HENRY COUNTY SCHOOLS

Unit 1

Conics

CCGPS Standards Addressed:
MCC9-12. G.GPE.3 (+)

Key Vocabulary

- Cone
- Coplanar
- Ellipse
- Focus
- Hyperbola
- Locus of Points
- Plane

KEY STANDARDS

Translate between the geometric description and the equation for a conic section

- **MCC9-12.G.GPE.3 (+)** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

RELATED STANDARDS

Translate between the geometric description and the equation for a conic section.

- ***MCC9-12.G.GPE.1*** *Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.*
- ***MCC9-12.G.GPE.2*** *Derive the equation of a parabola given a focus and directrix*

Essential Questions

- What role do foci play in the definition of conic quadratic relations?
- How can ellipses be defined in relation to their foci?
- How can hyperbolas be defined in relation to their foci?
- How can conic sections be identified by the A and C coefficients from the general form of quadratic relations?
- How can we solve real-world problems using what we know about conics?

Suggested Learning Resources/ Performance Tasks

GADOE CCGPS Frameworks
LearnZillion
Mathematics Assessment Project (www.map.mathshell.org)

Online Conics Text:
http://cms.gavirtualschool.org/Shared/Math/Math3/CirclesEtc/index.html

Enduring Understandings

Students will understand…

- Conic sections are quadratic relations that can be expressed generally by the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ and the comparison of the coefficients A and C reveal the specific type of conic.
- All conic sections are defined by the relationship of their locus of points to fixed points known as foci.
- Ellipses arise from a locus of points that represent a constant sum of distances from two fixed points (foci).
- Hyperbolas arise from a locus of points that represent a constant absolute value of difference of distances from two fixed points (foci).

Prerequisite Skills

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Factoring Quadratics
- Graphing/Cartesian plane
- Pythagorean Theorem
- Balancing equations
- Understanding circles
- Surface area and volume (evaluate the formula)
- Domain and range
- Diameter and radius
- Distance Formula
- Simplifying Radical Equations
- Quantitative reasoning
- Seeing the generalizability of relationships in building quadratic relations (and geometric concepts in general)
- Using algebraic methods, such as completing the square, to change forms of equations
- Seeing relationships between algebraic manipulation of equations and characteristics of corresponding graphs

HENRY COUNTY SCHOOLS

Unit 2
Trigonometric Functions

CCGPS Standards Addressed:
F.TF.6 (+) , F.TF.7 (+)

Key Vocabulary
- Co-terminal angle
- Even function
- Odd function
- Reference angle
- Special right triangles
- Terminal side of angle
- Unit circle

Prerequisite Skills
It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.
- Knowledge of unit circle in radians and degrees *
- Graphing trigonometric functions *

Enduring Understandings
Students will understand…
- the relationship between right triangle trigonometry and unit circle trigonometry
- how to use the unit circle to define trigonometric functions
- that there are many instances of periodic data in the world around us and trigonometric functions can be used to model real world data that is periodic in nature.
- the inverses of sine, cosine and tangent functions are not functions unless the domains are limited.

Essential Questions
- How can special right triangles help us find the coordinates of certain angles on the unit circle?
- How does symmetry help us extend our knowledge of the unit circle to an infinite number of angles?
- Why does the calculator only give one answer for an inverse trig function? Aren't there infinite answers?
- How do inverse trigonometric functions help us solve equations?

Suggested Learning Resources/ Performance Tasks

GADOE CCGPS Frameworks
LearnZillion
Mathematics Assessment Project (www.map.mathshell.org)

Online Trig Graphs Text:
http://cms.gavirtualschool.org/Shared/Math/Math4/05_InvestigateTrigGraphs_Lesson_Shared/index.html and
http://cms.gavirtualschool.org/Shared/Math/CCGPS_AdvancedAlgebra/TrigonometricFunctions/index.html and
http://cms.gavirtualschool.org/Shared/Math/CCGPS_AdvancedAlgebra/MathematicalModeling/index.html (for general inverse review)

FAL on Trig Functions:
http://map.mathshell.org/materials/download.php?fileid=1252

KEY STANDARDS
Model periodic phenomena with trigonometric functions
- **MCC9-12.F.TF.6 (+)** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows it's inverse to be constructed.
- **MCC9-12.F.TF.7 (+)** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

RELATED STANDARDS
MCC9-12.F.IF.7e *Graph trigonometric functions showing period, midline, and amplitude*
MCC9-12.F.TF.2 *Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers interpreted as radian measures of angles traversed counterclockwise around the unit circle.*
MCC9-12.F.TF.5 *Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* ★

* NOTE: Determine whether or not PreReq Skills were met in Accel. Analytic Geometry/Adv. Algebra (Unit 9).

HENRY COUNTY SCHOOLS

Unit 3
Trigonometry of General Triangles

CCGPS Standards Addressed:
G.SRT.9 (+), G.SRT.10 (+), G.SRT.11 (+)

Key Vocabulary
- **Altitude of a Triangle**
- **Hinge Theorem**
- **Included Angle**
- **Law of Cosines**
- **Law of Sines**
- **Oblique Triangle**
- **Vertex of a Triangle**

Essential Questions
- How can I calculate the area of any triangle given only two sides and a non-included angle?
- How can I apply trigonometric relationships to non-right triangles?
- What is the least amount of information that is sufficient to find all six parts of a triangle?

Prerequisite Skills
It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with whole numbers and decimals, including application of order of operations
- calculating the area of a triangle
- solving trigonometric equations
- using inverse trigonometric functions to solve problems
- constructing altitudes in a triangle
- performing operations with trigonometric functions

Suggested Learning Resources/ Performance Tasks

GADOE CCGPS Frameworks
LearnZillion
Mathematics Assessment Project (www.map.mathshell.org)

Online Extended Trig Text: http://cms.gavirtualschool.org/Shared/Math/Math4/07_ExtendedTrig_Lesson_Shared/index.html

Enduring Understandings
Students will understand…
- Derive a formula for the area of a triangle using two sides and a non-included angle
- Calculate the area of a general triangle using $A = \frac{1}{2}ab(Sin\,C)$
- Verify the Law of Sines and the Law of Cosines for a general triangle.
- Apply the Law of Sines and the Law of Cosines to solve problems.

KEY STANDARDS
Apply trigonometry to general triangles
- **MCC9-12.G.SRT.9 (+)** Derive the formula A= (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- **MCC9-12.G.SRT.10 (+)** Prove the Laws of Sines and Cosines and use them to solve problems.
- **MCC9-12.G.SRT.11 (+)** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

RELATED STANDARDS
Define trigonometric ratios and solve problems involving right triangles
MCC9-12.G.SRT.6 *Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.*
MCC9-12.G.SRT.7 *Explain and use the relationship between the sine and cosine of complementary angles.*
MCC9-12.G.SRT.8 *Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

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Unit 4
Trigonometric Identities

Key Vocabulary

- Addition Identity for Sine, Cosine, and Tangent
- Even & Odd Functions
- Identity
- Subtraction Identity for Sine, Cosine, and Tangent

CCGPS Standards Addressed:
F.TF.9 (+)

Suggested Learning Resources/ Performance Tasks

GADOE CCGPS Frameworks
LearnZillion
Mathematics Assessment Project (www.map.mathshell.org)

Online Trig Identities Text:
http://cms.gavirtualschool.org/Shared/Math/Math4/06_TrigIdentities_Lesson_Shared/index.html

Essential Question

- How can I add trigonometric functions?
- How can I subtract trigonometric functions?
- How can I prove the addition formula for trigonometric functions?
- How can I prove the subtraction formula for trigonometric functions?
- What is an identity?

Prerequisite Skills

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with whole numbers and decimals, including application of order of operations
- applications of the Pythagorean Theorem*
- operations with trigonometric ratios*
- operations with radians and degrees
- even and odd functions
- geometric constructions
- algebraic proofs*
- geometric proofs
- methods of proof

Enduring Understandings

Students will understand…

- Understand the concept of identity
- Prove the addition formula for sine, cosine and tangent
- Prove the subtraction formula for sine, cosine and tangent
- Use trigonometric functions to prove formulas

KEY STANDARDS

Prove and apply trigonometric identities

- **MCC9-12.F.TF.9 (+)** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

* NOTE: Determine whether or not PreReq Skills were met in Accel. Analytic Geometry/Adv. Algebra (Unit 9).

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Unit 5
Matrices

Key Vocabulary
- Determinant
- Dimensions or Order of a Matrix
- Identity Matrix
- Inverse Matrices
- Matrix
- Scalar
- Square Matrix
- Zero Matrix

CCGPS Standards Addressed:
N.VM.6(+), N.VM.7(+), N.VM.8(+), N.VM.9(+), N.VM.10(+), N.VM.12(+), A.REI.8(+), A.REI.9(+)

Prerequisite Skills

It is expected that students will have prior knowledge/ experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Commutative Property
- Associative Property
- Distributive Property
- Identity Properties of Addition and Multiplication
- Inverse Properties of Addition and Multiplication
- Solving Systems of Equations Graphically and Algebraically

Enduring Understandings

Students will understand…
- Matrices provide an organizational structure in which to represent and solve complex problems.
- The commutative property applies to matrix addition but does not extend to matrix multiplication.
- A zero matrix behaves in addition, subtraction, and multiplication much like 0 in the real number system.
- An identity matrix behaves much like the number 1 in the real number system.
- The determinant of a matrix is nonzero if and only if the matrix has an inverse.
- 2 X 2 matrices can be written as transformations of the plane and can be interpreted as absolute value of the determinant in terms of area.
- Solving systems of linear equations can be extended to matrices and the methods we use can be justified.

Essential Questions
- How can we represent data in matrix form?
- How do we add and subtract matrices and when are these operations defined?
- How do we perform scalar multiplication on matrices?
- How do we multiply matrices and when is this operation defined?
- How do the commutative, associative, and distributive properties apply to matrices?
- What is a zero matrix and how does it behave?
- What is an identity matrix and how does it behave?
- How do we find the determinant of a matrix and when is it nonzero?
- How do we find the inverse of a matrix and when does a matrix not have an inverse defined?
- How do we solve systems of equations using inverse matrices?
- How do we find the area of a plane using matrices?
- How do we write and use vertex-edge graphs to solve problems?

Suggested Learning Resources/ Performance Tasks

GADOE CCGPS Frameworks
LearnZillion
Mathematics Assessment Project (www.map.mathshell.org)
Online Matrices Text: http://cms.gavirtualschool.org/Shared/Math/Math3/OperationswithMatrices/index.html
Online Vertex Edge Text: http://cms.gavirtualschool.org/Shared/Math/Math3/LinearProgramming/index.html
Online Modeling With Matrices Text: http://cms.gavirtualschool.org/Shared/Math/Math3/ModelingwithMatricsAssessment/index.html

KEY STANDARDS

Perform operations on matrices and use matrices in applications.
- **MCC9-12.N.VM.6 (+)** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- **MCC9-12.N.VM.7 (+)** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- **MCC9-12.N.VM.8 (+)** Add, subtract, and multiply matrices of appropriate dimensions.
- **MCC9-12.N.VM.9 (+)** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- **MCC9-12.N.VM.10 (+)** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- **MCC9-12.N.VM.12 (+)** Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Solve systems of equations
- **MCC9-12.A.REI.8 (+)** Represent a system of linear equations as a single matrix equation in a vector variable.
- **MCC9-12.A.REI.9 (+)** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

HENRY COUNTY

Unit 6
Vectors

Key Vocabulary
- Vector
- Scalar
- Initial Point
- Terminal Point
- Magnitude of a Vector
- Components of a Vector
- Parallel Vectors
- Equivalent Vectors
- Zero Vector
- Resultant Vector
- Tail-to-Head & Parallelogram Representations (Rule)
- Velocity
- Complex Numbers
- Complex Plane
- Rectangular& Polar Forms of a Complex Number
- $cis(\theta)$
- Complex Conjugate
- Modulus of a Complex Number
- Argument of z, Re(z), Im(z)

Suggested Learning Resources/Performance Tasks

GADOE CCGPS Frameworks
LearnZillion
Mathematics Assessment Project (www.map.mathshell.org)
Online Vectors Text: http://cms.gavirtualschool.org/Shared/Math/Math4/07_ExtendedTrig_Lesson_Shared/index.html

CCGPS Standards Addressed:
N.CN.3, N.CN.4, N.CN.5, N.CN.6, N.VM.1, N.VM.2, N.VM.3, N.VM.4

Essential Question
- How can I represent complex numbers graphically?
- How does the complex plane show addition, subtraction, multiplication, and conjugation of complex numbers?
- What are two ways to represent a complex number, and what are the advantages of each form?
- How are operations on real numbers represented in the complex plane?
- When given two points on the complex plane, what does it mean to find the distance between them and the midpoint of the segment connecting them?
- How are vectors and scalars similar and different?
- How can I use vector operations to model, solve, and interpret real-world problems?
- How can I represent addition, subtraction, and scalar multiplication of vectors geometrically?
- How do geometric interpretations of addition, subtraction, and scalar multiplication of vectors help me perform computations efficiently?
- What are some different ways to add two vectors, and how are these representations related?
- In what ways can matrices transform vectors?

quisite Skills

edge/ experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Use the Pythagorean Theorem.
- Solve right triangles using trigonometry and inverse trigonometry.
- Use reference angles to find the measure of an angle in standard position.
- Apply transformations (translations, rotations, reflections) to objects on a coordinate grid.
- Use the triangle inequality: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- Use a ruler and protractor to measure lengths and angles.
- Multiply matrices.
- Perform operations on complex numbers—add, subtract, multiply, divide, and conjugate.
- Recognize multiplication as repeated addition and exponentiation as repeated multiplication.
- Use fractional exponents to write radicals.
- Find distance and midpoints in the plane.

KEY STANDARDS

Perform arithmetic operations with complex numbers.
- **MCC9-12.N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.
- **MCC9-12.N.CN.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- **MCC9-12.N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
- **MCC9-12.N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Represent and model with vector quantities.
- **MCC9-12.N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, *v*).
- **MCC9-12.N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- **MCC9-12.N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.
- **MCC9-12.N.VM.4** Add and subtract vectors.
- **MCC9-12.N.VM.4a** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- **MCC9-12.N.VM.4b** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- **MCC9-12.N.VM.4c** Understand vector subtraction **v** – **w** as **v** + **(–w)**, where **(–w)** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- **MCC9-12.N.VM.5** Multiply a vector by a scalar.
- **MCC9-12.N.VM.5a** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\mathbf{v}_x, \mathbf{v}_y) = (c\cdot\mathbf{v}_x, c\cdot\mathbf{v}_y)$.
- **MCC9-12.N.VM.5b** Compute the magnitude of a scalar multiple c**v** using ||c**v**|| = |c|**v** . Compute the direction of c**v** knowing that when |c|**v** ≠ 0, the direction of c**v** is either along **v** (for c > 0) or against **v** (for c < 0).

Perform operations on matrices and use matrices in applications.
- **MCC9-12.N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

HENRY COUNTY SCHOOLS

Unit 7
Probability

CCGPS Standards Addressed: CP.8(+), CP.9(+), S.MD.1(+), S.MD.2(+), S.MD.3(+), S.MD.4(+), S.MD.5(+), S.MD.6(+), S.MD.7(+),SP.8(+), S.CP.2(+), S.CP.3(+), S.CP.6(+), S.CP.7(+)

Key Vocabulary
- Conditional Probability
- Combinations
- Expected Value
- Odds
- Permutations
- Sample Space

Essential Question
- How do I use the General Multiplication Rule to calculate probabilities?
- How do I determine when to use a permutation or a combination to calculate a probability?
- How do I identify a random variable?
- How do I graphically display the probability distribution of a random variable?
- How do I calculate the expected value of a random variable?
- How do I calculate theoretical and empirical probabilities of probability distributions?
- How do I represent and calculate payoff values in a game of chance?
- How do I use expected values to make decisions?
- How do I explain the decisions I make using expected values?

Enduring Understandings
Students will…
- Understand how to calculate probabilities using the General Multiplication Rule and interpret the results in context.
- Understand how to use permutations and combinations in conjunction with other probability methods to calculate probabilities of compound events and solve problems.
- Know how to define random variables, assign probabilities to its sample space, and graphically display the distribution of the random variable.
- Understand how to calculate and interpret the expected value of random variables.
- Understand hot to develop the theoretical and empirical probability distribution and find expected values.
- Know how to set up a probability distribution for a random variable representing payoff values.

Prerequisite Skills
It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.
- Understand the basic nature of probability
- Determine probabilities of simple and compound events
- Understand the Fundamental Counting Principle
- Organize and model simple situations involving probability

KEY STANDARDS
Use the rules of probability to compute probabilities of compound events in a uniform probability model
- **MCC9-12.CP.8** (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B)=[P(A)] x [P(B | A)] = [P(B)] x [P(A | B)], and interpret the answer in terms of the model.
- **MCC9-12.S.CP.9** (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Calculate expected values and use them to solve problems
- **MCC9-12.S.MD.1**(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
- **MCC9-12.S.MD.2**(+) Calculate the expected value of a random variable; interpret it as the mean of a probability distribution.
- **MCC9-12.S.MD.3**(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value
- **MCC9-12.S.MD.4**(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value

Use probabilities to evaluate outcomes of decisions
- **MCC9-12.S.MD.5**(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
- **MCC9-12.S.MD.5a**(+) Find the expected payoff for a game of chance
- **MCC9-12.S.MD.5b**(+) Evaluate and compare strategies on the basis of expected values
- **MCC9-12.S.MD.6**(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- **MCC9-12.S.MD.7**(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

RELATED STANDARD
- **MCC7.SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
- **MCC9-12.S.CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★
- **MCC9-12.S.CP.3** Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. ★
- **MCC9-12.S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ★
- **MCC9-12.S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. ★

Suggested Learning Resources/ Performance Tasks

GADOE CCGPS Frameworks
LearnZillion
Mathematics Assessment Project (www.map.mathshell.org)

Normal Distribution Visual Tool

Online Probability Text: http://cms.gavirtualschool.org/Shared/Math/MathI/ProbabilityTasks/index.html