NAME: ____________________ PERIOD: ___

DATE: ______________ PRE-CALCULUS

MR. MELLINA

UNIT 1: REVIEW OF ALGEBRA
PART C: EXPONENTIAL & LOGARITHMIC FUNCTIONS

- *Lesson 1: Radicals & Rational Exponents*
- *Lesson 2: Exponential Functions*
- *Lesson 3: Applications of Exponential Functions*
- *Lesson 4: Common & Natural Logarithmic Functions*
- *Lesson 5: Properties and Laws of Logarithms*
- *Lesson 6: Solving Exponential & Logarithmic Equations*
- *Lesson 7: Exponential, Logarithmic, and Other Models*

Lesson 1: Polynomial Functions & Operations

Objectives:

- *Define and apply rational and irrational exponents*
- *Simplify expressions containing radicals or rational exponents.*

Warm Up 🔥

Simplify

a. $\dfrac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4}$

b. $\left(\dfrac{m^4n^2p^4}{2m^4n^{-2}p^3 \cdot m^2n^3p^{-1}}\right)^{-2}$

Example 1: Operations on nth Roots

Simplify each expression.

a. $\sqrt{8} \cdot \sqrt{12}$

b. $\sqrt{12} - \sqrt{75}$

c. $\sqrt[3]{8x^6y^4}$

d. $(5 + \sqrt{c})(5 - \sqrt{c})$

Rational Exponents

$$\sqrt[n]{x^m} =$$

Example 2: Simplifying Expressions with Rational Exponents

Write the expression using only positive exponents

a. $\left(8r^{\frac{3}{4}}s^{-3}\right)^{2/3}$

Example 3: Simplifying Expressions with Rational Exponents

Simplify

a. $x^{\frac{1}{2}}\left(x^{\frac{3}{4}} - x^{\frac{3}{2}}\right)$

b. $\left(x^{\frac{5}{2}}y^4\right)\left(xy^{\frac{7}{4}}\right)^{-2}$

Example 4: Simplifying Expressions with Rational Exponents

Let *k* be a positive rational number. Write the given expression without radicals, using only positive exponents.

a. $\sqrt[10]{c^{5k}}\sqrt{(c^{-k})^{1/2}}$

Example 5: Rationalizing the Denominator

Rationalize the denominator of each fraction.

a. $\frac{7}{\sqrt{5}}$

b. $\frac{2}{3+\sqrt{6}}$

Example 6: Rationalizing the Numerator

Rationalize the numerator given $h \neq 0$.

a. $\frac{\sqrt{x+h}-\sqrt{x}}{h}$

Lesson 2: Exponential Functions

Objectives:

- *Graph and identify transformations of exponential functions.*
- *Use exponential functions to solve application problems.*

Warm Up

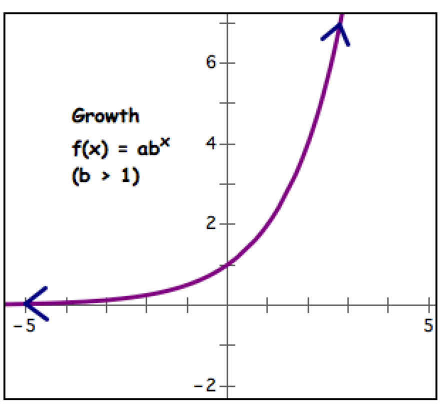
Simplify

a. 5^{-2} b. $8^{2/3}$ c. $-3 \cdot 4^{3/2}$

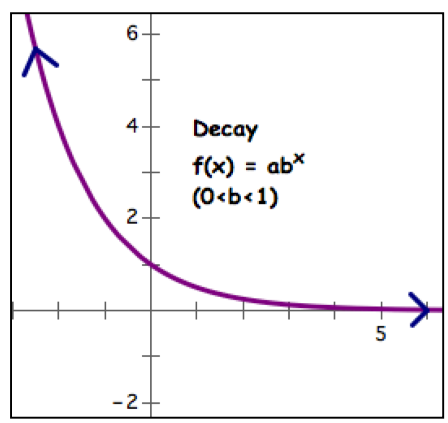
Exponential Functions

*Have the form ___________ where $a \neq 0$ and the base **b** is a positive number other than 1.*
***b** in the exponential function is called the ________________________.*

Exponential Growth:
*When **b** is greater than ______.*



Exponential Decay:
*When **b** is between ____ and ____.*



The x-axis is a horizontal ___________________ of the graph this is an imaginary line that a graph approaches more and more closely.

*The **Domain** consists of the _____- values of the function. **D:***
*The **Range** consists of the _____ - values of the function. **R:***

Example 1: Growth or Decay?
Without graphing, determine whether each function represents exponential <u>growth</u> or exponential <u>decay</u>.

a. $y = 129(1.63)^x$

b. $y = 12\left(\frac{17}{10}\right)^x$

c. $f(x) = 4\left(\frac{5}{6}\right)^x$

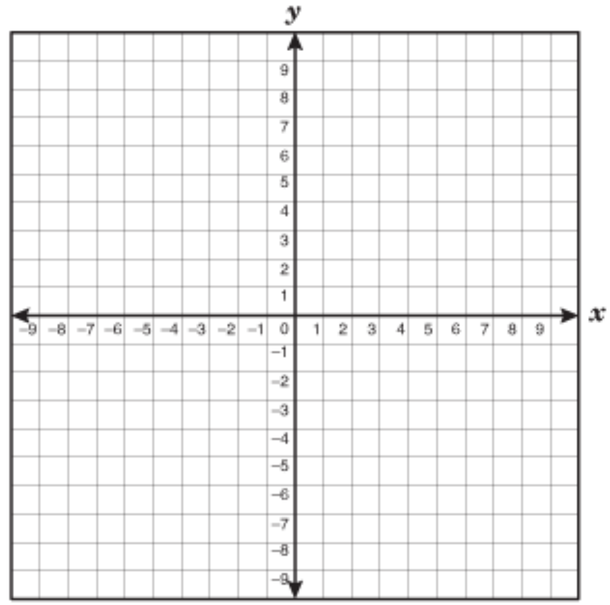
d. $g(x) = \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)^x$

e. $h(x) = \left(\frac{5}{4}\right)\left(\frac{1}{2}\right)^x$

f. $y = \left(\frac{1}{8}\right)^x\left(\frac{5}{4}\right)$

Example 2: Graphing
Graph the following exponential growth functions.

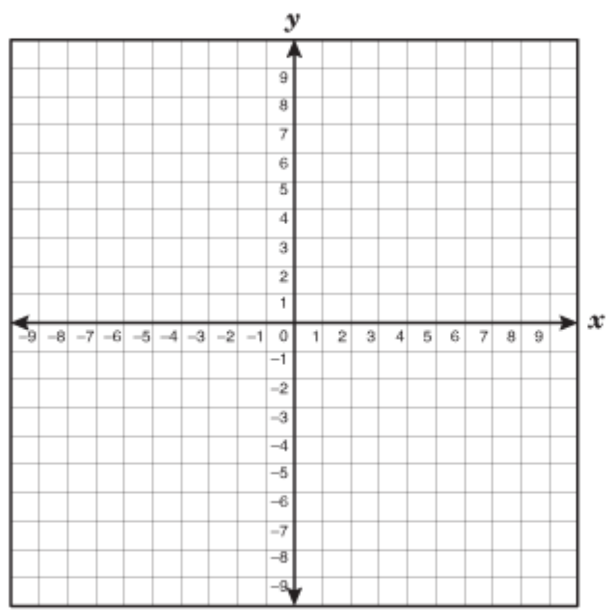
a. $y = 4 \cdot 2^{x-1} - 3$



Growth or Decay?

What transformations have occurred from the parent function?

e. $y = 3\left(\frac{1}{2}\right)^{x+1} - 2$



Growth or Decay?

What transformations have occurred from the parent function?

The Natural Base e

*The history of mathematics is marked by the discovery of special numbers such as π and i. Another special number is denoted by the letter **e**. The number is called the natural base e or the Euler number after its discoverer, Leonhard Euler.*

*The natural base **e** is _____________. It is defined as follows:*

$$\text{As } n \text{ approaches } +\infty, \left(1+\frac{1}{n}\right)^n \text{ approaches } e \approx 2.718281828$$

Example 3: Population Growth

If the popultion of the US continues to grow as it has since 1980, then the approximate population, in millions, of the US in year *t*, where *t* = 0 corresponds to the year 1980, will be given by the function $P(t) = 227e^{0.0093t}$.

a. Estimate the population in 2015.

b. When will the population reach half a billion?

Other Exponential Functions

*In most real-world applications, populations cannot grow infinitely large. A **logistic model**, which is designed to model situations that have limited future growth due to a fixed area, food supply, or other factors.*

Example 4: Logistic Model

The population of fish in a certain lake at time *t* months is given by the function below where $t \geq 0$. There is an upper limit on the fish population due to the oxygen supply, available food, etc. Find the upper limit on the fish population.

a. $p(t) = \frac{20{,}000}{1+24e^{-t/4}}$

b. $p(t) = \frac{2{,}000}{1+199e^{-0.5544t}}$

Lesson 3: Applications of Exponential Functions

Objectives:

- *Create and use exponential models for a variety of exponential growth and decay application problems.*

Warm Up 🔥

Without a graph, determine whether the function is even, odd, or neither

a. $f(x) = 10^x$

b. $f(x) = \frac{e^x + e^{-x}}{2}$

c. $f(x) = \frac{e^x - e^{-x}}{2}$

Compound Interest

*Consider an intial principal **P** (starting \$) deposited in an account that pays interest at an annual rate **r** (expressed as a decimal), compounded **n** times per year. The amount **A** in the account after **t** years is given by this equation:*

Example 1: Find the balance in an account using Compound interest.

You deposit \$4000 in an account that pays 2.92% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a. Quarterly

b. Daily

Example 2: Find the balance in an account using Compound interest.
You deposit $7,000 in an account that pays .3% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a. Monthly

b. Semi-Annually

c. Yearly

d. Weekly

Continuously Compounded Interest
*When interest is compounded ___________________, the amount **A** in an account after **t** years is given by the formula:*

*Where **P** is the principal and **r** is the annual interest rate expressed as a decimal.*

Example 3: Model Continuously Compounded Interest

a. You deposit $4,000 in an account that pays 6% annual interest compounded continuously. What is the balance after 1 year?

b. You deposit \$2,500 in an account that pays 5% annual interest compounded continuously. Find the balance after each amount of time.

❖ 2 years

❖ 5 years

❖ 7.5 years

Half Life

The amount of a radioactive substance that remains is given by the function

$$f(x) = P(0.5)^{x/h}$$

where P is the initial amount of the substance of the substance, x = 0 corresponds to the time when the radioactive decay began, and h is the half-life of the substance.

Example 4: Radioactive Decay

When a living organism dies, its carbon-14 decays exponentially. An archeologist determines that the skeleton of a mastodon has lost 64% of its carbon-14. The half-life of carbon-14 is 5730 years.

a. Estimate how long ago the mastodon died.

Lesson 4: Common and Natural Logarithmic Functions

Objectives:

- *Evaluate common and natural logarithms with and without a calculator.*
- *Solve common and natural exponential and logarithmic equations by using an equivalent equation.*
- *Graph and identify transformations of common and natural logarithmic functions.*

Warm Up 🔥

Solve for x.

a. $2^x = 4$

b. $10^x = \frac{1}{10}$

Example 1: Evaluating Common Logarithms

Without using a calculator, find each value.

a. $\log 1000$

b. $\log 1$

c. $\log \sqrt{10}$

d. $\log(-3)$

Example 2: Using Equivelant Statements

Solve each equation by using an equivelant statement.

a. $\log x = 2$

b. $10^x = 29$

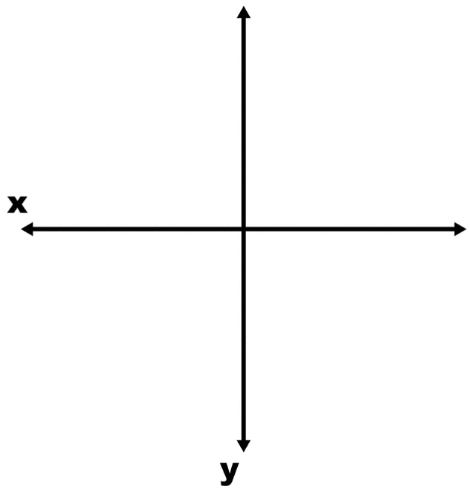
c. $\ln x = 4$

d. $e^x = 5$

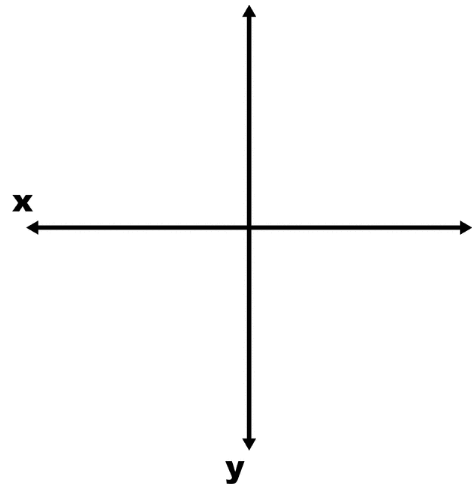
Example 3: Transforming Logarithmic Functions

Describe the transformations from the parent function to the function provided. Sketch the graph and give the domain and range of *h.*

a. $h(x) = 2\log(x-3)$



b. $h(x) = \ln(2-x) - 3$



Example 4: Solving Logarithmic Equations Graphically

If you invest money at an interest rate *r*, compounded annually, then *D(r)* gives the time in years that it would take to double.

$$D(r) = \frac{\ln 2}{\ln(1+r)}$$

a. How long will it take to double an investment of $2500 at 6.5% annual interst?

b. What annual interest rate is needed in order for the investment in part **a** to double in 6 years?

Lesson 5/5A: Properties and Laws of Logarithms/Other Bases

Objectives:

- *Use properties and laws of logarithms to simplify and evaluate expressions.*
- *Evaluate logarithms to any base with and without a calculator.*
- *Solve exponential equations to any base by using an equivalent equation.*
- *Use properties and laws of logarithms to simplify and evaluate logarithmic expressions to any base.*

Warm Up 🔥

Evaluate the logarithm

a. $\log 10$ b. $\ln e^2$ c. $\log 1000$

Example 1: Solving Equations by Using Properties of Logarithms

Use the basic properties of logarithms to solve.

a. $\ln(x+1) = 2$

Example 2: Using the Product Law of Logarithms

Use the Product Law of Logarithms to evaluate each logarithm.

a. Given $\log 3 = 0.4771, \log 11 = 1.0414$. Find $\log 33$

b. Given $\ln 7 = 1.9459, \ln 9 = 2.1972$. Find $\ln 63$

Example 3: Using the Quotient Law of Logarithms

Use the Quotient Law of Logarithms to evaluate each logarithm.

a. Given $\log 28 = 1.4472$ and $\log 7 = 0.8451$. Find $\log 4$

b. Given $\ln 18 = 2.8904, \ln 6 = 1.7918$. Find $\ln 3$

Example 4: Using the Power Law of Logarithms

Use the Power Law of Logarithms to evaluate each logarithm.

a. Given $\log 6 = 0.7782$, find $\log \sqrt{6}$

b. Given $\ln 50 = 3.9120$, find $\log \sqrt[3]{50}$

Example 5: Simplifying Expressions

Write as a single logarithm

a. $\ln 3x + 4 \ln x - \ln 3xy$

b. $\log_3(x + 2) + \log_3 y - \log_3(x^2 - 4)$

c. $3 - \log_5(125x)$

d. Simplify $\ln\left(\frac{\sqrt{x}}{x}\right) + \ln\left(\sqrt[4]{ex^2}\right)$

Example 6: Evaluating Logarithms to Other Bases

Without using a calculator, find each value.

a. $\log_2 16$

b. $\log_{1/3} 9$

c. $\log_5(-25)$

Example 7: Solving Logarithmic Equations

Solve each equation for x.

a. $\log_5 x = 3$

b. $\log_6 1 = x$

c. $\log_{1/6}(-3) = x$

d. $\log_6 6 = x$

e. $\log_3(x-1)=4$

Example 8: Change of Base Formula

Evaluate

a. $\log_8 9$

Lesson 6: Solving Exponential and Logarithmic Equations

Objectives:

- *Solve exponential and logarithmic equations*
- *Solve a variety of application problems by using exponential and logarithmic equations.*

Warm Up

Find the inverse.

a. $f(x) = \log(x-2)$

b. $g(x) = e^{2x-1}$

Example 1: Powers of the Same Base

Solve for x.

a. $2^x = 2^6$

b. $4^x = 64$

c. $9^{2x} = 27^{x-1}$

d. $100^{7x+1} = 1000^{3x-2}$

e. $4^x = \left(\frac{1}{2}\right)^{x-3}$

f. $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$

Example 2: Powers of Different Bases.

Solve for x

a. $4^x = 11$

b. $9^x = 35$

c. $11^{5x} = 33$

d. $2^{4x-1} = 3^{1-x}$

Example 3: Using Substitution

Solve

a. $e^x - e^{-x} = 4$

Example 4: Use an exponential model

You deposit $100 in an account that pays 6% annual interest. How long will it take for the balance to reach $1,000 for each given frequency of compounding.

a. Annual

b. Quarterly

Example 5: Equations with Logarithms (Mixed)

Solve

a. $\ln(5x-1)=3$

b. $\log_5(4x-7)=\log_5(x+5)$

c. $2\log_3 6-\frac{1}{4}\log_3 16=\log_3 x$

d. $\log_4(5x-1)=3$

e. $3\log_8 x=\log_8 27$

f. $\ln(3x-2)=2$

g. $\log_2 x + \log_2(x-2) = 3$

h. $2\ln 2x - \ln 4x = 5$

Example 6: Equations with Logarithms and Constant Terms

Solve

a. $\ln(x-3) = 5 - \ln(x-3)$

b. $\log(x-16) = 2 - \log(x-1)$

Lesson 7: Exponential, Logarithmic, & Other Models

Objectives:

- *Model real data sets with power, exponential, logarithmic, and logistic functions.*

Model	Equation	Examples	
Power	$y = ax^r$	$y = 5x^{2.7}$	$y = 3.5x^{-0.45}$
Exponential	$y = ab^x$ or $y = ae^{kx}$	$y = 2(1.64)^x$	$y = 2e^{0.4947x}$
Logarithmic	$y = a + b \ln x$	$y = 5 + 4.2 \ln x$	$y = 2 - 3 \ln x$
Logistic	$y = \dfrac{a}{1 + be^{-kx}}$	$y = \dfrac{20{,}000}{1 + 24e^{-0.25x}}$	$y = \dfrac{650}{1 + 6e^{-0.3x}}$

Example 1: Finding a Model

In the years before the Civil War, the population of the US grew rapidly, as shown in the following table. Find a model for this growth.

a.

Year	Population in millions
1790	3.93
1800	5.31
1810	7.24
1820	9.64
1830	12.86
1840	17.07
1850	23.19
1860	31.44

Example 2: Finding a Model

After the Civil War, the population of the US continued to increase, as shown in the following table. Find a model for this growth.

a.

Year	Population in millions
1870	38.56
1880	50.19
1890	62.98
1900	76.21
1910	92.23
1920	106.02
1930	123.20
1940	132.16
1950	151.13
1960	179.32
1970	202.30
1980	226.54
1990	248.72

Example 3: Finding a Model

The length of time that a planet takes to make one complete rotation around the sun is tha planet's "year". The table below shows the length of each planet's year, relative to an Earth year, and the average distance of that planet from the Sun in millions of miles. Find a model for this data in which x is the length of the year and y is the distance from the Sun.

a.

Planet	Year	Distance
Mercury	0.24	36.0
Venus	0.62	67.2
Earth	1.00	92.9
Mars	1.88	141.6
Jupiter	11.86	483.6
Saturn	29.46	886.7
Uranus	84.01	1783.0
Neptune	164.79	2794.0
Pluto	247.69	3674.5

Example 4: Finding a Model

Find a model for population growth in Anaheim California given the information in the following table.

a.

Year	1950	1970	1980	1990	1994
Population	14,556	166,408	219,494	266,406	282,133