Unit 1, Lesson 7: From Parallelograms to Triangles

Lesson Goals

- Understand and explain that any two identical triangles can be composed into a parallelogram.
- Describe how any parallelogram can be decomposed into two identical triangles by drawing a diagonal.

Required Materials

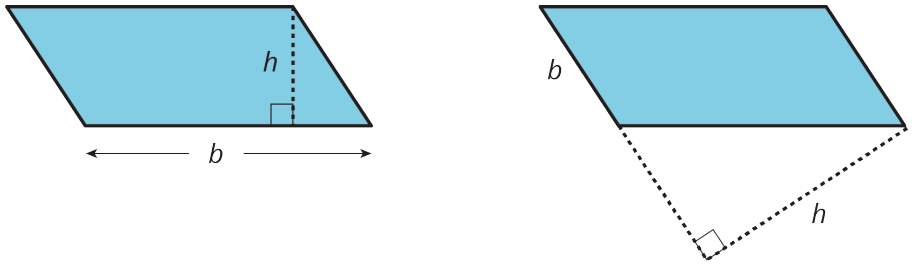
- rulers
- pre-printed slips, cut from copies of the blackline master
- geometry toolkits

7.1: Same Parallelograms, Different Bases (5 minutes)

Setup: 2 minutes of quiet work time. Access to geometry toolkits.

Student task statement

Here are two copies of a parallelogram. Each copy has one side labeled as the base b and a segment drawn for its corresponding height and labeled h.



1. The base of the parallelogram on the left is 2.4 centimeters; its corresponding height is 1 centimeter. Find its area in square centimeters.

2. The height of the parallelogram on the right is 2 centimeters. How long is the base of that parallelogram? Explain your reasoning.

Possible responses

1. The area is 2.4 square centimeters. $(2.4) \cdot 1 = 2.4$

2. The base is 1.2 centimeters because $(1.2) \cdot 2 = 2.4$

Anticipated misconceptions

Some students may not know how to begin answering the questions because measurements are not shown on the diagrams. Ask students to label the parallelograms based on the information in the task statement.

Students may say that there is not enough information to answer the second question because only one piece of information is known (the height). Ask them what additional information might be needed. Prompt them to revisit the task statement and see what it says about the two parallelograms. Ask what they know about the areas of two figures

that are identical.

Students may struggle to find the unknown base in the second question because the area of the parallelogram is a decimal and they are unsure how to divide a decimal. Ask them to explain how they would reason about it if the area was a whole number. If they understand that they need to divide the area by 2 (since the height is 2 cm), see if they could reason in terms of multiplication (i.e., 2 times what number is 2.4?) or if they could reason about the division using fractions (i.e., 2.4 can be seen as $2\frac{4}{10}$ or $\frac{24}{10}$; what is 24 tenths divided by 2?).

7.2: A Tale of Two Triangles (Part 1) (15 minutes)

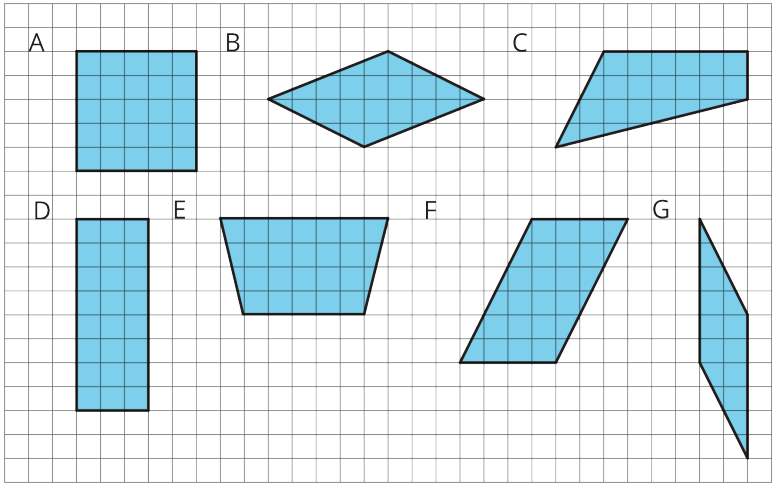
Setup:

Students into groups of 3–4. Access to geometry toolkits. 2 minutes of quiet think time for the first two questions, followed by group discussion, checking of triangles using tracing paper, and time for the last question.

Student task statement

Two polygons are identical if they match up exactly when placed one on top of the other.

1. Draw *one* line to decompose each of the following polygons into two identical triangles, if possible. Use a straightedge to draw your line.



2. Which quadrilaterals can be decomposed into two identical triangles?

   Pause here for a small-group discussion.

3. Study the quadrilaterals that can, in fact, be decomposed into two identical triangles. What do you notice about them? Write a couple of observations about what these quadrilaterals have in common.

Possible responses

1. Cutting lines vary, but each should be a diagonal connecting opposite vertices.

2. Quadrilaterals C and E cannot be decomposed into two identical triangles. A, B, D, F, and G can.

3. Answers vary. They are all parallelograms.

Anticipated misconceptions

It may not occur to students to rotate triangles to check congruence. If so, tell students that we still consider two triangles identical even when one needs to be rotated to match the other.

Are you ready for more?

On the grid, draw some other types of quadrilaterals that are not already shown. Try to decompose them into two identical triangles. Can you do it?

Come up with a rule about what must be true about a quadrilateral for it to be decomposed into two identical triangles.

Possible Responses

Answers vary.

7.3: A Tale of Two Triangles (Part 2) (15 minutes)

Setup:

Students in the same groups. One set of paper triangles per group; each set containing 2 copies of of triangles P—U. Access to scissors if the triangles are not pre-cut. 1–2 pairs of triangles per group member. 1–2 minutes of quiet think time for the first question, followed by 5 minutes for a group discussion and the second question.

Student task statement

Your teacher will give your group several pairs of triangles. Each group member should take 1–2 pairs.

1. a. Which pair(s) of triangles do you have?

   b. Can each pair be composed into a rectangle? A parallelogram?

2. Discuss with your group your responses to the first question. Then, complete each of the following statements with *all*, *some*, or *none*. Sketch 1–2 examples to illustrate each completed statement.

   a. _____ of these pairs of identical triangles can be composed into a *rectangle*.

   b. _____ of these pairs of identical triangles can be composed into a *parallelogram*.

Possible responses

1. a. Answers vary. Yes for triangles R and U, no for the rest.

   b. Yes for all triangles

2. a. *Some* can be composed into rectangles.

   b. *All* can be composed into parallelograms.

Anticipated misconceptions

Students may draw incorrect conclusions if certain pieces of their triangles are turned over (to face down), or if it did not occur to them that the pieces could be moved. Ask them to try manipulating the pieces in different ways.

Seeing that two copies of a triangle can always be composed into a parallelogram, students might mistakenly conclude that any two

copies of a triangle can *only* be composed into a parallelogram (i.e., no other quadrilaterals can be formed from joining two identical triangles). Showing a counterexample may be a simple way to help students see that this is not the case.

Lesson Synthesis (5 minutes)

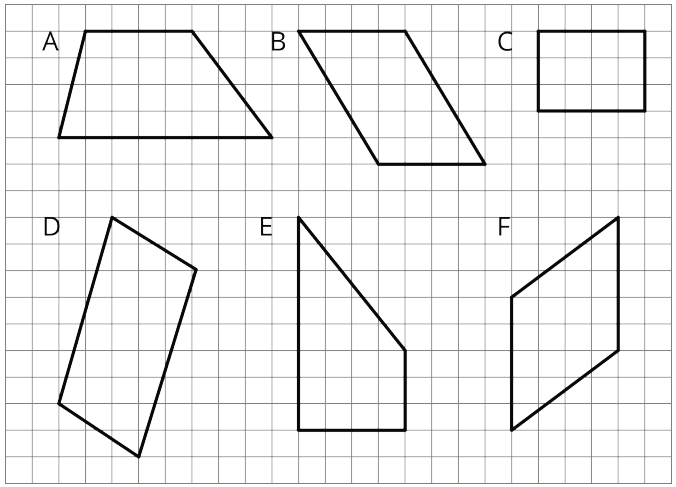
Draw out the special connections between triangles and parallelograms. What is unique about the relationship between triangles and parallelograms? Highlight that a parallelogram can always be decomposed into two identical triangles and that any two identical triangles can be composed into a parallelogram.

7.4: A Tale of Two Triangles (Part 3) (Cool-down, 5 minutes)

Setup: Access to geometry toolkits.

Student task statement

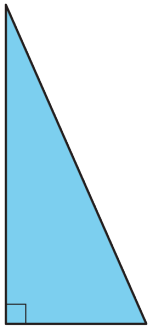
1. Here are some quadrilaterals.



   a. Circle all quadrilaterals that you think can be decomposed into two identical triangles using only one line.

   b. What characteristics do the quadrilaterals that you circled have in common?

2. Here is a right triangle. Show or briefly describe how two copies of it can be composed into a parallelogram.



Possible responses

1. a. Quadrilaterals B, C, D, and F should be circled.

   b. They all have two pairs of parallel sides. They are all parallelograms.

2. Answers vary. Three parallelograms are possible (one of them is a rectangle).