Name:
Teacher:

Unit 1&2 Mathematical Methods

Exam 1 2016

Wednesday November 9 (2.00 pm)

Reading time: 10 Minutes
Writing time: 60 Minutes

Instruction to candidates:
Students are only permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
No calculator or notes are allowed.

Materials Supplied:
8 page question and answer booklet, formula sheet can be removed at the end of paper.

Instructions:
- Write your name and that of your teacher in the spaces provided.
- Answer all short answer questions in this booklet where indicated.
- Always show your full working where spaces are provided.

Total exam
/52

Question 1

Solve the following equation for x   (2 marks)

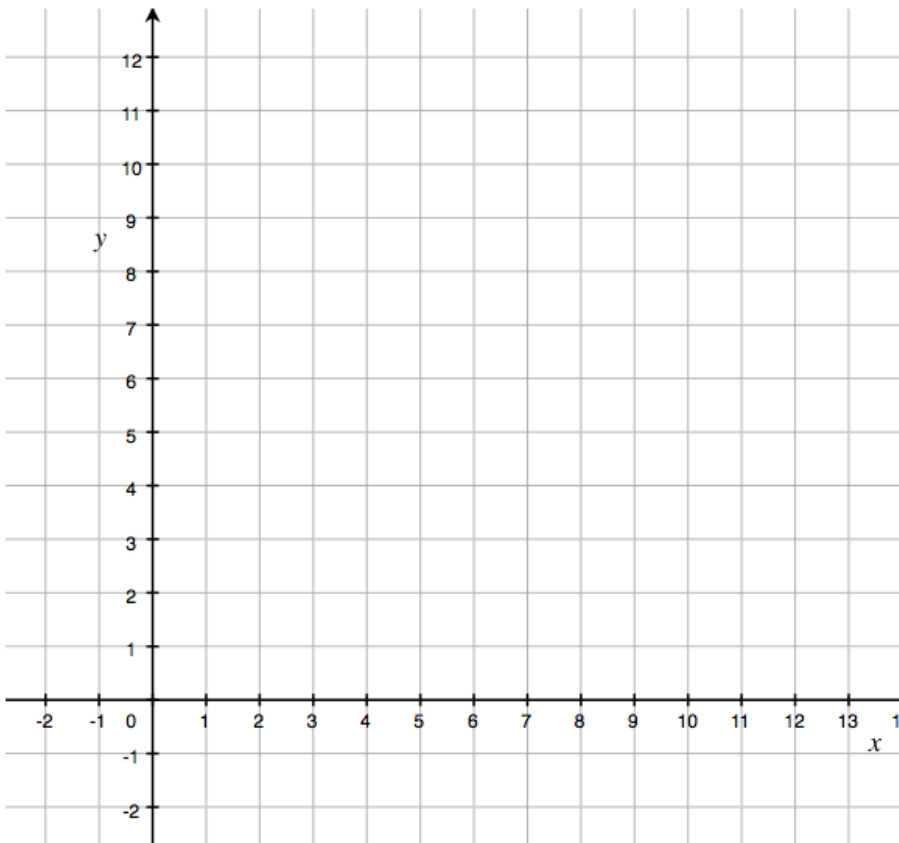   (a) $3 - 2x = 6$ (b) $3(58x - 24) + 10 < 25$

Question 2

a) On the axes below, draw the graph of the line $y = x - 1$.   (1 mark)

b) On the axes below, draw the graph of the line $3x + 2y = 18$ (2 marks)

c) Use simultaneous equations to find the co-ordinates of the point of intersection of the two lines.   (2 marks)

$(x, y) =$

Question 3

The quadratic function $y = 2x^2 + 8x - 24$ describes a parabola.

a) Fully factorise the quadratic function. (2 marks)

y =

b) Find the y intercept of the parabola. (1 marks)

y- intercept =

c) Find the two x intercepts (2 marks)

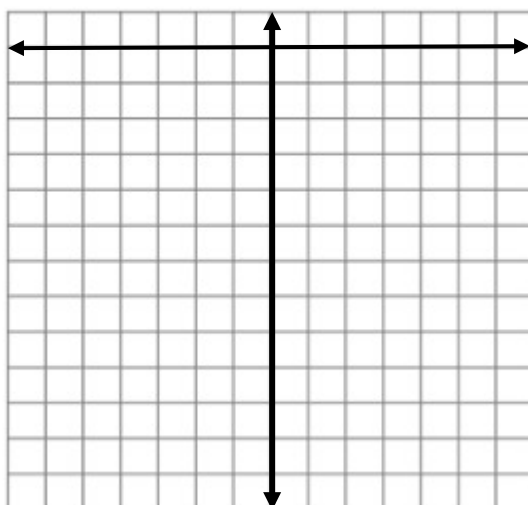| |
|---|
| x-intercept = |
| x-intercept = |

d) Find the turning point of the parabola (1 mark)

Turning Point =

e) Plot the parabola labelling the turning point and all intercepts (3 marks)

Question 4

(a) Using substitution show that (x-2) is a factor of the polynomial

$$P(x) = x^3 - 6x^2 - 13x + 42$$ (1 mark)

(b) Hence use long division, or otherwise, to fully factorise the polynomial

` $$P(x) = x^3 - 6x^2 - 13x + 42$$.   (2 marks)

(b) State the stationary point of inflection for $y = 3(x + 2)^3 - 5$ (2 marks)

Stationary Point of Inflection =

Question 5

For the cubic function $y = x^3 + 3x^2 + 2x + 1$

a) Find the y-value when $x = 2$ (1 mark)

b) Find the derivative of the function. (1 mark)

c) Find the gradient of the curve at the point where, $x = 2$. (1 mark)

d) Hence find the equation of the tangent to the curve at the point where, $x = 2$. (1 marks)

Question 6

Find the co-ordinates of the stationary points for $y = 9 + 12x - 2x^2$ (2 marks)

Anti-differentiation

Question 7

Find an antiderivative of: (2 marks)

   a. $3x^2 + 7x + 5$ b. $x^4 + \dfrac{1}{x^2}$

Question 8

(a) Solve the following equation for x.　　　　(b) Evaluate the expression　　　　(4 marks)

$$3^{x+1} - 27 = 0$$

$$x = \log_{10} 4 + 2 \log_{10} 3 - \log_{10} 6$$

Question 7

For the rule : $y = 2^x + 1$

a) State the implied domain of the function.　　　　　　　　　　　　　(1 mark)
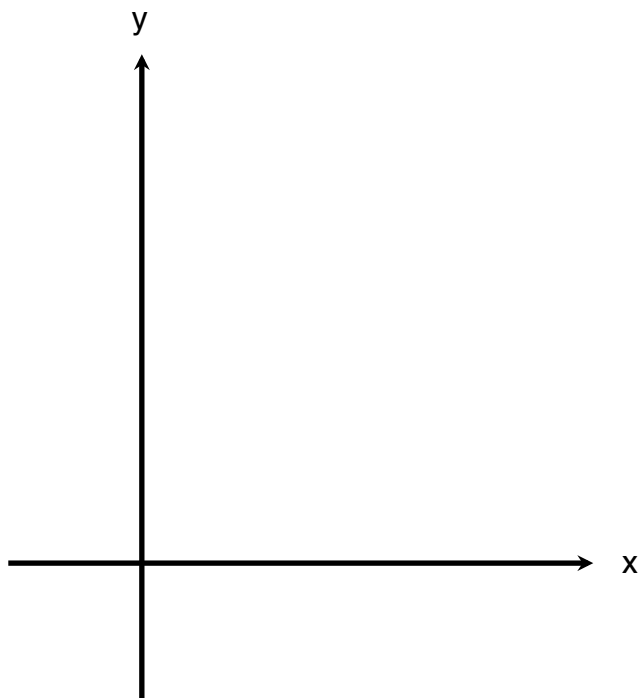

b) State the range of the function.　　　　　　　　　　　　　　　　(1 mark)

c) Sketch the graph of the function, including any asymptotes with equation and axis intercepts　　　(2 marks)

Question 10

For the circular function : $y = 2\cos(2x)$

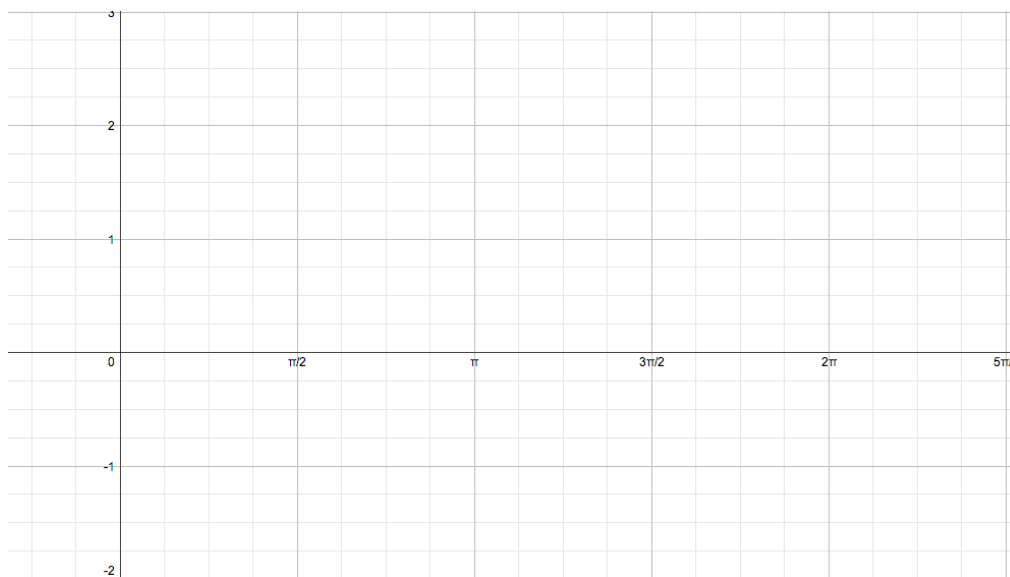a) State the period, amplitude, minimum and maximum values of the function. (4 marks)

Period:	
Amplitude:	
Minimum:	
Maximum:	

(b) Draw the graph of the function over the interval $[0, 2\pi]$ (2 marks)

Question 11

(a) Find the solution(s) to the equation $\sin(x) = -\dfrac{\sqrt{3}}{2}$ over the interval $[0, 2\pi]$. (2 marks)

Probability

Question 12

(a) A fair coin is tossed three times. Find the probability that exactly two tails are observed. (1 mark)

(c) In Troy's class of 20 students a total of 14 students play Xbox, a total of 8 students play Playstation and 7 play both. Draw a clearly labelled Venn diagram of Troy's class. (2 marks)

(d) Use the Venn diagram to find the probability that randomly chosen student plays Xbox given that they play Playstation. (1 marks)

(e) If one card is chosen from a well shuffled deck of playing cards, using the addition rule, what is probability that the card is a Queen (Q) or a Heart (H)? (1 marks)

(f) Suppose that the probability that a family in Panmure owns a television set (T) is 0.75, and the probability that a family owns a station wagon (S) is 0.25. If these events are independent, find the probability that

     i. a family chosen at random owns both a television set and a station wagon (1 mark)

     ii. a family chosen at random owns at least one of these items (1 mark)

Congratulations on completing the first exam, keep up the great effort into tomorrow's exam!

Formula Sheet

Differentiation

$$f(x) = x^n, \quad f'(x) = nx^{n-1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
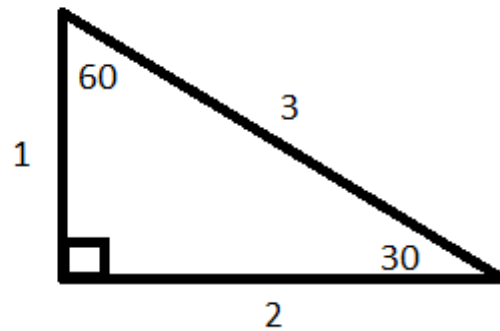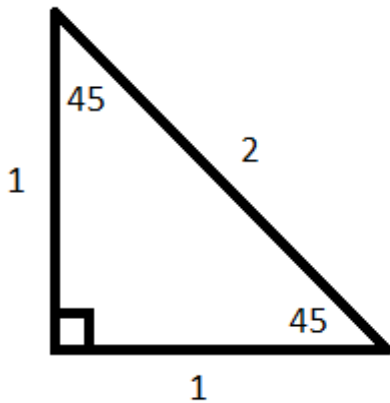
Anti Differentiation

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry

Probability

Addition rule:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(A) = 1 - \Pr(A')$$

Name:	**SOLUTIONS**
Teacher:	**BOT / PEC / THA / VIJ**

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Total exam
/52

Question 1

Solve the following equation for x   (2 marks)

  (a) $3 - 2x = 6$ (b) $3(58x - 24) + 10 < 25$

   - 2x = 2   3(58x – 24) < 15
                                                                              58x – 24 < 5
     x = -1   58x < 29
                                                                                   x < 1/2
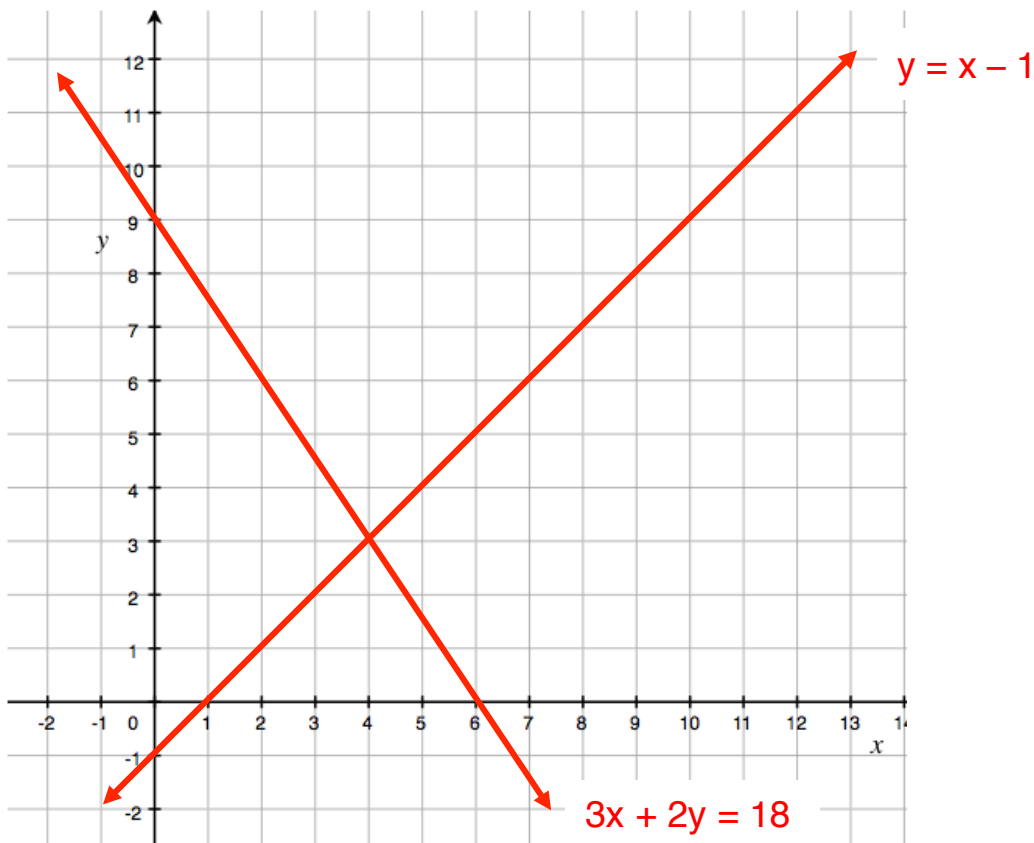
Question 2

a) On the axes below, draw the graph of the line $y = x - 1$.   (1 mark)

b) On the axes below, draw the graph of the line $3x + 2y = 18$ (2 marks)

y = x – 1

3x + 2y = 18

c) Use simultaneous equations to find the co-ordinates of the point of intersection of the two lines.   (2 marks)

   3x + 2(x – 1) = 18
   3x + 2x – 2 = 18
   5x = 20
   x = 4

   y = 4 – 1
   y = 3

(x, y) =   (4, 3)

2

Question 3

The quadratic function $y = 2x^2 + 8x - 24$ describes a parabola.

a) Fully factorise the quadratic function. (2 marks)

y = 2(x² + 4x − 12)
= 2(x + 6)(x - 2)

$$y = 2(x + 6)(x - 2)$$

b) Find the y intercept of the parabola. (1 marks)

Sub in (x=0)
y = -24

y- intercept = -24

c) Find the two x intercepts (2 marks)

Set y = 0
0 = 2(x + 6)(x − 2)
x +6 = 0 or x − 2 = 0
x = -6   or x = 2

x-intercept = 2

x-intercept = -6

d) Find the turning point of the parabola (1 mark)

y = 2(x² + 4x − 12)
y = 2(x + 2)² − 20
(h, k) = (-2, -20)

Turning Point = (-2, -20)

e) Plot the parabola labelling the turning point and all intercepts (3 marks)
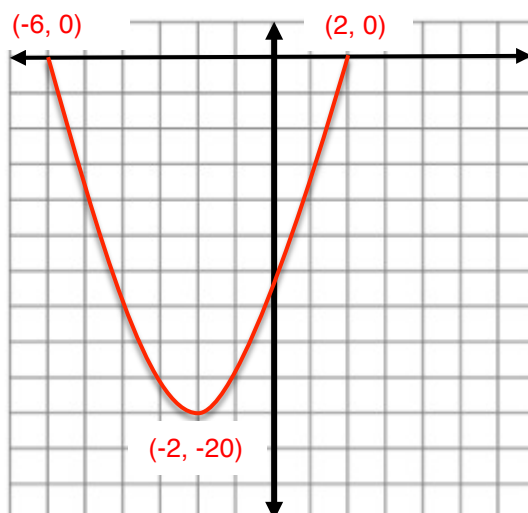
(-6, 0)   (2, 0)

(-2, -20)

Question 4

(a) Using substitution show that (x-2) is a factor of the polynomial

$$P(x) = x^3 - 6x^2 - 13x + 42$$ (1 mark)

For x = 2
P(2) = (2)³ − 6(2)² − 13(2) + 42
= 8 − 24 − 26 + 42
= 0, therefore a factor

(b) Hence use long division, or otherwise, to fully factorise the polynomial

` $$P(x) = x^3 - 6x^2 - 13x + 42$$. (2 marks)

```
                      x² – 4x – 21
        x – 2  ) x³ – 6x² – 13x + 42
                 x³ – 2x²
                  – 4x² – 13x
                  – 4x² + 8x
                       – 21x + 42
                       – 21x + 42
                             0
```

P(x) = (x – 2)(x² – 4x – 21)
    = (x – 2)(x – 7)(x + 3)

P(x) = (x − 2)(x − 7)(x + 3)

(b) State the stationary point of inflection for $y = 3(x + 2)^3 - 5$ (2 marks)

Stationary Point of Inflection = (-2, -5)

Differentiation

Question 5

For the cubic function $y = x^3 + 3x^2 + 2x + 1$

a) Find the y-value when $x = 2$ (1 mark)

$$y = (2)^3 + 2(2)^2 + 2(2) + 1 = 25$$

b) Find the derivative of the function.   (1 mark)

$$\frac{dy}{dx} = 3x^2 + 6x + 2$$

$$\boxed{\frac{dy}{dx} = 3x^2 + 6x + 2}$$

c) Find the gradient of the curve at the point where, $x = 2$.   (1 mark)

$$\frac{dy}{dx} = 12 + 12 + 2$$
$$= 26$$

$$\boxed{26}$$

d) Hence find the equation of the tangent to the curve at the point where, $x = 2$.   (1 marks)

y – y₁ = m (x – x₁)
y – 25 = 26 (x – 2)
y = 26x - 27

$$\boxed{y = 26x - 27}$$

Question 6

Find the co-ordinates of the stationary points for y = 9 + 12x – 2x²   (2 marks)

$$\frac{dy}{dx} = 12 - 4x = 0$$
12 = 4x
3 = x

y = 9 +12×3 – 18
y = -9 +36
y = 27

$$\boxed{(3, 27)}$$

Anti-differentiation

Question 7

Find an antiderivative of: (2 marks)

a. $3x^2 + 7x + 5$ b. $x^4 + \frac{1}{x^2}$

$F(x) = x^3 + \frac{7}{2}x^2 + 5x + c$

$$\int x^4 + x^{-2} dx = \frac{x^5}{5} - \frac{x^{-1}}{1} + c$$
$$= \frac{1}{5}x^5 - \frac{1}{x} + c$$

5

Question 8

(a) Solve the following equation for x. (b) Evaluate the expression (4 marks)

$$3^{x+1} - 27 = 0$$

$$x = \log_{10} 4 + 2\log_{10} 3 - \log_{10} 6$$

$$3^{x+1} = 27$$
$$3^{x+1} = 3^3$$
$$x + 1 = 3$$
$$x = 2$$

$$x = \log_{10} 4 + \log_{10} 9 - \log_{10} 6$$
$$x = \log_{10} \frac{36}{6}$$
$$x = \log_{10} 6$$

$x = 2$	$x = \log_{10} \dfrac{36}{6}$

Question 7

For the rule : $y = 2^x + 1$

a) State the implied domain of the function. (1 mark)
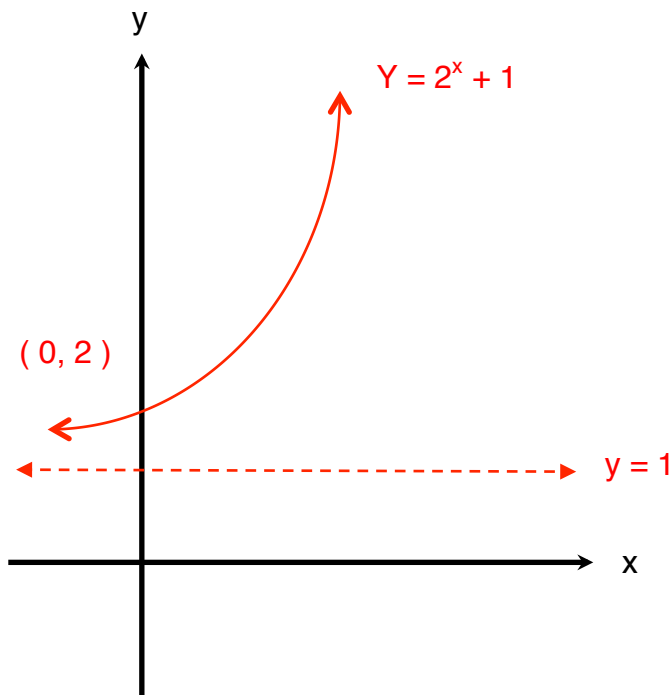
Domain: $x \in R$

b) State the range of the function. (1 mark)

Range: $y \in (1, \infty)$

c) Sketch the graph of the function, including any asymptotes with equation and axis intercepts (2 marks)

Trigonometric Functions

6
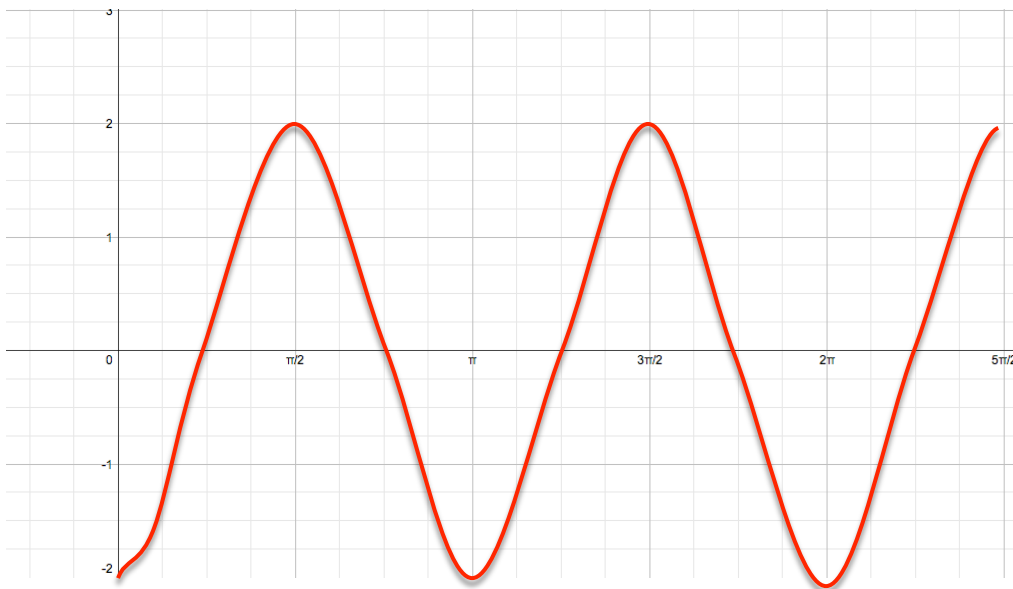
Question 10

For the circular function : $y = 2\cos(2x)$

a)   State the period, amplitude, minimum and maximum values of the function.   (4 marks)

Period:	π
Amplitude:	2
Minimum:	-2
Maximum:	2

(b) Draw the graph of the function over the interval $[0, 2\pi]$ (2 marks)

Question 11

(a) Find the solution(s) to the equation $\sin(x) = -\dfrac{\sqrt{3}}{2}$ over the interval $[0, 2\pi]$.   (2 marks)

$\text{Sin } \theta = \dfrac{\sqrt{3}}{2}$ $\text{Sin }(x) = -\dfrac{\sqrt{3}}{2}$

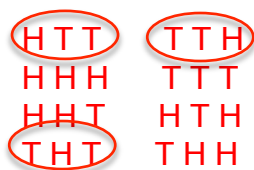$\theta = \dfrac{\pi}{3}$ $x = \dfrac{5\pi}{3} \text{ and } x = \dfrac{4\pi}{3}$

$x = \dfrac{5\pi}{3} \text{ and } x = \dfrac{4\pi}{3}$

7
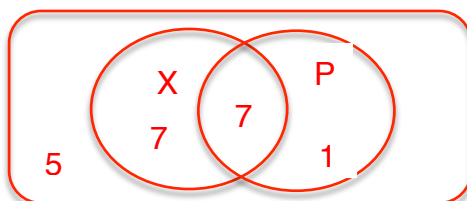
Question 12

(a) A fair coin is tossed three times. Find the probability that exactly two tails are observed (1 mark)

H T T   T T H
H H H   T T T
H H T   H T H
T H T   T H H

$$Pr = \frac{3}{8}$$

(c) In Troy's class of 20 students a total of 14 students play Xbox, a total of 8 students play Playstation and 7 play both. Draw a clearly labelled Venn diagram of Troy's class. (2 marks)

(d) Use the Venn diagram to find the probability that randomly chosen student plays Xbox given that they play Playstation. (1 marks)

$$Pr (X \mid N) = \frac{7}{8}$$

(e) If one card is chosen from a well shuffled deck of playing cards, using the addition rule, what is probability that the card is a Queen (Q) or a Heart (H)? (1 marks)

$$Pr (Q \cup H) = Pr (Q) + Pr (H) - Pr (Q \cap H)$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

(f) Suppose that the probability that a family in Panmure owns a television set (T) is 0.75, and the probability that a family owns a station wagon (S) is 0.25. If these events are independent, find the probability that

   i. a family chosen at random owns both a television set and a station wagon (1 mark)

$$Pr (T \cap S) = Pr (T) \times Pr (S)$$
$$= 0.75 \times 0.25$$
$$= 0.1875 \text{ or } \frac{3}{16}$$

   ii. a family chosen at random owns at least one of these items (1 mark)

$$1 - Pr (T' \cap S') = 1 - (0.25 \times 0.75)$$
$$= \frac{13}{16}$$

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8

Formula Sheet

Differentiation

$$f(x) = x^n, \quad f'(x) = nx^{n-1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
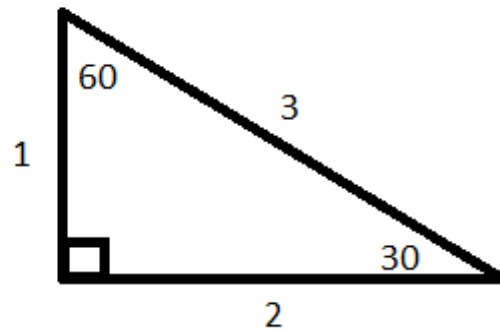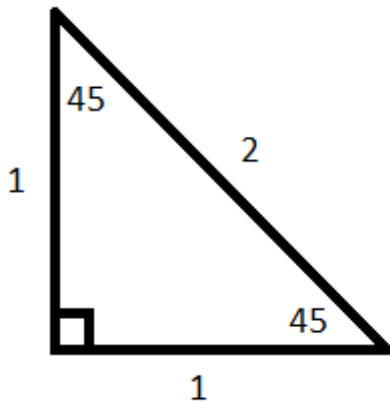
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Probability

Addition rule:

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$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(A) = 1 - \Pr(A')$$