Precalculus and PreAP PreCalculus Scope and Sequence
with NMSI's Laying the Foundation lessons

Unit 1: Trigonometric Functions-
Graphing, Inverses, and Equations
21 days of instruction plus assessment time- 3.5 weeks

Unit Note: The committee decided to start the course with the last concept suggested in the Algebra II course. It should be something familiar and then quickly they add on to what they know to move the graphs. Because there are so many concepts that need to be covered in this course, the committee believed that the review would be best if covered when needed. The teacher notes will give advice as to when the review topics would be best considered. There are no LTF lessons for inverse functions, but there are suggested activities in the additional resource folder. The last two LTF lessons would be good to use as a long term project use, but class time could be better spent in other ways.

Teacher Led Instruction – Unit Circle (1 day)

Teacher Note: Students <u>should</u> know unit circle. Khan Academy –video explains symmetry, illustrativemathematics.org-F-TF "Properties of Trigonometric Functions" has questions involving symmetry. There is a blank Unit Circle and Answer Key in the Additional Resources.

AL COS PRECALCULUS	Common Core Standard	Common Core
30	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	F-TF

NMSI's Laying the Foundation lesson: Window Pane Graphing of Trigonometric Functions (2 days)

Teacher Note: In Algebra II, students are introduced to the graphs of sine and cosine. This lesson provides a review of graphing the sine and cosine parent functions and then introduces translations of trig functions by looking at one period of the graph. Students being familiar with translations of other parent functions will allow them to see the same type of translations happening with the trig functions.

Day 1: In the Teacher Notes, teachers are encouraged to model how to make a "pane" with the parent functions. Using the values of the quadrantals in the unit circle are the guides and connection to the unit circle. The teacher is then asked to show students how to do the same "pane" with the translated graph and then translate it onto the coordinate plane. There are several examples given and the "pane" format is on the teacher page which may need to be given to students. Questions 1-5 give equations and ask students to graph on a coordinate plane. Question 6 asks the student to then graph a window "pane" which is just one period. This might be redundant. It might be easier to do the pane and translate it onto a coordinate plane for a few and then just use the coordinate plane for the rest.

Day 2: Question 7 asks students to graph the cosecant graph. This is an excellent opening for discussing the reciprocal graphs of sine and cosine and their translations. There is an interactive applet from NCTM that allows students to change different values and see what it does to the graph sin x, cos x, tan x, cot x, csc x, sec x can all be manipulated. The link to this applet listed on the Internet Resource Page for Unit 1. Questions

8-10 are situational problems in which students have to write a trigonometric function that models the situation.

** Use Unit 1 Checkpoint: 2 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases… d) Graph trigonometric functions, showing period, midline, and amplitude.	F-IF7
26	Determine the amplitude, period, phase shift, domain, and range of trigonometric functions and ~~their inverses.~~	AL

NMSI's Laying the Foundation lesson: Fitting Trigonometric Models to Data (1 day)

Teacher Note: Students should be familiar with trigonometric parent functions, transformations of trigonometric functions, relative maximum/minimum, domain, range.   Questions 1-10 are about a Ferris Wheel problem. #11 is optional if you have data collection software.

** Use Unit 1 Checkpoint: 9 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither.	F-IF4
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases… d) Graph trigonometric functions, showing period, midline, and amplitude.	F-IF7
26	Determine the amplitude, period, phase shift, domain, and range of trigonometric functions and their inverses.	AL

NMSI's Laying the Foundation lesson: Linking Trigonometry and Statistics (1 day)

Teacher Note: Use this lesson if more practice is needed. Students are given a table of data and asked to use graphing calculator to write a trigonometric function to fit. Students are then asked to plot the points on the graphing calculator and use the regression capabilities to check their equation.

** Use Unit 1 Checkpoint: 11,13 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases… d) Graph trigonometric functions, showing period, midline, and amplitude.	F-IF7
26	Determine the amplitude, period, phase shift, domain, and range of trigonometric functions and their inverses.	AL

NMSI's Laying the Foundation lesson: Adaptation of AP Calculus 1998 AB-5(2 days)

Teacher Note: This lesson can be worked by all students, but should definitely be worked with PreAP students. Students will have to know about RHR-Right Hand Rectangle, LHR-Left Hand Rectangle, Trapezoid, and MPR-Midpoint Rectangle for this lesson. They need to be able to calculate the area of rectangles and trapezoids.

AL COS PRECALCULUS	Common Core Standard	Common Core
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases… d) Graph trigonometric functions, showing period, midline, and amplitude.	F-IF7
26	Determine the amplitude, period, phase shift, domain, and range of trigonometric functions and their inverses.	AL

Teacher Led Instruction- Graphs of Trigonometric Inverse Functions (2 days)

Teacher Note: Students should know that a function has an inverse if and only if it is *one-to-one*. If it is not one-to-one, the domain may be restricted so that the restricted function is one-to-one. The textbook provides adequate resource for these objectives. Khan academy website has several videos that talk about inverse trig functions. There are additional activities in the additional resources folder.

AL COS PRECALCULUS	Common Core Standard	Common Core
20	Determine the inverse of a function and a relation.	
31	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed	F-TF6
23	Produce an invertible function from a non-invertible function by restricting its domain.	F-BF4d
22	Read values of an inverse function from a graph or a table, given that the function has an inverse.	F-BF4c
21	Verify by composition that one function is the inverse of another.	F-BF4b
26	Determine the amplitude, period, phase shift, domain, and range of trigonometric functions and their inverses.	

Teacher Led Instruction- Solving Trigonometric Equations (2-4 days)

Teacher Note: Teachers will need to provide students with some practice solving simple trigonometric equations with and without the unit circle. The problems that are without the unit circle will require students to use the inverse function to solve equations. This should be followed with introduction of all of the identities and how they are used in solving equations that have the different identities in them. Students are asked to "use" sum and difference formulas. Teachers can supply the formulas and ask students to use them to solve problems. These formulas will not be prevalent in AP Calculus. The Pythagorean Identities and Double Angle formulas are used quite a bit in AP Calculus. The equations do not need to be the most difficult problems, but should include working with Pythagorean Identity substitution, factoring of trigonometric functions using GCF and normal factoring of quadratics, and solving double angle equations. The traditional textbook provides lessons on these objectives. Illustrativemathematics.org has 3 great lessons: "Trig Ratios and Pythagorean Thm.", "Finding Trig Values", and "Calculations with sine and cosine". There are additional activities in the additional resources folder.

** Use Unit 1 Checkpoint: 1,3,4,5,6,7,8 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
33	Prove the Pythagorean identity $sin^2\vartheta + cos^2\vartheta = 1$, and find $\sin(\vartheta), \cos(\vartheta), \tan(\vartheta)$,given $\sin(\vartheta), \cos(\vartheta), or \tan(\vartheta)$ and the quadrant of the angle.	F-TF8
27	Use the sum, difference, and half-angle identities to find the exact value of a trigonometric function.	AL
32	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	F-TF7
34	Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.	F-TF9

NMSI's Laying the Foundation lesson: Investigating Double Argument Trigonometric/ Circular Equations (2 days)
Teacher Note: The beginning example1-14 is a good way to encourage students not to divide by a factor or they will possibly lose an answer. Most of these problems are more difficult than needed- #7 is okay. Students should be familiar with graphing trig/circular functions, factoring quadratics, solving quadratic equations, and solving trig/circular equations.

AL COS PRECALCULUS	Common Core Standard	Common Core
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases… d) Graph trigonometric functions, showing period, midline, and amplitude.	F-IF7
27	Use the sum, difference, and half-angle identities to find the exact value of a trigonometric function.	
32	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	F-TF7
34	Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.	F-TF9

Teacher Led Instruction- Derive the Area of Triangle Formula (.5 days)
Teacher Note: This can be done very easily and quickly. This actually should be done in Geometry

AL COS PRECALCULUS	Common Core Standard	Common Core
35	Derive the formula $A = \left(\dfrac{1}{2}\right)ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	G-SRT9

NMSI's Laying the Foundation lesson: Related Rates- Triangle Applications (Project)

Teacher Note: These problems might be used for project problems, but may not be the best use of class time with all that must be covered. This lesson takes a while to only cover two Precalculus objectives, but it does a lot of average rate of change. Students should be familiar with the Pythagorean Theorem, right triangle trigonometric rations, the Law of Cosines, algebraic manipulation, and equation solving.

AL COS PRECALCULUS	Common Core Standard	Common Core
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	F-IF6
29	Use special triangles to determine geometrically the values of sine, cosine, and tangent for and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x, \pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.	F-TF3

NMSI's Laying the Foundation lesson: What is Best? (Project)

Teacher Note: These problems might be used for project problems, but may not be the best use of class time with all that must be covered. Students should have experience in setting up equations from word problems and with solving systems of equations by substitution methods. The primary objective of the activity is to provide students with experience in creating functions based on a given scenario. Allow students to work cooperatively in setting up questions. Offer hints, but try not to set up functions for them. Suggested hints for each question are provided.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither.	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	F-IF6
29	Use special triangles to determine geometrically the values of sine, cosine, and tangent for and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x, \pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.	F-TF3
32	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	F-TF7

** Use Unit 1 Checkpoint: 10,12 as a review before test.

Precalculus and PreAP PreCalculus Scope and Sequence
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Unit 2: Vectors and Parametric Equations
10 days of instruction plus assessment time- 2.5 weeks

Unit Note: The Alabama Course of Study requires teaching vector addition using end-to-end, component-wise and the parallelogram rule. Both NMSI lessons can be reused to teach all three methods although the diagramed examples show preferred methods. Also, the AP Exam requires answers to be expressed to 3 decimal places after keeping numbers in the calculator until the very end. The answers in *Vectors in Geometry* are not consistent with this policy. Many of the answers represent ill-advised rounding in intermediate calculations. Finally, please refer to additional resources folder for links to some helpful applets for visual representations of vectors at NCTM illuminations for material that teaches AL-COS 6, 9-11. The concepts of parametric equations are very important in AP Calculus BC, but are not as relevant in AP Calculus AB.

Teacher Led Instruction- Components of Vectors by Subtracting Coordinates (Part of 2-5 days below)

Teacher Note: The following concepts are not specifically covered in the above LTF lessons. This can be covered quickly.

AL COS PRECALCULUS	Common Core Standard	Common Core
6	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	N-VM2

NMSI's Laying the Foundation lesson: Vectors in Geometry (2 Days)

Teacher Note: This lesson is the first introduction to vectors. A great deal of vocabulary is needed for this introduction. Vector notation is also is a little different and should be introduced. The parallelogram rule is interesting, but maybe not the most efficient. Because this is leading to parametric equations, an emphasis of writing the vector in component form and tying it into the end-to end drawing of vectors is a good approach. There is a great simulation of this https://phet.colorado.edu/en/simulation/vector-addition
Finding the angle of the resultant vector from the x-axis ties back to what students know about the unit circle. Using the examples in the Teacher Notes would be a great introduction drawing vectors by hand.
In #1 a-g the students are only going directly north, south, east or west and writing the component form as $\langle horizontal\ component, vertical\ component \rangle$ will get them ready for more complicated problems. In problems h, it might be helpful to show the two vectors on two different axes and find the horizontal and vertical component. Eventually the students should recognize that each vector can be found by $\langle r\cos\theta, r\sin\theta \rangle$ where r is the resultant of the vector and θ is the angle from the x-axis. To show that the magnitude of the resultant vector is not the sum of the two vector magnitudes, use the simulation above. It does a great visual to get students to see that the sum must be found another way. To find the resultant of the two vectors, add the horizontal components to find the horizontal component of the resultant and to add the vertical components of each vector to find the vertical component of the resultant vector. In order to find the length of the resultant vector, the Pythagorean Theorem can be used and to find the angle of the resultant vector

$\tan^{-1}\left(\dfrac{\text{resultant vertical component}}{\text{resultant horizontal component}}\right)$. Make sure to ask students if the answer makes sense to the problem.

Because of restrictions of inverse tangent being from $-\dfrac{\pi}{2}\ to\ \dfrac{\pi}{2}$, students may have to manipulate their answer to get an answer that makes sense to the problem (they may have to add π to a negative answer that is really in the 2ⁿᵈ quadrant).   In using the Pythagorean Theorem and inverse tangent, students will have a much easier time when you begin polar and rectangular conversions. You can also utilize the store feature on your graphing calculator to store your horizontal and vertical components to make computations easier. Questions 2-6 could be given as partner/group work.

** **Use Unit 2 Checkpoint: 1 after completing this lesson.**

AL COS PRECALCULUS	Common Core Standard	Common Core
5	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v).	N-VM1
7	(+) Solve problems involving velocity and other quantities that can be represented by vectors.	N-VM3
8	(+) Add and subtract vectors. a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. (+) Understand vector subtraction **v** − **w** as **v** + (−**w**), where −**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	N-VM4 N-VM4a N-VM4b N-VM4c

NMSI's Laying the Foundation lesson: Applications of Vectors (2 Days)

Teacher Note: After going through the Vectors in Geometry, this lesson should be fun! There is one complication- Bearing clockwise from north. Students will need to figure out how to use the bearing given to find the angle that they need to use from the horizontal axis. Once this is decided, these problems are set up and solved similarly previous lesson. This would be a great lesson to have students put on large post-its or posterboard/butcher paper and hang up in the hall!

** Use Unit 2 Checkpoint: 4 after completing this lesson.**

AL COS PRECALCULUS	Common Core Standard	Common Core
5	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v).	N-VM1
7	(+) Solve problems involving velocity and other quantities that can be represented by vectors.	N-VM3
8	(+) Add and subtract vectors. a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. (+) Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	N-VM4 N-VM4a N-VM4b N-VM4c

Teacher Led Instruction- Vectors as Matrices (2 days)

Teacher Note: The following concepts are not specifically covered in the above LTF lessons. #10,11,14 are new in that they relate vectors and matrices. The Khan Academy video links on the internet resource page are very helpful in tying these two concepts together and a few of them would be good for flipped classroom.

AL COS PRECALCULUS	Common Core Standard	Common Core
9	Multiply a vector by a scalar. a.(+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. (+) Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = \|c\|v$. Compute the direction of $c\mathbf{v}$ knowing that when $\|c\|v \neq 0$, the direction of $c\mathbf{v}$ is either along **v** (for $c > 0$) or against **v** (for $c < 0$).	N-VM5 N-VM5a N-VM5b
10	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	N-VM11
11	Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	N-VM12
14	(+) Represent a system of linear equations as a single matrix equation in a vector variable.	A-REI8

NMSI's Laying the Foundation lesson: Parametric Equations (1 day)

Teacher Note: This is a good introduction to parametric equations. Students have not seen the third dimension of time in equations. Use of graphing calculators is very helpful for this lesson.
Additional lessons are in the additional resources folder.

AL COS PRECALCULUS	Common Core Standard	Common Core
28	Utilize parametric equations by graphing and by converting to rectangular form.	AL

NMSI's Laying the Foundation lesson: Motion Defined Parametrically (2-3 days)

Teacher Note: You may want to check student answers to 3a before they continue with the rest of the problem so they do not work the entire problem with the wrong formula. Students may also need another day with more application based problems (ball going through the air, football path with time, etc). Additional lessons are in the additional resources folder.
** Use Unit 2 Checkpoint: 2,3 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
28	Utilize parametric equations by graphing and by converting to rectangular form. a. a. Solve application-based problems involving parametric equations.	AL

Unit 3: Complex Numbers and Polar Points and Graphs
6 days of instruction plus assessment time- 1 ½ weeks

Unit Note: There are not many NMSI LTF lessons that coincide with these topics. Linked graphic organizers and/or activities can also be found in the additional resources folder for this unit. Complex numbers should have been introduced in the Algebra II course, but this is the first time that students have experienced polar curves and parametric equations. The standards in the Alabama Precalculus Course of Study only mention polar in reference to a point being defined as both polar and rectangular. There is no mention of polar equations or graphs. The teacher will need to decide how much time to spend on this topic. The LTF lessons in this unit support polar points through application of graphing polar curves and solving polar equations to find intersections. The concepts of polar and parametric are very important in AP Calculus BC, but are not as relevant in AP Calculus AB.

Teacher Led Instruction: Complex Numbers (1 – 2 days)
Teacher Note: Your textbook should have an adequate lesson for this topic. There are additional resources in the additional resources folder for this unit. The story John and Betty's Journey into Complex Numbers would be helpful as a homework assignment the night before starting this topic. It is a quick review of what they learned in Algebra II about complex numbers. The link to the website is in the additional resource folder.

AL COS PRECALCULUS	Common Core Standard	Common Core
1	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	[N-CN4]
2	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. Example: $(-1 + \sqrt{3i}\,)^3 = 8$ because $(-1 + \sqrt{3i}\,)$ has modulus 2 and argument 120°.	[N-CN5]
3	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	[N-CN6]

Teacher Led Instruction: Graphing Polar Coordinates and Conversion between Polar Equations and rectangular form (1 day)
Teacher Note: Your textbook should provide a good lesson for this. This introduction will be necessary before using the following LTF lesson. There is a graphic organizer and Polar Battleship Applet in the additional resources folder.
** Use Unit 3 Checkpoint: 2 after completing this lesson.

NMSI's Laying the Foundation lesson: Graphing Polar Equations (2 days)
Teacher Note: For Calculus BC, students should be able to identify and sketch lines, circles, **limaçons**, roses, and lemniscates in polar form. There are additional lessons in the additional resource folder for extra practice.

NMSI's Laying the Foundation lesson: Special Points on Polar Curves and Intersections of Two Polar Graphs **(1 day)**

This can provide a good review of polar graphing as well as introducing finding the intersection points. Good application of solving trig equations for a reason. AP Calculus BC students need this skill.

Note: Example 2 may be confusing for students after finding the angle of the tips of the petals in the previous lesson. Also, on the examples there are some that intersect at the pole. This answer will not show up when solving the equations and can only be seen when graphed. The thetas may not even be the same (but they will describe the same point).

** Use Unit 3 Checkpoint: 1 after completing this lesson.

Precalculus and PreAP PreCalculus Scope and Sequence
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Unit 4: Conics
8-10 days of instruction plus assessment time- 2-2.5weeks

Unit Note: Student should have seen mastered graphing vertical parabolas in Algebra I, sideways parabolas in Algebra II and circles in Geometry and Algebra II. They may have seen ellipses and hyperbolas before, but this is their first formal introduction to them.

Teacher Led Instruction- -Introduction to all Conic Sections (1 day)
There is a link to Desmos (a cool graphing calculator app) on the internet resource page that allows students to see all of the conics when the sliders go through different values. There are notes pages in the additional resources that allow students to either take notes and draw graphs or have graphs provided and encourage them to label. There is a name that conic activity so that students can differentiate between forms of conical sections. At this point, you may also want to briefly mention the degenerate conics. If there is access to Algebra in Motion, the conic tab on it is very helpful for the visual students.

Teacher Led Instruction: Conic Sections and Deriving the Formulas- Parabolas (1-2 days)
Teacher Note: Students should be familiar with graphing parabolas, but have not been formally introduced to the different "parts" of the parabola or the formal definition. There are notes pages in the additional resources to help with labeling the parts of the conics. When giving the equation of a parabola, books tend to promote a non-translated form $4p(y-k)=(x-h)^2$. This is not how the form of parabola that was taught in Algebra I and Algebra II, so it is suggested that teachers show students that this is the same as

$y=\dfrac{1}{4p}(x-h)^2+k$ (where p is the distance from the vertex to the focus and the vertex to the directrix) and

use this form to keep consistent with the graphing transformations that are taught in previous grades. It will also be the same transformation technique that we will use with the other conic sections. There is a link to deriving the parabola formula on the internet resource page for this unit. Application problems involving paraboloids (satellite dishes, car lights, microphones on ESPN, Angry Birds) would be appropriate here and would help students tie parabolas to their world. The Algebra in Motion software shows students how parabolas work and how all points when strike the parabola bounce to the focus. When working with problems that involve vocabulary, encourage students to draw a picture of all of the parts given and then write the equation. There is no mention of the latus rectum in the standards, so teachers will decide if that is an important vocabulary of the parabola to share with students.

AL COS PRECALCULUS	Common Core Standard	Common Core
36	(+) Derive the equations of a parabola given a focus and directrix	G-GPE2
15	Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations. a. Formulate equations of conic sections from their determining characteristics.	

Teacher Led Instruction: Conic Sections and Deriving the Formulas- Ellipse (1-2 day)

Teacher Note: Because students have not been exposed to ellipses, teachers will need to introduce vocabulary, derive the formula, introduce the general form, relate how translation affects the ellipse, complete the square to put into translated form and explain the application of ellipses. The students have worked with translated circles, so the idea of moving the center is not foreign to students. There is a link to the derivation of the formula on the internet resource page. The students have never had to complete the square twice or worry about a leading coefficient being factored out and may need a little practice with this. The applications including a lithotriptor and the whispering gallery in Washington DC are some things that may be of interest to the students. Algebra In Motion has a great visual using the idea of a lithotripter and an elliptical pool table.

** Use Unit 4 Checkpoint: 1 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
15	Create graphs of conic sections, including ~~parabolas~~, hyperbolas, <u>ellipses</u>, ~~circles~~, and degenerate conics, from second-degree equations. a. Formulate equations of conic sections from their determining characteristics.	G-GPE2
37	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	G-GPE3

Teacher Led Instruction: Conic Sections and Deriving the Formulas- Hyperbola (1-2 days)

Teacher Note: Teachers will need to introduce the new vocabulary, pointing out the similarities and differences with ellipses, derive the formula, introduce the general form and relate how the translation affects hyperbolas and the similarities to ellipses, complete the square and application of hyperbolas. In the additional resources folder there are sheets to help organize vocabulary. There is a link to the derivation of the formula on the internet resource page. After ellipses, graphing hyperbolas are a little easier. When completing the square, the negative is a stumbling block. The application of hyperbolas include LORAN, orbits of comets and the shape is found in several architectural designs.

AL COS PRECALCULUS	Common Core Standard	Common Core
15	Create graphs of conic sections, including ~~parabolas~~, hyperbolas, <u>ellipses</u>, ~~circles~~, and degenerate conics, from second-degree equations. a. Formulate equations of conic sections from their determining characteristics.	G-GPE2
37	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	G-GPE3

NMSI's Laying the Foundation lesson: Transformations of Graphs of Conic Sections- Hyperbolas

Teacher Note: Students should have worked through the parabola and circle problems in Algebra II. The hyperbola section 1-5 solves the hyperbola for y and ties in the transformational work done in all of the previous work.

AL COS PRECALCULUS	Common Core Standard	Common Core
15	Create graphs of conic sections, including ~~parabolas~~, hyperbolas, ellipses, ~~circles~~, and degenerate conics, from second-degree equations. a. Formulate equations of conic sections from their determining characteristics.	

NMSI's Laying the Foundation lesson: Conics in Parametric Form (optional) **(1 – 1.5 days)**

Teacher Note: If time allows, this is an interesting lesson about converting conics to polar form, and how to transform them in polar form. You could divide the questions among different groups and ask them to present.

AL COS PRECALCULUS	Common Core Standard	Common Core
15	Create graphs of conic sections, including ~~parabolas~~, hyperbolas, ellipses, ~~circles~~, and degenerate conics, from second-degree equations. a. Formulate equations of conic sections from their determining characteristics.	

NMSI's Laying the Foundation lesson: Planets, Parametric Curves, and Ellipses (optional)

Teacher Note: If time allows, this is an interesting lesson about relating ellipses to the orbit of planets. This might be a good project lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
15	Create graphs of conic sections, including ~~parabolas~~, hyperbolas, ellipses, ~~circles~~, and degenerate conics, from second-degree equations. a. Formulate equations of conic sections from their determining characteristics.	

Teacher Led Instruction: Review/Wrap up of (1/2 day)

Teacher Note: An Activity named Conic cards is in the additional resource folder and would serve as a great review activity.

** Use Unit 4 Checkpoint: 2 after completing this lesson.

Precalculus and PreAP PreCalculus Scope and Sequence
with NMSI's Laying the Foundation lessons

Unit 5: Statistics and Probability
11 days of instruction plus assessment time- 2.5 weeks

Unit Note: Students should have experienced a good foundation in working with statistical data starting in the 6th grade and the lessons that follow build on this experience. Students have been working with probability starting in the 7th grade and this unit takes them a little further with some application and putting together statistical data and probability. Talking with the AP Statistics teacher in your school might help you have a better perspective on how this material affects that class.
** Use Unit 5 Checkpoint: 1 and 2 as a preassessment.

NMSI's Laying the Foundation lesson: Comparing Boxplots (1 day)
Teacher Note: If students have not been as exposed to statistical graphing, middle school LTF lesson *Box-and-Whisker Plots* can be used to introduce vocabulary for boxplots

AL COS PRECALCULUS	Common Core Standard	Common Core
39	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *(Focus on increasing rigor using standard deviation.)*	S-ID2
40	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers*). (Identify uniform, skewed, and normal distributions in a set of data. Determine the quartiles and interquartile range for a set of data.)*	S-ID3

NMSI's Laying the Foundation lesson: Empirical Rule and Normal Distributions (2 days)
Teacher Note: This lesson is the next lesson that follows the *Describing Distributions: Standard Deviation* lesson from Algebra I. If students did not work through *Describing Distributions: Standard Deviation* in Algebra I, this lesson would be need to be worked. This lesson introduces the empirical rule and z-scores. The use of graphing calculator is optional, but fun.

AL COS PRECALCULUS	Common Core Standard	Common Core
41	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	S-ID4
45	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. Example:A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?	S-IC2
39	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *(Focus on increasing rigor using standard deviation.)*	S-ID2
40	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers*). (Identify uniform, skewed, and normal distributions in a set of data. Determine the quartiles and*	S-ID3

NMSI's Laying the Foundation lesson: Applying the Binomial Expansion to Probabilities (3 days)

Teacher Note: You will need to spend a day discussion the binomial expansion, remind them of Pascal's Triangle, or use the LTF lesson *Family Fun* to remind them. In Algebra II they did a lesson on Permutations and Combinations that can be connected to Pascal's triangle numbers and to the numbers in the binomial expansions. Day 2- The teacher notes give a good example of how to connect the binomial expansion of normal binomials to that of binomial probability that would be a great example to work with them and then let them work #1. Day 3 #2 and #3 are good problems to practice the new skill.

** Use Unit 5 Checkpoint: 3,4,5,6 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
13	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)	A-APR5
50	(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	S-MD1
51	(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	S-MD2
53	(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.	S-MD4

NMSI's Laying the Foundation lesson: Let's Take a quiz (1 day)

Teacher Note: This lesson asks that the students use simulation to find the probabilities of success for guessing on a quiz. After the Binomial Expansion lesson, students can actually find the theoretical answer for additional practice of binomial probability. If time is limited, you could use this lesson to just do the theoretical probability.

AL COS PRECALCULUS	Common Core Standard	Common Core
13	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)	A-APR5
50	(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	S-MD1
51	(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	S-MD2
52	(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. Example: Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.	S-MD3

NMSI's Laying the Foundation lesson: How is my driving? (2 days)

Teacher Note: This lesson focuses on random sampling and using sample data to predict different aspects of a population. It discusses the data gathered when drivers are on cell phone versus talking to passenger while driving and to either prove or disprove the research.

AL COS PRECALCULUS	Common Core Standard	Common Core
44	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	S-IC1
47	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	S-IC4
48	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	S-IC5
49	Evaluate reports based on data.	S-IC6

Teacher Led Instruction- Making decisions with probability and different ways to gather data (2 days) **Teacher Note:** There are notes and problems for the following standards in the additional resources folder.

AL COS PRECALCULUS	Common Core Standard	Common Core
54	(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.	S-MD5
54a	a. Find the expected payoff for a game of chance. Examples: Find the expected winnings from a state lottery ticket or a game at a fast- food restaurant.	S-MD5a
54b	b. Evaluate and compare strategies on the basis of expected values. Example: Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major	S-MD5b
46	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	S-IC3

There are three more LTF lessons that might be used in this section:
I Want Candy – Students determine if a candy manufacturer's published net weight for a package of candy is accurate.
The Jury- Uses Spinner to create random outcomes to approximate probabilities and compare to theoretical probabilities to determine if a jury is biased.
Take a Sample Please – Students experience some of the underlying principles of inferential statistics.

Precalculus and PreAP PreCalculus Scope and Sequence
with NMSI's Laying the Foundation lessons
Unit 6: Functions
11 days of instruction plus assessment time- 2.5 weeks

Unit Note: This unit pulls together all of the transformation that students have learned with the trigonometric functions included. There are two games in this unit. If time is short, one of the games could be used as a semester review. There are several other lessons that could be used as semester exam reviews and there is an indication of this in the teacher notes.

NMSI's Laying the Foundation lesson: Variations on Detecting Linear Motion (1.5 days)
Teacher Note: You must have access to a CBR (Calculator Based Ranger) and a graphing calculator. The students are asked to walk different situations including discussion about concavity.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table)over a specified interval. Estimate the rate of change from a graph.	F-IF6
42	Compute (using technology) and interpret the correlation coefficient of a linear fit.	S-ID8

NMSI's Laying the Foundation lesson: Linear Regression with Coded Data (1-1.5 days)
Teacher Note: You need access to a graphing calculator. There is a LTF video clip on creating scatter plots on the TI-84.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table)over a specified interval. Estimate the rate of change from a graph.	F-IF6
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	F-IF7b

NMSI's Laying the Foundation lesson: A Study of Population Growth (1 day)

Teacher Note: In order to cover standard #47 students must use the regression capabilities of their graphing calculator to get a value of r that shows the correlation of the data to the regression function given. The student must have the DIAGNOSTIC ON (found in catalog) in order to see the value of r. The most efficient use of this lesson is to assign each state's population to different pairs/groups of students and display and analyze the results as a whole class. This lesson analyzes functions that are linear, exponential and neither.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table)over a specified interval. Estimate the rate of change from a graph.	F-IF6
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	F-IF7b
42	Compute (using technology) and interpret the correlation coefficient of a linear fit.	S-ID8

NMSI's Laying the Foundation lesson: Power and Root Functions-A Look at Concavity (1 day)

Teacher Note: (Inverse Functions) A graphing calculator is needed. This activity could be done quickly by giving different pairs of inverse functions and present and display.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table)over a specified interval. Estimate the rate of change from a graph.	F-IF6
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	F-IF7b
20	Determine the inverse of a function and a relation.	AL
21	(+) Verify by composition that one function is the inverse of another.	F-BF4b
22	(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	F-BF4c

NMSI's Laying the Foundation lesson: Parent Functions Charades (1 day)

Teacher Note: Good reivew of transformations. This game includes all functions that students have been exposed to at this point. Students must act out the transformation and other students have to guess the equations with transformations.

AL COS PRECALCULUS	Common Core Standard *Quality Core Identifier. Quality Core Standard(s)*	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	F-IF7b

NMSI's Laying the Foundation lesson: Frantic Functions (.5-1 day)

Teacher Note: Great review for transformations. Absolute Value, square root, rational functions in translated form, quadratics, cubic, step function and linear functions are included. Students must match equation, description of function, table from Graphing calculator, and graph and put answers in a table to check.
** Use Unit 6 Checkpoint: 13-15 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	F-IF7b

NMSI's Laying the Foundation lesson: A Piecewise Function with a Discontinuous Domain (1.5-2 days)

Teacher Note: This lesson asks students to work with a discontinuous piecewise function that has linear, cosine, quadratic and constant function and translate and discuss new features. The students are then given a description of different pieces and asked to graph and write equation and then translate and discuss different features.
** Use Unit 6 Checkpoint: 1-10 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing	F-IF4

	key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	F-IF7b
19	Compose functions.	F-BF1c

NMSI's Laying the Foundation lesson: Piecewise Puzzle (1 day)

Teacher Note: Teachers will need to copy and cut out the grids and the coordinate mat at the end of the lesson and provide to students. This lesson would probably be best if done in pairs. It might take a while for the students to get started, but it is a good exercise in patience and thinking. If time is short, you could use this as a semester review.

** Use Unit 6 Checkpoint: 11 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard *Quality Core Identifier. Quality Core Standard(s)*	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	F-IF7b

**NMSI's Laying the Foundation lesson: Graphing From the Inside Out: Composition of Functions (2 days)

Teacher Note: Composition is one of the standards that could be adequately covered when discussing the composition of inverse functions resulting in x. This lesson's approach to composition is more of a graphing approach and gives students the skill to have an educated guess as to what a very difficult looking equation would look like by breaking it into pieces. This can be introduced to any Precalculus students, but is definitely a concept that PreAP teachers would want to cover considering the importance of this topic in AP Calculus. Compositions of functions is very important for students to learn in order to perform the chain rule in derivatives and do u-substitution in integration.

** Use Unit 6 Checkpoint: 16 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard *Quality Core Identifier. Quality Core Standard(s)*	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or*	F-IF4

	negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	F-IF7b
19	Compose functions.	F-BF1c

** Use Unit 6 Checkpoint: 12,14,17-22 as a review of unit.

Precalculus and PreAP PreCalculus Scope and Sequence
with NMSI's Laying the Foundation lessons
Unit 7: Position, Velocity, and Acceleration (PVA)
10 days of instruction plus assessment time- 2.5 weeks

Unit Note: This unit is a great introduction to the idea of particle motion. Students are given equations of functions that represent the position, velocity and acceleration of particles and asked to make conclusions about the motion. The use of polynomials as the position and velocity graphs lends itself to a great application of the use of polynomials. This can be introduced to any Precalculus students, but is definitely a concept that PreAP teachers would want to cover considering the importance of this topic in AP Calculus.

NMSI's Laying the Foundation lesson: Particle Motion for Precalculus (2 days)
Teacher Note: Teachers will need graphing calculators, CBR, stopwatch, masking tape and measuring device (Yard stick or measuring tape). This is a great review of the characteristics of polynomial functions. How many maximum and minimum because of highest power, the end behavior, etc.

In Question 1 students look at a polynomial graph that represents position and answer questions concerning the motion and times that this motion occurred. In Question 2 the students are then encouraged to use the CBR to mimic the motion using the CBR. Even if the instructor does not have access to a CBR, The students could answer the questions about #1 and do a mock walking of the graph to get the feel of the motion. Questions 3 and 4 deal with finding average rates of change and writing a piecewise function to mimic the motion with the ARC in each interval. Question 5 leads students to the idea of instantaneous rates of change.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	F-IF6
18b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	F-IF7 F-IF7b

NMSI's Laying the Foundation lesson: Distance, Speed, and Velocity Functions (1 day)
Teacher Note: Students will examine the ball's height in a height vs. time graph and explore the change in height over time introducing them to velocity. The students will be asked to discuss the difference in positive and negative velocities and explore that relationship to the speed of the object. Question 1 asks students to discuss position, velocity and speed given the position function, graph, and table. Question 2 gives students the position and speed graphs and asks them to make some connections. Question 3 gives students the velocity function graph and asks them to make connections with the position and speed graph. Question 4 involves parametric which could be discussed at a different time or as a review if needed.
** Use Unit 7 Checkpoint: 16 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	F-IF6
18b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	F-IF7 F-IF7b

NMSI's Laying the Foundation lesson: Particle Motion Along a Horizontal Line (1 day)

Teacher Note: A graphing calculator is needed for this assignment. This lessons is sequenced to allow the students to discover what is happening to the particle when the velocity is zero, positive and negative. It introduces students to vocabulary relative max and relative min. Questions 1-18 deal with putting the velocity and position functions in parametric mode with the functions in X1 and X2 and a constant in for Y1 and Y2 so that they can truly see horizontal motion. Questions 19-23 encourage them to change back into function mode and put the velocity and position functions into Y1 and Y2 and make the comparisons/connections with the parametric motion. The last questions asks them to sum up their findings about particle motion.
** Use Unit 7 Checkpoint: 14,17,18,19 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	F-IF6
18b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* b.Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	F-IF7 F-IF7b

NMSI's Laying the Foundation lesson: Reading the Graphs of Velocity Functions (1 day)

Teacher Note: This lesson takes the next step in relating velocity and acceleration to the notion of a particle speeding up or slowing down. Students are supplied with the velocity graph and table of values and asked to discover speeding up and slowing down. They are then asked to draw the speed graph and make connections. The last few questions ask the students to summarize what they have discovered.

** Use Unit 7 Checkpoint: 5,6,7,12,15,20 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	F-IF6
18b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* b.Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	F-IF7 F-IF7b

NMSI's Laying the Foundation lesson: Connecting to AP Calculus: Position/Velocity/ Acceleration (.5 day)

Teacher Note: In Example 1, a graph of a bug's velocity crawling along a wire is given and students are asked to make a lot of connections about changing directions, acceleration. The answers are at the bottom of page- might want to cover up if this is for discovery. Example 2, equations of position, velocity and acceleration are given and students are asked questions about the particle. The idea of total distance is introduced in this problem. Teachers will need to have a discussion about the difference between displacement and total distance at this point.

** Use Unit 7 Checkpoint:2, 3,4,8 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	F-IF6
18b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* b.Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	F-IF7 F-IF7b

NMSI's Laying the Foundation lesson: Sign analysis (2 days)

Teacher Note: This lesson encourages students to make notecards with the problems given on them. You can do this or just work the problems in groups and have students present their findings. Questions 1-4 are polynomial problems that require one factoring a cubic by grouping and the rest are in factored form. The idea of double roots not leading to a sign change therefore the particle does not change directions is represented in several of these problems. Question 5 requires students to solve a natural log equation and use

values of e to evaluate in intervals- yummy! Questions 6 and 7 ask students to solve trig equations and find trig values that are easy to evaluate to determine sign in each interval. This is so helpful for the AP Calculus teacher!

** Use Unit 7 Checkpoint: 1 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	F-IF4
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	F-IF6
18b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* b.Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	F-IF7 F-IF7b

NMSI's Laying the Foundation lesson: Accumulation (2 days)

Teacher Note: A graphing calculator is needed for this assignment. Question 1 presents a constant function that builds the conceptual understanding of accumulating a rate of change (velocity) to determine distance. Students need to understand that d= r x t and use the graph to find the distance. In Question 2, students are given a set of points that represent velocity at given times and asked to graph them and connect them. 2c then takes them from an area of one rectangle used in #1 to the area of 6 rectangles using the lowest velocity value as the height of the rectangle. The introduction of overestimating and underestimating and the word accumulation is then introduced. Question 3 a graphing calculator is needed to graph a complicated function for rate of change of yeast culture and the trapezoidal approximation is introduced. It is very important that the teacher gets the students to read the statement several times in 3a which states that the interval width is the height of the trapezoid.

** Use Unit 7 Checkpoint: 9, 10,11,13 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	F-IF4
18b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* b.Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	F-IF7 F-IF7b

** Use Unit 7 Checkpoint:21,22 as a review of the unit.

Precalculus and PreAP PreCalculus Scope and Sequence
with NMSI's Laying the Foundation lessons

Unit 8: Exponential and Logarithmic Functions
10 days of instruction plus assessment time-2.5 weeks

Unit Note: Students have been introduced to exponential functions in Algebra I and in Algebra II students are introduced to exponential and logarithmic laws. Students should have solved exponential and logarithmic equations by rewriting statements in the other form. For example, $2^x = 7$ can be written as $\log_2 7 = x$ and by using change of base could get an approximation of $x = \dfrac{\log 7}{\log 2} \approx 2.807$. In this unit, students will discuss the inverse relationship of exponential and logarithmic functions and use the inverse functions to solve equations. Students will explore different types of exponential equations including unlimited growth and limited growth. Students will use their knowledge of geometric sequences introduced in Algebra I and the idea of finite sums for geometric introduced in Algebra II and expand it to the idea of infinite geometric series and their sums.

Teacher Led Instruction- Inverse Function Relationship for Exponential and Logs (.5 days)
Teacher Note: The textbook provides lessons that will adequately teach these objectives. Talking about functions graphically makes this an easy way to present material. They have already discussed inverses in the trig unit and in the function unit, so this should not be hard to convey. The idea that e^x and $\ln x$ are inverses allows students to use them to solve equations. This is a new concept and teachers should show how to solve an exponential equation by changing it into logs like they did in Algebra II and by taking the log or ln of both sides to solve.
** **Use Unit 8 Checkpoint: 1,2,3,4,5,6,7,after completing this lesson.**

AL COS PRECALCULUS	Common Core Standard	Common Core
20	Determine the inverse of a function and a relation.	
21	Verify by composition that one function is the inverse of another.	F-BF4b
22	Read values of an inverse function from a graph or a table, given that the function has an inverse.	F-BF4c
23	Produce an invertible function from a non-invertible function by restricting the domain.	F-BF4d
24	Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents.	F-BF5

NMSI's Laying the Foundation lesson: "E"xponential Growth(1-2 days)
Teacher Note: Lesson covers different models of growth. Students should be familiar with exponential and logarithmic functions. This lesson introduces the limit definition of e and does it using the calculator
** **Use Unit 8 Checkpoint: 8 after completing this lesson.**

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and*	F-IF4

	periodicity.	
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	F-IF6
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior.	F-IF7
24	Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents.	F-BF5
28	Solve applied problems that include sequences with recurrence relations.	

NMSI's Laying the Foundation lesson: Exponential Growth and Decay (1-2 days)

Teacher Note: Students should be familiar with solving exponential growth and decay problems using the universal growth formula $y = Ce^{kt}$. This lesson broadens the range of applications to any situation where the rate of growth in something is directly proportional to the amount of that something present. Problem #7 is tricky. The last two questions involve transformations.

** Use Unit 8 Checkpoint:19,21 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior.	F-IF7
25	Compare effects of parameter changes on graphs of transcendental functions.	

**NMSI's Laying the Foundation lesson: Rumor Has It (Optional)

Teacher Note: This lesson is optional, but might want to be covered with students who are going to BC Calculus. Students should be able to create a scatterplot and use a calculator to determine a regression equation.

** Use Unit 8 Checkpoint: 9 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF4

NMSI's Laying the Foundation lesson: Logistic and Gompertz Curves(1.5-2 days)

Teacher Note: Students should be familiar with regression and with modeling data. Students should have access to calculators. If time is a factor, 1-4 could be given to groups of students and allow them to present. Students should work together to solve the problems.

** Use Unit 8 Checkpoint: 17 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table)over a specified interval. Estimate the rate of change from a graph.	F-IF6
18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior.	F-IF7

NMSI's Laying the Foundation lesson: Infinite Summing(1-2 days)

Teacher Note: Students should already be familiar with geometric sequences. Activity 4 leads students through a discovery of the formula for the sum of infinite converging geometric series. 1-9 could be done in one day with #10 possible homework problem. Day 2 Activity 5 needs to have teacher instruction on one of d-f and one of 2-5, the rest the students should be able to do.

** Use Unit 8 Checkpoint: 10,11 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
12	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. * *(Extend to infinite geometric series*.)	A-SSE4
4	Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity. a. Apply limits in problems involving convergence and divergence	AL

NMSI's Laying the Foundation lesson: Getting Serious About Series (1-2 days)

Teacher Note: Students should have experience with patterns, arithmetic and geometric sequences and series, ratios, convergent, divergent. Students should know about factorials.   This is probably the first time that students have seen the sigma notation and the notation $\cos(\pi n)\ and\ \sin\dfrac{(2n-1)\pi}{2}$ being used to show alternating signs instead of $(-1)^n$. This is helpful when working with series in BC Calculus. **21-25 can be introduced to any Precalculus students, but is definitely a concept that PreAP teachers would want to cover considering the importance of this topic in AP Calculus BC.

** Use Unit 8 Checkpoint: 12,13,14,15,16 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
12	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. * *(Extend to infinite geometric series*.)	A-SSE4
28	Solve applied problems that include sequences with recurrence relations.	AL
4	Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity. a. Apply limits in problems involving convergence and divergence	AL

** Use Unit 8 Checkpoint:20,23,24 as a review for the unit.

Unit 9: Rational Functions
6 days of instruction plus assessment time- 1.5 weeks

Unit Note: Students have graphed rational functions in Algebra II, but only in translated form. This unit will allow students to graph rational functions by factoring and looking at the factors to determine different characteristics. It is important that students are exposed to rational functions that have and do not have vertical asymptotes. The $\lim_{x \to \#} f(x) = \infty$ represents a vertical asymptote and $\lim_{x \to \infty} f(x) = \#$ represents a horizontal asymptote is introduce and/or reinforced from Algebra II.

NMSI's Laying the Foundation lesson: Rational Functions with Removable Discontinuities (1 day)

Teacher Note: A nice discussion of holes that could also tie in to a discussion of end behavior, along with graphing calculator practice. Note that there are a few questions that use the vocabulary of "discontinuous" and "continuous," but it does not involve the formal definition.
 ** Use Unit 9 Checkpoint: 7 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither.)	F-IF4
18.c	(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	F-IF7d

NMSI's Laying the Foundation lesson: End Behavior (1 days)

Teacher note: This lesson will go quickly if students use graphing calculators and come up with the pattern themselves. The use of limits as x-> infinity is continued from Algebra II.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither.)	F-IF4
18.c	(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	F-IF7d
4	Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity.	AL

NMSI's Laying the Foundation lesson: Rational Functions and Their Asymptotes (1 days)

Teacher note: This lesson will go quickly if students use graphing calculators and get familiar with rational functions.

** Use Unit 9 Checkpoint: 4 and 6 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither.)	F-IF4
18.c	(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	F-IF7d
4	Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity.	AL

NMSI's Laying the Foundation lesson: RATEY- A Graphing Organizer (2 days)

Teacher Note: The students should have worked the LTF lessons Short Run Behavior and Long Run Behavior. If not, these need to be worked before RATEY to explain what tangency at root and togetherness at asymptote mean. If a factor in the numerator is raised to an even power then the graph will not go through the axis, it will touch and go back in the direction (tangency). If the factor in the numerator has a power of two then the the graph looks like x^2 at that root. If the factor of the denominator is even, then the graph will go in the same direction as it approached the vertical asymptote (togetherness). If the factor in the denominator is odd, the graph will go in opposite directions as it approaches the asymptote. All of this is discovered in the two lessons listed above.

RATEY is an mnemonic for finding out all the important information of a rational graph. Some time should be spent explaining each part of RATEY, what it tells us and why. By this point, they have seen a great deal of rational functions using their graphing calculators and now it is time to graph some by hand. Discussion of accuracy needs to be handled here.

** Use Unit 9 Checkpoint: 1,2,3 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither.)	F-IF4
18.c	(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	F-IF7d
4	Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity.	AL

** Use Unit 9 Checkpoint: 5,8,9 as a review for this unit.

Precalculus and PreAP PreCalculus Scope and Sequence
with NMSI's Laying the Foundation lessons

Unit 10: Limits and Derivatives
13 days of instruction plus assessment time- 3 weeks

Unit Note: The students should be familiar with the idea of limits after their use in the rational function discussion starting in Algebra II and then in Precalculus and through the use of limits when discussing exponential functions and the applications. If limits are covered well in the exponential and rational units, the COS #4 standard could be adequately covered. If teachers want to introduce the students to derivatives, then a little practice with the algebraic idea of limits would help facilitate the progression into the definition of the derivative using limits and possibly some derivative rules. The second part of this unit is designed to introduce students to the meaning of derivatives by relating a derivative to the slope of the secant line and the slope of the tangent line. They are required to examine a function to make connections between the average rate of change and the slope of the secant. The activities gently move to the concept of the instantaneous rate of change. A graphing calculator is required to complete this unit in the time frame listed above. Communication with the AP Calculus teacher is suggested in order to decide how much of the derivative rules should be covered at this time. This can be introduced to any Precalculus students, but is definitely a concept that PreAP teachers would want to cover considering the importance of this topic in AP Calculus.

Teacher Led Instruction (3 days) -Graphical and Algebraic Limits - More time can be spent on this topic if time permits.

Teacher Note: Students should have a big understanding of which family of functions have "easy" limits and other functions that have challenges. For example, a student understanding that polynomial functions are smooth and continuous means that to find the limit as x approaches a number is found by evaluating the polynomial at the x being approached. In contrast, a rational function may have some points of discontinuity and finding a limit may be more difficult at some x values. Ensuring that students understand the graphs of the 19 basic functions will help them to visualize how to find limits as x approaches infinity for these functions and their translations. Even though students should have seen all of the functions by the end of Precalculus, some functions like the step function and semicircle may need to be reviewed during instruction. There are not specific LTF lessons on the mechanics of finding limits graphically or algebraically.

Students should find limits as x approaches a number graphically using the idea of looking at the point from the left and the right. This should lead to the definition of continuity using limits and is natural here. Tying in the graphical limit to the algebraic limit is the next concept and needs to include piecewise function in the discussion. Use of the graphing calculators to "see" the value of the limit is very appropriate. The use of tables allows students to zoom in on a x value on a function or zoom out and look at what happens as x approaches infinity. There are notes and problems to help facilitate in the additional resource folder.

** Use Unit 10 Checkpoint: 1 after completing this lesson.

AL COS PRECALCULUS	Common Core Standard	Common Core
4	Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity. a. ~~Apply limits in problems involving convergence and divergence.~~	AL

NMSI's Laying the Foundation lesson: Average Rate of Change Versus Instantaneous Rate of Change. (2 days)

Teacher Note: If available, consider using a motion detector (Go!Motion or CBR2)to demonstrate the motion of the two students at the beginning of the lesson. The teacher might also need to review changing units (eg. from minutes to hours or miles/minute to miles/hour).

AL COS PRECALCULUS	Common Core Standard	Common Core
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	F-IF.6
16	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	F-IF.4

NMSI's Laying the Foundation lesson: Investigating Average Rate of Change (2 days)

Teacher Note: It will be important to discuss how to change " $y = f(x)$ on the interval $[a,b]$" to two ordered pairs to use in the difference quotient. The teacher might also want to consider requiring colored pencils or provide several blank graphs for the students. If a regular pencil is used, the graph gets messy, and the student can easily become confused.

AL COS PRECALCULUS	Common Core Standard	Common Core
17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	F-IF.6

NMSI's Laying the Foundation lesson: Slope of the Secant Line and Limits (2 days)

Teacher Note: This lesson provides examples of how we use limits in AP Calculus. It does an excellent job of introducing the limit definition of the derivative to precalculus students. This is a tough concept and is done well.

AL COS PRECALCULUS	Common Core Standard	Common Core
4	Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity.	AL

Teacher Led Instruction – Mechanics of Derivatives (5days)

Teacher Note: Teachers will lead the students from the definition of the derivative using the limit definition to the power rule by discovering the pattern. After the power rule, the product, quotient, simple chain rule and even introduce the derivatives of trig functions, e^x and $\ln x$. Have a discussion with AP Calculus teacher as to how much to do and even resources to use. Master Math Mentor has some excellent materials for this. There are also some teacher made resources in the additional resource folder for this unit.