Unit 1 Review, pages 100–107

Knowledge

1. (c)
2. (c)
3. (b)
4. (d)
5. (b)
6. (c)
7. (d)
8. (b)
9. (d)
10. (b)
11. (b)
12. True
13. True
14. False. The average velocity of an object is the change in *displacement* divided by the change in *time*.
15. True
16. False. A runner that is veering left to pass another runner *is accelerating leftward*.
17. False. Since 1987, the annual number of automobile accident fatalities in Canada has *decreased* by 33%.
18. False. A diagram where 150 m in real life is represented as 1 cm on the diagram would have a scale of *1 cm : 150 m*.
19. False. When given only the *x*- component and *y*-component vectors, *the Pythagorean theorem* should be used to determine the magnitude of the displacement vector.
20. True
21. True
22. False. If a bowling ball and a feather are dropped from the same height in a vacuum at the same time, then *they will both hit the ground at the same time*.
23. True
24. (a) (vii)
(b) (iv)
(c) (v)
(d) (ii)
(e) (iii)
(f) (i)
(g) (vi)
25. Answers may vary. Sample answer:
Position is the location of an object relative to a given reference point. Displacement is the change in position of an object, while distance is the total length of the path travelled by an object. For example, an object that starts at 0 m, moves 20 m [E] and then 10 m [W] has moved a total distance of 30 m, but the displacement of the object from its starting point is only 10 m [E].
26. Answers may vary. Sample answer:
(a) A scalar is a quantity that only has magnitude, whereas a vector is a quantity that has magnitude and direction. For example 20 m/s is a scalar since it has no direction. 20 m/s [E] is a vector since it has magnitude (20 m/s) and direction (east).
(b) A vector is drawn as a directed line segment, which is a line segment between two points with an arrow at one end. The end with the arrow is called the tip and the end without the arrow is called the tail. When two vectors are added on a diagram, one vector is drawn from its starting point and the second vector is drawn, keeping its size and direction, but with its tail starting at the tip of the first vector. The tail of the first vector to the tip of the second vector is the sum of the two vectors.
27. Answers may vary. Sample answer:
One argument for using speed limiters for teenage drivers is that teenage drivers are more likely to speed and cause accidents. Using speed limiters would lower speeds and decrease the number of accidents. One of the arguments against using speed limiters is that by limiting the speed of some cars they could disrupt the flow of traffic and cause more problems.

Understanding

28. Given: $\vec{d}_{\text{initial}} = 1750\text{ m [W]}$; $\vec{d}_{\text{final}} = 3250\text{ m [W]}$

Required: $\Delta\vec{d}_{\text{T}}$

Analysis: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{initial}}$

Solution:
$$\begin{aligned}\Delta\vec{d}_{\text{T}} &= \vec{d}_{\text{final}} - \vec{d}_{\text{initial}}\\ &= 3250\text{ m [W]} - 1750\text{ m [W]}\\ \Delta\vec{d}_{\text{T}} &= 1500\text{ m [W]}\end{aligned}$$

Statement: My displacement is 1500 m [W].

29. Given: $\vec{d}_{\text{initial}} = 2620\text{ m [E]}$; $\vec{d}_{\text{final}} = 3250\text{ m [W]}$

Required: $\Delta\vec{d}_{\text{T}}$

Analysis: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{initial}}$

Solution:
$$\begin{aligned}\Delta\vec{d}_{\text{T}} &= \vec{d}_{\text{final}} - \vec{d}_{\text{initial}}\\ &= 3250\text{ m [W]} - 2620\text{ m [E]}\\ &= 3250\text{ m [W]} + 2620\text{ m [W]}\\ \Delta\vec{d}_{\text{T}} &= 5870\text{ m [W]}\end{aligned}$$

Statement: My displacement is 5870 m [W].

30. Given: $\vec{d}_{\text{inital}}$ = 1750 m [W]; $\vec{d}_{\text{final}}$ = 0 m (ignore the detail about the market because the girl's displacement only involves her initial and final positions)
Required: $\Delta\vec{d}_{\text{T}}$
Analysis: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}$
Solution:
$$\begin{aligned}\Delta\vec{d}_{\text{T}} &= \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}\\ &= 0\text{ m} - 1750\text{ m [W]}\\ &= 0\text{ m} + 1750\text{ m [E]}\\ \Delta\vec{d}_{\text{T}} &= 1750\text{ m [E]}\end{aligned}$$
Statement: The girl's displacement is 1750 m [E].
31. Given: $\vec{d}_{\text{inital}}$ = 121 m [W]; $\vec{d}_{\text{final}}$ = 64 m [E]
Required: $\Delta\vec{d}_{\text{T}}$
Analysis: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}$
Solution:
$$\begin{aligned}\Delta\vec{d}_{\text{T}} &= \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}\\ &= 64\text{ m [E]} - 121\text{ m [W]}\\ &= 64\text{ m [E]} + 121\text{ m [E]}\\ \Delta\vec{d}_{\text{T}} &= 185\text{ m [E]}\end{aligned}$$
Statement: The bird's displacement is 185 m [E].
32. Given: Δd = 280 m; Δt = 4.3 s
Required: v_{av}
Analysis: $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$
Solution:
$$\begin{aligned}v_{\text{av}} &= \frac{\Delta d}{\Delta t}\\ &= \frac{280\text{ m}}{4.3\text{ s}}\\ v_{\text{av}} &= 65\text{ m/s}\end{aligned}$$
Statement: The race car's average speed is 65 m/s.
33. Given: $\Delta\vec{d}$ = 420 m [E]; Δt = 14.4 s
Required: $\vec{v}_{\text{av}}$
Analysis: $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$
Solution:
$$\begin{aligned}\vec{v}_{\text{av}} &= \frac{\Delta\vec{d}}{\Delta t}\\ &= \frac{420\text{ m [E]}}{14.4\text{ s}}\\ \vec{v}_{\text{av}} &= 29\text{ m/s [E]}\end{aligned}$$
Statement: The average velocity of the bird is 29 m/s [E].
34. Given: $\vec{d}_{\text{inital}}$ = 32 km [W]; $\vec{d}_{\text{final}}$ = 27 km [E]; Δt = 1.8 h
Required: $\vec{v}_{\text{av}}$
Analysis:
$$\begin{aligned}\vec{v}_{\text{av}} &= \frac{\Delta\vec{d}}{\Delta t}\\ \vec{v}_{\text{av}} &= \frac{\vec{d}_{\text{final}} - \vec{d}_{\text{initial}}}{\Delta t}\end{aligned}$$
Solution:
$$\begin{aligned}\vec{v}_{\text{av}} &= \frac{\vec{d}_{\text{final}} - \vec{d}_{\text{initial}}}{\Delta t}\\ &= \frac{27\text{ km [E]} - 32\text{ km [W]}}{1.8\text{ h}}\\ &= \frac{27\text{ km [E]} + 32\text{ km [E]}}{1.8\text{ h}}\\ &= \left(\frac{59\ \cancel{\text{km}}\text{ [E]}}{1.8\text{ h}}\right)\left(\frac{1000\text{ m}}{1\ \cancel{\text{km}}}\right)\\ &= \left(\frac{59\ 000\text{ m [E]}}{1.8\ \cancel{\text{h}}}\right)\left(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\text{ s}}\right)\\ \vec{v}_{\text{av}} &= 9.1\text{ m/s [E]}\end{aligned}$$
Statement: The velocity of the car is 9.1 m/s [E].
35. Given: v_{av} = 263 km/h; Δt = 13.7 s
Required: Δd
Analysis:
$$\begin{aligned}v_{\text{av}} &= \frac{\Delta d}{\Delta t}\\ \Delta d &= v_{\text{av}}\,\Delta t\end{aligned}$$
Solution:
$$\begin{aligned}\Delta d &= v_{\text{av}}\,\Delta t\\ &= \left(263\ \frac{\text{km}}{\cancel{\text{h}}}\right)(13.7\ \cancel{\text{s}})\left(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\right)\\ \Delta d &= 1.00\text{ km}\end{aligned}$$
Statement: The length of the track is 1.00 km or 1000 m.
36. (a) The position–time graph is curved, so the object has non-uniform velocity. In the first half of the time, the displacement is more than twice the displacement in the second half of the time.
(b) The slope at all points of the position–time graph is negative, so the velocity is always negative. The slope is becoming less steep, so the magnitude of the velocity must also be decreasing. The object is slowing down as it heads west.
37. (a) Average velocity is the total distance divided by the total time. Instantaneous velocity is the velocity at a specific moment. It is possible for these two values to be different whenever an object has non-uniform velocity.
(b) Given a position–time graph, I would calculate the slope between two points to determine the average velocity between them. I would look at the slope of a tangent to the curve to determine the instantaneous velocity at that point.

38. Given: $v_i = 0$ m/s; $\Delta t = 1.6$ s; $v_f = 2.8$ m/s
Required: a_{av}

Analysis: $a_{av} = \dfrac{\Delta v}{\Delta t}$

$$a_{av} = \frac{v_f - v_i}{\Delta t}$$

Solution: $a_{av} = \dfrac{v_f - v_i}{\Delta t}$

$$= \frac{2.8 \text{ m/s} - 0 \text{ m/s}}{1.6 \text{ s}}$$

$$a_{av} = 1.8 \text{ m/s}^2$$

Statement: The average acceleration of the runner is 1.8 m/s^2.

39. Given: $v_i = 0$ m/s; $a_{av} = 7.10$ m/s^2; $\Delta t = 2.20$ s
Required: v_f

Analysis: $a_{av} = \dfrac{v_f - v_i}{\Delta t}$

$$a_{av}\,\Delta t = v_f - v_i$$

$$v_f = v_i + a_{av}\,\Delta t$$

Solution:

$$v_f = v_i + a_{av}\,\Delta t$$

$$= 0 \text{ m/s} + \left(7.10 \ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(2.20 \text{ s})$$

$$\vec{v}_f = 15.6 \text{ m/s}$$

Statement: The horse's final speed is 15.6 m/s.

40. Given: $v_i = 0$ m/s; $v_f = 152$ m/s; $a_{av} = 1.35 \times 10^4$ m/s^2
Required: Δt

Analysis: $a_{av} = \dfrac{v_f - v_i}{\Delta t}$

$$\Delta t = \frac{v_f - v_i}{a_{av}}$$

Solution: $\Delta t = \dfrac{v_f - v_i}{a_{av}}$

$$= \frac{152 \ \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}} - 0 \ \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}}{1.35 \times 10^4 \ \dfrac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}}$$

$$\Delta t = 1.13 \times 10^{-2} \text{ s}$$

Statement: The arrow will take 1.13×10^{-2} s or 11.3 ms to accelerate from rest to a speed of 152 m/s.

41. Given: $b = 4.0$ s; $h = 2.0$ m/s [E]; $l = 4.0$ s; $w = 2.0$ m/s [E]
Required: $\Delta\vec{d}$

Analysis: Use the area under the graph to determine the position at $t = 4.0$ s:

$$\Delta\vec{d} = A_{\text{triangle}} + A_{\text{rectangle}}$$

Solution:

$$\Delta\vec{d} = A_{\text{triangle}} + A_{\text{rectangle}}$$

$$= \frac{1}{2}bh + lw$$

$$= \frac{1}{2}(4.0 \ \cancel{\text{s}})\left(2.0 \ \frac{\text{m}}{\cancel{\text{s}}} \text{ [E]}\right) + (4.0 \ \cancel{\text{s}})\left(2.0 \ \frac{\text{m}}{\cancel{\text{s}}} \text{ [E]}\right)$$

$$= 4.0 \text{ m [E]} + 8.0 \text{ m [E]}$$

$$\Delta\vec{d} = 12 \text{ m [E]}$$

Statement: The object has travelled 12 m [E] after 4.0 s.

42. (a) Given: $\vec{v}_i = 0$ m/s; $\Delta\vec{d} = 15.0$ m [down]; $\vec{a} = \vec{g} = 9.8$ m/s^2 [down]
Required: Δt

Analysis: $\Delta\vec{d} = \vec{v}_i\,\Delta t + \dfrac{1}{2}\vec{a}(\Delta t)^2$

$$= (0 \text{ m/s})\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$(\Delta t)^2 = \frac{2\Delta\vec{d}}{\vec{a}}$$

$$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$$

Solution: $\Delta t = \sqrt{\dfrac{2\Delta\vec{d}}{\vec{a}}}$

$$= \sqrt{\frac{2(15.0 \ \cancel{\text{m}})}{\left(9.8 \ \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$

$$\Delta t = 1.7 \text{ s}$$

Statement: The camera takes 1.7 s to hit the ground.

(b) Given: $\vec{v}_i = 0$ m/s; $\Delta\vec{d} = 10.0$ m [down]; $\vec{a} = \vec{g} = 9.8$ m/s^2 [down]
Required: $\vec{v}_f$

Analysis: $v_f^{\,2} = v_i^{\,2} + 2a\,\Delta d$

$$v_f = \sqrt{v_i^{\,2} + 2a\,\Delta d}$$

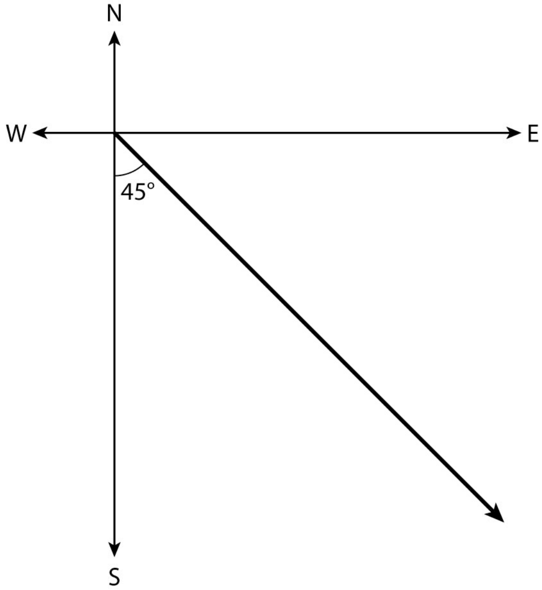
Solution: $v_f = \sqrt{v_i^{\,2} + 2a\,\Delta d}$

$$= \sqrt{\left(0 \ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(15.0 \text{ m})}$$

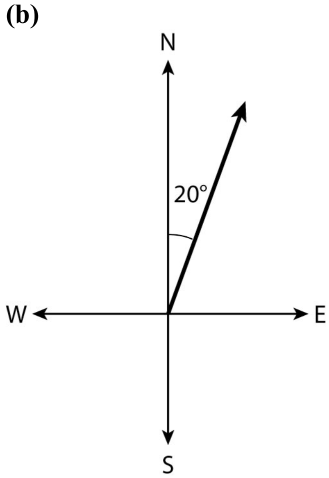
$$v_f = 17 \text{ m/s}$$

Statement: The final velocity of the camera is 17 m/s.

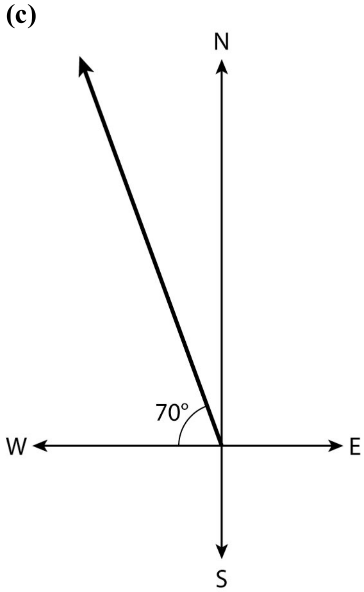
43. (a)



(b)



(c)



44. For each vector, determine the complementary angle, then reverse the order of the directions.

(a) 90° – 8° = 82°

$\Delta\vec{d}$ = 86 m [E 8° N]

$\Delta\vec{d}$ = 86 m [N 82° E]

(b) 90° – 23° = 67°

$\Delta\vec{d}$ = 97 cm [E 23° S]

$\Delta\vec{d}$ = 97 cm [S 67° E]

(c) 90° – 68° = 22°

$\Delta\vec{d}$ = 3190 km [S 68° W]

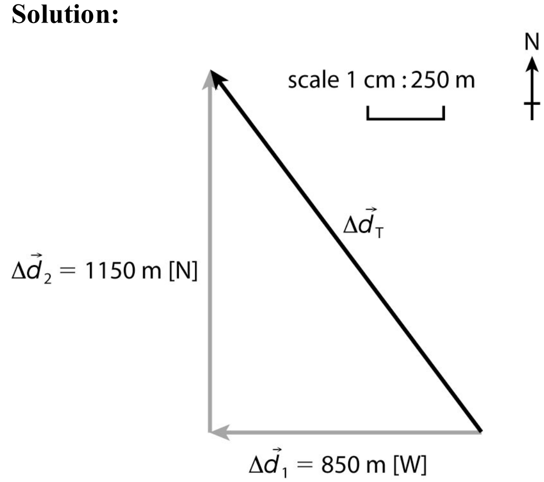
$\Delta\vec{d}$ = 3190 km [W 22° S]

45. Given: $\Delta\vec{d}_1$ = 850 m [W]; $\Delta\vec{d}_2$ = 1150 m [N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [W 54° N]. $\Delta\vec{d}_T$ measures 5.7 cm in length, so using the scale of 1 cm : 250 m, the actual magnitude of $\Delta\vec{d}_T$ is 1400 m.

Statement: The student's net displacement is 1400 m [W 54° N].

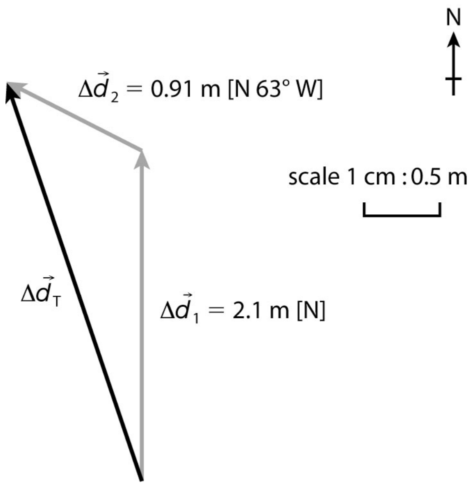
46. Given: $\Delta\vec{d}_1$ = 2.1 m [N];

$\Delta\vec{d}_2$ = 0.91 m [N 63° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [N 18° E]. $\Delta\vec{d}_T$ measures 5.2 cm in length, so using the scale of 1 cm : 0.5 m, the actual magnitude of $\Delta\vec{d}_T$ is 2.6 m.

Statement: The net displacement of the cue ball is 2.6 m [N 18° E].

47.

d_x	d_y	d_T
6	8	**10**
5.0	**12**	13
8.0	15	17
2.0	**7.0**	7.3
6.0	6.7	**9.0**

Use the Pythagorean theorem to determine teach missing magnitude: $d_x^2 + d_y^2 = d_T^2$.

Row 1:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_T = \sqrt{d_x^2 + d_y^2} = \sqrt{(6)^2 + (8)^2}$$
$$d_T = 10$$

Row 2:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_y = \sqrt{d_T^2 - d_x^2} = \sqrt{(13)^2 - (5.0)^2}$$
$$d_y = 12$$

Row 3:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_x = \sqrt{d_T^2 - d_y^2} = \sqrt{(17)^2 - (15)^2}$$
$$d_x = 8.0$$

Row 4:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_y = \sqrt{d_T^2 - d_x^2} = \sqrt{(7.3)^2 - (2.0)^2}$$
$$d_y = 7.0$$

Row 5:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_T = \sqrt{d_x^2 + d_y^2} = \sqrt{(6.0)^2 + (6.7)^2}$$
$$d_T = 9.0$$

48.

$\vec{d}_x$	$\vec{d}_y$	ϕ
5.0 [E]	12.0 [N]	**[E 67° N]**
15.00 [W]	8.00 [N]	**[W 28° N]**
91.0 [E]	**151 [S]**	[E 58.9° S]
640 [W]	213 [N]	[W 18.4° N]
0.051 [W]	**0.10 [S]**	[W 63° S]

Use the tangent function: $\tan\phi = \dfrac{d_y}{d_x}$.

Row 1: Find the missing angle.

$$\text{an}\,\phi = \frac{d_y}{d_x}$$
$$\tan\phi = \frac{12.0}{5.0}$$
$$\tan\phi = 2.4$$
$$\phi = \tan^{-1}(2.4)$$
$$\phi = 67°$$

Row 2: Find the missing angle.

$$\text{an}\,\phi = \frac{d_y}{d_x}$$
$$\tan\phi = \frac{8.00}{15.00}$$
$$\tan\phi = 5.33$$
$$\phi = \tan^{-1}(5.33)$$
$$\phi = 28°$$

Row 3: Find the missing component vector.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\ 58.9° = \frac{d_y}{91.0}$$
$$(1.659)(91.0) = d_y \quad \text{(two extra digits caried)}$$
$$151 = d_y$$

Row 4: Find the missing component vector.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\ 18.4° = \frac{213}{d_x}$$
$$d_x = \frac{213}{0.33} \quad \text{(one extra digit carried)}$$
$$d_x = 640$$

Row 5: Find the missing component vector.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\ 63° = \frac{d_y}{0.051}$$
$$(1.96078)(0.051) = d_y \quad \text{(two extra digits caried)}$$
$$0.10 = d_y$$

49. (a) Given: $\Delta\vec{d}_{\text{T}} = 82$ m [W 76° S]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_{\text{T}} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_{\text{T}}$ is between west and south, the direction of $\Delta\vec{d}_x$ is [W] and the direction of $\Delta\vec{d}_y$ is [S].

$$\sin\theta = \frac{\Delta d_y}{\Delta d_{\text{T}}}$$
$$\begin{aligned}\Delta d_y &= \Delta d_{\text{T}} \sin\theta \\ &= (82\text{ m})(\sin 76°) \\ \Delta d_y &= 80\text{ m}\end{aligned}$$

$$\cos\theta = \frac{\Delta d_x}{\Delta d_{\text{T}}}$$
$$\begin{aligned}\Delta d_x &= \Delta d_{\text{T}} \cos\theta \\ &= (82\text{ m})(\cos 76°) \\ \Delta d_x &= 20\text{ m}\end{aligned}$$

Statement: The vector has a horizontal or x-component of 20 m [W] and a vertical or y-component of 80 m [S].

(b) Given: $\Delta\vec{d}_{\text{T}} = 34$ m [E 13° N]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_{\text{T}} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_{\text{T}}$ is between east and north, the direction of $\Delta\vec{d}_x$ is [E] and the direction of $\Delta\vec{d}_y$ is [N].

$$\sin\theta = \frac{\Delta d_y}{\Delta d_{\text{T}}}$$
$$\begin{aligned}\Delta d_y &= \Delta d_{\text{T}} \sin\theta \\ &= (34\text{ m})(\sin 13°) \\ \Delta d_y &= 7.6\text{ m}\end{aligned}$$

$$\cos\theta = \frac{\Delta d_x}{\Delta d_{\text{T}}}$$
$$\begin{aligned}\Delta d_x &= \Delta d_{\text{T}} \cos\theta \\ &= (34\text{ m})(\cos 13°) \\ \Delta d_x &= 33\text{ m}\end{aligned}$$

Statement: The vector has a horizontal or x-component of 33 m [E] and a vertical or y-component of 7.6 m [N].

(c) Given: $\Delta\vec{d}_{\text{T}} = 97$ m [S 65° W]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_{\text{T}} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_{\text{T}}$ is between south and west, the direction of $\Delta\vec{d}_x$ is [W] and the direction of $\Delta\vec{d}_y$ is [S].

$$\cos\theta = \frac{\Delta d_x}{\Delta d_{\text{T}}}$$
$$\begin{aligned}\Delta d_x &= \Delta d_{\text{T}} \cos\theta \\ &= (97\text{ m})(\cos 65°) \\ \Delta d_x &= 41\text{ m}\end{aligned}$$

$$\sin\theta = \frac{\Delta d_y}{\Delta d_{\text{T}}}$$
$$\begin{aligned}\Delta d_y &= \Delta d_{\text{T}} \sin\theta \\ &= (97\text{ m})(\sin 65°) \\ \Delta d_y &= 88\text{ m}\end{aligned}$$

Statement: The vector has a horizontal or x-component of 41 m [W] and a vertical or y-component of 88 m [S].

50. (a) Given: $\Delta\vec{d}_x = 4.0$ m [W]; $\Delta\vec{d}_y = 1.9$ m [S]

Required: $\Delta\vec{d}_\text{T}$

Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_\text{T}$ makes with the x-axis.

$$\begin{aligned}
\Delta\vec{d}_\text{T} &= \Delta\vec{d}_x + \Delta\vec{d}_y \\
\Delta d_\text{T}^{\,2} &= \Delta d_x^{\,2} + \Delta d_y^{\,2} \\
\Delta d_\text{T} &= \sqrt{\Delta d_x^{\,2} + \Delta d_y^{\,2}} \\
&= \sqrt{(4.0\text{ m})^2 + (1.9\text{ m})^2} \\
\Delta d_\text{T} &= 4.4\text{ m}
\end{aligned}$$

$$\begin{aligned}
\tan\phi &= \frac{\Delta d_y}{\Delta d_x} \\
\tan\phi &= \frac{1.9\ \cancel{\text{m}}}{4.0\ \cancel{\text{m}}} \\
\phi &= 25°
\end{aligned}$$

Statement: The sum of the two vectors is 4.4 m [W 25° S].

(b) Given: $\Delta\vec{d}_x = 1.9$ m [E]; $\Delta\vec{d}_y = 7.6$ m [N]

Required: $\Delta\vec{d}_\text{T}$

Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_\text{T}$ makes with the x-axis.

$$\begin{aligned}
\Delta\vec{d}_\text{T} &= \Delta\vec{d}_x + \Delta\vec{d}_y \\
\Delta d_\text{T}^{\,2} &= \Delta d_x^{\,2} + \Delta d_y^{\,2} \\
\Delta d_\text{T} &= \sqrt{\Delta d_x^{\,2} + \Delta d_y^{\,2}} \\
&= \sqrt{(1.9\text{ m})^2 + (7.6\text{ m})^2} \\
\Delta d_\text{T} &= 7.8\text{ m}
\end{aligned}$$

$$\begin{aligned}
\tan\phi &= \frac{\Delta d_y}{\Delta d_x} \\
\tan\phi &= \frac{7.6\ \cancel{\text{m}}}{1.9\ \cancel{\text{m}}} \\
\phi &= 76°
\end{aligned}$$

Statement: The sum of the two vectors is 7.8 m [E 76° N].

(c) Given: $\Delta\vec{d}_x = 72$ m [W]; $\Delta\vec{d}_y = 15$ m [N]

Required: $\Delta\vec{d}_\text{T}$

Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_\text{T}$ makes with the x-axis.

$$\begin{aligned}
\Delta\vec{d}_\text{T} &= \Delta\vec{d}_x + \Delta\vec{d}_y \\
\Delta d_\text{T}^{\,2} &= \Delta d_x^{\,2} + \Delta d_y^{\,2} \\
\Delta d_\text{T} &= \sqrt{\Delta d_x^{\,2} + \Delta d_y^{\,2}} \\
&= \sqrt{(72\text{ m})^2 + (15\text{ m})^2} \\
\Delta d_\text{T} &= 74\text{ m}
\end{aligned}$$

$$\begin{aligned}
\tan\phi &= \frac{\Delta d_y}{\Delta d_x} \\
\tan\phi &= \frac{15\ \cancel{\text{m}}}{72\ \cancel{\text{m}}} \\
\phi &= 12°
\end{aligned}$$

Statement: The sum of the two vectors is 74 m [W 12° N].

51. Given: $\Delta\vec{d}_1 = 32$ m [W 14° S]; $\Delta\vec{d}_2 = 15$ m [E 62° S]

Required: $\Delta\vec{d}_\text{T}$

Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_\text{T}$:

$$\begin{aligned}
\vec{d}_{\text{T}x} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x} \\
&= (32\text{ m})(\cos 14°)\text{ [W]} + (15\text{ m})(\cos 62°)\text{ [E]} \\
&= 31.05\text{ m [W]} + 7.04\text{ m [E]} \\
&= 31.05\text{ m [W]} - 7.04\text{ m [W]} \\
&= 24.01\text{ m [W]} \\
\vec{d}_{\text{T}x} &= 24\text{ m [W]}
\end{aligned}$$

$$\begin{aligned}
\vec{d}_{\text{T}y} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y} \\
&= (32\text{ m})(\sin 14°)\text{ [S]} + (15\text{ m})(\sin 62°)\text{ [S]} \\
&= 7.74\text{ m [S]} + 13.24\text{ m [S]} \\
&= 20.98\text{ m [S]} \\
\vec{d}_{\text{T}y} &= 21\text{ m [S]}
\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\Delta d_T^{\ 2} = d_{Tx}^{\ 2} + d_{Ty}^{\ 2}$$
$$\Delta d_T = \sqrt{d_{Tx}^{\ 2} + d_{Ty}^{\ 2}}$$
$$= \sqrt{(24.01\text{ m})^2 + (20.98\text{ m})^2} \text{ (two extra digits carried)}$$
$$\Delta d_T = 32\text{ m}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\tan\phi = \frac{\Delta d_{Ty}}{\Delta d_{Tx}}$$
$$\tan\phi = \frac{20.98\ \cancel{\text{m}}}{24.01\ \cancel{\text{m}}} \text{ (two extra digits carried)}$$
$$\phi = 41°$$

Statement: The net displacement of the disc is 32 m [W 41° S].

52. (a) Given: Δd = 64 m; v = 0.2 m/s

Required: Δt

Analysis: $v = \frac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \frac{\Delta d}{v}$

$$= \frac{64\ \cancel{\text{m}}}{0.2\ \frac{\cancel{\text{m}}}{\text{s}}}$$
$$\Delta t = 3.2\times10^2\text{ s}$$

Statement: It takes the fish 3.2×10^2 s to cross the river.

(b) Given: $\vec{v}_1$ = 0.90 m/s [S]; $\vec{v}_2$ = 0.2 m/s [E]

Required: $\vec{v}_T$

Analysis: $\vec{v}_T = \vec{v}_1 + \vec{v}_2$

Solution: Let ϕ represent the angle $\Delta\vec{v}_T$ makes with the y-axis.

$$\vec{v}_T = \vec{v}_1 + \vec{v}_2$$
$$v_T^{\ 2} = v_1^{\ 2} + v_2^{\ 2}$$
$$v_T = \sqrt{v_1^{\ 2} + v_2^{\ 2}}$$
$$= \sqrt{(0.90\text{ m/s})^2 + (0.2\text{ m/s})^2}$$
$$v_T = 0.9\text{ m/s}$$

$$\tan\phi = \frac{v_2}{v_1}$$
$$\tan\phi = \frac{0.2\text{ m/s}}{0.9\text{ m/s}}$$
$$\phi = 13°$$

Statement: The resulting velocity of the fish is 0.9 m/s [S 13° E].

(c) Given: $\vec{v}_x$ = 0.90 m/s [S], $\Delta t = 3.2 \times 10^2$ s

Required: $\Delta\vec{d}_x$

Analysis: $\vec{v}_x = \frac{\Delta\vec{d}_x}{\Delta t}$

$$\Delta\vec{d}_x = \vec{v}_x\,\Delta t$$

Solution: $\Delta\vec{d}_x = \vec{v}_x\,\Delta t$

$$= \left(0.90\ \frac{\text{m}}{\cancel{\text{s}}}\ [\text{S}]\right)(320\ \cancel{\text{s}})$$
$$= 288\text{ m [S]}$$
$$\Delta\vec{d}_x = 2.9\times10^2\text{ m [S]}$$

Statement: The fish arrives 2.9×10^2 m downstream from being directly across from where it started.

53. The pens will hit the ground at the same time. Both pens have no vertical component to their initial velocities: the first starts with no velocity at all and the second starts with only horizontal velocity. Since both accelerate down at the same rate (gravity), both pens will land at the same time.

54. Given: v_i = 22 m/s; θ = 62°

Required: $\vec{v}_{ix}$; $\vec{v}_{iy}$

Analysis: $\vec{v}_i = \vec{v}_{ix} + \vec{v}_{iy}$

Solution: $\sin\theta = \frac{\vec{v}_{iy}}{\vec{v}_i}$

$$\vec{v}_{iy} = \vec{v}_i\sin\theta$$
$$= (22\text{ m/s})(\sin 62°)$$
$$\vec{v}_{iy} = 19\text{ m/s}$$

$$\cos\theta = \frac{\vec{v}_{ix}}{\vec{v}_i}$$
$$\vec{v}_{ix} = \vec{v}_i\cos\theta$$
$$= (22\text{ m/s})(\cos 62°)$$
$$\vec{v}_{ix} = 10\text{ m/s}$$

Statement: The initial velocity has a horizontal or x-component of 10 m/s and a vertical or y-component of 19 m/s.

55. (a) Given: $\Delta d_y = -1.2$ m; $a_y = -9.8$ m/s^2; $v_y = 0$ m/s
Required: Δt

Analysis: $\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$

$$\Delta d_y = 0 + \frac{1}{2} a_y \Delta t^2$$

$$\Delta t^2 = \frac{2\Delta d_y}{a_y}$$

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

Solution: $\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$

$$= \sqrt{\frac{2(-1.2\ \cancel{\text{m}})}{\left(-9.8\ \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$

$$= 0.4949 \text{ s}$$

$$\Delta t = 0.49 \text{ s}$$

Statement: The time of flight of the tennis ball will be 0.49 s.

(b) Given: $a_x = 0$ m/s^2; $v_x = 5.3$ m/s
Required: Δd_x
Analysis: $\Delta d_x = v_x \Delta t$
Solution:

$$\Delta d_x = v_x \Delta t$$

$$= \left(5.3\ \frac{\text{m}}{\cancel{\text{s}}}\right)(0.4949\ \cancel{\text{s}}) \text{ (two extra digits carried)}$$

$$\Delta d_x = 2.6 \text{ m}$$

Statement: The range of the tennis ball will be 2.6 m.

Analysis and Application

56. Given: $\vec{v}_i = 0.60$ m/s [up]; $\vec{v}_f = 27.0$ m/s [down]; $\Delta t = 5.50$ s

Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

Solution: $\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$= \frac{27.0 \text{ m/s [down]} - 0.60 \text{ m/s [up]}}{5.50 \text{ s}}$$

$$= \frac{27.0 \text{ m/s [down]} + 0.60 \text{ m/s [down]}}{5.50 \text{ s}}$$

$$\vec{a}_{av} = 5.0 \text{ m/s}^2 \text{ [down]}$$

Statement: The average acceleration of the roller coaster is 5.0 m/s^2 [down].

57. (a) Given: $\Delta\vec{v} = 6.0$ m/s [S]; $\Delta t = 3.0$ s
Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$

Solution: $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$

$$= \frac{6.0 \text{ m/s [S]}}{3.0 \text{ s}}$$

$$\vec{a}_{av} = 2.0 \text{ m/s}^2 \text{ [S]}$$

Statement: The average acceleration from 0 s to 3.0 s is 2.0 m/s^2 [S].

(b) Given: $\Delta\vec{v} = 6.0$ m/s; $\Delta t = 4.0$ s
Required: $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$

Solution: $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$

$$= \frac{6.0 \text{ m/s [S]}}{4.0 \text{ s}}$$

$$\vec{a}_{av} = 1.5 \text{ m/s}^2 \text{ [S]}$$

Statement: The average acceleration from 2.0 s to 6.0 s is 1.5 m/s^2 [S].

(c) Given: $b = 5.0$ s; $h = 10.0$ m/s [W]; $l = 1.0$ s
Required: $\Delta\vec{d}$
Analysis: $\Delta\vec{d} = A_{triangle} + A_{rectangle}$
Solution:

$$\Delta\vec{d} = A_{triangle} + A_{rectangle}$$

$$= \frac{1}{2}bh + lh$$

$$= \frac{1}{2}(5.0\ \cancel{\text{s}})\left(10.0\ \frac{\text{m}}{\cancel{\text{s}}} \text{ [S]}\right) + (1.0\ \cancel{\text{s}})\left(10.0\ \frac{\text{m}}{\cancel{\text{s}}} \text{ [S]}\right)$$

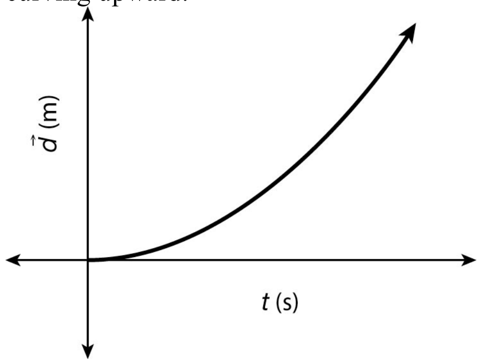
$$= 25 \text{ m [S]} + 10 \text{ m [S]}$$

$$\Delta\vec{d} = 35 \text{ m [S]}$$

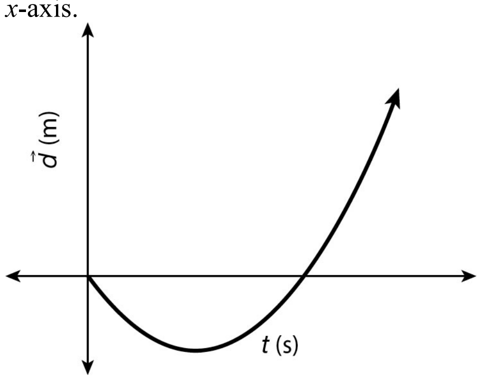
Statement: The object has travelled 35 m [S] after 6.0 s.

58. Graph (a) shows uniformly increasing velocity. Graph (b) shows uniformly decreasing velocity. Every second, the velocity in graph (a) changes by 1 m/s while the velocity in graph (b) changes by 1.5 m/s^2. So, graph (b) must have the greater acceleration in magnitude since its velocity is changing faster.

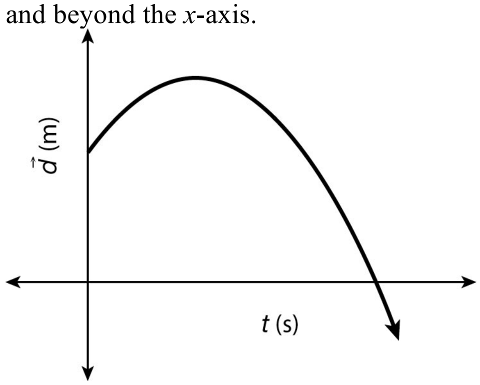
59. (a) The object has constant positive acceleration, so the graph will be curved up. Since the object started from rest at the reference point, the graph will start at the origin and increase, curving upward.



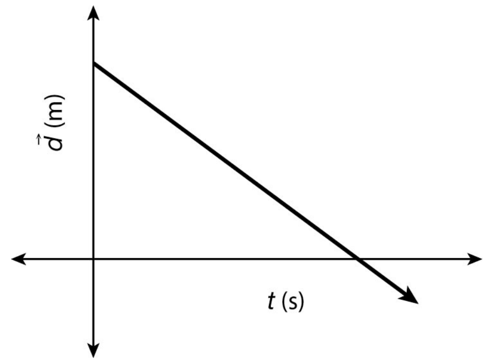
(b) The object has constant positive acceleration, so the graph will be curved up. Since the object started with negative velocity at the reference point, the graph will start at the origin, decrease, level out, then increase up to and beyond the *x*-axis.



(c) The object has constant negative acceleration, so the graph will be curved down. Since the object started with positive velocity away from the reference point, the graph will start high on the *y*-axis, increase, level out, then decrease down to and beyond the *x*-axis.



(d) The object has no acceleration, so the graph will be a straight line. Since the object started with negative velocity away from the reference point, the graph will start high on the *y*-axis, then decrease down to and beyond the *x*-axis.



60. (a) Given: v_i = 145 km/h; v_f = 0 m/s; a = 10.4 m/s^2

Required: Δt

Analysis: $v_f = v_i + a_{av}\Delta t$

$$\Delta t = \frac{v_f - v_i}{a_{av}}$$

Solution: Convert v_i to metres per second:

$$v_i = \left(145\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)$$

v_i = 40.278 m/s

$$\begin{aligned}\Delta t &= \frac{v_f - v_i}{a_{av}} \\ &= \frac{0\ \text{km/h} - 40.278\ \text{m/s}}{-10.4\ \text{m/s}^2}\ \text{(two extra digits carried)} \\ &= \frac{-40.278\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{-10.4\ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}}\end{aligned}$$

Δt = 3.87 s

Statement: The car takes 3.87 s to stop.

(b) Given: v_f = 0 m/s; a = 11.0 m/s^2

Required: Δd

Analysis: $v_f^2 = v_i^2 + 2a\Delta d$

$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$

Solution:

$$\begin{aligned}\Delta d &= \frac{v_f^2 - v_i^2}{2a} \\ &= \frac{(0\ \text{m/s})^2 - (40.278\ \text{m/s})^2}{2(-10.4\ \text{m/s}^2)}\ \text{(two extra digits carried)} \\ &= \frac{-1622.3\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{-20.8\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}\end{aligned}$$

Δd = 78.0 m

Statement: The car travels 78.0 m while slowing down.

61. (a) Given: v_i = 54.0 km/h; a_{av} = 1.90 m/s^2; Δt = 7.50 s

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \vec{v}_i\,\Delta t + \frac{1}{2}\vec{a}_{av}\,\Delta t^2$

Solution: Convert v_i to metres per second:

$$v_i = \left(54.0\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)$$

$$v_i = 15\ \text{m/s}$$

$$\begin{aligned}\Delta d &= v_i\,\Delta t + \frac{1}{2}a_{av}\,\Delta t^2\\ &= \left(15\ \frac{\text{m}}{\cancel{\text{s}}}\right)(7.50\ \cancel{\text{s}}) + \frac{1}{2}\left(1.90\ \frac{\text{m}}{\cancel{\text{s}^2}}\right)(7.50\ \cancel{\text{s}})^2\end{aligned}$$

$$\Delta d = 166\ \text{m}$$

Statement: The displacement of the vehicle is 166 m.

(b) Given: v_i = 15 m/s; a = 1.90 m/s^2; Δt = 7.50 s

Required: v_f

Analysis: $v_f = v_i + a\,\Delta t$

Solution:

$$\begin{aligned}v_f &= v_i + a\,\Delta t\\ &= 15\ \frac{\text{m}}{\text{s}} + \left(1.90\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(7.50\ \cancel{\text{s}})\\ &= \left(29.25\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{1\ \text{km}}{1000\ \cancel{\text{m}}}\right)\left(\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\ \text{h}}\right)\end{aligned}$$

$$\vec{v}_f = 105\ \text{km/h}$$

Statement: The van's final velocity is 105 km/h.

62.

	Equation	Uses
Equation 1	$\Delta\vec{d} = \left(\frac{\vec{v}_1 + \vec{v}_2}{2}\right)\Delta t$	Solving for $\Delta\vec{d}$, $\vec{v}_1$, $\vec{v}_2$, or Δt when the other three are known and acceleration is not known.
Equation 2	$\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t$	Solving for $\vec{v}_i$, $\vec{v}_f$, $\vec{a}$, or Δt when the other three are known and displacement is not known.
Equation 3	$\Delta\vec{d} = \vec{v}_i\,\Delta t + \frac{1}{2}\vec{a}_{av}\,\Delta t^2$	Solving for $\Delta\vec{d}$, $\vec{v}_i$, $\vec{a}$, or Δt when the other three are known and the final velocity is not known.
Equation 4	$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}_{av}\,\Delta d$	Solving for $\vec{v}_i$, $\vec{v}_f$, $\vec{a}$, or $\Delta\vec{d}$ when the other three are known and the time is not known.
Equation 5	$\Delta\vec{d} = \vec{v}_f\,\Delta t - \frac{1}{2}\vec{a}_{av}\,\Delta t^2$	Solving for $\Delta\vec{d}$, $\vec{v}_f$, $\vec{a}$, or Δt when the other three are known and the initial velocity is not known.

63. Given: $\Delta \vec{d}_1 = 240$ m [W 11° N];

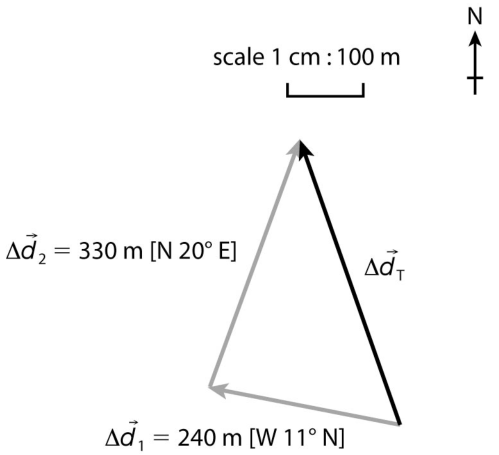
$\Delta \vec{d}_2 = 330$ m [N 20° E]; $\Delta t = 22$ s

Determine the displacement of the boat:

Required: $\Delta \vec{d}_T$

Analysis: $\Delta \vec{d}_T = \Delta \vec{d}_1 + \Delta \vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta \vec{d}_1$ joined to the tail of $\Delta \vec{d}_2$. The resultant vector $\Delta \vec{d}_T$ is drawn in black from the tail of $\Delta \vec{d}_1$ to the tip of $\Delta \vec{d}_2$. Using a compass, the direction of $\Delta \vec{d}_T$ is [N 1.4° W]. $\Delta \vec{d}_T$ measures 3.8 cm in length, so using the scale of 1 cm : 100 m, the actual magnitude of $\Delta \vec{d}_T$ is 380 m.

Statement: Her displacement is 380 m [W 71° N].

Determine the average velocity of the boat:

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \dfrac{\Delta \vec{d}}{\Delta t}$

Solution:
$$\begin{aligned}
\vec{v}_{av} &= \frac{\Delta \vec{d}}{\Delta t} \\
&= \frac{380 \text{ m [W 71° N]}}{22 \text{ s}} \\
&= 17.27 \text{ m/s [W 71° N]} \\
\vec{v}_{av} &= 17 \text{ m/s [W 71° N]}
\end{aligned}$$

Statement: Her average velocity is 17 m/s [W 71° N].

64. Answers may vary. Sample answer:
I would draw what I knew of the vector addition to determine the magnitude and direction of the missing vector. For example, I would put the tips of the two vectors together, and the missing component vector would be the vector from the tail of the other component vector to the tip of the resultant vector.

65. Answers may vary. Sample answer:

(a) Vectors can be added using a scale diagram or algebraically.

To add vectors using a scale diagram in one or two dimensions, draw the second vector starting at the tip of the first vector. The distance from the tail of the first to the tip of the second is the resultant vector.

Adding vectors algebraically is different depending on the number of dimensions. With one dimension, just add the values together once they have the same direction. In two dimensions, add the x-components and y-components separately, then use the tangent function to determine the direction of the resultant.

(b) Adding vectors using a scale diagram is the same in one and two dimensions. Using algebra, you must break the vectors into their components, then solve each dimension by adding as you would in one dimension.

66. Given: $\Delta d_x = 1250$ m; $\Delta d_T = 1550$ m

Determine how far north the ranger travelled:

Required: Δd_x

Analysis:
$$\begin{aligned}
\Delta d_T^2 &= \Delta d_x^2 + \Delta d_y^2 \\
\Delta d_y^2 &= \Delta d_T^2 - \Delta d_x^2 \\
\Delta d_y &= \sqrt{\Delta d_T^2 - \Delta d_x^2}
\end{aligned}$$

Solution:
$$\begin{aligned}
\Delta d_y &= \sqrt{\Delta d_T^2 - \Delta d_x^2} \\
&= \sqrt{(1550 \text{ m})^2 - (1250 \text{ m})^2} \\
\Delta d_y &= 917 \text{ m}
\end{aligned}$$

Statement: The ranger travelled 917 m north.

Determine the direction the ranger travelled:

Required: θ

Analysis: $\sin\theta = \dfrac{\Delta d_x}{\Delta d_T}$

Solution: Let θ represent the angle $\Delta \vec{d}_T$ makes with the y-axis.
$$\begin{aligned}
\sin\theta &= \frac{\Delta d_x}{\Delta d_T} \\
&= \frac{1250 \cancel{\text{m}}}{1550 \cancel{\text{m}}} \\
\theta &= 53.8°
\end{aligned}$$

Statement: The ranger travelled in the direction [N 53.8° E].

67. Given: $\Delta d_x = 11$ m; θ = [N 28° W]
Required: Δd_T

Analysis: $\sin\theta = \dfrac{\Delta d_x}{\Delta d_T}$

$$\Delta d_T = \frac{\Delta d_x}{\sin\theta}$$

Solution: $\Delta d_T = \dfrac{\Delta d_x}{\sin\theta}$

$$= \frac{11\text{ m}}{\sin 28°}$$

$$\Delta d_T = 23\text{ m}$$

Statement: The archer is 23 m from her target.

68. (a) Given: $\Delta t = 3.0$ s; $\Delta\vec{d}_x = 9.0$ m [E];
$\vec{v}_y = 4.0$ m/s [S]

Required: $\vec{v}_T$

Analysis: $\vec{v}_T = \vec{v}_x + \vec{v}_y$

Solution: Determine $\vec{v}_x$, which is constant:

$$\vec{v}_x = \frac{\Delta\vec{d}_x}{\Delta t} = \frac{9.0\text{ m [E]}}{3.0\text{ s}}$$

$$\vec{v}_x = 3.0\text{ m/s [E]}$$

Use the Pythagorean theorem:

$$v_T^{\,2} = v_x^{\,2} + v_y^{\,2}$$

$$v_T = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{(3.0\text{ m/s})^2 + (4.0\text{ m/s})^2}$$

$$v_T = 5.0\text{ m/s}$$

Let ϕ represent the angle $\vec{v}_T$ makes with the x-axis.

$$\tan\phi = \frac{v_y}{v_x} = \frac{4.0\ \cancel{\text{m/s}}}{3.0\ \cancel{\text{m/s}}}$$

$$\phi = 53°$$

Statement: The velocity of the object at $t = 3.0$ s is 5.0 m/s [E 53° S].

(b) Given: $\Delta t = 3.0$ s; $\Delta\vec{d}_x = 6.0$ m [E];
$\vec{a}_y = 2.0$ m/s^2 [S]

Required: $\vec{v}_T$

Analysis: $\vec{v}_T = \vec{v}_x + \vec{v}_y$

Solution: Determine $\vec{v}_x$, which is constant:

$$\vec{v}_x = \frac{\Delta\vec{d}_x}{\Delta t} = \frac{6.0\text{ m [E]}}{3.0\text{ s}}$$

$$\vec{v}_x = 2.0\text{ m/s [E]}$$

Determine $\vec{v}_y$:

$$\vec{v}_y = \vec{a}_y\,\Delta t = \left(2.0\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\ \text{[S]}\right)(3.0\ \cancel{\text{s}})$$

$$\vec{v}_y = 6.0\text{ m/s [S]}$$

Use the Pythagorean theorem:

$$v_T^{\,2} = v_x^{\,2} + v_y^{\,2}$$

$$v_T = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{(2.0\text{ m/s})^2 + (6.0\text{ m/s})^2}$$

$$v_T = 6.3\text{ m/s}$$

Let ϕ represent the angle $\vec{v}_T$ makes with the x-axis.

$$\tan\phi = \frac{v_y}{v_x} = \frac{6.0\ \cancel{\text{m/s}}}{2.0\ \cancel{\text{m/s}}}$$

$$\phi = 72°$$

Statement: The velocity of the object at $t = 3.0$ s is 6.3 m/s [E 72° S].

69. (a) Given: $a_y = -9.8$ m/s^2; $v_i = 27.5$ m/s;
$\theta = 41°$
Determine the time of flight:
Required: Δt

Analysis: $\Delta d_y = v_y\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$

Solution:

$$\Delta d_y = v_y\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$$

$$= v_i(\sin\theta)\Delta t + \frac{1}{2}a_y\,\Delta t^2$$

$$0 = (27.5\text{ m/s})(\sin 41°)\Delta t + \frac{1}{2}(-9.8\text{ m/s}^2)\Delta t^2$$

$$0 = (18.04\text{ m/s})\Delta t - (4.9\text{ m/s}^2)\Delta t^2$$

$$0 = (18.04\text{ m/s}) - (4.9\text{ m/s}^2)\Delta t\ \ (\Delta t \neq 0)$$

$$\Delta t = \frac{18.04\ \frac{\cancel{m}}{\cancel{s}}}{4.9\ \frac{\cancel{m}}{s^{\cancel{2}}}}$$
$$= 3.682\ s$$
$$\Delta t = 3.7\ s$$

Statement: The football's time of flight is 3.7 s.

Determine the range:

Required: Δd_x

Analysis: $\Delta d_x = v_x \Delta t$

Solution:

$$\Delta d_x = v_x \Delta t$$
$$= v_i \cos\theta \Delta t$$
$$= \left(27.5\ \frac{m}{\cancel{s}}\right)(\cos 41°)(3.682\ \cancel{s})\ \text{(two extra digits carried)}$$
$$\Delta d_x = 76\ m$$

Statement: The football's range is 76 m.

Determine the maximum height:

Required: Δd_y

Analysis: $v_{fy}{}^2 = v_{iy}{}^2 + 2a_y \Delta d_y$

$$\Delta d_y = \frac{v_{fy}{}^2 - v_{iy}{}^2}{2a_y}$$

Solution: $\Delta d_y = \frac{v_{fy}{}^2 - v_{iy}{}^2}{2a_y}$

$$= \frac{0 - (v_i \sin 41°)^2}{2(-9.8\ m/s^2)}$$
$$= \frac{0 - [(27.5\ m/s)(\sin 41°)]^2}{-19.6\ m/s^2}$$
$$= \frac{325.5\ \frac{m^{\cancel{2}}}{\cancel{s^2}}}{19.6\ \frac{\cancel{m}}{\cancel{s^2}}}$$
$$\Delta d_y = 17\ m$$

Statement: The football reached a maximum height of 17 m.

(b) Given: a_y = –9.8 m/s^2; Δt = 3.2 s; Δd_x = 29 m

Determine the initial velocity:

Required: v_i

Analysis: $\vec{v}_i = \vec{v}_{ix} + \vec{v}_{iy}$

Solution: Determine the x-component:

$$v_{ix} = \frac{\Delta d_x}{\Delta t}$$
$$= \frac{29\ m}{3.2\ s}$$
$$= 9.0625\ m/s$$
$$v_{ix} = 9.1\ m/s$$

Determine the y-component:

$$\Delta d_y = v_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2$$
$$v_{iy} = \frac{\Delta d_y - \frac{1}{2} a_y \Delta t^2}{\Delta t}$$
$$= \frac{0 - \frac{1}{2}(-9.8\ m/s^2)(3.2\ s)^2}{3.2\ s}$$
$$= \frac{\left(4.9\ \frac{m}{\cancel{s^2}}\right)(10.24\ \cancel{s^2})}{3.2\ s}$$
$$= 15.68\ m/s$$
$$v_{iy} = 16\ m/s$$

Use the Pythagorean theorem:

$$v_i^2 = v_{ix}{}^2 + v_{iy}{}^2$$
$$v_i = \sqrt{v_{ix}{}^2 + v_{iy}{}^2}$$
$$= \sqrt{(9.062\ m/s)^2 + (15.68\ m/s)^2}\ \text{(two extra digits carried)}$$
$$v_f = 18\ m/s$$

Statement: The football is kicked with an initial velocity of 18 m/s.

Determine the initial angle:

Required: θ

Analysis: $\tan\theta = \frac{v_{iy}}{v_{ix}}$

$$\tan\theta = \frac{v_{iy}}{v_{ix}}$$
$$= \frac{15.68\ \frac{\cancel{m}}{\cancel{s}}}{9.062\ \frac{\cancel{m}}{\cancel{s}}}\ \text{(two extra digits carried)}$$
$$\theta = 60°$$

Statement: The football is kicked at an angle of 60°.

70. (a) If two objects were dropped at the same time from the same height, but one was on the Moon and the other here on Earth, then the one that is dropped on Earth will hit the ground first since the acceleration due to Earth's gravity is greater than the acceleration due to the Moon's gravity.
(b) If one beanbag was launched horizontally and another of equal mass was dropped from the same height at the same time, then they would both hit the ground at the same time. If this experiment were performed on the Moon the results would be the same since both beanbags would still experience the same vertical acceleration.
(c) For the beanbags launched horizontally in part (b), the beanbag on the Moon would have the larger range. Since both beanbags are launched with the same horizontal velocity, the one with the larger time of flight will have the larger range. Since the Moon has less gravity, the beanbag on the Moon will fall more slowly and have a longer time of flight than the one on Earth.
71. Suppose a projectile in a parabolic path was surrounded by a room. Within that room, the projectile would appear to be free-floating without gravity. So, by flying an airplane in the parabolic path of a projectile, the people in the airplane will experience what feels like a gravity-free environment inside the airplane.

Evaluation

72. Answers may vary. Sample answer:
(a) Drawing vectors in three dimensions would use the same directed line segments, except that instead of only pointing in directions on a surface, they would point in any direction in a space.
(b) Adding two vectors on a three dimensional diagram would apply the same methods used for one and two dimensions. The first vector would be drawn and the second added by joining its tail to the tip of the first vector. The resultant vector would then be the vector drawn from the tail of the first to the tip of the second.
(c) Three dimensional vectors would have three component vectors. These components would correspond to the normal x- and y-axes vectors: east and west for the x-directions, north and south for the y-axis vectors, and then there would be an additional direction on a vertical z-axis corresponding to up and down.
73. Answers may vary. Sample answer:
Yes, Galileo's experiments had a major impact and lead to the discovery of modern kinematics. Not only did the experiments lead to important scientific discoveries that enabled Newton to formulate his theories, but he was also one of the first to challenge the scientific notions of the day. Without his leadership, people may have been left with their false understandings for another century.
74. Answers may vary. Sample answer:
Accelerometers could be used in wireless mice to adjust the power needed for the signal. Wireless mice already use blue tooth technology to allow you to control your computer remotely, but this is usually limited to a short distance. Accelerometers could be used in the mouse to measure its position from the receiver. If the distance increases, the mouse could draw more power. If the distance decreases, this could also be used as a power saving technology because the mouse is close to the receiver and does not need as strong of a signal.

Reflect on Your Learning

75. Answers may vary.
(a) Students should discuss any material in the unit that they found illuminating or insightful.
(b) Students should discuss their understanding of trigonometry and its application in direction and projectiles.
76. Answers may vary. Students should discuss their understanding of gravity and projectiles.

Research

77. Answers may vary. Students' answers should discuss the use of high-speed rails around the world. They may wish to include which countries have the most high-speed trains, and which countries have the fastest trains. They will probably discuss Canada's proposed high-speed rail locations and the issues involved.
78. Answers may vary. Students' answers should discuss a sport and the world record speeds and distances involved. They should also include their group work and calculations performed to determine how their values compare with the record values.
79. Answers may vary. Students' answers should discuss the future of space travel and any new technologies that may make it more easily accessible. Topics include but are not limited to space elevators, warp drives, the future of private space flight, and designs for space shuttles that carry their own fuel.