UNIT 1
FLUID PROPERTIES AND FLUID STATICS
PART – A (2 MARKS)

1. Define fluids.
   Fluid may be defined as a substance which is capable of flowing. It has no definite shape of its own, but confirms to the shape of the containing vessel.
2. What are the properties of ideal fluid?
   Ideal fluids have following properties
   i) It is incompressible
   ii) It has zero viscosity
   iii) Shear force is zero
3. What are the properties of real fluid?
   Real fluids have following properties
   i) It is compressible
   ii) They are viscous in nature
   iii) Shear force exists always in such fluids.
4. Define densityand specific weight.
   Density is defined as mass per unit volume (kg/m^3)
   Specific weight is defined as weight possessed per unit volume (N/m^3)
5. Define Specific volume and Specific Gravity.
   Specific volume is defined as volume of fluid occupied by unit mass (m^3/kg) Specific gravity is defined as the ratio of specific weight of fluid to the specific weight of standard fluid.
6. Define Surface tension and Capillarity.
   Surface tension is due to the force of cohesion between the liquid particles at the free surface.
   Capillary is a phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid.
7. Define Viscosity.
   It is defined as the property of a liquid due to which it offers resistance to the movement of one layer of liquid over another adjacent layer.
8. Define kinematic viscosity.
   It is defined as the ratio of dynamic viscosity to mass density. (m^2/sec)
9. Define Relative or Specific viscosity.
   It is the ratio of dynamic viscosity of fluid to dynamic viscosity of water at 20°C.
10. Define Compressibility.
    It is the property by virtue of which fluids undergoes a change in volume under the action of external pressure.
11. Define Newtonian law of Viscosity.
    According to Newton's law of viscosity the shear force F acting between two layers of fluid is proportional to the difference in their velocities du

and area A of the plate and inversely proportional to the distance between them.

12. What is cohesion and adhesion in fluids?

    Cohesion is due to the force of attraction between the molecules of the same liquid.

    Adhesion is due to the force of attraction between the molecules of two different liquids or between the molecules of the liquid and molecules of the solid boundary surface.

13. State momentum of momentum equation?

    It states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum

14. What is momentum equation

    It is based on the law of conservation of momentum or on the momentum principle It states that,the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

PART – B(16 MARKS)

1. What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of $1.53 x 10^3$ kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury? The specific gravity of mercury is 13.6 and density of water is 1000 kg/m^3.

Solution

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 x 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$= 0.75$ m of Hg

Atmospheric pressure, $P_{atm} = \rho_0 \text{ x } g \text{ x } Z_0$

$= (13.6 \text{ x } 1000) \text{ x } 9.81 \text{ x } 0.75$

$= 100062$ N/m^2

Pressure at a point at a depth of 3 m from the free surface of a liquid

$P = \rho_1 \text{ x } g \text{ x } Z_1$

$= (1.53 \text{ x } 1000) \text{ x } 9.81 \text{ x } 3$

$= 45028$ N/m^2

Gauge pressure, $P = 45028$ N/m^2

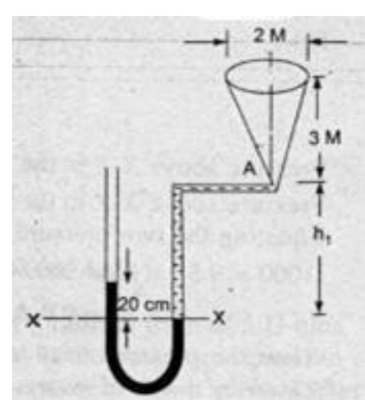
Absolute pressure, = Gauge pressure + Atmospheric pressure

$= 45028 + 100062$

= 145090 N/m²

2. Figure shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer given in the figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.

Solution



Difference of mercury level, $h_2 = 20$ cm

Sp. Gravity of mercury , $S_2 = 13.6$

Sp. Gravity of water, $S_1 = 1$

Density of mercury, $\rho_2 = 13.6 \text{ x } 1000$

Density of water, $\rho_1 = 1 \text{ x } 1000$

Equating the pressure above the datum line X-X

$$P_2 \times g \times h_2 = \rho_1 \times g \times h_1$$
$$(13.6 \times 1000) \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$
$$h_1 = 2.72 \text{ m of water}$$
Pressure in left limb = Pressure in right limb
$$13.6 \times 1000 \times 9.81 \times (0.2 + 2y/100) = 1000 \times 9.81 \times (3 + h_1 + y/100)$$
$$(27.2y - y)/100 = 3.0$$
$$y = 11.45 \text{ cm}$$
The difference of mercury level in two limbs,
= (20 + 2y) cm of mercury level
= 20 + (2 x 11.45)
= 42.90 cm of mercury

Reading of manometer = 42.90 cm

3. Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the center of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.

Solution

Diameter of plate, d = 1.5 m
Area, $A = (\Pi/4) \times 1.5^2 = 1.767 \text{ m}^2$
$\bar{h} = 3.0$ m
Total pressure, $F = \rho \times g \times A \times \bar{h}$
= 1000 x 9.81 x 1.767 x 3
= 52002.81 N

Position of centre of pressure, $h^* = (I_G/A\bar{h}) + \bar{h}$
$I_G = (\Pi \times d^4)/64$
$= 0.2485 \text{ m}^4$
$h^* = (0.2485/(1.767 \times 3)) + 3$

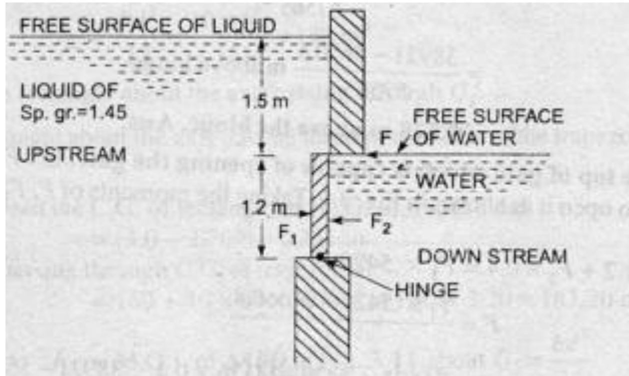
Position of centre of pressure = 3.0468 m

4. A vertical sluice gate is used to cover an opening in a da m. The opening is 2 m wide and 1.2 m high. On the upstream of the gate, the liquid of Sp.gr 1.45 lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.

Solution

Width of gate, b = 2 m
Depth of gate, d = 1.2 m
Area, $A = (2 \times 1.2) = 2.4 \text{ m}^2$
Sp. gr of liquid = 1.45
Density of Density, $\rho_1 = 1.45 \times 1000 = 1450 \text{ kg/m}^3$

Let F_1 = Force exerted by the fluid of sp. gr 1.45 on gate
F_2 = Force exerted by water on gate
$F_1 = \rho_1 \times g \times A \times \bar{h}_1$
$\bar{h}_1$ = Depth of C.G of gate from free surface of liquid
= 1.5 + (1.2/2)
= 2.1 m



$F_1 = 1450 \times 9.81 \times 2.4 \times 2.1 = 71691$ N

$F_2 = \rho_2 \times g \times A \times h_2$

$P_2 = 1 \times 1000$ kg/m^3

$h_2 = 1.2/2 = 0.6$ m

$F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 71691$ N

(i) Resultant force on the gate = $F_1 - F_2 = 71691 – 14126 = 57565$ N

(ii) Position of centre of pressure of resultant force

The force F_1 will be acting at a depth of h_1^* from free surface of liquid, given by the relation,

$h_1^* = (I_G/A\ h_1) + h_1$

$I_G = bd^3/12 = 2 \times 1.2^3/12 = 0.288$ m^4

$h_1^* = 0.288 / (2.4 \times 2.1) + 2.1 = 2.1571$ m

Distance of F_1 from hinge,

$= (1.5 \pm 1.2) - h_1^* = 2.7 – 2.1571 = 0.5429$ m

$h_2^* = (I_G/A\ h_2) + h_2$

$= (0.288/2.4 \times 0.6) + 0.6 = 0.8$ m

Distance of F_2 from hinge $= 1.2 – 0.8 = 0.4$ m

The resultant force 57565 N will be acting at a distance given by

$= ((71691 \times 0.5429) – (14126\ 0.4))/57565$

$= 0.578$ m above the hinge

(iii) Force at the top of gate which is capable of opening the gate

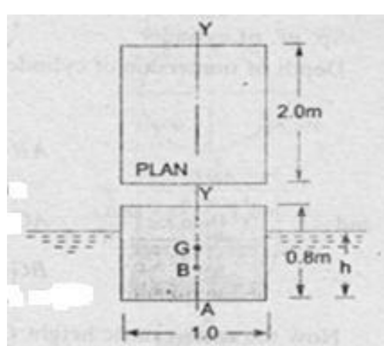
$F \times 1.2 + F_2 \times 0.4 = F_1 \times 0.5429$

$F = ((71691 \times 0.5429) - (14126 \times 0.4))/1.2$

$F = 27725.5$ N

5. A block of wood of specific gravity 0.7 floats in water. Determine the meta-centric height of the block if its size is 2 m x 1 m x 0.8 m.

Solution



Dimension of block = 2 x 1 x 0.8
Depth of immersion = h m
Sp. gr of wood = 0.7
Weight of wooden piece = Weight density of wood x Volume
= 0.7 x 1000 x 9.81 x 2 x 1 x 0.8

Weight of water displaced = Weight density of water x Volume of the wood submerged in water
= 1000 x 9.81 x 2 x 1 x h

Weight of wooden piece = Weight of water displaced
0.7 x 1000 x 9.81 x 2 x 1 x 0.8 = 1000 x 9.81 x 2 x 1 x h
h = 0.56 m

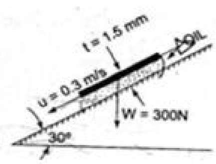
Distance of centre of buoyancy from bottom,
AB = (h/2) = 0.56/2 = 0.28 m
AG = 0.8/2.0 = 0.4 m
BG = AG – AB = 0.4 – 0.28 = 0.12 m

The meta-centric height, GM = (I/A) – BG
$= (2 \times 1^3 / 12) / (2 \times 1 \times 0.56)$
= 0.0288 m

6. Calculate the dynamic viscosity of oil which is used for lubrication between a square plate of size 0.8 mx0.8 m and an inclined angle of inclination 30 as shown in figure. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of the oil film is 1.5 mm.

Solution

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$
Angle of plane, $\theta = 30°$
Weight of plate, $W = 300$ N
Velocity of plate, $u = 0.3$ m/s



Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Let the viscosity of fluid between plate and inclined plane is μ.
Component of weight W, along the plane = $W \cos 60° = 300 \cos 60° = 150$ N
Thus the shear force, F, on the bottom surface of the plate = 150 N

and shear stress,
$$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$$

Now using equation
$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity = $u - 0 = u = 0.3$ m/s
$dy = t = 1.5 \times 10^{-3}$ m

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \textbf{11.7 poise}$$

7. Find the kinematic viscosity of an oil having density 981 kg/m. The shear stress at a point in oil is 0.2452 N/m and velocity gradient at that point is 0.2 per second.

Solution

Mass density, $\rho = 981 \text{ kg/m}^3$

Shear stress, $\tau = 0.2452 \text{ N/m}^2$

Velocity gradient, $\frac{du}{dy} = 0.2 \text{ s}$

Using the equation $\tau = \mu \frac{du}{dy}$ or $0.2452 = \mu \times 0.2$

$\therefore \quad \mu = \frac{0.245}{0.200} = 1.226 \text{ Ns/m}^2$

Kinematic viscosity ν is given by

$$\therefore \quad \nu = \frac{\mu}{\rho} = \frac{1.226}{981} = .125 \times 10^{-2} \text{ m}^2\text{/sec}$$

$$= 0.125 \times 10^{-2} \times 10^4 \text{ cm}^2\text{/s} = 0.125 \times 10^2 \text{ cm}^2\text{/s}$$

$$= 12.5 \text{ cm}^2\text{/s} = \mathbf{12.5 \text{ stoke}}$$

8. The dynamic viscosity of oil used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Solution

Viscosity $\mu = 6$ poise

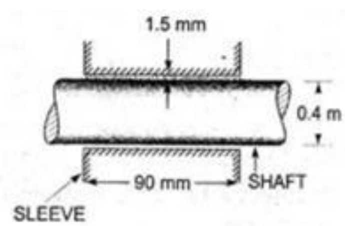
$$= \frac{6}{10}\frac{\text{Ns}}{\text{m}^2} = 0.6 \frac{\text{Ns}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90 \text{ mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m



Tangential velocity of shaft, $u = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \frac{du}{dy}$

where du = Change of velocity $= u - 0 = u = 3.98$ m/s

dy = Change of distance $= t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

∴ Shear force on the shaft, F = Shear stress × Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $T = \text{Force} \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01$ Nm

∴ *Power lost $= \frac{2\pi NT}{60} = \frac{2\pi \times 190 \times 36.01}{60} = \mathbf{716.48 \text{ W}}$

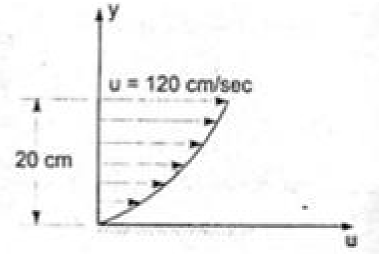
9. If the velocity profile of a fluid over a plate is a parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. calculate the velocity gradients and shear stresses at a distance of 0, 10, 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.

Solution

Distance of vertex from plate $= 20$ cm

Velocity at vertex, $u = 120$ cm/sec

Viscosity, $\mu = 8.5 \text{ poise} = \frac{8.5}{10} \frac{\text{N s}}{\text{m}^2} = 0.85.$



The velocity profile is given parabolic and equation of velocity profile is

$$u = ay^2 + by + c \qquad ...(i)$$

where a, b and c are constants. Their values are determined from boundary conditions as :

(a) at $y = 0$, $u = 0$

(b) at $y = 20$ cm, $u = 120$ cm/sec

(c) at $y = 20$ cm, $\frac{du}{dy} = 0$.

Substituting boundary condition (a) in equation (i), we get

$$c = 0.$$

Boundary condition (b) on substitution in (i) gives

$$120 = a(20)^2 + b(20) = 400a + 20b \qquad ...(ii)$$

Boundary condition (c) on substitution in equation (i) gives

$$\frac{du}{dy} = 2ay + b \qquad ...(iii)$$

or $$0 = 2 \times a \times 20 + b = 40a + b$$

Solving equations (ii) and (iii) for a and b

From equation (iii), $b = -40a$

Substituting this value in equation (ii), we get

$$120 = 400a + 20 \times (-40a) = 400a - 800a = -400a$$

$$\therefore \quad a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$\therefore \quad b = -40 \times (-0.3) = 12.0$$

Substituting the values of a, b and c in equation (i),

$$u = -0.3y^2 + 12y.$$

Velocity Gradient

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

at $y = 0$, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 = \mathbf{12/s.}$

at $y = 10$ cm, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 = \mathbf{6/s.}$

at $y = 20$ cm, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 = \mathbf{0.}$

Shear Stresses

Shear stress is given by, $\tau = \mu \frac{du}{dy}$

(*i*) Shear stress at $y = 0$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2 \text{ N/m}^2.$

(*ii*) Shear stress at $y = 10$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1 \text{ N/m}^2.$

(*iii*) Shear stress at $y = 20$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 = \mathbf{0}.$

10. What is the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of 0.0125 m^3 at 80 N/cm^2 pressure to a volume of 0.0124 m^3 at 150 N/cm^2 pressure?

Solution

Initial volume, $\forall = 0.0125 \text{ m}^3$

Final volume $= 0.0124 \text{ m}^3$

$\therefore$ Decrease in volume, $d\forall = .0125 - .0124 = .0001 \text{ m}^3$

$$\therefore \quad -\frac{d\forall}{\forall} = \frac{.0001}{.0125}$$

Initial pressure $= 80 \text{ N/cm}^2$

Final pressure $= 150 \text{ N/cm}^2$

$\therefore$ Increase in pressure, $dp = (150 - 80) = 70 \text{ N/cm}^2$

Bulk modulus is given by equation (1.10) as

$$K = \frac{dp}{-\frac{d\forall}{\forall}} = \frac{70}{\frac{.0001}{.0125}} = 70 \times 125 \text{ N/cm}^2$$

$$= \mathbf{8.75 \times 10^3 \text{ N/cm}^2}$$

11. The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2 (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Solution

Dia. of droplet, $d = 0.04\ \text{mm} = .04 \times 10^{-3}\ \text{m}$

Pressure outside the droplet $= 10.32\ \text{N/cm}^2 = 10.32 \times 10^4\ \text{N/m}^2$

Surface tension, $\sigma = 0.0725\ \text{N/m}$

The pressure inside the droplet, in excess of outside pressure is given by equation

$$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250\ \text{N/m}^2 = \frac{7250\ \text{N}}{10^4\ \text{cm}^2} = 0.725\ \text{N/cm}^2$$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$= 0.725 + 10.32 =$ **11.045 N/cm²**

12. Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tension is 0.0725 N/m for water and 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact is 130°.

Solution

Dia. of tube, $d = 2.5\ \text{mm} = 2.5 \times 10^{-3}\ \text{m}$

Surface tenstion, σ for water $= 0.0725\ \text{N/m}$

σ for mercury $= 0.52\ \text{N/m}$

Sp. gr. of mercury $= 13.6$

$\therefore$ Density $= 13.6 \times 1000\ \text{kg/m}^3$.

(*a*) **Capillary rise for water ($\theta = 0$)**

Using equation
$$h = \frac{4\sigma}{\rho \times g \times d} = \frac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$= .0118\ \text{m} =$ **1.18 cm.**

(*b*) **For mercury**

Angle of constant between mercury and glass tube, $\theta = 130°$

Using equation
$$h = \frac{4\sigma \cos\theta}{\rho \times g \times d} = \frac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$= -.004\ \text{m} =$ **– 0.4 cm.**

The negative sign indicates the capillary depression.

13. Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Solution

Capillary rise, $h = 2.0 \text{ mm} = 2.0 \times 10^{-3}$ m

Surface tension, $\sigma = 0.073575$ N/m

Let dia. of tube $= d$

The angle θ for water $= 0$

The density for water, $\rho = 1000 \text{ kg/m}^3$

Using equation

$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

$$\therefore \quad d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m} = \mathbf{1.5 \text{ cm.}}$$

Thus minimum diameter of the tube should be 1.5.cm.

UNIT 2
FLUID KINEMATICS AND DYNAMICS
PART – A (2 MARKS)

1. Define local acceleration?

   It is defined as the rate of increase of velocity with respect to time at a given point in a flow field.

2. Define convective acceleration?

   It is defined as the rate of change of velocity due to the change of position of fluid particle in a fluid flow.

3. Define velocity potential function?

   It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is denoted by □.

   $$U = -\partial\Phi/\partial x, v = -\partial\Phi/\partial y, w = -\partial\Phi/\partial z.$$

   U,v,w are the velocity in x,y,z direction.

4. Mention the properties of potential function?
   1. If velocity potential exists, The flow should be irrotational.
   2. If velocity potential satisfies the Laplace equation, It represents the possible steady incompressible irrotational flow.

5. Define stream function

   It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction.

6. Mention the properties of stream function?

   1. If stream function exists, it is a possible case of fluid flow which may be rotational.

   2. If stream function satisfies Laplace equation, It is a possible case of an irrotational flow.

7. What is equipotential line?

   A line along which the velocity potential □ is constant is called equipotential line.

8. Give the relation between stream function and velocity potential function?

   $$u = -\partial\Phi/\partial x \quad \text{and} \quad v = -\partial\Phi/\partial y$$
   $$u = -\partial\Psi/\partial y \quad \text{and} \quad v = -\partial\Psi/\partial x$$
   $$u = -\partial\Phi/\partial x = -\partial\Psi/\partial y \quad \text{and} \quad v = -\partial\Phi/\partial y = -\partial\Psi/\partial x$$

   Hence $\partial\Phi/\partial x = \partial\Psi/\partial y$

   $$\partial\Phi/\partial y = -\partial\Psi/\partial x$$

9. What is flow net?

   A grid obtained by drawing a series of equipment lines and steam lines is called a flow net. The flow net is an important tool is analysis two dimensional. Irrotational flow problems.

10. What are the types of motion of fluid particle?

    i. Linear translation or pure translation

    ii. Linear Deformation

    iii. Angular Deformation

iv. Rotation.

11. What is linear translation?

It is defined as the movement of a fluid element in such a way that it moves bodily from one position to represents in new position by a'b'&c'd' are parallel.

12. What is linear deformation?

It is defined as the deformation of a fluid element in linear direction when the element moves the axes of the element in the deformation position and un deformation position are parallel but their lengths changes.

13. Define rotation of fluid element?

It is defined as the movement of a fluid element in such a way that both of Rotate in same direction. It is equal to1/2(□□v/□x-□u/□y) for a two-dimensional element x, y plane.

□x=1/2(□w/□y-□v/□z)

□y=1/2(□u/□z-□w/□x)

□z=1/2(□u/□x-□u/□y)

14. Define vortex flow mention its types?

Vortex flow is defined as the flow of a fluid along a curved path or the flow of a rotating mass of fluid is known as vortex flow.

i. Forced vortex flow.
ii. Free vortex flow.

15. Define free vortex flow?

When no external torque is required to rotate the fluid mass that type of flow Is called free vortex flow.

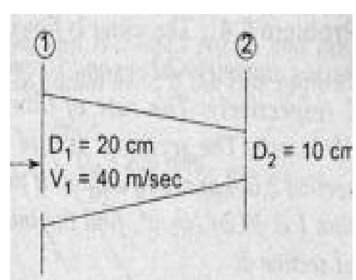
16. Define forced vortex flow?

Forced vortex flow is defined as that type of vortex flow in which some external Torque is required to rotate the fluid mass. The fluid mass in the type of flow rotates at constant Angular velocity 'w'.The tangential velocity of any fluid particle is given by v=cosr.

PART – B(16 MARKS)

1. A pipe through which water is flowing is having diameters 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.

Solution



D_1 = 20 cm = 0.2 m

$A_1 = (\pi/4) \times .2^2 = 0.0314\ m^2$

V_1 = 4 m/s

D_2 = 10 cm = 0.1 m

$A_1 = (\pi/4) \times .1^2 = 0.00785\ m^2$

(i) Velocity head at section 1 = $(V_1^2/2g)$

= (4 x 4)/ 2 x 9.81

= 0.815 m

(ii) Velocity head at section 2 = $(V_2^2/2g)$

To find V_2 apply continuity equation at 1 and 2

$A_1 \times V_1 = A_2 \times V_2$

$V_2 = (A_1 \text{ x } V_1)/ A_2$
$= (0.0314 \text{ x } 4)/0.00785$
$= 16$ m/s

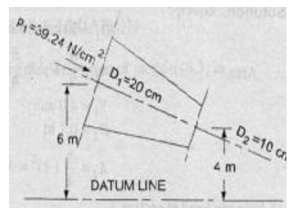
Velocity head at section 2 = $(V_2^2/2g) = (16 \text{ x } 16)/2 \text{ x } 9.81$

$= 83.047$ m

(iii) Rate of discharge $= A_1 \text{ x } V_1$ (or) $A_2 \text{ x } V_2$
$= (0.0314 \text{ x } 4)$
$= 125.6$ litres

2. The water is flowing through a pipe having diameter 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm^2, find the intensity of pressure at section 2.

Solution



At section 1, $D_1 = 20$ cm $= 0.2$ m
$A_1 = (\Pi/4) \text{ x } 0.2^2 = 0.314\ m^2$
$p_1 = 39.24\ N/cm^2 = 39.24 \text{ x } 10^4\ N/m^2$
$z_1 = 6.0$ m

At section 2, $D_2 = 10$ cm $= 0.1$ m
$A_1 = (\Pi/4) \text{ x } 0.1^2 = 0.00785\ m^2$
$z_1 = 6.0$ m

Rate of flow, $Q = 35$lit/s $= (35/1000) = 0.035\ m^3/s$
$Q = A_1 V_1 = A_2 V_2$
$V_1 = (Q/ A_1) = (0.035/0.0314) = 1.114$ m/s
$V_2 = (Q/ A_2) = (0.035/0.00785) = 4.456$ m/s

By Bernoulli's equation at section 1 and 2,

$(p_1/\rho g) + (V_1^2/2g) + z_1 = (p_2/\rho g) + (V_2^2/2g) + z_2$

$(39.24 \text{ x } 10^4/ 1000 \text{ x } 9.81) + (1.114^2/ 2 \text{ x } 9.81) + 6.0$
$= (p_2/ 1000 \text{ x } 9.81) + (4.456^2/ 2 \text{ x } 9.81) + 4.0$

$p_2 = 40.27\ N/cm^2$

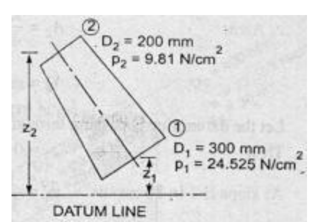
3. Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.

Solution

At section 1, $D_1 = 300$ cm $= 0.3$ m
$p_1 = 24.525\ N/cm^2 = 24.525 \text{ x } 10^4\ N/m^2$

At section 2, $D_2 = 200$ cm $= 0.2$ m
$p_2 = 9.81\ N/cm^2 = 9.81 \text{ x } 10^4\ N/m^2$

Rate of flow, $Q = 40$ lit/s $= (40/1000) = 0.040\ m^3/s$



$Q = A_1 V_1 = A_2 V_2$
$V_1 = (Q/ A_1) = (0.04/(\Pi/4) \text{ x } 0.3^2) = 0.566$ m/s
$V_2 = (Q/ A_2) = (0.04/ \Pi/4) \text{ x } 0.2^2) = 1.274$ m/s

By Bernoulli's equation at section 1 and 2,

$(p_1/\rho g) + (V_1^2/2g) + z_1 = (p_2/\rho g) + (V_2^2/2g) + z_2$

$(24.525 \times 10^4 / 1000 \times 9.81) + (0.566^2 / 2 \times 9.81) + z_1 =$

$(9.81 \times 10^4 / 1000 \times 9.81) + (1.274^2 / 2 \times 9.81) + z_2$

$z_2 - z_1 = 25.32 - 11.623 = 13.70$ m

Difference in datum head = 13.70 m

4. In a two dimensional incompressible flow the fluid velocity components are given by u = x – 4y and v = -y – 4x, Where u and v are x and y components of flow velocity. Show that the flow satisfies the continuity equation and obtain the expression for stream function. If the flow is potential, obtain also the expression for the velocity potential.

Solution:

$u = x - 4y$ and $v = -y - 4x$

$(\partial u / \partial x) = 1$ and $(\partial v / \partial y) = -1$

$(\partial u / \partial x) + (\partial v / \partial y) = 1 - 1 = 0.$

Hence it satisfies continuity equation and the flow is continuous and velocity potential exists.

Let φ be the velocity potential.

$(\partial \varphi / \partial x) = -u = -(x - 4y) = -x + 4y$ (1)

$(\partial \varphi / \partial y) = -v = -(-y - 4x) = y + 4x$ (2)

Integrating Eq. 1, we get

$\varphi = (-x^2 / 2) + 4xy + C$ (3)

Differentiating Eq. 3 w.r.t. y, we get

$(\partial \varphi / \partial y) = 0 + 4x + (\partial C / \partial y) \Rightarrow y + 4x$

Hence, we get $(\partial C / \partial y) = y$

Integrating the above expression, we get $C = y^2 / 2$

Substituting the value of C in Eq. 3, we get the general expression as

$\varphi = (-x^2 / 2) + 4xy + y^2 / 2$

Stream Function

Let ψ be the velocity potential.

$(\partial \psi / \partial x) = v = (-y - 4x) = -y - 4x$ (4)

$(\partial \psi / \partial y) = u = -(x - 4y) = -x + 4y$ (5)

Integrating Eq. 4, we get

$\psi = - y x - 4 (x2/2) + K$ (6)

Differentiating Eq. 6 w.r.t. y, we get

$(\partial \psi / \partial y) = - x - 0 + (\partial K / \partial y) \Rightarrow -x + 4 y$

Hence, we get $(\partial K / \partial y) = 4 y$

Integrating the above expression, we get $C = 4 y^2 / 2 = 2 y^2$

Substituting the value of K in Eq. 6, we get the general expression as

$\psi = - y x - 2 x^2 + 2 y^2$

5. The stream function and velocity potential for a flow are given by $\psi = 2xy$ and $\varphi = x^2 - y^2$. Show that the conditions for continuity and irrotational flow are satisfied.

Solution:

From the properties of Stream function, the existence of stream function shows the possible case of flow and if it satisfies Laplace equation, then the flow is irrotational.

(i) $\psi = 2xy$

$(\partial \psi /\partial x) = 2y$ and $(\partial \psi /\partial y) = 2x$

$(\partial^2 \psi /\partial x^2) = 0$ and $(\partial^2 \psi /\partial y^2) = 0$

$(\partial^2 \psi /\partial x \partial y) = 2$ and $(\partial^2 \psi /\partial y \partial x) = 2$

$(\partial^2 \psi /\partial x \partial y) = (\partial^2 \psi /\partial y \partial x)$

Hence the flow is Continuous.

$(\partial^2 \psi /\partial x^2) + (\partial^2 \psi /\partial y^2) = 0$

As it satisfies the Laplace equation, the flow is irrotational. From the properties of Velocity potential, the existence of Velocity potential shows the flow is irrotational and if it satisfies Laplace equation, then it is a possible case of flow

(ii) $\varphi = x^2 - y^2$

$(\partial \varphi /\partial x) = 2x$ and $(\partial \varphi /\partial y) = -2y$

$(\partial^2 \varphi /\partial x^2) = 2$ and $(\partial^2 \varphi /\partial y^2) = -2$

$(\partial^2 \varphi /\partial x \partial y) = 0$ and $(\partial^2 \varphi /\partial y \partial x) = 0$

$(\partial^2 \varphi /\partial x \partial y) = (\partial^2 \varphi /\partial y \partial x)$

Hence the flow is irrotational

$(\partial^2 \varphi /\partial x^2) + (\partial^2 \varphi /\partial y^2) = 0$

As it satisfies the Laplace equation, the flow is Continuous.

6. Derive the Bernoulli's equation from the Euler equation

As sum ptions

1. The fluid is ideal. i. e. the viscosity is zero
2. The flow is steady
3. The flow is incompressible
4. The flow is irrotational or the flow is along a stream line

From Euler's equation of motion, we have

$$\frac{dp}{\rho} + g\,dz + v\,dv = 0 \qquad \ldots(01)$$

Integrating the above expression

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{Constant}$$

As the flow is incompressible, ρ is constant and we get

$$\frac{p}{\rho} + g\,z + \frac{v^2}{2} = \text{Constant} \qquad \div g$$

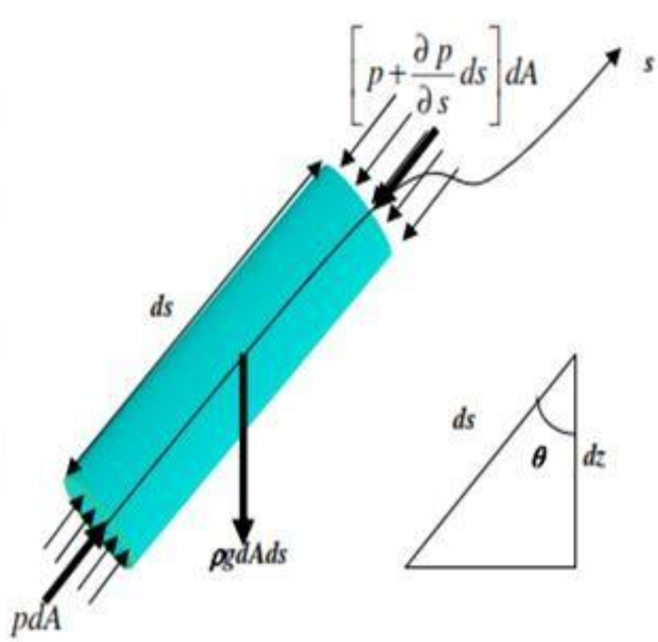
$$\frac{p}{\rho g}+\frac{v^2}{2g}+z=\text{Constant} \quad ...(02)$$

The above equation is known as Bernoulli's energy equation in which

$\frac{p}{\rho g} \Rightarrow$ *Pressure energy per unit weight or Pressure head*

$\frac{v^2}{2g} \Rightarrow$ *Kinetic energy per unit weight or kinetic/velocity head*

$z \Rightarrow$ *Potential energy per unit weight or potential/datum head*



7. Explain about the types of flow

1. Steady and unsteady flows

A flow is said to be steady if the properties (P) of the fluid and flow do not change with time (t) at any section or point in a fluid flow.

$$\frac{\partial}{\partial t}(P) = 0$$

A flow is said to be unsteady if the properties (P) of the fluid and flow change with

$$\frac{\partial}{\partial t}(P) \neq 0$$

time (t) at any section or point in a fluid flow.

Eg: Flow observed at a dam section during rainy season, wherein, there will be lot of inflow with which the flow properties like depth, velocity etc.. will change at the dam section over a period of time representing it as unsteady flow.

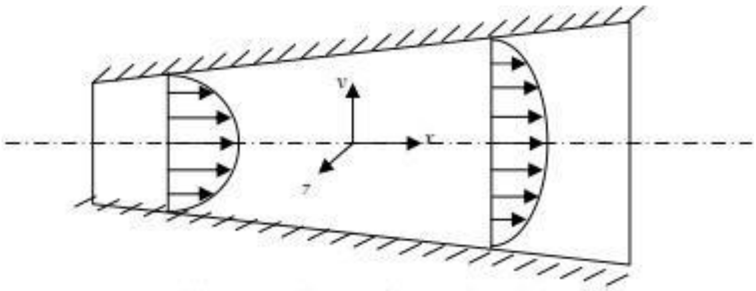
2. Uniform and non-uniform flows

A flow is said to be uniform if the properties (P) of the fluid and flow do not change (with direction) over a length of flow considered along the flow at any instant.

A flow is said to be non-uniform if the properties (P) of the fluid and flow change (with direction) over a length of flow considered along the flow at any instant.

Eg: Flow observed at any instant, at the dam section during rainy season, wherein, the flow varies from the top of the overflow section to the foot of the dam and the flow properties like depth, velocity etc., will change at the dam section at any instant between two sections, representin_g it as non-uniform flow.

Consider a fluid flow as shown above in a channel. The flow is said to be steady at sections 1 and 2 as the flow does not change with respect to time at the respective sections ($y_1=y_2$ and $v_1=v_2$). The flow between sections 1 and 2 is said to be uniform as the properties does not change between the sections at any instant ($y_1 = y_2$ and $v_1 = v_2$). The flow between sections 2 and 3 is said to be non-uniform flow as the properties vary over the length between the sections.



Non-uniform flow can be further classified as Gradually varied flow and Rapidly varied flow. As the name itself indicates, Gradually varied flow is a non-uniform flow wherein the flow/fluid properties vary gradually over a long length (Eg: between sections 2 and 3).

Rapidly varied flow is a non-uniform flow wherein the flow/fluid properties vary rapidly within a very short distance. (Eg: between sections 4 and 5).

Combination of steady and unsteady flows and uniform and non-uniform flows can be classified as steady-uniform flow (S ections 1 and 2), unsteady-uniform flow, steady-non-uniform flow (Sections 2 and 3) and unsteady-non-uniform flow (Sections 4 and 5).

3. One, two and three dimensional flows

Flow is said to be one-dimensional if the properties vary only along one axis / direction and will be constant with respect to other two directions of a three-dimensional axis system.

Flow is said to be two-dimensional if the properties vary only along two axes / directions and will be constant with respect to other direction of a three-dimensional axis system.

Flow is said to be three-dim ensional if the properties vary along all the axes /directions of a three-dimensional axis system.

4. Laminar and Turbulent flows

When the flow occurs like sheets or laminates and the fluid elements flowing in a layer does not mix with other layers, then the flow is said to be laminar. The Reynolds number (R_e) for the flow will be less than 2000.

$$R_e = \frac{\rho v D}{\mu}$$

When the flow velocity increases, the sheet like flow gets mixed up and the fluid elements mix with other layers thereby causing turbulence. There will be eddy currents generated and flow reversal takes place. This flow is said to be Turbulent.

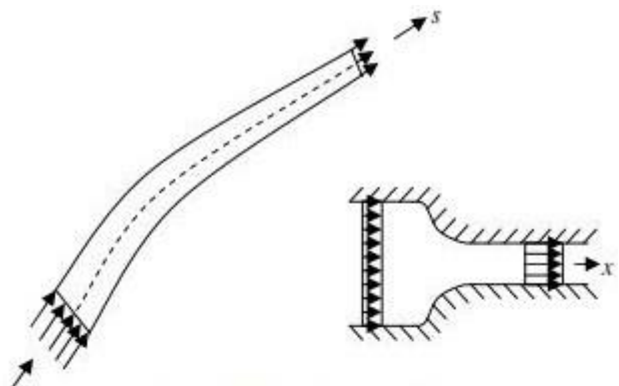


Fig. 4 a) One dimensional flow

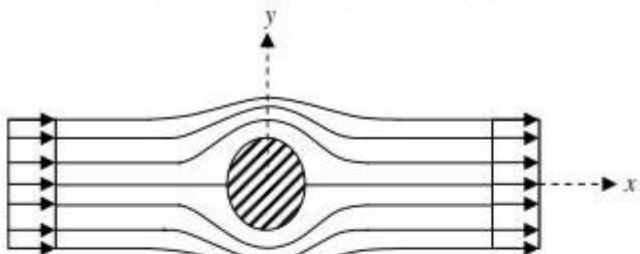


Fig. 4 b) Two dimensional flow

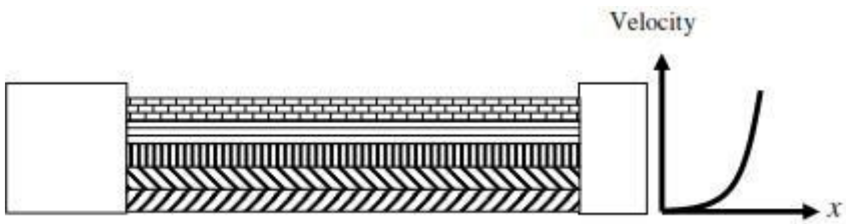


Fig. 5 Laminar flow

The Reynolds number for the flow will be greater than 4000. For flows with Reynolds number between 2000 to 4000 is said to be transition flow.

5. Compressible and Incompressible flows

Flow is said to be Incom pressible if the fluid density does not change (constant) along the flow direction and is Compressible if the fluid density varies along the flow direction

ρ = Constant (incompressible) and $\rho \neq$ Constant (compressible)

6. Rotational and Irrotational flows

Flow is said to be Rotational if the fluid elements does not rotate about their own axis as they move along the flow and is Rotational if the fluid elements rotate along their axis as they move along the flow direction

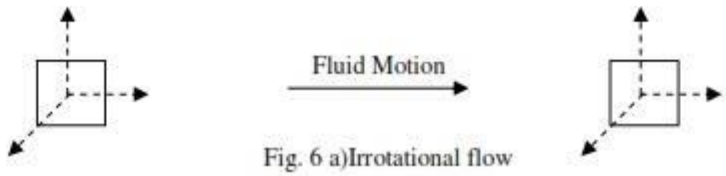


Fig. 6 a)Irrotational flow

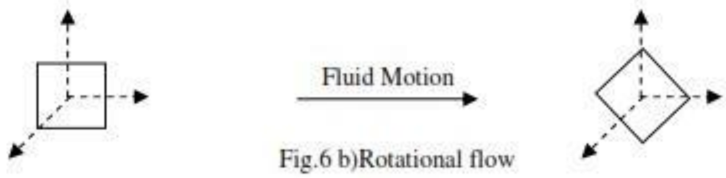


Fig.6 b)Rotational flow

8. A 250 mm diameter pipe carries oil of specific gravity 0.9 at a velocity of 3 m/s. At another section the diameter is 200 mm. Find the velocity at this section and the mass rate of flow of oil.

Solution

$D_1 = 0.25$ m, $D_2 = 0.2$ m

$S_o = 0.9$

$V_1 = 3$ m/s

$\rho = 1000$ kg/m^3

From discharge continuity equation for steady incompressible flow, we have

$Q = A_1V_1 = A_2\ V_2$

$A_1 = (\pi/4)D_1^2 = (\pi/4)0.25^2 = 0.0499\ m^2$

$A_2 = (\pi/4)D_2^2 = (\pi/4)0.20^2 = 0.0314\ m^2$

$Q = 0.0499 \times 3 = 0.1473\ m^3/s$

Mass rate of flow = ρ Q = 0.1479 x 1000 = 147.9 kg/m^3 (Ans)

$V_2 = (A_1 / A_2) \times V_1$

$= (D_1 / D_2)^2 \times V_1$

$V_2 = (0.25/0.2)^2 \times 3 = 4.6875$ m/s

9. A 30 cm x 15 cm venturimeter is inserted in a vertical pipe carring water, flowing in a upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 20 cm. Find the discharge. Take $C_d = 0.98$.

Solution

Dia at inlet, $D_1 = 30$ cm

$a_1 = (\Pi/4) \times 30^2 = 706.85\ cm^2$

Dia at throat, $D_2 = 15$ cm

$a_2 = (\Pi/4) \times 15^2 = 176.7\ cm^2$

$h = x\ (S_1/S_0 - 1) = 20(13.6/1 - 1)$

$= 20 \times 12.6 = 252$ cm of water

$Q = C_d \times (a_1 \times a_2)/ \sqrt{(a_1^2 - a_2^2)} \times \sqrt{(2 \times g \times h)}$

$= (0.98 \times 706.85 \times 176.7)/\sqrt{(706.85^2 - 176.7^2)} \times \sqrt{(2 \times 981 \times 252)}$

$= 125756\ cm^3/s$ = 125.756 lit/s

10. State the momentum equation. How will you apply momentum equation for determining the force exerted by a flowing fluid on a pipe bend.

It is base d on the law of c onser vation of mome ntum or on mome ntu m princ ip le, which st ates that the net f orce acting on a fluid mass is eq ual to the rate of cha nge of momentum of flow in that direc ti on. If the f orce ac ting on a mass of fl uid **m** is $\mathbf{F_x}$ along **x** direction, the net force along t he direction is give n b y N ewt on's second law of moti on as $\mathbf{F_x = m\, a_x}$

Where $\mathbf{a_x}$ is the a ccelera tion produced d ue to th e f orc e $\mathbf{F_x}$ al ong the sa me direction.

But $a_x = \frac{du}{dt}$

Hence $F_x = ma_x = m\frac{du}{dt} = \frac{d(mu)}{dt}$ (as m is constant for incompressible flow)

The abo ve equation is called moment um pri nciple. The sa me equa ti on can also be written as

$F_x\, dt = d\,(mu)$ whic h i s know n as **im puls e m om entum principle** an d can be state d as "The impulse of a f orce acting on a f luid of ma ss **m** in a s hor t i nte r val of ti me **dt** alon g a n y d irecti on is gi ven b y the rate of cha nge of mome ntu m **d(m u)** along the sa me dire ction.

Force e xer ted by a Fl owi n g fl uid on a pipe Bend

Consider a flow occurring in a pipe be nd w hic h i s changi ng its cr os s s ectional area al ong the be nd as sh ow n in t he Fig. Let θ be the angl e of bend a nd F_x and F_y be the f orce exe rted b y the f luid on the be nd alon g the x and y direct ion s re s pecti vel y. The f orc e exerted b y t he bend on th e mass of fluid is $-F_x$ and $-F_y$. The other f orces acti ng on t he mass of f luid are h yd r os tatic pres s ure f orces at the tw o sections 1 an d 2 $p_1 A_1$ alon g the flow direction and $p_2 A_2$ against the flow direct ion respectively. Fr om t he momentum e quati on, th e net force acting on t he f luid mass al on g x direction is given b y the rate of c ha nge of momentum in x direction.

$p_1 A_1 - p_2 A_2 \cos\theta - F_x$ =(Mas s p er second)

(change in veloc it y)

$= \rho Q$ (Final vel oc it y – initial velocit y) alon g x

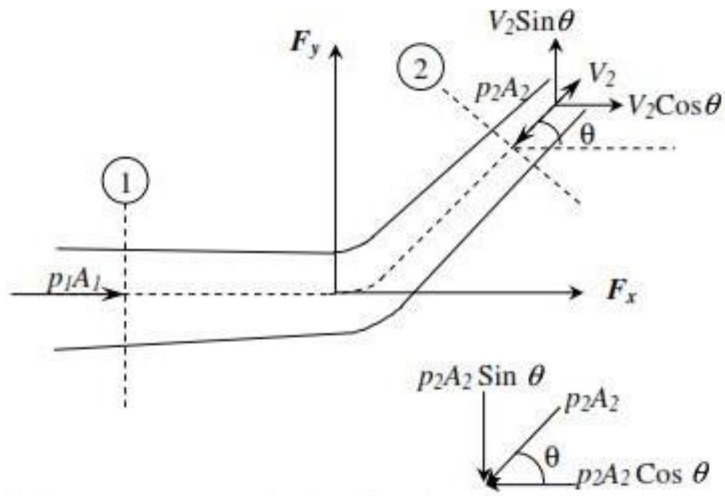
$= \rho Q\,(V_2 \cos\theta - V_1)$

$$F_x = \rho Q\,(V_1 - V_2 \cos\theta) + p_1 A_1 - p_2 A_2 \cos\theta \quad \text{---(01)}$$

Similarl y the mome ntu m e quation i n y d irection gi ves

$0 - p_2 A_2 \sin\theta - F_y = \rho Q\,(V_2 \sin\theta - 0)$

$F_y = \rho Q\,(-V_2 \sin\theta) - p_2 A_2 \sin\theta \quad \ldots(02)$


V_2Sinθ
F_y
2
p_2A_2
V_2
V_2Cosθ
θ
1
p_1A_1
F_x
p_2A_2 Sin θ
p_2A_2
θ
p_2A_2 Cos θ

UNIT III DIMENSIONAL ANALYSIS AND MODEL STUDIES

DIMENSIONAL ANALYSIS AND MODEL STUDIES
PART – A(2 MARKS)

1. Define dimensional analysis.

   Dimensional analysis is a mathematical technique which makes use of the study of dimensions as an aid to solution of several engineering problems. It plays an important role in research work.

2. Write the uses of dimension analysis?
   - It helps in testing the dimensional homogeneity of any equation of fluid motion.
   - It helps in deriving equations expressed in terms of non-dimensional parameters.
   - It helps in planning model tests and presenting experimental results in a systematic manner.

3. Define dimensional homogeneity.

   An equation is said to be dimensionally homogeneous if the dimensions of the terms on its LHS are same as the dimensions of the terms on its RHS.

4. Mention the methods available for dimensional analysis.

   Rayleigh method,
   Buckinghum π method

5. State Buckingham's π theorem.

   It states that "if there are 'n' variables (both independent & dependent variables) in a physical phenomenon and if these variables contain 'm' functional dimensions and are related by a dimensionally homogeneous equation, then the variables are arranged into n-m dimensionless terms. Each term is called π term".

6. List the repeating variables used in Buckingham π theorem.

   Geometrical Properties – l, d, H, h, etc,
   Flow Properties – v, a, g, ω, Q, etc,
   Fluid Properties – ρ, μ, γ, etc.

7. Define model and prototype.

   The small scale replica of an actual structure or the machine is known as its Model, while the actual structure or machine is called as its Prototype. Mostly models are much smaller than the corresponding prototype.

8. Write the advantages of model analysis.
   - Model test are quite economical and convenient.
   - Alterations can be continued until most suitable design is obtained.
   - Modification of prototype based on the model results.
   - The information about the performance of prototype can be obtained well in advance.

9. List the types of similarities or similitude used in model anlaysis.

   Geometric similarities, Kinematic similarities, Dynamic similarities

10. Define geometric similarities

    It exists between the model and prototype if the ratio of corresponding lengths, dimensions in the model and the prototype are equal. Such a ratio is known as "Scale Ratio".

11. Define kinematic similarities

    It exists between the model and prototype if the paths of the homogeneous moving particles are geometrically similar and if the ratio of the flow properties is equal.

12. Define dynamic similarities

It exits between model and the prototype which are geometrically and kinematic similar and if the ratio of all forces acting on the model and prototype are equal.

13. Mention the various forces considered in fluid flow.
    - Inertia force,
    - Viscous force,
    - Gravity force,
    - Pressure force,
    - Surface Tension force,
    - Elasticity force
14. Define model law or similarity law.

    The condition for existence of completely dynamic similarity between a model and its prototype are denoted by equation obtained from dimensionless numbers. The laws on which the models are designed for dynamic similarity are called Model laws or Laws of Similarity.
15. List the various model laws applied in model analysis.
    - Reynold's Model Law,
    - Froude's Model Law,
    - Euler's Model Law,
    - Weber Model Law,
    - Mach Model Law
16. State Euler's model law

    In a fluid system where supplied pressures are the controlling forces in addition to inertia forces and other forces are either entirely absent or in-significant the Euler's number for both the model and prototype which known as Euler Model Law.
17. State Weber's model law

    When surface tension effect predominates in addition to inertia force then the dynamic similarity is obtained by equating the Weber's number for both model and its prototype, which is called as Weber Model Law.
18. State Mach's model law

    If in any phenomenon only the forces resulting from elastic compression are significant in addition to inertia forces and all other forces may be neglected, then the dynamic similarity between model and its prototype may be achieved by equating the Mach's number for both the systems. This is known Mach Model Law.
19. Classify the hydraulic models.

    The hydraulic models are classified as: Undistorted model & Distorted model
20. Define undistorted model

    An undistorted model is that which is geometrically similar to its prototype, i.e. the scale ratio for corresponding linear dimensions of the model and its prototype are same.
21. Define distorted model

    Distorted models are those in which one or more terms of the model are not identical with their counterparts in the prototype.
22. Define Scale effect

    An effect in fluid flow that results from changing the scale, but not the shape, of a body around which the flow passes.

PART B

1. Explain about the types of Similarity

Hydraulic models may be either true or distorted models. True models reproduce features of the prototypebut at a scale - that is they are *geometrically* similar.

(i) Geometric similarity

Geometric similarity exists between model and prototype if the ratio of all corresponding dimensions inthe model and prototype are equal.

$$\frac{L_{model}}{L_{prototype}} = \frac{L_m}{L_p} = \lambda_L$$

where λ_L is the scale factor for length.

For area

$$\frac{A_{model}}{A_{prototype}} = \frac{L_m^2}{L_p^2} = \lambda_L^2$$

All corresponding angles are the same.

(ii) Kinematic similarity

Kinematic similarity is the similarity of time as well as geometry. It exists between model and prototype

i. If the paths of moving particles are geometrically similar
ii. If the rations of the velocities of particles are similar

Some useful ratios are:

Velocity $$\frac{V_m}{V_p} = \frac{L_m / T_m}{L_p / T_p} = \frac{\lambda_L}{\lambda_T} = \lambda_u$$

Acceleration $$\frac{a_m}{a_p} = \frac{L_m / T_m^2}{L_p / T_p^2} = \frac{\lambda_L}{\lambda_T^2} = \lambda_a$$

Discharge $$\frac{Q_m}{Q_p} = \frac{L_m^3 / T_m}{L_p^3 / T_p} = \frac{\lambda_L^3}{\lambda_T} = \lambda_Q$$

This has the consequence that streamline patterns are the same.

(iii) Dynamic similarity

Dynamic similarity exists between geometrically and kinematically similar systems if the ratios of all forces in the model and prototype are the same.

Force ratio $\frac{F_m}{F_p} = \frac{M_m a_m}{M_p a_p} = \frac{\rho_m L_m^3}{\rho_p L_p^3} \times \frac{\lambda_L}{\lambda_T^2} = \lambda_\rho \lambda_L^2 \left(\frac{\lambda_L}{\lambda_T}\right)^2 = \lambda_\rho \lambda_L^2 \lambda_u^2$

This occurs when the controlling dimensionless group on the right hand side of the defining equation isthe same for model and prototype.

2. The discharge Q through an orifice is a function of the diameter d, the pressure difference p, the density ρ,and the viscosity μ, show that $Q = \frac{d^2 p^{1/2}}{\rho^{1/2}} \phi\left(\frac{d\rho^{1/2} p^{1/2}}{\mu}\right)$, where ϕ is some unknown function.

p:(force/area) $ML^{-1}T^{-2}$

We are told from the question that there are 5 variables involved in the problem: *d, p,* ρ, μ and *Q*.

Choose the three recurring (governing) variables; *Q, d,* ρ.

From Buckingham's π theorem we have m-n = 5 - 3 = 2 non-dimensional groups.

Write out the dimensions of the variables

ρ:	ML^{-3}	*u:*	LT^{-1}
d:	L	μ:	$ML^{-1}T^{-1}$

$c_1 = -1$

L] $0 = 3a_1 + b_1 - 3c_1 - 1$

$-2 = 3a_1 + b_1$

T] $0 = -a_1 - 1$

$a_1 = -1$

$b_1 = 1$

$\phi(Q, d, \rho, \mu, p) = 0$

$\phi(\pi_1, \pi_2) = 0$

$\pi_1 = Q^{a_1} d^{b_1} \rho^{c_1} \mu$

$\pi_1 = Q^{-1} d^{1} \rho^{-1} \mu$

$= \frac{d\mu}{\rho Q}$

$\pi_2 = Q^{a_2} d^{b_2} \rho^{c_2} p$

For the first group, π_1:

$M^0 L^0 T^0 = (L^3 T^{-1})^{a_1} (L)^{b_1} (ML^{-3})^{c_1} ML^{-1}T^{-1}$

M] $0 = c_1 +$

And the second group π_2 :

(note p is a pressure (force/area) with dimensions $ML^{-1}T^{-2}$)

$$M^0 L^0 T^0 = \left(L^3 T^{-1}\right)^{a_1} (L)^{b_1} \left(ML^{-3}\right)^{c_1} MT^{-2} L^{-1}$$

M] $0 = c_2 + 1$

$c_2 = -1$

L] $0 = 3a_2 + b_2 - 3c_2 - 1$

$-2 = 3a_2 + b_2$

T] $0 = -a_2 - 2$

$a_2 = -2$

$b_2 = 4$

$$\pi_2 = Q^{-2} d^4 \rho^{-1} p = \frac{d^4 p}{\rho Q^2}$$

$$\pi_{1a} = \pi_1 \pi_{2a} = \frac{d\mu}{Q\rho} \frac{\rho^{1/2} Q}{d^2 p^{1/2}} = \frac{\mu}{d\rho^{1/2} p^{1/2}}$$

So the physical situation is described by this function of non-dimensional numbers, then we can say

$$\phi(\pi_1, \pi_2) = \phi\left(\frac{d\mu}{Q\rho}, \frac{d^4 p}{\rho Q^2}\right) = 0$$

or

$$\phi(1/\pi_{1a}, \pi_{2a}) = \phi\left(\frac{d\rho^{1/2} p^{1/2}}{\mu}, \frac{d^2 p^{1/2}}{Q\rho^{1/2}}\right) = 0$$

or

$$\frac{d\mu}{Q\rho} = \phi_1\left(\frac{d^4 p}{\rho Q^2}\right)$$

$$Q = \frac{d^2 p^{1/2}}{\rho^{1/2}} \phi\left(\frac{d\rho^{1/2} p^{1/2}}{\mu}\right)$$

The question wants us to show : $Q = \frac{d^2 p^{1/2}}{\rho^{1/2}} \phi\left(\frac{d\rho^{1/2} p^{1/2}}{\mu}\right)$

Take the reciprocal of square root of π_2: $\frac{1}{\sqrt{\pi_2}} = \frac{\rho^{1/2} Q}{d^2 p^{1/2}} = \pi_{2a}$,

Convert π_1 by multiplying by this new group, π_{2a}

3. A model aeroplane is built at 1/10 scale and is to be tested in a wind tunnel operating at a pressure of 20times atmospheric. The aeroplane will fly at 500km/h. At what speed should the wind tunnel operate togive dynamic similarity between the model and prototype? If the drag measure on the model is 337.5 Nwhat will be the drag on the plane?

From earlier we derived the equation for resistance on a body moving through air:

$$R = \rho u^2 l^2 \phi\left(\frac{\rho u l}{\mu}\right) = \rho u^2 l^2 \phi(\mathrm{Re})$$

For dynamic similarity $\mathrm{Re}_m = \mathrm{Re}_p$, so

$$u_m = u_p \frac{\rho_p}{\rho_m} \frac{d_p}{d_m} \frac{\mu_m}{\mu_p}$$

The value of μ does not change much with pressure so $\mu_m = \mu_p$

The equation of state for an ideal gas is $p = \rho RT$. As temperature is the same then the density of the air in the model can be obtained from

$$\frac{p_m}{p_p} = \frac{\rho_m RT}{\rho_p RT} = \frac{\rho_m}{\rho_p}$$

$$\frac{20\, p_p}{p_p} = \frac{\rho_m}{\rho_p}$$

$$\rho_m = 20\rho_p$$

So the model velocity is found to be

$$u_m = u_p \frac{1}{20} \frac{1}{1/10} = 0.5u_p$$

$$u_m = 250\ km/h$$

The ratio of forces is found from

$$\frac{R_m}{R_p} = \frac{(\rho u^2 l^2)_m}{(\rho u^2 l^2)_p}$$

$$\frac{R_m}{R_p} = \frac{20}{1} \frac{(0.5)^2}{1} \frac{(0.1)^2}{1} = 0.05$$

So the drag force on the prototype will be

$$R_p = \frac{1}{0.05} R_m = 20 \times 337.5 = 6750N$$

4. A 1:100 scale model has a total drag of $F = 0.9$ N when towed at a speed of $U = 1.0$ m/s in fresh water. The smooth hull has its leading edge roughened, a wetted length of 0.60 m and a wetted surface area of 0.10 m^2. Calculate for the prototype in salt water the corresponding speed, total drag force and power required to overcome the drag force.

Solution: The model surface drag calculation requires a Reynolds number.

$$Re = \frac{UL}{\nu} = \frac{(1.0)\,(0.6)}{1.31 \times 10^{-6}} = 4.6 \times 10^5$$

Since the rough leading edge ensures a completely turbulent boundary layer, Figure 8.4 gives

$$C_D = 0.0052$$

This allows an approximate calculation of the model surface drag.

$$F_{\text{sur}} = C_D A \rho \frac{U^2}{2} = (0.0052)\,(0.10)\,(1000)\,\frac{(1.0)^2}{2} = 0.26 \text{ N}$$

Thus, the pressure drag for the model is

$$F_{\text{pres}} = F - F_{\text{sur}} = 0.9 - 0.26 = 0.64 \text{ N}$$

The corresponding prototype speed is obtained by requiring Froude number similarity.

$$\left(\frac{U}{\sqrt{g\ell}}\right)_p = \left(\frac{U}{\sqrt{g\ell}}\right)_m \quad \text{or} \quad U_p = U_m\sqrt{\frac{\ell_p}{\ell_m}} = 1.0\sqrt{\frac{100}{1}} = \boxed{10 \text{ m/s} = 36 \text{ km/hr}}$$

Euler number similarity gives the prototype pressure drag.

$$\left(\frac{F/\ell^2}{\rho U^2/2}\right)_p = \left(\frac{F/\ell^2}{\rho U^2/2}\right)_m \quad \text{or} \quad F_p = F_m \frac{\rho_p}{\rho_m}\left(\frac{U_p}{U_m}\right)^2\left(\frac{\ell_p}{\ell_m}\right)^2$$

Since $U_p/U_m = \sqrt{\ell_p/\ell_m}$, this gives

$$F_p = F_m \frac{\rho_p}{\rho_m}\left(\frac{\ell_p}{\ell_m}\right)^3 = (0.64)\,(1.025)\left(\frac{100}{1}\right)^3 = 6.56 \times 10^5 \text{ N}$$

The prototype Reynolds number is

$$Re = \frac{UL}{\nu} = \frac{(10)\,(0.60 \times 100)}{1.31 \times 10^{-6}} = 4.6 \times 10^{8}$$

in which L has been computed from the product of the model wetted length with the scale ratio. Figure 8.4 gives

$$C_D = 0.0017$$

Thus, the prototype surface drag is

$$F_{sur} = C_D A \rho \frac{U^2}{2} = (0.0017)\left(0.10 \times 100^2\right)(1{,}025)\,\frac{10^2}{2} = 8.71 \times 10^4 \text{ N}$$

Adding the prototype surface and pressure drag forces gives the total drag.

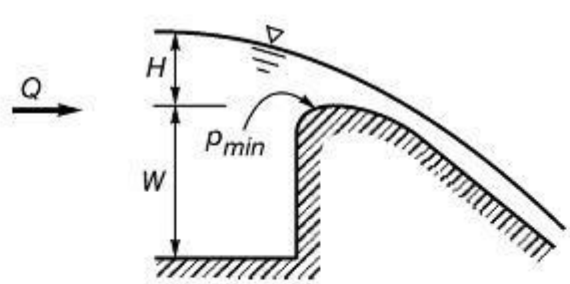
$$F = F_{pres} + F_{sur} = 6.56 \times 10^5 + 8.71 \times 10^4 = \boxed{7.43 \times 10^5 \text{ N}}$$

Finally, the power required to overcome this drag is calculated from the product of the drag force and speed.

$$\text{Power} = FU = \left(7.43 \times 10^5\right)(10) = 7.43 \times 10^6 \text{ watts}$$

$$= \boxed{7.43 \text{ megawatts}}$$

5. A model study of the spillway shown in the sketch is to be carried out in a laboratory by constructing a geometrically similar model at a reduced scale.



Solution: The flow rate, Q, over the spillway is a function of the following independent variables:

$$Q = f(W, H, g, \epsilon, \nu, \rho)$$

W = spillway height, H = reservoir height above the spillway crest, g = gravitational constant, ϵ = spillway surface roughness, ν = kinematic viscosity and ρ = fluid mass density. Dimensions of these variables follow:

$$Q \sim \frac{L^3}{T}, \quad W \sim L, \quad H \sim L, \quad g \sim \frac{L}{T^2}, \quad \epsilon \sim L, \quad \nu \sim \frac{L^2}{T}, \quad \rho \sim \frac{M}{L^3}$$

Combining W and g with Q gives

$$Q \times \frac{1}{\sqrt{g}} \times \frac{1}{W^{5/2}} = \frac{Q}{W^2\sqrt{gW}}$$

$$\frac{L^3}{T} \times \frac{T}{L^{1/2}} \times \frac{1}{L^{5/2}} \sim 1$$

This dimensionless flow rate is a Froude number. Combining W and g with H obviously gives H/W, and from ϵ we obtain a relative roughness term ϵ/W. The kinematic viscosity can be combined with W and g to obtain

$$\frac{1}{\nu} \times \sqrt{g} \times W^{3/2} = \frac{W\sqrt{gW}}{\nu}$$

$$\frac{T}{L^2} \times \frac{L^{1/2}}{T} \times L^{3/2} \sim 1$$

Since any analysis that contains a viscosity, velocity and length can be expected to yield a Reynolds number, we will replace this last variable by its product with the dimensionless flow rate to obtain

$$\boxed{\frac{Q}{W^2\sqrt{gW}} = f_1\left(\frac{H}{W}, \frac{\epsilon}{W}, \frac{Q}{\nu W}\right)}$$

The result contains both a Froude and Reynolds number, but there is a fundamental difficulty in obtaining both Froude and Reynolds number similarity when the same fluid is used for model and prototype. In particular, this requires

$$\left(\frac{Q}{W^2\sqrt{gW}}\right)_p = \left(\frac{Q}{W^2\sqrt{gW}}\right)_m \quad \text{and} \quad \left(\frac{Q}{\nu W}\right)_p = \left(\frac{Q}{\nu W}\right)_m$$

If g and ν are the same for both model and prototype, this gives two conflicting requirements:

$$\frac{Q_p}{Q_m} = \left(\frac{W_p}{W_m}\right)^{5/2} \quad \text{and} \quad \frac{Q_p}{Q_m} = \frac{W_p}{W_m}$$

$$\boxed{\frac{Q}{W^2\sqrt{gW}} = f_1\left(\frac{H}{W}\right)}$$

and similarity requirements become

$$\boxed{\left(\frac{Q}{W^2\sqrt{gW}}\right)_p = \left(\frac{Q}{W^2\sqrt{gW}}\right)_m} \quad \text{and} \quad \boxed{\left(\frac{H}{W}\right)_p = \left(\frac{H}{W}\right)_m}$$

Minimum spillway crest pressures depend upon the following variables:

$$p_{\min} = f(W, H, g, \rho)$$

in which boundary layer effects have been neglected. Dimensions of $p_{\min}$ are

$$p_{\min} \sim \frac{M}{LT^2}$$

$$p_{\min} \times \frac{1}{g} \times \frac{1}{\rho} \times \frac{1}{W} = \frac{p_{\min}}{\rho g W}$$

$$\frac{M}{LT^2} \times \frac{T^2}{L} \times \frac{L^3}{M} \times \frac{1}{L} \sim 1$$

Combining W, g and ρ with H gives H/W, and we obtain the following end result:

$$\frac{p_{\min}}{\rho g W} = f\left(\frac{H}{W}\right)$$

Thus, for similarity we have the requirements

$$\left(\frac{p_{\min}}{\rho g W}\right)_p = \left(\frac{p_{\min}}{\rho g W}\right)_m \quad \text{and} \quad \left(\frac{H}{W}\right)_p = \left(\frac{H}{W}\right)_m$$

6. The drag force on a large structure can be measured in a laboratory with a greatly reduced scale model. The model and prototype must be geometrically similar and must be orientated with respect to the flow in the same way. Then the drag coefficient is a function only of the Reynolds number and relative roughness

$$C_D = f\left(\frac{UL}{\nu}, \frac{\epsilon}{L}\right)$$

Solution

U = approach velocity, L = characteristic length, ν = kinematic viscosity and ϵ = roughness height. The unknown function f is to be determined from measurements, and it will be the same function for both model and prototype. Thus, if the Reynolds number and relative roughness are the same for model and prototype,

$$\left(\frac{UL}{\nu}\right)_m = \left(\frac{UL}{\nu}\right)_p \quad \text{and} \quad \left(\frac{\epsilon}{L}\right)_m = \left(\frac{\epsilon}{L}\right)_p$$

then C_D will be the same for model and prototype.

$$(C_D)_m = (C_D)_p \quad \text{or} \quad \left(\frac{F/A}{\rho U^2/2}\right)_m = \left(\frac{F/A}{\rho U^2/2}\right)_p$$

The Reynolds number similarity requirement can be rewritten in the form

$$\frac{U_m}{U_p} = \frac{L_p}{L_m}\frac{\nu_m}{\nu_p}$$

in which $L_p/L_m > 1$. If the same fluid is used for model and prototype, then $\nu_m/\nu_p = 1$ and we must have

$$\frac{U_m}{U_p} = \frac{L_p}{L_m} > 1$$

This is not usually practical. For example, the drag force on a 50 m high building in a 20 m/s wind could be measured in a wind tunnel with a 0.5 m high model only if

$$U_m = \frac{L_p}{L_m} U_p = \frac{50}{0.5}(20) = 2{,}000 \text{ m/s}$$

If water is used as the fluid for the model, then $\nu_m/\nu_p \approx 1/10$ and

$$U_m = \frac{L_p}{L_m}\frac{\nu_m}{\nu_p} U_p = \frac{50}{0.5}\frac{1}{10}(20) = 200 \text{ m/s}$$

7. An underwater missile, diameter 2m and length 10m is tested in a water tunnel to determine the forcesacting on the real prototype. A 1/20th scale model is to be used. If the maximum allowable speed of theprototype missile is 10 m/s, what should be the speed of the water in the tunnel to achieve dynamicsimilarity?

For dynamic similarity the Reynolds number of the model and prototype must be equal:

$$\text{Re}_m = \text{Re}_p$$

$$\left(\frac{\rho u d}{\mu}\right)_m = \left(\frac{\rho u d}{\mu}\right)_p$$

So the model velocity should be

$$u_m = u_p \frac{\rho_p d_p \mu_m}{\rho_m d_m \mu_p}$$

As both the model and prototype are in water then, $\mu_m = \mu_p$ and $\rho_m = \rho_p$

UNIT IV FLOW THROUGH PIPES

PART – A(2 MARKS)

1. What is meant by energy loss in a pipe?

   When the fluid flows through a pipe, it loses some energy or head due to frictional resistance and other reasons. It is called energy loss. The losses are classified as; Major losses and Minor losses

2. Explain the major losses in a pipe.

   The major energy losses in a pipe is mainly due to the frictional resistance caused by the shear force between the fluid particles and boundary walls of the pipe and also due to viscosity of the fluid.

3. Explain minor losses in a pipe.

   The loss of energy or head due to change of velocity of the flowing fluid in magnitude or direction is called minor losses. It includes: sudden expansion of the pipe, sudden contraction of the pipe, bend in a pipe, pipe fittings and obstruction in the pipe, etc.

4. State Darcy-Weisbach equation **OR** What is the expression for head loss due to friction?

$$h_f = 4flv^2 / 2gd$$

where, h_f = Head loss due to friction (m), L = Length of the pipe (m),

d = Diameter of the pipe (m),

V = Velocity of flow (m/sec)

f = Coefficient of friction

where V varies from 1.5 to 2.0.

5. What are the factors influencing the frictional loss in pipe flow? Frictional resistance for the turbulent flow is,

   i. Proportional to v^n

   ii. Proportional to the density of fluid.

   iiiProportional to the area of surface in contact.

   iv. Independent of pressure.

   v. Depend on the nature of the surface in contact.

6. What is compound pipe or pipes in series?

   When the pipes of different length and different diameters are connected end to end, then the pipes are called as compound pipes or pipes in series.

7. What is mean by parallel pipe and write the governing equations.

   When the pipe divides into two or more branches and again join together downstream to form a single pipe then it is called as pipes in parallel. The governing equations are:

$$Q_1 = Q_2 + Q_3 \quad h_{f1} = h_{f2}$$

8. Define equivalent pipe and write the equation to obtain equivalent pipe diameter.

   The single pipe replacing the compound pipe with same diameter without change in discharge and head loss is known as equivalent pipe.

9. What is meant by Moody's chart and what are the uses of Moody's chart?

   The basic chart plotted against Darcy-Weisbach friction factor against Reynold's Number (Re) for the variety of relative roughness and flow regimes. The relative roughness is the ratio of the mean height of roughness of the pipe and its diameter (ε/D).

   Moody's diagram is accurate to about 15% for design calculations and used for a large number of applications. It can be used for non-circular conduits and also for open channels.

10. Define the terms a) Hydraulic gradient line [HGL] b) Total Energy line [TEL]

Hydraulic gradient line: It is defined as the line which gives the sum of pressure head and datum head of a flowing fluid in a pipe with respect the reference line.

HGL = Sum of Pressure Head and Datum head

Total energy line: Total energy line is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

TEL = Sum of Pressure Head, Datum head and Velocity head

11. What do you understand by the terms a) major energy losses , b) minor energy losses

Major energy losses : -

This loss due to friction and it is calculated by Darcy weis bach formula and chezy's formula .

Minor energy losses :- This is due to

i. Sudden expansion in pipe .
ii. Sudden contraction in pipe .
iii. Bend in pipe .
iv. Due to obstruction in pipe .

12. . Give an expression for loss of head due to sudden enlargement of the pipe :- he = $(V1-V2)^2 /2g$

Where he = Loss of head due to sudden enlargement of pipe .

V1 = Velocity of flow at section 1-1

V2 = Velocity of flow at section 2-2

13. Give an expression for loss of head due to sudden contraction : -

hc =$0.5 V^2/2g$

Where hc = Loss of head due to sudden contraction .

V = Velocity at outlet of pipe.

14. Give an expression for loss of head at the entrance of the pipe: -

hi =$0.5V^2/2g$

where hi = Loss of head at entrance of pipe .

15. What are the basic educations to solve the problems in flow through branched pipes?

i. Continuity equation .
ii. Bernoulli's formula .
iii. Darcy weisbach equation .

16. Mention the general characteristics of laminar flow.

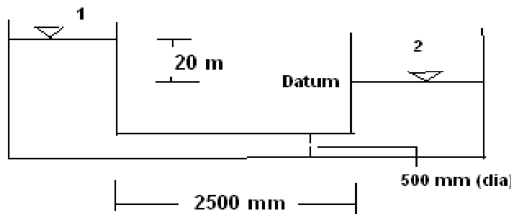
- There is a shear stress between fluid layers
- 'No slip' at the boundary
- The flow is rotational
- There is a continuous dissipation of energy due to viscous shear

PART - B(16 MARKS)

1. Two tanks are connected by a 500mm diameter 2500mm long pipe. Find the rate of flow if the difference in water levels between the tanks is 20m. Take f=0.016. Neglect minor losses.

Solution:-

Applying Bernoulli's equation between (1) & (2) with (2) as datum & considering head loss due to friction hf only,



$$Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_f \;---(1)$$

Z1 = 20m, Z2 = 0 (Datum);

V1=V2 = 0 (tanks are very large)

p1=p2=0 (atmospheric pressure)

Therefore From (1)

20+0+0 = 0+0+0+hf

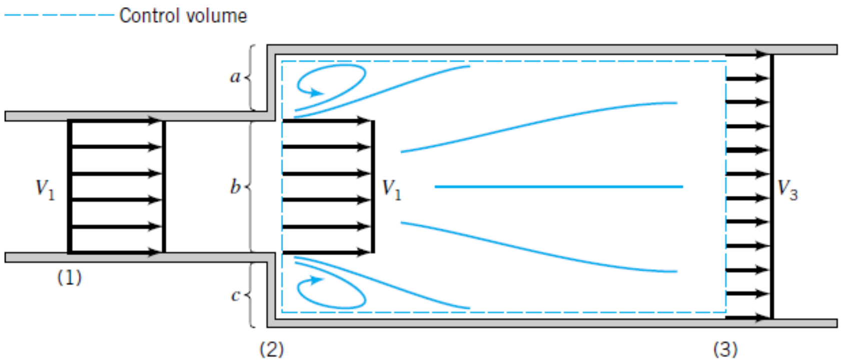
Or hf = 20m.

But $h_f = \frac{8 fLQ^2}{g\pi D^5}$

$$Q = \left[\frac{20x9.81x\pi^2 x0.5^5}{8x0.016x2500}\right]^{\frac{1}{2}}$$

$$Q = 0.4348 m^3/\sec = 434.8 lps$$

2. Derive the expression for head lost and power lost due to Suddden Expansion



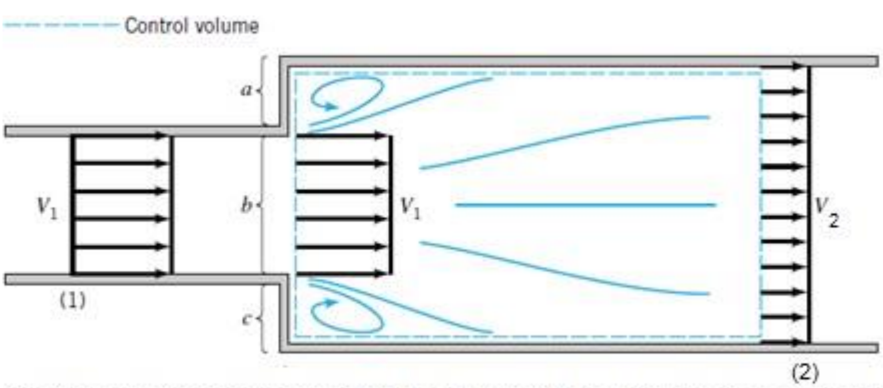
Control volume used to calculate the loss coefficient for a sudden expansion.

Flow in a sudden expansion is similar to exit flow.
Referring Fig., the fluid leaves the smaller pipe and initially forms a jet-type structure as it enters the larger pipe.
Within a few diameters downstream of the expansion, the jet becomes dispersed across the pipe.

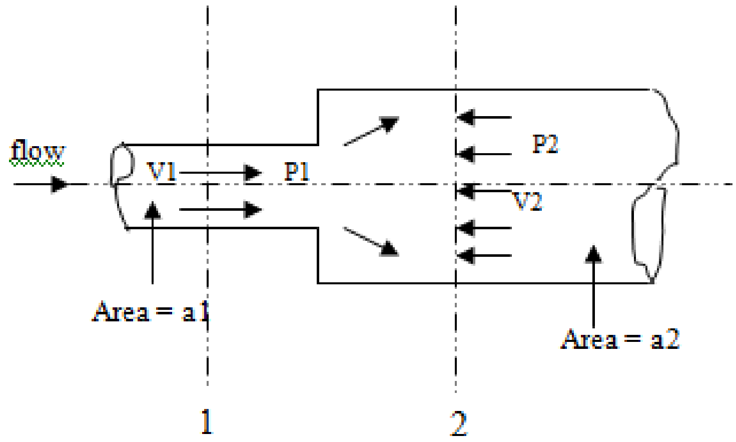
In this process of dispersion [between sections (2) and (3)], a portion of the kinetic energy of the fluid is dissipated as a result of viscous effects.
Loss coefficient for sudden expansion can be obtained by means of a simple analysis based on continuity and momentum equations for the control volume shown in figure

Assumption: Flow is uniform at sections (1), (2), and (3) and the pressure is constant across the left-hand side of the control volume (pa = pb = pc = p1)

Consider the sections as shown in figure
P1 & P2 are the pressure acting at (1) (1) and (2) (2)
From experiments, it is proved that pressure P1 acts on the area (a2 – a1) i.e. at the point of sudden expansion.



Control volume used to calculate the loss coefficient for a sudden expansion.



V1 and V2 are the velocities.
From II Law of Newton Force = Mass x Acceleration.
The forces acting on the control volume (LHS)

$$\sum forces = +p_1a_1 - p_2a_2 + p_1(a_2 - a_1) --- (i)$$

$$or, \sum forces = a_2(p_1 - p_2) --- (ii)$$

RHS of Neton's second law,
Mass x acceleration = ρ x vol x change in velocity /time

= ρ xvolume/time x change in velocity

$$= \rho x Q x (V_1 - V_2) --- (iii)$$

Substitution (ii) & (iii) in newton's Equation

$$a_2(p_1 - p_2) = pQ(V_1 - V_2)$$

Divide both sides by sp.weight

$$\therefore \left(\frac{p_1 - p_2}{\gamma}\right) = \frac{V_2(V_1 - V_2)}{g} --- (iv)$$

Applying Bernoulli's equation between (1) and (2) with the centre line of the pipe as datum and considering head loss due to sudden expansion hLonly.

$$Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$Z_1 = Z_2$ because pipe is horizontal

$$\therefore \left(\frac{p_1 - p_2}{\gamma}\right) + \left(\frac{V_1^2 - V_2^2}{2g}\right) = h_L \qquad \text{----------- v}$$

Repalcing $(p_1-p_2)/\gamma$ by Eq. (iv) in Eq. (v)

$$h_L = \frac{2V_2(V_2 - V_1) + (V_1^2 - V_2^2)}{2g}$$

$$h_L = \frac{2V_1V_2 - 2V_2^2 + V_1^2 - V_2^2}{2g}$$

$$h_L = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$h_L = \frac{V_2^2 + V_1^2 - 2V_1V_2}{2g}$$

$$h_L = \frac{(V_1 - V_2)^2}{2g} \qquad \text{.....vi}$$

The Equation (vi) represents the loss due to sudden expansion.

Loss of Power

The loss of power in overcoming the head loss in the transmission of fluid is given by

$$P = \gamma Q h_f --- (vi)$$

3. Derive the DARCY – WEISBACH Equation

Force = Mass x accn. But acceleration = 0, as there is no change in velocity, the reason that pipe diameter is uniform or same throughout.

$$\therefore \Sigma forces = 0$$

$$i.e. \quad +P_1\frac{\pi D^2}{4} - P_2\frac{\pi D^2}{4} - \tau_0 x \pi DL = 0$$

$$(P_1 - P_2)\frac{\pi D^2}{4} = \tau_0 \pi DL$$

$$or (P_1 - P_2) = \frac{4\tau_0 L}{D} --- (1)$$

Applying Bernoulli's equation between (1) & (2) with the centre line of the pipe as datum & considering head loss due to friction hf,.

$$Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_f$$

$Z_1 = Z_2$ ∵ Pipe is horizontal

$V_1 = V_2$ ∵ Pipe diameter is same throughout

$$\therefore \frac{P_1 - P_2}{\gamma} = h_f --- (2)$$

Substituting eq (2) in eq.(1) $h_f x \gamma = \frac{4\tau_0 L}{D}$

or $\tau_0 = \frac{h_f \gamma D}{4L} --- (3)$

From Experiments, Darcy Found that

$$\tau_0 = \frac{f}{8}\rho V^2 --- (4)$$

f=Darcy's friction factor (property of the pipe materials Mass density of the liquid.

V = average velocity

Substituting eq (4) in eq.(3)

$$\frac{f}{8}\rho V^2 = \frac{h_f \gamma D}{4L} \quad \text{or} \quad h_f = \frac{4Lf\rho V^2}{8\gamma D}$$

But $\frac{\gamma}{\rho} = g$

$$\therefore h_f = \left(\frac{fLV^2}{2gD}\right) --- (5)$$

From Continuity equation, Q= AxV, V= Q/A

$$V = \frac{4Q}{\pi D^2}$$

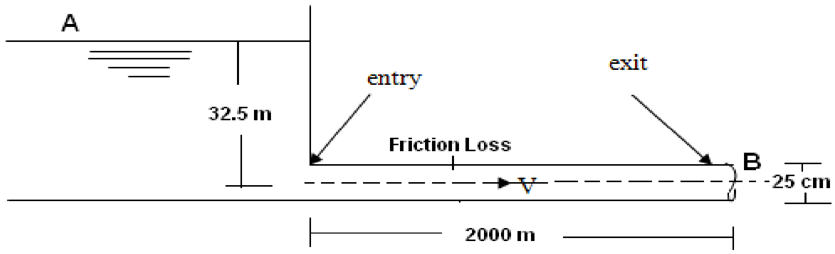
Substituting for V in Eq. 5,

$$\therefore h_f = \left(\frac{8fLQ^2}{g\pi^2 D^5}\right) --- (6)$$

Equations (5) & (6) are known as DARCY – WEISBACH Equation

4. A 25cm diameter, 2km long horizontal pipe is connected to a water tank. The pipe discharges freely into atmosphere on the downstream side. The head over the centre line of the pipe is 32.5m, f=0.0185. Find the discharge through the pipe

Solution:



,

Applying Bernoulli's equation between (A) and (B) with (B) as datum & considering all losses.

$$Z_A + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} = Z_B + \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + entryloss + frictionloss + exitloss$$

The tank surface and the outlet are exposed to atmospheric condition and hence, $P_A = P_B$.
When tank area is compared with the pipe area, it is very much grater than the pipe and hence the variation of velocity in the tank can be neglected. Therefore, $V_A = 0$.
The above equation now can be written as,

$$32.5 + 0 + 0 = 0 + 0 + \frac{V^2}{2g} + \frac{0.5V^2}{2g} + \frac{fLV^2}{2gD} + \frac{V^2}{2g}$$

$$32.5 = \frac{V^2}{2g}\left[1 + 0.5 + \frac{0.0185 X\, 2000}{0.25} + 1\right]$$

$$32.5 = 7.67V^2$$

$$V = 2.06 m / s$$

The discharge is calculated using continuity equation.

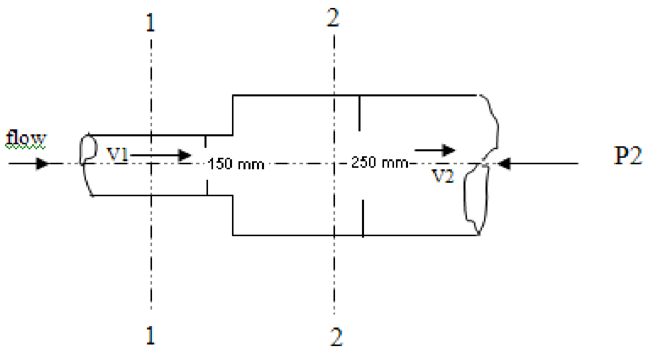
$$Q = \frac{\pi D^2}{4} V$$

$$\pi x\, 0.25^2 / 4 \; x 2.06 = 0.101 m^3 / \sec$$

$$Q = 101 lps$$

5. The discharge through a pipe is 225lps. Find the loss of head when the pipe is suddenly enlarged from 150mm to 250mm diameter.

Solution:
D1=0.15m, D2 = 0.25m Q=225lps = 225m3/sec



Head loss due to sudden expansion is

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

Writing the above equation in terms of discharge,,

$$hL = \left(\frac{4Q}{\pi D_1^2} - \frac{4Q}{\pi D_2^2}\right)^2 X \frac{1}{2g}$$

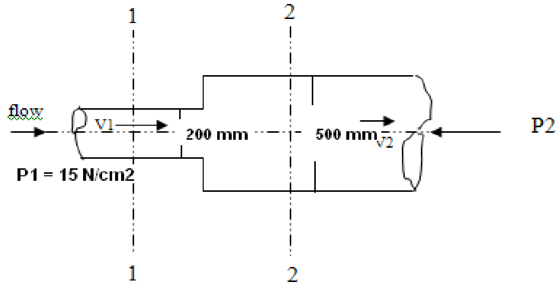
$$h_L = \frac{16Q^2}{2g\pi^2}\left(\frac{1}{D_1^2} - \frac{1}{D_2^2}\right)^2$$

$$= \frac{16x0.225^2}{2x9.81x\pi^2}\left(\frac{1}{0.15^2} - \frac{1}{0.25^2}\right)^2$$

$$h_L = 3.385m$$

6. The rate of flow of water through a horizontal pipe is 350lps. The diameter of the pipe is suddenly enlarge from 200mm to 500mm. The pressure intensity in the smaller pipe is 15N/cm². Determine (i) loss of head due to sudden enlargement. (ii) pressure intensity in the larger pipe (iii) power lost due to enlargement.

Solution:
Q=350lps=0.35m^3/s
D1=0.2m, D2=0.5m,
P1=15N/cm2
hL=?, p2=?, P=?



From continuity equation

$$V_2 = \frac{4Q}{\pi D_2^2} = \frac{4x0.35}{\pi x 0.5^2} = 1.78 m/s$$

$$h_L = \frac{(V_1 - V_2)^2}{2g} = \frac{(11.14 - 1.78)^2}{2x9.81} = 4.463 m of water$$

Applying Bernoulli's equation between (1) (1) and (2) (2) with the central line of the pipe as datum and considering head loss due to sudden expansion hL only,

$$Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h$$

$$0 + \frac{150}{9.81} + \frac{11.14^2}{19.62} = 0 + \frac{p_2}{9.81} + \frac{1.78^2}{19.62} + 4.463$$

$$Z_1 = Z_2 = 0 (pipe horizontal)$$

$$p_2 = 166.68 kN/m^2 = 16.67 N/cm^2$$

Power Loss;

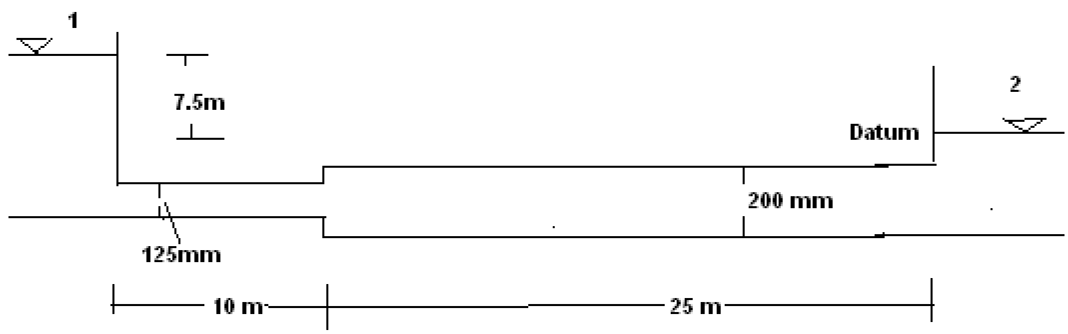
$$P = \gamma Q h_L$$

$$= 9.81x0.35x4.463$$

$$P = 15.32 kW$$

7. Two reservoirs are connected by a pipe line which is 125mm diameter for the first 10m and 200mm in diameter for the remaining 25m. The entrance and exit are sharp and the change of section is sudden. The water surface in the upper reservoir is 7.5m above that in the lower reservoir. Determine the rate of flow, assuming f=0.001 for each of the types.

Solution;



From continuity equation

$$\frac{\pi x 0.125^2}{4} V_1 = \frac{\pi x 0.2^2}{4} V_2$$

$$\therefore V_1 = 2.56 V_2$$

$$7.5+0+0\} = \left[0+0+0+\frac{0.5V_1^2}{2g}+\frac{fL_1V_1^2}{2g}+\frac{(V_1-V_2)^2}{2g}+\frac{fL_2V_2^2}{2g}+\frac{V_2^2}{2g}\right]$$

$$7.5+0+0\} = \left[\frac{0.5(2.56V_2)^2}{2g}+\frac{0.01x10x(2.56V_2)^2}{2g}+\frac{(2.56V_2-V_2)^2}{2g}+\frac{0.01x25x(V_2)^2}{2g}+\frac{V_2^2}{2g}\right]$$

$$Z_A+\frac{p_A}{\gamma}+\frac{V_A^2}{2g}=Z_B+\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+entryloss+frictionloss+sudden\ expansionloss+frictionloss+exitloss$$

The tank surfaces are exposed to atmospheric condition and hence, $P_1 = P_2$.
When tank area is compared with the pipe area, it is very much grater than the pipe and hence the variation of velocities in the tanks can be neglected. Therefore, $V_1 = V_2 = 0$.
The above equation now can be written as,

$$147.15 = 3.2768 (V_2)^2 + 0.65536(V_2)^2 + 2.4336 (V_2)^2 + 0.25(V_2)^2 + (V_2)^2$$
$$147.15 = 7.61576 (V_2)^2$$

$$V_2 = (147.15/7.61576)^{0.5}$$

V_2 = 4.4 m/s

Q= (π(0.2)²/4)x4.4 = 0.138 m³/s

8. Derive the expression for pipes in series and parallel connection

(i)Pipes in Series

The compound pipe (or pipe n series) is an arrangement made by connecting different diameters of pipe with a common axis as shown in figure.

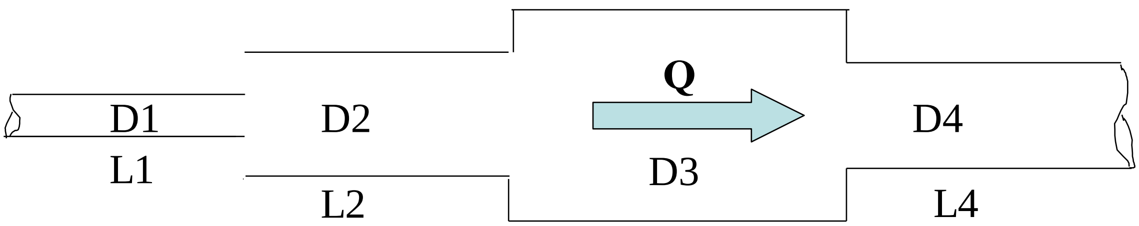


Fig. 6 Compound Pipe OR Pipes in Series

Let D1, D2, D3, D4 be the diameters of the pies as shown in figure..
Let L1, L2,L3, L4 are lengths of a number of Pipes connected in series
The (hf)1, (hf)2, (hf)3 & (hf)4 are the head loss due to friction for each pipe.

The total head loss due to friction, h_f , for the entire pipe system is the summation of each of the head loss occurring in all he pipes, which is given by,

$$h_f = hf_1 + hf_2 + hf_3 + hf_4$$

I.e.,

Or

$$h_f = \frac{8\,fL_1Q^2}{g\pi^2 D_1^5} + \frac{8\,fL_2Q^2}{g\pi^2 D_2^5} + \frac{8\,fL_3Q^2}{g\pi^2 D_3^5} + \frac{8\,fL_4Q^2}{g\pi^2 D_4^5}$$

(ii)Pipes in parallel

The below figure shows the arrangement of pipes in parallel. As it can be seen from the figure, the pipes are parallel to each other.

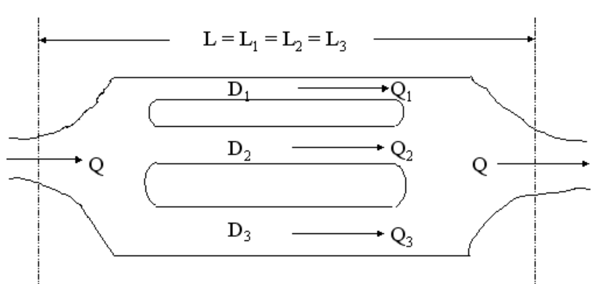


Fig. 7 Pipes in Parallel Arrangement

In this arrangement, the length of the each pipe is same and discharge is distributed in the parallely connected pipes a shown in figure.

Let D1, D2, D3be the diameter of the pipe.
Let L1=L2=L3= L, is length of Pipe connected in parallel
The (hf)1, (hf)2, (hf)3 are the head loss due to friction for each pipe.
The condition for the parallel pipe is
h_f = hf)1= (hf)2= (hf)3
i.e.,

$$h_f = \frac{fL_1V_1^2}{2gD_1} = \frac{fL_2V_2^2}{2gD_2} = \frac{fL_3V_3^2}{2gD_3}$$

Or

$$\frac{8fL_1Q_1^2}{g\pi^2D_1^5} = \frac{8fL_2Q_2^2}{g\pi^2D_2^5} = \frac{8fL_3Q_3^2}{g\pi^2D_3^5}$$

or

$$\frac{Q_1^2}{D_1^5} = \frac{Q_2^2}{D_2^5} = \frac{Q_3^2}{D_3^5}$$

From continuity equation Q= Q1+Q2+Q3

UNIT V BOUNDRY LAYER

PART – A (2 MARKS)

1. Mention the range of Reynold's number for laminar and turbulent flow in a pipe.

   If the Reynolds number is less than 2000, the flow is laminar. But if the Reynold's number is greater than 4000, the flow is turbulent flow.

2. What does Haigen-Poiseulle equation refer to?

   The equation refers to the value of loss of head in a pipe of length 'L' due to viscosity in a laminar flow.

3. What are the factors to be determined when viscous fluid flows through the circular pipe?

   The factors to be determined are:

   i. Velocity distribution across the section.
   ii. Ratio of maximum velocity to the average velocity.
   iii. Shear stress distribution.
   iv. Drop of pressure for a given length

4. Define kinetic energy correction factor?

   Kinetic energy factor is defined as the ratio of the kinetic energy of the flow per sec based on actual velocity across a section to the kinetic energy of the flow per sec based on average velocity across the same section. It is denoted by (α).

   K. E factor (α) = K.E per sec based on actual velocity / K.E per sec based on Average velocity

5. Define momentum correction factor (β):

   It is defined as the ratio of momentum of the flow per sec based on actual velocity to the momentum of the flow per sec based on average velocity across the section.

   β= Momentum per sec based on actual velocity/Momentum Per sec based on average velocity

6. Define Boundary layer.

   When a real fluid flow passed a solid boundary, fluid layer is adhered to the solid boundary. Due to adhesion fluid undergoes retardation thereby developing a small region in the immediate vicinity of the boundary. This region is known as boundary layer.

7. What is mean by boundary layer growth?

   At subsequent points downstream of the leading edge, the boundary layer region increases because the retarded fluid is further retarded. This is referred as growth of boundary layer.

8. Classification of boundary layer.

   (i) Laminar boundary layer,
   (ii) Transition zone,
   (iii)Turbulent boundary layer.

9. Define Laminar boundary layer.

   Near the leading edge of the surface of the plate the thickness of boundary layer is small and flow is laminar. This layer of fluid is said to be laminar boundary layer.

   The length of the plate from the leading edge, upto which laminar boundary layer exists is called as laminar zone. In this zone the velocity profile is parabolic.

10. Define transition zone.

    After laminar zone, the laminar boundary layer becomes unstable and the fluid motion transformed to turbulent boundary layer. This short length over which the changes taking place is called as transition zone.

11. Define Turbulent boundary.

Further downstream of transition zone, the boundary layer is turbulent and continuous to grow in thickness. This layer of boundary is called turbulent boundary layer.

12. Define Laminar sub Layer

In the turbulent boundary layer zone, adjacent to the solid surface of the plate the velocity variation is influenced by viscous effects. Due to very small thickness, the velocity distribution is almost linear. This region is known as laminar sub layer.

13. Define Boundary layer Thickness.

It is defined as the distance from the solid boundary measured in y-direction to the point, where the velocity of fluid is approximately equal to 0.99 times the free stream velocity (U) of the fluid. It is denoted by δ.

14. List the various types of boundary layer thickness.

Displacement thickness(δ*),

Momentum thickness(θ),

Energy thickness(δ**)

15. Define displacement thickness.

The displacement thickness (δ) is defined as the distance by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation.

$$\delta^* = \int [\, 1 - (u/U) \,]\, dy$$

16. Define momentum thickness.

The momentum thickness (θ) is defined as the distance by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation.

$$\theta = \int [\, (u/U) - (u/U)^2 \,]\, dy$$

17. Define energy thickness

The energy thickness (δ**) is defined as the distance by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation.

PART - B(16 MARKS)

1. Explain about the boundary layer separationBoundary layer separation

(i) Convergent flows: Negative pressure gradients

If flow over a boundary occurs when there is a pressure decrease in the direction of flow, the fluid willaccelerate and the boundary layer will become thinner.

This is the case for convergent flows.

u_1 u_2

p_1 p_2

$p_1 > p_2$ $u_1 < u_2$

The accelerating fluid maintains the fluid close to the wall in motion. Hence the flow remains stable and turbulence reduces. Boundary layer separation does not occur.

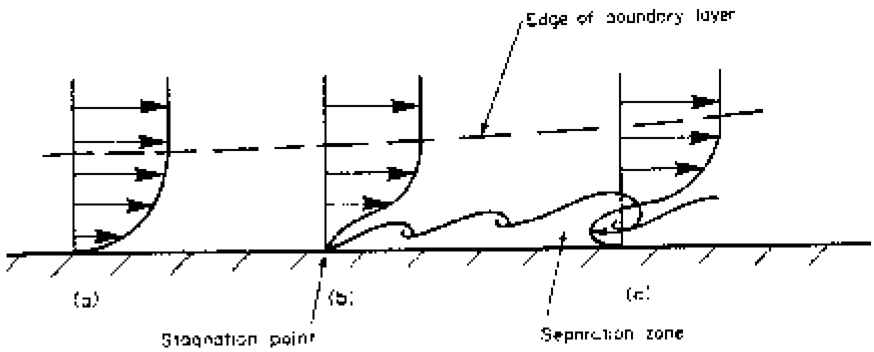
(ii) Divergent flows: Positive pressure gradients

When the pressure increases in the direction of flow the situation is very different. Fluid outside theboundary layer has enough momentum to overcome this pressure which is trying to push it backwards.The fluid within the boundary layer has so little momentum that it will very quickly be brought to rest,and possibly reversed in direction. If this reversal occurs it lifts the boundary layer away from the surfaceas shown below.

u_1 u_2

p_1 p_2

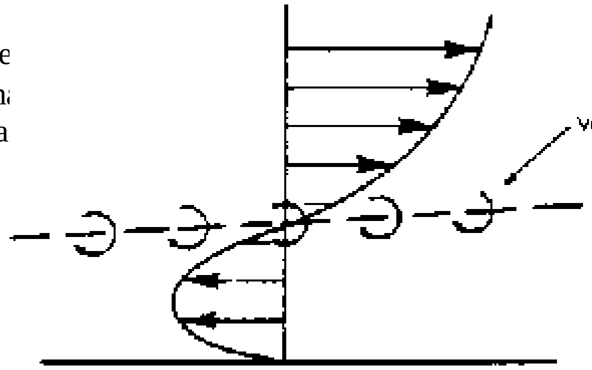
$p_1 < p_2$ $u_1 > u_2$



This phenomenon is known as **boundary layer separation**.

At the edge of the separated boundary layer, where the velocities change direction, a line of vortices occur (known as a vortex sheet). This happens because fluid to either side is moving in the opposite direction.

Increasing the angle of the ... paration. In a Venturi meter it has been found th... etween length of meter and danger of boundary la... e energy losses.

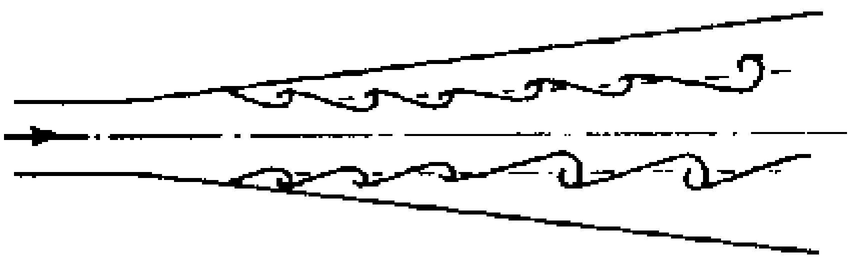


This boundary layer separation and increase in the turbulence because of the vortices results in very large energy losses in the flow.

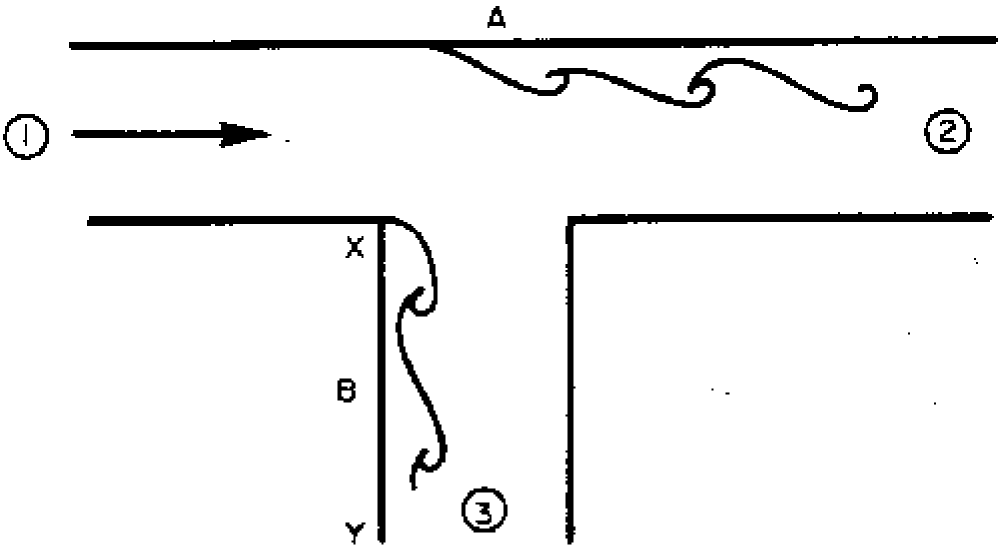
These separating / divergent flows are inherently unstable and far more energy is lost than in parallel or convergent flow.

(iii) A divergent duct or diffuser

The increasing area of flow causes a velocity drop (according to continuity) and hence a pressure rise(according to the Bernoulli equation).



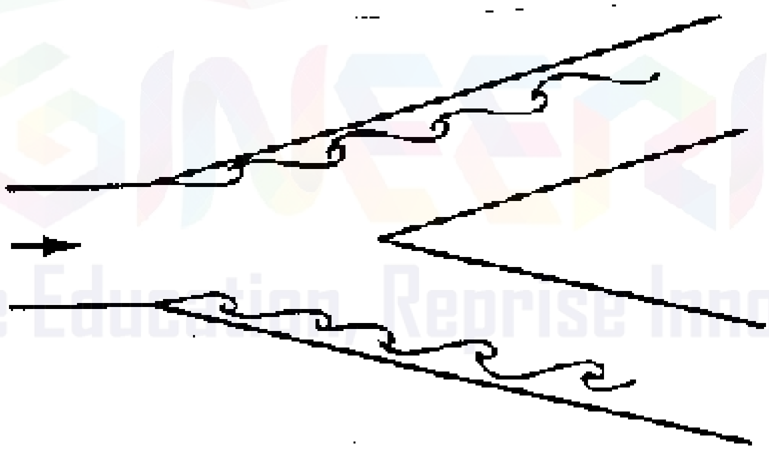
(iv) Tee-Junctions



Assuming equal sized pipes, as fluid is removed, the velocities at 2 and 3 are smaller than at 1, the entrance to the tee. Thus the pressure at 2 and 3 are higher than at 1. These two adverse pressure gradients can cause the two separations shown in the diagram above.

(v) Y-Junctions

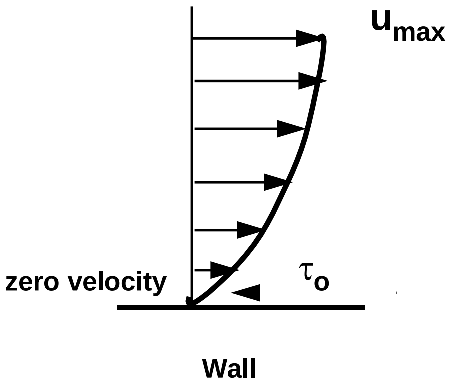
Tee junctions are special cases of the Y-junction with similar separation zones occurring. See the diagram below.



Downstream, away from the junction, the boundary layer reattaches and normal flow occurs i.e. the effect of the boundary layer separation is only local. Nevertheless fluid downstream of the junction will have lost energy.

2. Explain about Boundary Layers

When a fluid flows over a stationary surface, e.g. the bed of a river, or the wall of a pipe, the fluidtouching the surface is brought to rest by the shear stress τ_o at the wall. The velocity increases from thewall to a maximum in the main stream of the flow.



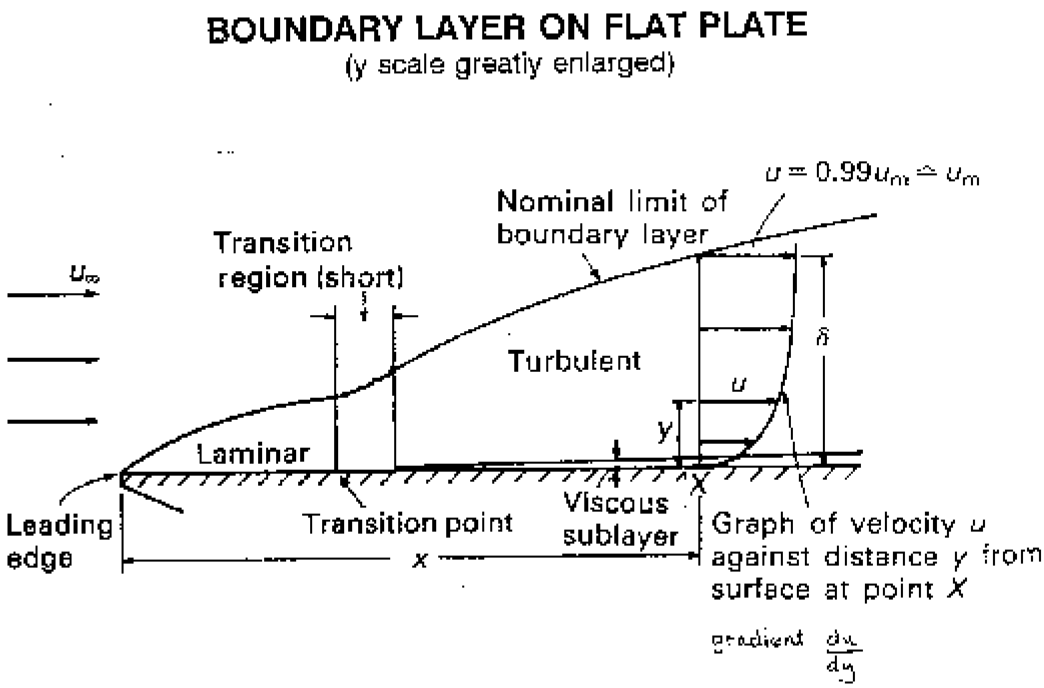
Looking at this two-dimensionally we get the above velocity profile from the wall to the centre of the flow.

This profile doesn't just exit, it must build up gradually from the point where the fluid starts to flow past the surface - e.g. when it enters a pipe.

If we consider a flat plate in the middle of a fluid, we will look at the build up of the velocity profile as the fluid moves over the plate.

Upstream the velocity profile is uniform, (free stream flow) a long way downstream we have the velocity profile we have talked about above. This is the known as **fully developed flow**. But how do we get to that state?

This region, where there is a velocity profile in the flow due to the shear stress at the wall, we call the **boundary layer**. The stages of the formation of the boundary layer are shown in the figure below:



We define the thickness of this boundary layer as the distance from the wall to the point where the velocity is 99% of the "free stream" velocity, the velocity in the middle of the pipe or river.

boundary layer thickness, δ = distance from wall to point where $u = 0.99\, u_{mainstream}$

The value of δ will increase with distance from the point where the fluid first starts to pass over the boundary - the flat plate in our example. It increases to a maximum in fully developed flow.

Correspondingly, the drag force D on the fluid due to shear stress τ_o at the wall increases from zero at the start of the plate to a maximum in the fully developed flow region where it remains constant. We can calculate the magnitude of the drag force by using the momentum equation. But this complex and not necessary for this course.

Our interest in the boundary layer is that its presence greatly affects the flow through or round an object. So here we will examine some of the phenomena associated with the boundary layer and discuss why these occur.

3. Explain the effects of Formation of the boundary layer

Above we noted that the boundary layer grows from zero when a fluid starts to flow over a solid surface. As is passes over a greater length more fluid is slowed by friction between the fluid layers close to the boundary. Hence the thickness of the slower layer increases.

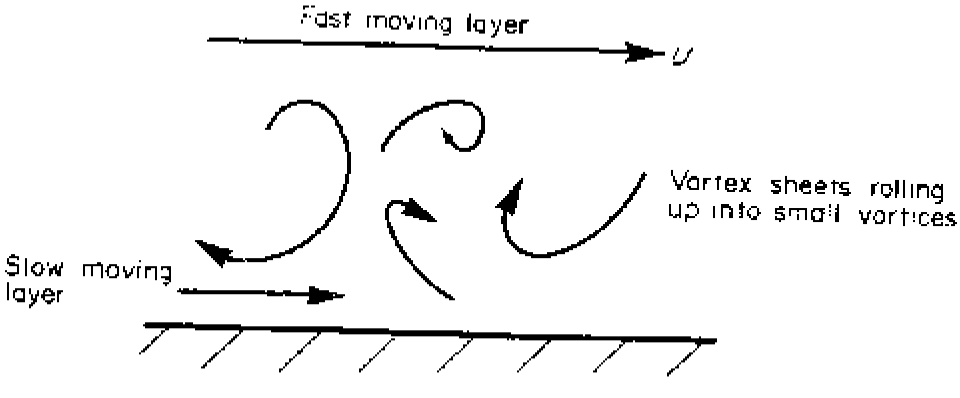
The fluid near the top of the boundary layer is dragging the fluid nearer to the solid surface along. The mechanism for this dragging may be one of two types:

The first type occurs when the normal viscous forces (the forces which hold the fluid together) are large enough to exert drag effects on the slower moving fluid close to the solid boundary. If the boundary layer is thin then the velocity gradient normal to the surface, (du/dy), is large so by Newton's law of viscosity the shear stress, $\tau = \mu\, (du/dy)$, is also large. The corresponding force may then be large enough to exert drag on the fluid close to the surface.

As the boundary layer thickness becomes greater, so the velocity gradient become smaller and the shear stress decreases until it is no longer enough to drag the slow fluid near the surface along. If this viscous force was the only action then the fluid would come to a rest.

It, of course, does not come to rest but the second mechanism comes into play. Up to this point the flow has been **laminar** and Newton's law of viscosity has applied. This part of the boundary layer is

The viscous shear stresses have held the fluid particles in a constant motion within layers. They become small as the boundary layer increases in thickness and the velocity gradient gets smaller. Eventually they are no longer able to hold the flow in layers and the fluid starts to rotate.



This causes the fluid motion to rapidly becomes turbulent. Fluid from the fast moving region moves to the slower zone transferring momentum and thus maintaining the fluid by the wall in motion. Conversely, slow moving fluid moves to the faster moving region slowing it down. The net effect is an increase in momentum in the boundary layer. We call the part of the boundary layer the **turbulent boundary layer**.

At points very close to the boundary the velocity gradients become very large and the velocity gradients become very large with the viscous shear forces again becoming large enough to maintain the fluid in laminar motion. This region is known as the **laminar sub-layer**. This layer occurs within the turbulent zone and is next to the wall and very thin – a few hundredths of a mm.

Surface roughness effect

Despite its thinness, the laminar sub-layer can play a vital role in the friction characteristics of the surface.

This is particularly relevant when defining pipe friction - as will be seen in more detail in the level 2 module. In **turbulent** flow if the height of the roughness of a pipe is greater than the thickness of the laminar sub-layer then this increases the amount of turbulence and energy losses in the flow. If the height of roughness is less than the thickness of the laminar sub-layer the pipe is said to be smooth and it has little effect on the boundary layer.

In **laminar** flow the height of roughness has very little effect

Boundary layers in pipes

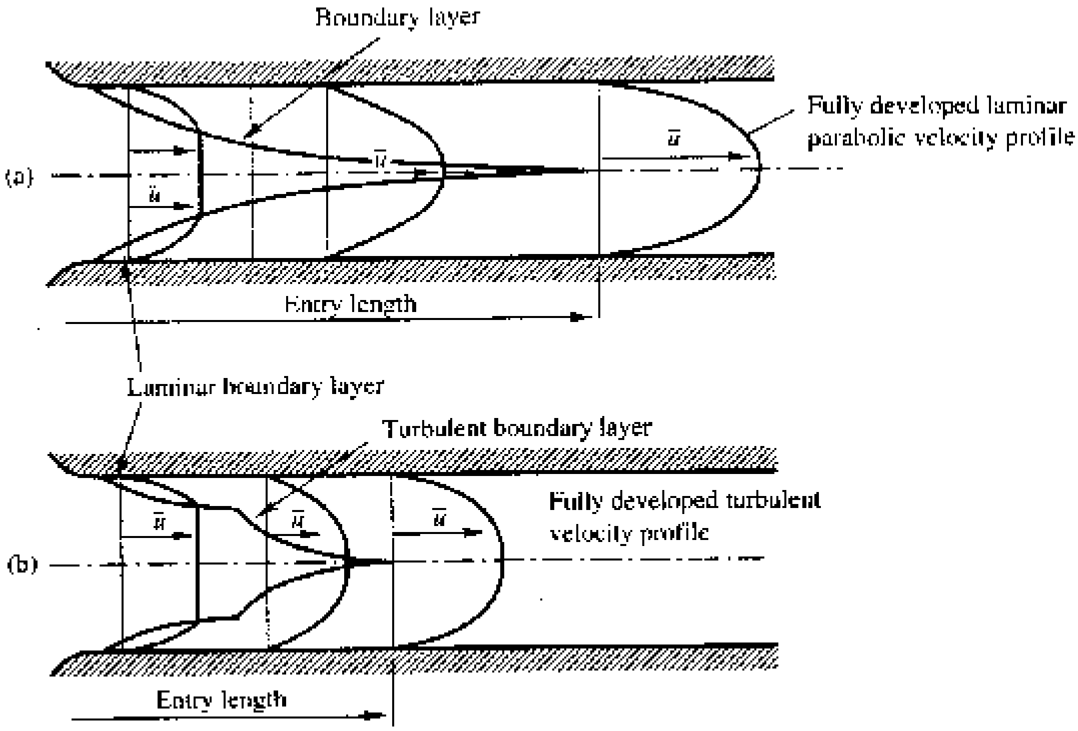
As flow enters a pipe the boundary layer will initially be of the laminar form. This will change depending on the ration of inertial and viscous forces; i.e. whether we have laminar (viscous forces high) orturbulent flow (inertial forces high).

From earlier we saw how we could calculate whether a particular flow in a pipe is laminar or turbulent using the Reynolds number.

$$\mathrm{Re} = \frac{\rho u d}{\mu}$$

(ρ = *density* u = *velocity* μ = *viscosity* d = *pipe diameter*)

Laminar flow:	$\mathrm{Re} < 2000$
Transitional flow:	$2000 < \mathrm{Re} < 4000$
Turbulent flow:	$\mathrm{Re} > 4000$



If we only have laminar flow the profile is parabolic – as proved in earlier lectures – as only the first part of the boundary layer growth diagram is used. So we get the top diagram in the above figure.

If turbulent (or transitional), both the laminar and the turbulent (transitional) zones of the boundary layer growth diagram are used. The growth of the velocity profile is thus like the bottom diagram in the above figure.

Once the boundary layer has reached the centre of the pipe the flow is said to be **fully developed**. (Note that at this point the whole of the fluid is now affected by the boundary friction.)

The length of pipe before fully developed flow is achieved is different for the two types of flow. The length is known as the **entry length**.

Laminar flow entry length $\approx 120 \times$ diameter

Turbulent flow entry length $\approx 60 \times$ diameter

4. From the given calculate the solution for a laminar boundary layer along a flat plate. Since fully developed laminar flow between two flat plates has a velocity distribution given by a parabola, we will assume that

$$u(x, y) = a(x) + b(x)y + c(x)y^2$$

Physics requires the boundary conditions

$$u(x, 0) = 0 \quad , \quad u(x, \delta) = U \quad , \quad \frac{\partial u(x, \delta)}{\partial y} = 0$$

These three equations determine a, b and c and lead to the result

$$u(x, y) = U(2\xi - \xi^2) \text{ in which } \xi = \frac{y}{\delta(x)}$$

Thus, the integral on the right sides of

$$\int_0^\delta (U - u)u\,dy = U^2 \int_0^\delta (1 - 2\xi + \xi^2)(2\xi - \xi^2)\,dy = U^2\delta \int_0^1 (2\xi - 5\xi^2 + 4\xi^3 - \xi^4)\,d\xi$$

$$= \frac{2}{15} U^2 \delta(x)$$

$$\tau_0 = \mu \frac{\partial u(x, 0)}{\partial y} = \mu \left(\frac{du}{d\xi} \right)_{\xi = 0} \frac{\partial \xi}{\partial y} = \mu \frac{2U}{\delta}$$

Thus, Eq. (8.7) becomes

$$\frac{\mu}{\rho} \frac{2U}{\delta} = \frac{2}{15} U^2 \frac{d\delta}{dx}$$

Separating variables gives

$$\int_0^{\delta(x)} \delta\,d\delta = 15 \frac{\nu}{U} \int_0^x dx$$

and integration gives

$$\frac{1}{2}\delta^2 = 15 \frac{\nu}{U} x$$

This can be written dimensionlessly as

$$\frac{\delta}{x} = \sqrt{\frac{30}{Ux/\nu}} = \frac{5.48}{\sqrt{Re_x}}$$

The force per unit width on one side of the plate is calculated from (8.9).

$$F = \rho \int_0^{\delta(L)} (U-u)\,u\,dy = \rho \frac{2}{15} U^2 \delta(L) = \rho \frac{2}{15} U^2 \frac{5.48}{\sqrt{Re_L}} L$$

This can be put in the more significant form

$$F = C_D A \rho \frac{U^2}{2} \quad , \quad A = L \times 1 \quad , \quad C_D = \frac{1.46}{\sqrt{Re_L}} \quad , \quad Re_L = \frac{UL}{\nu}$$

The exact solution of (5.23 a, b, c) that was obtained by Blasius gave

$$\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}} \quad , \quad C_D = \frac{1.33}{\sqrt{Re_L}}$$

5. From the given equation calculate an approximate solution for a turbulent boundary layer along a smooth flat plate. We will use the one seventh power law given by Eq. (7.43)

$$u(x,y) = U\xi^{1/7} \quad , \quad \xi = \frac{y}{\delta(x)}$$

Thus, the integral on the right sides

$$\int_0^{\delta} (U-u)\,u\,dy = U^2 \delta \int_0^1 \left(1-\xi^{1/7}\right)\xi^{1/7}\,d\xi = \frac{7}{72} U^2 \delta(x)$$

$$\tau_0 = 0.0225 \left(\frac{\nu}{U\delta} \right)^{1/4} \rho U^2$$

$$0.0225 \left(\frac{\nu}{U\delta} \right)^{1/4} U^2 = \frac{7}{72} U^2 \frac{d\delta}{dx}$$

Integration of this differential equation gives

$$\frac{\delta}{x} = \frac{0.37}{Re_x^{1/5}}$$

This allows the drag force to be computed

$$F = \rho \int_0^{\delta(L)} (U - u)\, u\, dy = \rho \frac{7}{72} U^2 \delta(L) = \rho \frac{7}{72} U^2 \frac{0.37}{Re_L^{1/5}} L$$

This result can be rewritten in the following standard form:

$$F = C_D A \rho \frac{U^2}{2} \quad , \quad A = L \times 1 \quad , \quad C_D = \frac{0.072}{Re_L^{1/5}} \quad , \quad Re_L = \frac{UL}{\nu}$$

In practice, experimental data shows that this holds for $5 \times 10^5 < Re_L < 10^7$ when 0.072 is changed to 0.074 in the formula for C_D.

6. From the given equation assume that the boundary layer is either entirely laminar or entirely turbulent from the leading edge of the flat plate. More generally, the boundary layer will change from laminar to turbulent at $x = x_c$ when $0 < x_c < L$. It is possible to calculate a solution for this case using the same techniques that were used in examples 8.1 and 8.2. In practice, a simpler approximation suggested by Prandtl is used in which the laminar drag force for $0 < x < x_c$ is added to the turbulent drag force for $x_c < x < L$. Prandtl's approximation assumes that the forces on each of these two intervals are identical with the forces that would occur if the boundary layer were entirely laminar or entirely turbulent, respectively, from the leading edge. Thus, if we set

$$Re_c = \frac{U x_c}{\nu}$$

then the total drag force is approximated with

$$F = \frac{1.33}{\sqrt{Re_c}} x_c \rho \frac{U^2}{2} + \frac{0.074}{Re_L^{1/5}} L \rho \frac{U^2}{2} - \frac{0.074}{Re_c^{1/5}} x_c \rho \frac{U^2}{2}$$

$$F = C_D A \rho \frac{U^2}{2}$$

in which

$$C_D = \frac{1/33}{\sqrt{Re_c}} \frac{x_c}{L} + \frac{0.074}{Re_L^{1/5}} - \frac{0.074}{Re_c^{1/5}} \frac{x_c}{L}$$

However, $x_c / L = Re_c / Re_L$ and this becomes

$$C_D = \frac{0.074}{Re_L^{1/5}} - \frac{C_1}{Re_L}$$

in which

$$C_1 = Re_c \left(\frac{0.074}{Re_c^{1/5}} - \frac{1.33}{\sqrt{Re_c}} \right)$$

An average value of $Re_c = 5 \times 10^5$ is usually used to calculate $C_1 = 1741$ in applications.