Unfired pressure vessels- Part 3: Design

Analysis performed by:		**Analysis performed by:**	
Analysis version:		**According to procedure:**	
Calculation case:	Unfired pressure vessels	**EDMS Reference:**	EF EN 13445-3 V1

Introduction:

This Mathcad template is made for the purpose of aiding, automating and simplifiying the calculations of various parameters concerning unfired pressure vessels, according to the standard NF EN 13445-3. Please note that the worksheet is intended as a supplementary tool to the standard, and should therefore always be used alongside the standard, and never alone. To ensure minimal risk of error, it is recommended (although not necessary) to open a new version of the worksheet for each new calculation. When printing the worksheet, it is recommended to save as pdf and print in A4 size. Due to the nature of hidden areas in Mathcad, it may be necessary to add a few lines to ensure no text or calcultations gets split in two between pages.

To use the worksheet, all relevant input parameters should be entered first. The worksheet uses two sets of input parameters: Global input parameters and local input parameters. The global input variables are values concerning the materials of the vessel and the inteded pressure, and are found under this introduction. The local input variables concern the specific geometry of the vessel. Each section contains its own relevant input parameters at the top of its hidden area. Every input parameter is coloured $blue$. After all relevant input parameters have been entered, the worksheet will automatically calculate the output values further down. The most relevant output parameters are $green$. An $orange$ value might be of interest to the reader, but it is not the end result.

Global input parameters:

0.2% proof strenght:	$R_{p0.2;T} := 200\ MPa$	[MPa]
1.0% proof strenght at temperature T:	$R_{p1.0;T} := 98\ MPa$	[MPa]
1.0% proof strenght at test temperature:	$R_{p1.0;Ttest} := 110\ MPa$	[MPa]
0.2% proof strenght at test temperature:	$R_{p0.2;Ttest} := 211\ MPa$	[MPa]
Tensile strenght at temperature T:	$R_{m;T} := 100\ MPa$	[MPa]
Tensile strenght at test temperature:	$R_{m;Ttest} := 100\ MPa$	[MPa]
Tensile strenght at temperature 20C:	$R_{m;20} := 120\ MPa$	[MPa]

Calculation of design stress as shown in 6.6:

The value of f_{dn} above, where n is the desired steel designation, should be entered here as f	$f := 154\ MPa$	[MPa]
Joint coefficient:	$z := 1$	[unitless]
Calculation pressure:	$P := 4\ MPa$	[MPa]

Clause 7 - Shells under internal pressure

7.4.2 Cylindrical shells

Local input parameters:

Internal diameter of shell: $D_i := 25\ mm$ [mm]

External diameter of shell: $D_e := 28.64\ mm$ [mm]

Calculation values:

Mean diameter of cylinder part:

$$D_m := \frac{1}{2} \cdot (D_e + D_i) = 26.82\ mm$$

Analysis thickness:

$$e_a := \frac{1}{2} \cdot (D_e - D_i) = 1.82\ mm$$

Output parameters:

Minimum required thickness of shell:

$$e := \frac{P \cdot D_i}{2\ f \cdot z - P} = 0.329\ mm$$ 7.4-1

OR $$e := \frac{P \cdot D_e}{2\ f \cdot z + P} = 0.367\ mm$$ 7.4-2

For a given geometry, the maximum internal pressure is:

$$P_{max} := \frac{2\ f \cdot z \cdot e_a}{D_m} = 20.901\ MPa$$ 7.4-3

Conditions of applicability:

$status\_R1 :=$ if $\frac{e}{D_e} \le 0.16$ return "OK" "Not Valid"

if only the internal diameter is known: $status\_R1_i :=$ if $\frac{e}{D_i + 2\ e} \le 0.16$ return "OK" "Not Valid"

Requirements		**Results**
R1:	$\frac{e}{D_e} \le 0.16$	$status\_R1 =$ "OK"
		$status\_R1_i =$ "OK"

7.4.3 Spherical shells

Local input parameters:

Internal diameter of shell: $D_i := 45\ mm$ [mm]

External diameter of shell: $D_e := 45.89\ mm$ [mm]

Calculation values:

Mean diameter of cylinder part:

$$D_m := \frac{1}{2} \cdot (D_e + D_i) = 45.445\ mm$$

Analysis thickness:

$$e_a := \frac{1}{2} \cdot (D_e - D_i) = 0.445\ mm$$

Output parameters:

Minimum required thickness of shell:

$$e := \frac{P \cdot D_i}{4\ f \cdot z - P} = 0.294\ mm$$ 7.4-4

OR $$e := \frac{P \cdot D_e}{4\ f \cdot z + P} = 0.296\ mm$$ 7.4-5

For a given geometry, the maximum internal pressure is:

$$P_{max} := \frac{4\ f \cdot z \cdot e_a}{D_m} = 6.032\ MPa$$ 7.4-6

Conditions of applicability:

$$status\_R1 := \begin{array}{l} \text{if } \frac{e}{D_e} \le 0.16 \\ \quad \text{return "OK"} \\ \text{"Not Valid"} \end{array}$$

if only the internal diameter is known:

$$status\_R1_i := \begin{array}{l} \text{if } \frac{e}{D_i + 2\ e} \le 0.16 \\ \quad \text{return "OK"} \\ \text{"Not Valid"} \end{array}$$

Requirements		Results	
R1:	$\frac{e}{D_e} \le 0.16$	$status\_R1 = \text{"OK"}$	$status\_R1_i = \text{"OK"}$

7.6.4 Conical shells

Local input parameters:

Internal diameter of cone at point under consideration:	$D_i := 20\ mm$	[mm]
External diameter of cone at point under consideration:	$D_e := 22\ mm$	[mm]
Mean diameter of cone at point under consideration:	$D_m := 21.03\ mm$	[mm]
Mean diameter of cylinder at junction with cone:	$D_c := 300\ mm$	[mm]
Analysis thickness of cone at point under consideration:	$e_{con.a} := 1.36\ mm$	[mm]
Semi angle of cone at apex:	$\alpha := 30\ °$	[degrees]

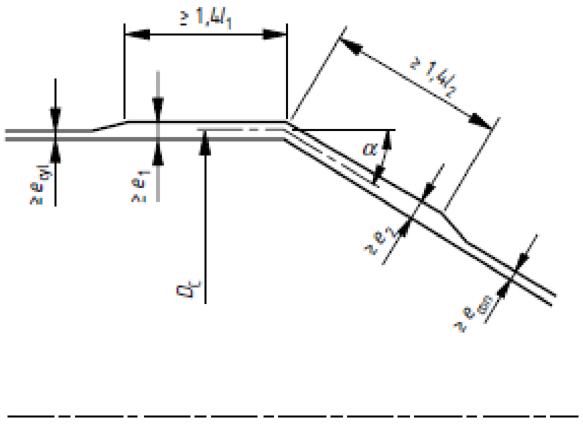

Figure 7.6-1 — Geometry of cone/cylinder intersection without knuckle — Large end

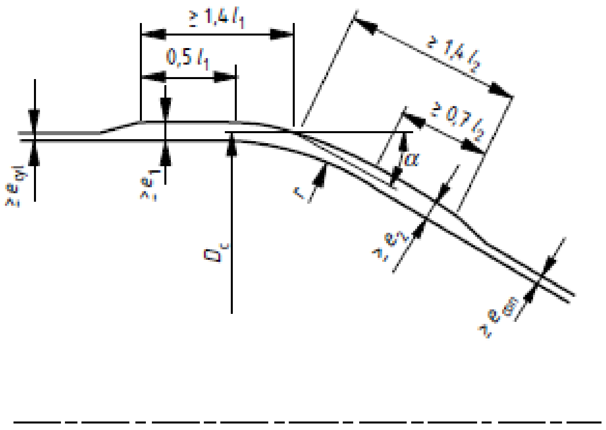

Figure 7.6-2 — Geometry of cone/cylinder intersection with knuckle — Large end

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Output parameters:

Minimum required thickness of the conical part is:

$$e := \frac{P \cdot D_i}{2\ f \cdot z - P} \cdot \frac{1}{\cos(\alpha)} = 0.304\ mm$$ 7.6-2

OR

$$e := \frac{P \cdot D_e}{2\ f \cdot z + P} \cdot \frac{1}{\cos(\alpha)} = 0.326\ mm$$ 7.6-3

For a given geometry, the maximum internal pressure is:

$$P_{max} := \frac{2\ f \cdot z \cdot e_{con.a} \cdot \cos(\alpha)}{D_m} = 17.25\ MPa$$ 7.6-4

NOTE: both the input and output parameters in this section only considers one specific point on the cone, so multiple calculations may be required

Conditions of applicability:

$status\_R1 :=$ if $\alpha \le 75\ °$
  return "OK"
 "Not Valid"

$status\_R2 :=$ if $\frac{e_{con.a} \cdot \cos(\alpha)}{D_c} > 0.001$
  return "OK"
 "Not Valid"

Requirements		**Results**
R1:	$r \le 0.2\ D_i$	$status\_R1 =$ "OK"
R2:	$r \ge 0.06 \cdot D_i$	$status\_R2 =$ "OK"

7.5.3 Torispherical ends

Input parameters:

Inside radius torispherical end:	$R := 600\ mm$	[mm]
Inside radius of curvature of a knuckle:	$r := 40\ mm$	[mm]
Internal diameter of cylindrical flange:	$D_i := 430\ mm$	[mm]
Analysis thickness:	$e_a := 2\ mm$	[mm]

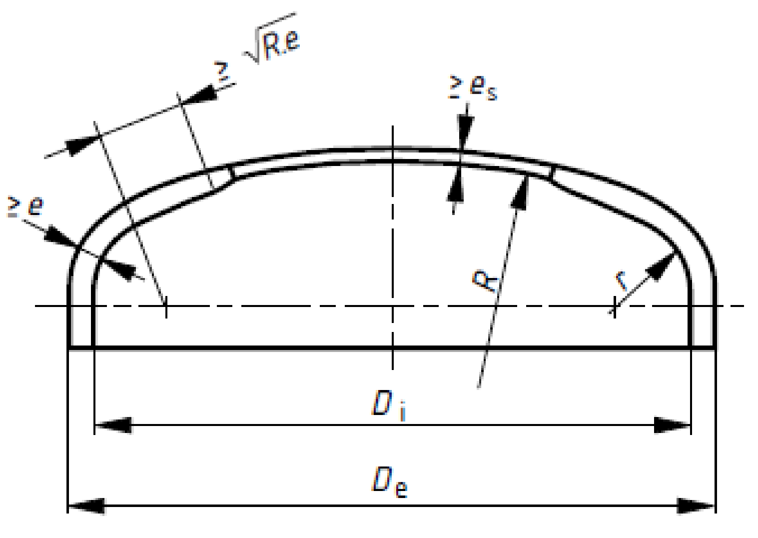


Figure 7.5-3 — Geometry of torispherical end

7.5.3.2 Design

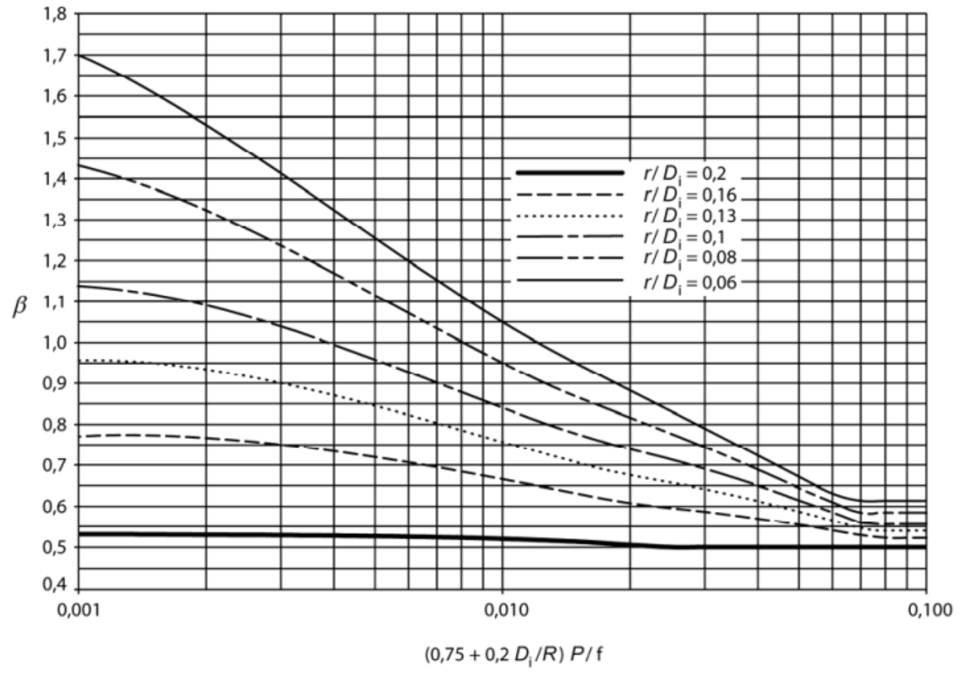


Figure 7.5-1 — Parameter β for torispherical end – Design

$\frac{r}{D_i}=0.093$

$\left(0.75+0.2\ \frac{D_i}{R}\right)\cdot\frac{P}{f}=0.023$

The value of β_a can be read from the chart above or calculated through the iterative process shown in 3.5.3.5, a) $\beta_a:=$ [unitless]

Calculation values:

$$f_{b1}:=\frac{R_{p0.2;T}}{1.5}=133.333\ MPa$$

7.5-4

OR, for cold spun seamless austentic stainless steel:

$$f_{b2}:=\frac{1.6\ R_{p0.2;T}}{1.5}=213.333\ MPa$$

7.5-5

The correct value for f_b should be entered with regard to the material used: $f_b:=133\ MPa$ [MPa]

(at test conditions the value 1.5 in the f_b equations shall be replaced by 1.05)

The required tickness e shall be the greatest of e_s, e_y and e_b, where:

$$e_s:=\frac{P\cdot R}{2\ f\cdot z-0.5\ P}=7.843\ mm$$

7.5-1

$$e_y:=\beta_a\cdot P\cdot\frac{(0.75\ R+0.2\ D_i)}{f}=?\ mm$$

7.5-2

$$e_b:=(0.75\ R+0.2\ D_i)\cdot\left(\frac{P}{111\ f_b}\cdot\left(\frac{D_i}{r}\right)^{0.825}\right)^{\left(\frac{1}{1.5}\right)}=8.286\ mm$$

7.5-3

Output parameters:

$$e:=\max\left(e_s,e_y,e_b\right)=?\ mm$$

NOTE 1 For stainless steel ends that are not cold spun, f_b will be less than f.

NOTE 2 The 1.6 factor for cold spun ends takes account of strain hardening.

NOTE 3 The inside height of a torispherical end is given by:

$$h_i := R - \sqrt{\left(R - \frac{D_i}{2}\right) \cdot \left(R + \frac{D_i}{2} - 2\ r\right)} = 68.046\ mm$$

7.5.3.3 Rating

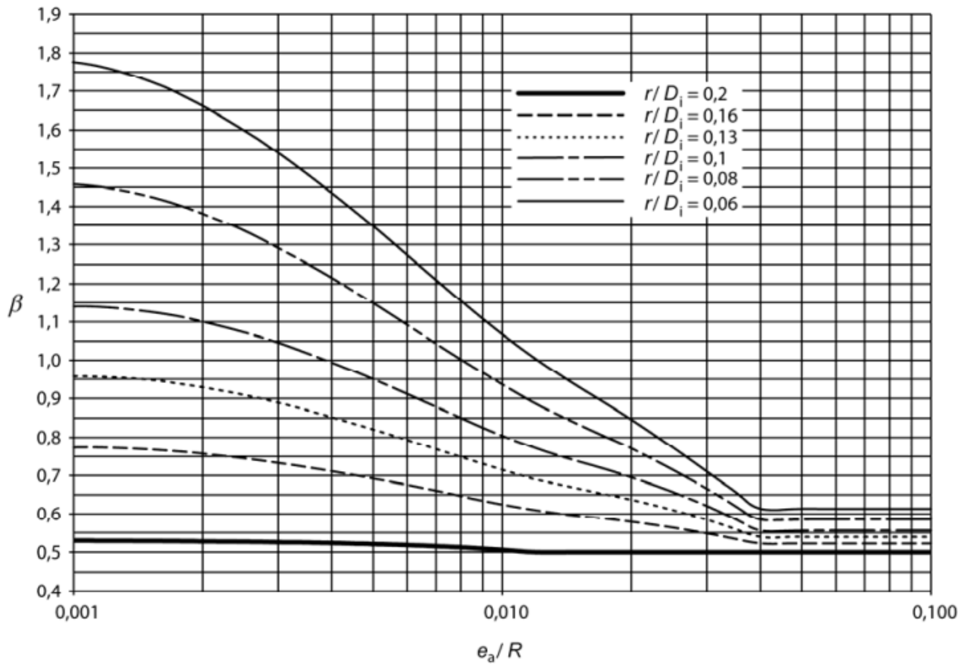


Figure 7.5-2 — Parameter ß for torispherical end - rating

$$\frac{e_a}{R} = 0.003$$

$$\frac{r}{D_i} = 0.093$$

The value of β_b can be read from the chart above or calculated through the process shown in 3.5.3.5, b) $\beta_b :=$ ▯ [unitless]

Calculation values:

$$f_{b1} := \frac{R_{p0.2;T}}{1.5} = 133.333\ MPa$$

7.5-4

OR, for cold spun seamless austentic stainless steel:

$$f_{b2} := \frac{1.6\ R_{p0.2;T}}{1.5} = 213.333\ MPa$$ 7.5-5

The correct value for f_b should be entered with regard to the material used: $f_b := 133\ MPa$ [MPa]

(at test conditions the value 1.5 in the fb equations shall be replaced by 1.05)

For a given geometry Pmax shall be the least of Ps, Py and Pb, where:

$$P_s := \frac{(2\ f \cdot z \cdot e_a)}{R + 0.5\ e_a} = 1.025\ MPa$$ 7.5-6

$$P_y := \frac{f \cdot e_a}{\beta_b \cdot (0.75\ R + 0.2\ D_i)} = ?\ MPa$$ 7.5-7

$$P_b := 111\ f_b \cdot \left(\frac{e_a}{0.75\ R + 0.2\ D_i}\right)^{1.5} \left(\frac{r}{D_i}\right)^{0.825} = 0.474\ MPa$$ 7.5-8

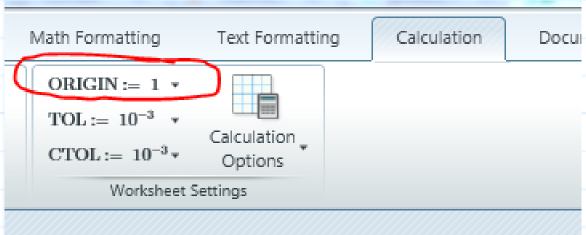
Output parameters:

$$P_{max} := min\left(P_s, P_y, P_b\right) = ?\ MPa$$

7.5.3.4 Exceptions

7.5.3.5 Formulae for calculation of factor β

a) iterative process for finding the minimum thickness



NB! Origin has to be 1

Functions

$Y(e_y) := min\left(\frac{e_y}{R}, 0.04\right)$ $Z(e_y) := \log\left(\frac{1}{Y(e_y)}\right)$ $N(Y) := 1.006 - \frac{1}{\left(6.2 + (90\ Y)^4\right)}$

$e_y(\beta) := \beta \cdot P \cdot \frac{(0.75\ R + 0.2\ D_i)}{f}$ $f_\Delta(x, y) := \frac{y - x}{x}$

Calculation quantities

$X := \frac{r}{D_i} = 0.093$

Initial beta (β): $\beta_0 := 1$ $0.4 \le \beta \le 1.8$ (The initial value does not have any impact on the result)

Convergence limit: $\Delta_{lim} := 0.001$ (Percentage change)

Beta Code

```
β_code := | Δ ← 1
          | i ← 1
          | β_i ← β_0
          | while Δ > Δ_lim
          |   | e_i ← e_y(β_i)
          |   | Y ← Y(e_i)
          |   | Z ← Z(e_i)
          |   | N ← N(Y)
          |   | β_(i+1) ← | β_0.06 ← N·(−0.3635·Z^3 + 2.2124·Z + 1.8873)
          |   |           | β_0.1 ← N·(−0.1833·Z^3 + 1.0383·Z^2 − 1.2943·Z + 0.837)
          |   |           | β_0.2 ← max(0.95 (0.56 − 1.94 Y − 82.5 Y^2), 0.5)
          |   |           | β_A ← 25 ((0.1 − X)·β_0.06 + (X − 0.06)·β_0.1)
          |   |           | β_B ← 10·((0.2 − X)·β_0.1 + (X − 0.1)·β_0.2)
          |   |           | if X = 0.06
          |   |           |   | β_0.06
          |   |           | if 0.06 < X < 0.1
          |   |           |   | β_A
          |   |           | if X = 0.1
          |   |           |   | β_0.1
          |   |           | if 0.1 < X < 0.2
          |   |           |   | β_B
          |   |           | if X = 0.2
          |   |           |   | β_0.2
          |   | Δ ← |f_Δ(β_i, β_(i+1))|
          |   | i ← i + 1
          | return β
```

Outputs

From all iterations:

$\beta_{code} = \begin{bmatrix} 1.0000 \\ 1.2145 \\ 1.2035 \\ 1.2044 \end{bmatrix}$ $e_y(\beta_{code}) = \begin{bmatrix} 13.922 \\ 16.908 \\ 16.756 \\ 16.767 \end{bmatrix} mm$

Number of iterations: $n := length(\beta_{code}) = 4$

Resulting values:

$\beta := \beta_{code_n} = 1.2044$ $e_y := e_y(\beta) = 16.767\ mm$

The minimum required thickness is then:

$e := \max(e_s, e_y, e_b) = 16.767\ mm$

b) non-iterative process for finding maximum pressure

$Y := min\left(\frac{e_a}{R}, 0.04\right) = 0.003$ 7.5-9

$Z := \log\left(\frac{1}{Y}\right) = 2.477$ 7.5-10

$X := \frac{r}{D_i} = 0.093$ 7.5-11

$N := 1.006 - \frac{1}{\left(6.2 + (90\ Y)^4\right)} = 0.845$ 7.5-12

$\beta_{0.06} := N \cdot \left(-0.3635 \cdot Z^3 + 2.2124 \cdot Z + 1.8873\right)$ 7.5-13

$\beta_{0.1} := N \cdot \left(-0.1833 \cdot Z^3 + 1.0383 \cdot Z^2 - 1.2943 \cdot Z + 0.837\right)$ 7.5-15

$\beta_{0.2} := \max\left(0.95\ \left(0.56 - 1.94\ Y - 82.5\ Y^2\right), 0.5\right)$ 7.5-17

For 0.06<X<0.1 $\beta_A := 25\ \left((0.1 - X) \cdot \beta_{0.06} + (X - 0.06) \cdot \beta_{0.1}\right)$ 7.5-14

For 0.1<X<0.2 $\beta_B := 10 \cdot \left((0.2 - X) \cdot \beta_{0.1} + (X - 0.1) \cdot \beta_{0.2}\right)$ 7.5-16

$\beta_b :=$
```
if X = 0.06
    β ← β_0.06
if 0.06 < X < 0.1
    β ← β_A
if X = 0.1
    β ← β_0.1
if 0.1 < X < 0.2
    β ← β_B
if X = 0.2
    β ← β_0.2
```

$\beta_b = 1.12$ => $P_y := \frac{f \cdot e_a}{\beta_b \cdot (0.75\ R + 0.2\ D_i)} = 0.513\ MPa$

$P_{max} := min\left(P_s, P_y, P_b\right) = 0.474\ MPa$

7.5.3.1 Conditions of applicabillity

$status\_R1 :=$
```
if r ≤ 0.2 D_i
    return "R1 OK"
"R1 Not Valid"
```

$status\_R3 :=$
```
if r ≥ 2·e
    return "R3 OK"
"R3 Not Valid"
```

$status\_R5 :=$
```
if e_a ≥ 0.001·D_e
    return "R5 OK"
"R5 Not Valid"
```

$status\_R2 :=$
```
if r ≥ 0.06·D_i
    return "R2 OK"
"R2 Not Valid"
```

$status\_R4 :=$
```
if e ≤ 0.08·D_e
    return "R4 OK"
"R4 Not Valid"
```

$status\_R6 :=$
```
if R ≤ D_e
    return "R6 OK"
"R6 Not Valid"
```

Requirements		Results
R1:	$r \le 0.2\ D_i$	$status\_R1 =$ "R1 OK"
R2:	$r \ge 0.06 \cdot D_i$	$status\_R2 =$ "R2 OK"
R3:	$r \ge 2 \cdot e$	$status\_R3 =$ "R3 OK"
R4:	$e \le 0.08 \cdot D_e$	$status\_R4 =$ "R4 Not Valid"

R5: $e_a \geq 0.001 \cdot D_e$ $status\_R5 = \text{"R5 OK"}$

R6: $R \leq D_e$ $status\_R6 = \text{"R6 Not Valid"}$

Clause 15 - Pressure vessels of rectangular section

15.5 Unreinforced vessels

Input parameters:

The inside corner radius: $a := 50\ mm$ [mm]

The dimensions of the vessel: $l_1 := 400\ mm$ [mm]
(see figure 15.5-1)

$L := 367\ mm$ [mm]

The thickness of the vessel: $e := 3\ mm$ [mm]

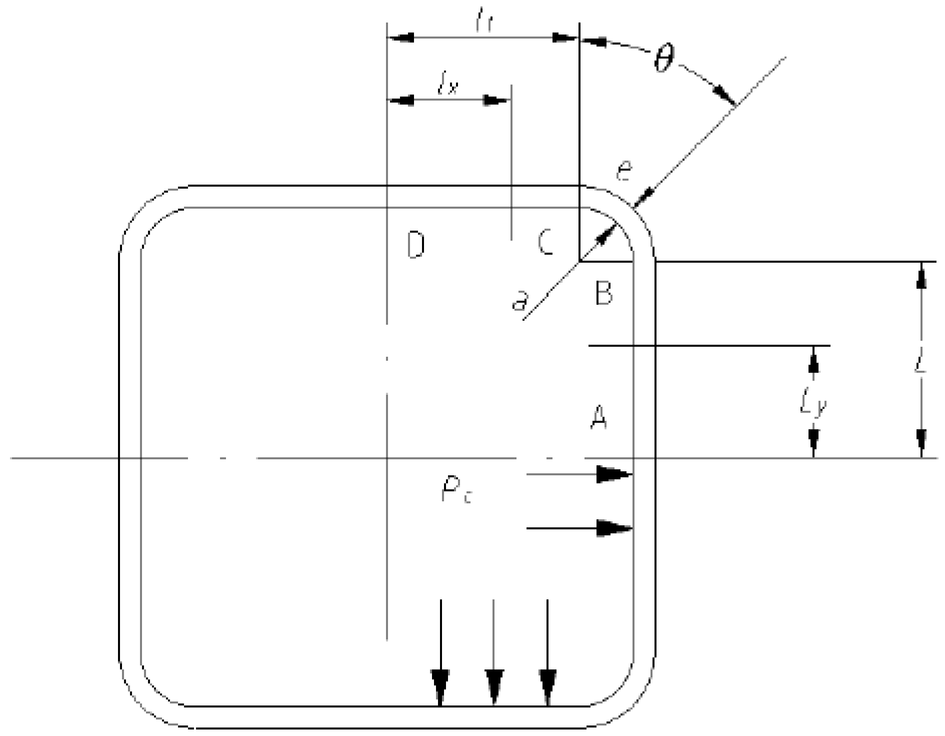


Figure 15.5-1 — Unreinforced vessels

Calculation values:

The membrane stresses at each area is determined by:

At C:

$$\sigma_{m.C} := \frac{P \cdot (a + L)}{e} = 556\ MPa$$ 15.5.1.2-1

At D:

$$\sigma_{m.D} := \sigma_{m.C} = 556\ MPa$$

At B:

$$\sigma_{m.B} := \frac{P \cdot (a + l_1)}{e} = 600\ MPa$$ 15.5.1.2-2

At A:

$$\sigma_{m.A} := \sigma_{m.B} = 600\ MPa$$

At a corner:

$$\sigma_{m.BC} := \frac{P}{e} \cdot \left(a + \sqrt{L^2 + l_1^2}\right) = 790.47\ MPa$$ 15.5.1.2-3

The second moment of area is given by:

$$I_1 := \frac{e^3}{12} = 2.25\ mm^3$$ 15.5.1.2-4

$$I_2 := I_1 = 2.25\ mm^3$$

The following equations apply to calculate the bending stresses (eq. 15.5.1.2-5 - eq. 15.5.1.2-9)

$$\theta := \text{atan}\left(\frac{l_1}{L}\right) = 0.828$$ 15.5.1.2-10

NOTE: the value of θ varies thorughout the corner, but the value given here will give the highest bending stress

$$\phi := \frac{a}{l_1} = 0.125$$ 15.5.1.2-15

$$\alpha_3 := \frac{L}{l_1} = 0.918$$ 15.5.1.2-14

$$K_3 := \frac{l_1^2 \cdot \left(6\ \phi^2 \cdot \alpha_3^2 - 3\ \pi \cdot \phi^2 + 6\ \phi^2 + \alpha_3^2 - 6\ \phi - 2 + 1.5 \cdot \pi \cdot \alpha_3^2 \cdot \phi + 6\ \phi \cdot \alpha_3\right)}{3 \cdot \left(2\ \alpha_3 + \pi \cdot \phi + 2\right)} = -8.815 \cdot 10^3\ mm^2$$ 15.5.1.2-12

$$M_A := P \cdot \left(-K_3\right) = \left(3.526 \cdot 10^4\right) \frac{N \cdot mm}{mm}$$ 15.5.1.2-10

The bending stresses of each area is determined by:

At C:

$$\pm \quad \sigma_{b.C} := \frac{e}{4\ I_1}\left(2\ M_A + P \cdot \left(2\ a \cdot L - 2\ a \cdot l_1 + L^2\right)\right) = \left(1.987 \cdot 10^5\right)\ MPa$$ 15.5.1.2-5

At D:

$$\pm \quad \sigma_{b.D} := \text{abs}\left(\frac{e}{4\ I_1}\left(2\ M_A + P \cdot \left(2\ a \cdot L - 2\ a \cdot l_1 + L^2 - l_1^2\right)\right)\right) = ?\ MPa$$ 15.5.1.2-6

At A:

$$\pm \quad \sigma_{b.A} := \text{abs}\left(\frac{M_A \cdot e}{2\ I_1}\right) = ?\ MPa$$ 15.5.1.2-7

At B:

$$\pm \quad \sigma_{b.B} := \text{abs}\left(\frac{e}{4\ I_1} \cdot \left(2\ M_A + P \cdot L^2\right)\right) = \left(2.031 \cdot 10^5\right)\ MPa$$ 15.5.1.2-8

At a corner:

$$\pm \quad \sigma_{b.BC} := \text{abs}\left(\frac{e}{4\ I_1} \cdot \left(2\ M_A + P \cdot \left(2\ a \cdot \left(L \cdot \cos(\theta) - l_1 \cdot \left(1 - \sin(\theta)\right)\right) + L^2\right)\right)\right) = \left(2.221 \cdot 10^5\right)\ MPa$$ 15.5.1.2-9

Output parameters::

The maximum stress at any location is equal to the sum of the bending and membrane stresses at that location (as stated in 15.4). The maximum stress is therefore:

At C:

$$\sigma_{max.C} := \sigma_{m.C} + \sigma_{b.C} = \left(1.992 \cdot 10^5\right)\ MPa$$

At D:

$\sigma_{max.D} := \sigma_{m.D} + \sigma_{b.D} = ?\ MPa$

At A:

$\sigma_{max.A} := \sigma_{m.A} + \sigma_{b.A} = ?\ MPa$

At B:

$\sigma_{max.B} := \sigma_{m.B} + \sigma_{b.B} = (2.037 \cdot 10^{5})\ MPa$

At the corners:

$\sigma_{max.BC} := \sigma_{m.BC} + \sigma_{b.BC} = (2.229 \cdot 10^{5})\ MPa$

15.5.3 Allowable stresses for unreinforced vessels

The membrane stresses shall be limited as follows:

$\sigma_m \le f \cdot z$ 15.5.3-1

$status\_\sigma_{m.C} :=$ if $\sigma_{m.C} \le f \cdot z$ return "OK" "Not Valid"

$status\_\sigma_{m.A} :=$ if $\sigma_{m.A} \le f \cdot z$ return "OK" "Not Valid"

$status\_\sigma_{m.BC} :=$ if $\sigma_{m.BC} \le f \cdot z$ return "OK" "Not Valid"

$status\_\sigma_{m.D} :=$ if $\sigma_{m.D} \le f \cdot z$ return "OK" "Not Valid"

$status\_\sigma_{m.B} :=$ if $\sigma_{m.B} \le f \cdot z$ return "OK" "Not Valid"

This gives:

$status\_\sigma_{m.C} =$ "Not Valid"

$status\_\sigma_{m.D} =$ "Not Valid"

$status\_\sigma_{m.A} =$ "Not Valid"

$status\_\sigma_{m.B} =$ "Not Valid"

$status\_\sigma_{m.BC} =$ "Not Valid"

The sum of the membrane stresses and the bending stresses shall conform to:

$\sigma_m + \sigma_b \le 1.5 \cdot f \cdot z$ 15.5.3-2

$status\_\sigma_C :=$ if $\sigma_{max.C} \le 1.5 \cdot f \cdot z$ return "OK" "Not Valid"

$status\_\sigma_D :=$ if $\sigma_{max.D} \le 1.5 \cdot f \cdot z$ return "OK" "Not Valid"

$status\_\sigma_{BC} :=$ if $\sigma_{max.BC} \le 1.5 \cdot f \cdot z$ return "OK" "Not Valid"

$status\_\sigma_A :=$ if $\sigma_{max.A} \le 1.5 \cdot f \cdot z$ return "OK" "Not Valid"

$status\_\sigma_B :=$ if $\sigma_{max.B} \le 1.5 \cdot f \cdot z$ return "OK" "Not Valid"

This gives:

$status\_\sigma_C =$ "Not Valid"

$status\_\sigma_D = ?$

$status\_\sigma_A = ?$

$status\_\sigma_B =$ "Not Valid"

$status\_\sigma_{BC} =$ "Not Valid"

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