only exception is the series of unemployment rates of Slovakia, where the data starts in 2006 – in this case we roll the forecasts one step at a time forward, each time re-estimating the model by extending the estimation window.

Finally, for all series and forecasts we calculate different forecasting errors and identify the models with the lowest errors. We also divide whole forecasts origin into increases and decreases in unemployment rates and examine if there are any differences between forecasting errors in these two states.

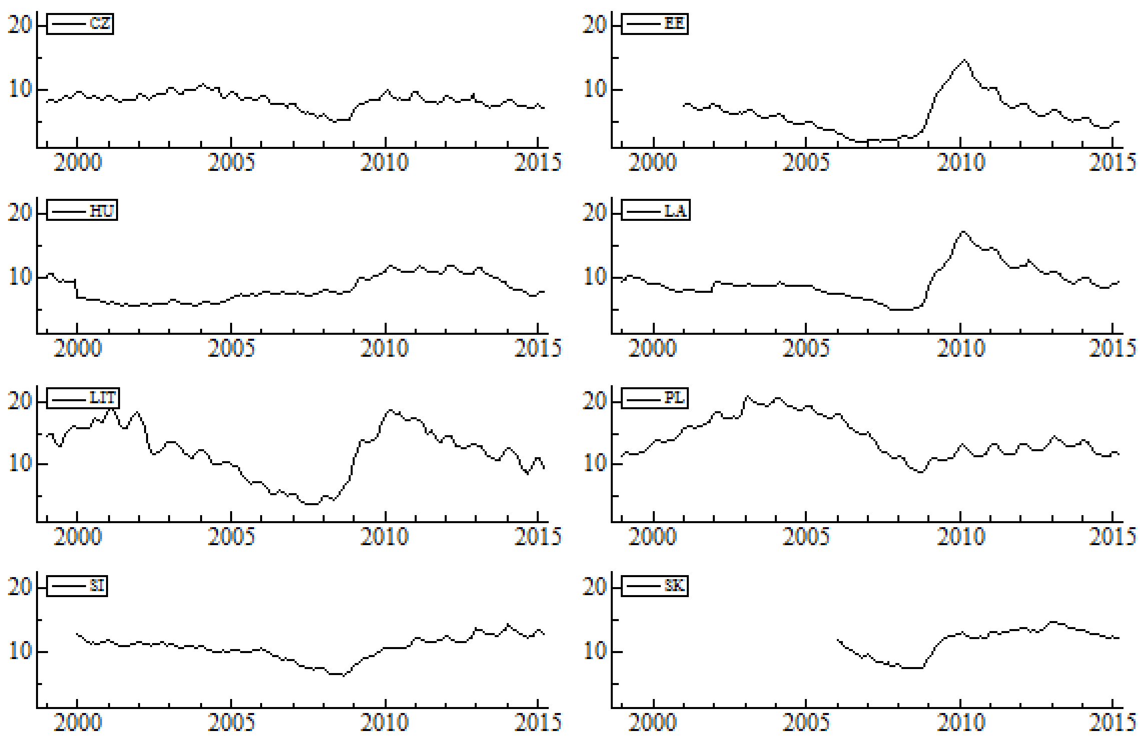
Data

Our sample data consists of monthly unemployment rates from eight first-wave accession Central and Eastern European countries that joined European Union in May 2004. There are (in alphabetical order): Czech Republic (CZ), Estonia (EE), Hungary (HU), Latvia (LA), Lithuania (LIT), Poland (PL), Slovenia (SI) and Slovakia (SK). We consider logarithms of monthly seasonally unadjusted series. The seasonality is included in the models: in the unobserved component models seasonal component is modelled as a stochastic one.

The data source is CEIC database (www.ceic.com).The sample starts in January 1999 and ends in March 2015 with some minor exceptions. The data for Estonian unemployment rate starts in 2001, for Slovenia starts in 2000, and for Slovakia in 2006 (in all cases the first month of the available data is January). In case of the series that are available since January 1999 starting from that date each model is estimated and forecasts from 1 month till 12 months are computed. The process is repeated until the end of sample is reached. In case of Estonian and Slovenian unemployment rate the pre-forecasts period is extended until it reaches 108 observations and then the rolling window procedure is applied. The experiment provides in total 75 forecasts for horizons from one-month to one-year for each model and each series. In case of unemployment rate of Slovakia pre-forecasts period is extended each time with a new information till March 2014 when the last forecast are generated.

The forecasts origin consists of the period of more or less rapid increase in the unemployment rates as well as the gradual decrease what give the possibility of observing the forecasting accuracy in different business cycle phases.

Figure 1 Unemployment rates in the first-wave EU accession CEE countries within 1999.01-2015.03



CZ stands for Czech Republic, EE for Estonia, HU for Hungary, LV for Latvia, LIT for Lithuania, PL for Poland, SI for Slovenia and SK for Slovakia.

The calculations and graphics are done in OxMetrics STAMP7 [Koopman, Harvey, Doornik and Shephard 2006, Doornik and Hendry 2005].

Figure 1 shows how unemployment rates changes within the sample period. There is no single tendency for the unemployment rates in the region at that time, but some common features are recognizable. At the beginning of the sample some unemployment rates are increasing and some decreasing. Starting from 2001 the unemployment rates in the region are decreasing (with Hungarian rate as the exception). There is also a visible change in all series as they start to increase sharply in the beginning or the mid of 2008 and start to decrease in the mid 2010 (with the exception of Slovenia). Zooming it the single series behave differently, some having huge differences between the lowest and the highest point. In the whole sample the highest unemployment rate was observed in Poland in March 2003 and the lowest in Estonia in December 2006. The common feature is the dynamic asymmetry which is observable in all series: the decrease in unemployment rates is rather gradual, whereas the increase is very steep.

Empirical results

The comparative performance of a rolling forecast experiment is presented in three steps. In the first one an out-of-sample test of forecast accuracy for the whole forecasts origin is shown. Then we compare the forecasts errors in two cases: increase and decrease in the series. In the third step, the errors are depicted together with the series in order to illustrate in which periods we observe the biggest and the lowest errors.

We report comparative performance of the rolling forecasts in the models used in the study and described earlier. Tables 1 presents the different forecasting errors for each series whereby: $\tilde{y}_{t+l|t}$ is the *l*-ahead forecast for a given model, the Mean Error (ME) is obtained as an average of forecasts errors, $y_t - \tilde{y}_{t+l|t}$, the Mean Square Forecast Error (MSFE) is calculated as square root of averages of $(y_t - \tilde{y}_{t+l|t})^2$, and the Mean Absolute Percentage Error, MAPE, is obtained as an average of $\left| y_t - \tilde{y}_{t+l|t} \right| / y_t * 100\%$. These errors are reported for 1-month and 1-year horizon.

Table 1. Comparison of forecasts performance in the test period 2008.1-2015.3 for unemployment rates CEE countries

	CZ			EE			HU			LA		
1 month	ME	RMSE	MAPE	ME	RMSE	MAPE	ME	RMSE	MAPE	ME	RMSE	MAPE
SARIMA1	0.0058	**0.0206**	**1.0120**	0.0080	0.0364	2.1943	**-0.0020**	0.0151	0.6683	0.0022	0.0272	1.1926
SARIMA2	0.0020	0.0210	1.0257	0.0043	0.0211	1.3709	-0.0029	**0.0151**	**0.6684**	**0.0002**	**0.0183**	**0.8225**
BSM	-0.0029	0.0220	1.0781	-0.0009	0.0206	1.1983	0.0021	0.0157	0.6962	-0.0011	0.0192	0.8552
SMC	**-0.0019**	0.0228	1.1146	**-0.0001**	0.0223	1.2923	0.0025	0.0152	0.6728	**0.0001**	0.0203	0.9006
ARSM	-0.0029	0.0220	1.0781	-0.0009	**0.0206**	**1.1932**	0.0021	0.0157	0.6961	-0.0011	0.0192	0.8539
12 months												
SARIMA1	0.0831	**0.1100**	**4.5645**	-0.0411	0.3245	13.8566	-0.0518	0.0911	3.4156	0.0210	0.2244	7.9093
SARIMA2	0.0309	0.1270	5.2363	**0.0248**	0.2329	9.8575	-0.0558	0.0880	3.2697	0.0085	**0.1793**	**6.3304**
BSM	-0.0327	0.1716	7.0297	0.0556	0.2197	8.6889	0.0251	0.0904	3.4197	-0.0035	0.1885	6.6558
SMC	**-0.0007**	0.1798	7.3484	0.1326	0.2729	10.9082	0.0366	**0.0840**	**3.1354**	0.0151	0.2016	7.0545
ARSM	-0.0316	0.1725	7.0636	0.0560	**0.2193**	**8.6662**	**0.0250**	0.0904	3.4184	**-0.0033**	0.1888	6.6646

	LIT			PL			SI			SK		
1 month	ME	RMSE	MAPE	ME	RMSE	MAPE	ME	RMSE	MAPE	ME	RMSE	MAPE
SARIMA1	0.0052	0.0326	1.4241	0.0044	0.0116	0.4765	0.0034	0.0133	0.5862	-0.0030	0.0110	0.4269
SARIMA2	0.0046	0.0276	1.1976	0.0008	**0.0091**	**0.3698**	0.0006	**0.0111**	**0.4842**	-0.0006	**0.0097**	**0.3759**
BSM	**0.0002**	0.0274	1.1946	-0.0008	0.0103	0.4230	-0.0003	0.0126	0.5497	0.0003	0.0115	0.4695
SMC	0.0007	**0.0261**	**1.1086**	**-0.0008**	0.0099	0.4051	-0.0008	0.0124	0.5364	**-0.0001**	0.0113	0.4369
ARM	0.0012	0.0268	1.1757	-0.0008	0.0103	0.4247	**-0.0003**	0.0126	0.5503	0.0003	0.0116	0.4771
12 months												
SARIMA1	**0.0002**	0.2193	7.5631	0.0669	0.0805	2.8133	0.0461	**0.0927**	**3.4312**	-0.0504	0.0544	1.8580
SARIMA2	0.0301	**0.1992**	7.0236	0.0267	**0.0612**	**2.1209**	0.0129	0.0962	3.4656	-0.0319	**0.0542**	**1.8284**
BSM	0.0721	0.2190	7.3663	-0.0179	0.0821	2.8268	-0.0086	0.1122	4.0394	-0.0079	0.1210	4.0687
SMC	0.0677	0.2000	**6.5930**	**-0.0176**	0.0761	2.6076	**0.0010**	0.1138	4.0938	0.0227	0.0699	2.3577
ARSM	0.0687	0.2181	7.3264	-0.0177	0.0827	2.8453	-0.0083	0.1124	4.0467	**-0.0066**	0.1218	4.0972

The bolded values are the lowest in a given horizon.

In most cases the lowest forecasts errors are obtained from the same model for 1 month and 12 months horizon. The average difference between forecasting errors from different models are rather small. In terms of considered forecasting errors, the greatest accuracy is provided by one of the seasonal ARIMA models (for CZ, LA, PL, SI and SK for both horizons, whereas for HU for 1 month horizon). On aggregate the seasonal ARIMA models outperform unobserved component models. The empirical evidence speaks strongly against BSM model as it is the only one which is outperformed by other models for all series. There is a trade-off between a Mean Error and the Mean Square Forecast Error or Mean Absolute Percentage Error: the lowest forecasts' bias measured by Mean Error is observed for the models that have higher forecasts' variability.

In the next step the forecasts origin is divided into two subsamples depending on increase or decrease (or remaining at the same level) of unemployment rates. The formerly described errors are calculated separately for these two states. Table 2 presents the results of *t*-test of equality of two sample means [Snedecor and Cochran 1989].

Table 2 Two-sample *t*-Test for equal means of errors in time of unemployment rates' increase or decrease

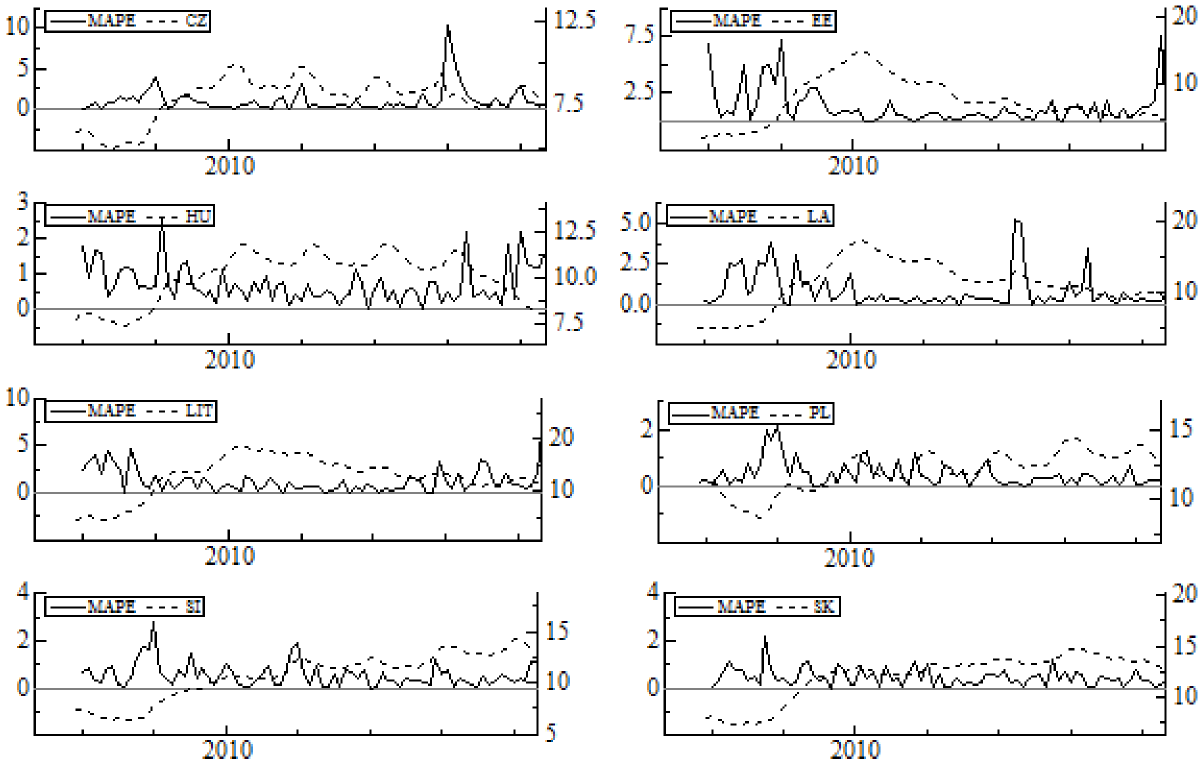
	CZ	EE	HU	LA	LIT	PL	SI	SK
1 month	-0.6726	**-3.0655**	1.1359	**-2.4533**	-1.1712	-1.6840	**-3.3242**	-0.4870
12 months	-1.8739	**-4.8674**	0.7862	**-2.9217**	-1.7194	0.2594	-1.4220	-0.2920

Bolded values are statistically significant at significance level $\alpha = 0.05$. The statistics are presented for seasonal ARIMA(2,1,0)(0,1,1) model and MAPE errors, however the results of the statistical interference are not changed for other models as well as for ME or MSFE.

According to the numbers presented in Table 2, in case of Estonian, Latvian and Slovenian one-month forecasts of unemployment rates, errors coming from the forecasts generated for the time of increase in unemployment rates are systematically higher than errors obtained in case of decrease in unemployment rates. This result holds also for Estonian and Latvian 12-month forecasts.

We also present one-step ahead Mean Absolute Percentage Error from SARIMA2 model for one month and 12 months horizon in 75 consecutive periods in forecasts origin together with the unemployment series. Figure 2 shows that the forecasting accuracy scores better in periods of gradual decrease or increase in unemployment rates and deteriorates in the beginning of the periods of rapid increase or decrease in the series. Similar behavior, although not presented here, characterizes the MAPE of multistep forecasts and other errors taken into account in the study (ME and MSFE). It is contrary to what was found in Proietti (2003) with respect to US unemployment rate.

Figure 2 Unemployment rates and one month MAPE for eight first-wave EU accession CEE countries within 1999.01-2015.03



CZ stands for Czech Republic, EE for Estonia, HU for Hungary, LV for Latvia, LIT for Lithuania, PL for Poland, SI for Slovenia and SK for Slovakia.

Conclusion

In this paper we have examined the out-of-sample performance of two alternative specifications that are used to represent the dynamic properties of time series, namely linear models for unemployment rates of eight CEE countries that have accessed European Union in May 2004. As the main interest is to select the best forecasting models according to their post-sample performance, we have used rolling forecasts experiment and examine, which model generated the best forecasts. Starting in January 1999 and ending in March 2015 our sample consists of the periods of decrease and increase in unemployment rates.

We find that for the monthly data in majority of cases seasonal ARIMA models perform better than unobserved component models considered in the study. The forecasting ability across different series is surprisingly differential. Generally speaking ARIMA models prove to be a very useful forecasting tool, both for 1 month and 12 months horizon. Only for two series in the sample, the Estonian and the Hungarian unemployment rates, the structural time series models give better forecasts.

When periods of increases and decreases in the unemployment rates are considered separately, forecasting errors for these two states are significantly different only in three cases. Last but not least the forecasting accuracy deteriorates in periods of rapid upward and downward movement and improves in periods of gradual change in the unemployment rates.

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