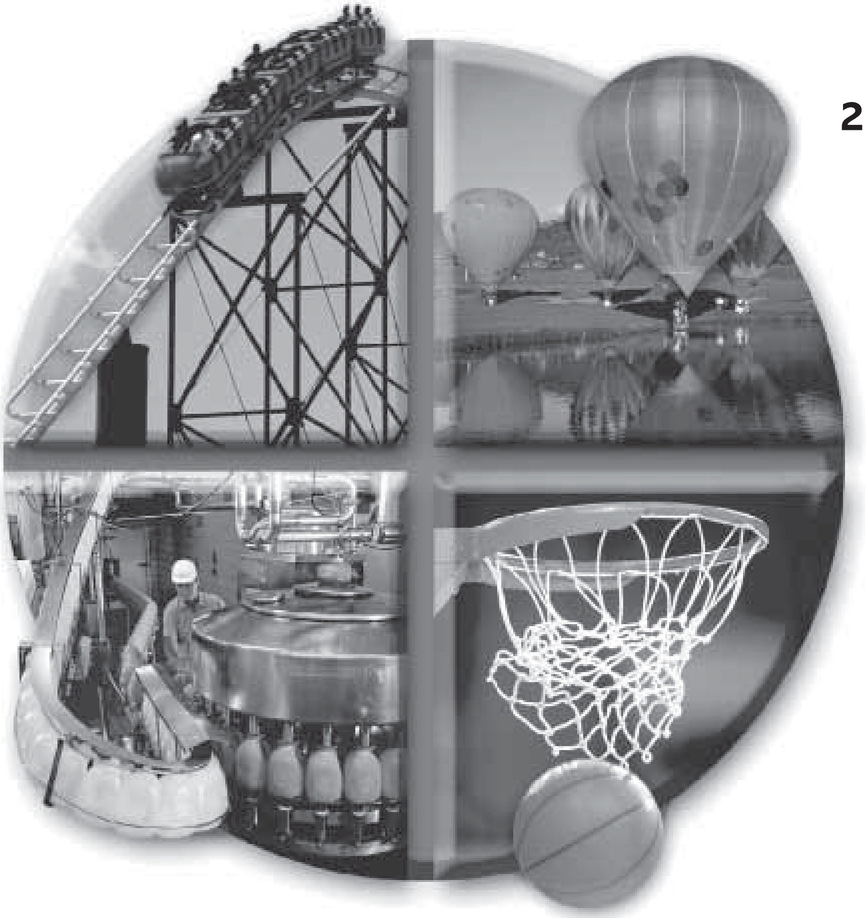
Course 3

Core-Plus Mathematics

Contemporary Mathematics in Context



2nd Edition

Unit 1
Resource Masters
Reasoning and Proof

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Core-Plus Mathematics
Contemporary Mathematics in Context
Course 3 Unit 1 Resource Masters

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Core-Plus Mathematics
Using the Unit Resource Masters

Overview of Unit Resource Masters

To assist you as you teach Course 3 of *Core-Plus Mathematics*, unit-specific resource books have been developed. The unit resources provided can help you focus student attention on the important mathematics being developed. They can be used to help students organize their results related to specific problems, synthesize what they are learning, and practice for standardized tests.

Unit resource books provide the following types of masters in the order that they are used in the unit.

- *Transparency Masters*
  1. Think About This Situation (TATS) masters to help launch the lesson
  2. Masters to collect class results
  3. Summarize the Mathematics (STM) masters to help facilitate the synthesis of mathematical ideas from the investigation (To guide your planning, sample discussion scenarios called "Promoting Mathematical Discourse" are provided in the Teacher's Guide for selected TATS and STM discussions.)

- *Student Masters*
  1. Masters to help students organize or display their results or thinking.
  2. Masters to help students develop their proof-writing abilities (differentiation)
  3. Technology Tips to facilitate learning technology features of graphing calculators, *CPMP-Tools* software, and computer algebra systems (CAS)
  4. Templates for manipulatives
  5. Unit Summary masters to provide a starting point for pulling together the main mathematical ideas of a unit
  6. Practicing for Standardized Tests masters provide an opportunity for students to complete tasks presented in the format of most high-stakes tests and to consider test-taking strategies. (Solutions to these tasks are printed in the Teacher's Guide following the final unit Summarize the Mathematics. This allows you the option of providing or not providing the solutions to students.)

- *Assessment Masters*
  1. Quizzes (two forms for each lesson)
  2. In-class tests (two forms for each unit)
  3. Take-home assessment items (three items for each unit)
  4. Projects (two for each unit except Unit 1)
  5. Midterm and end-of-course assessment items (Unit 4 and Unit 8 contain a bank of assessment items from which to design cumulative exams.)

All of the items in this book are included for viewing and printing from the *Core-Plus Mathematics* TeacherWorks Plus CD-ROM. Custom tailoring of assessment items in this book, as well as creation of additional items, can be accomplished by using the ExamView Assessment Suite.

Assessment in Core-Plus Mathematics

Throughout the *Core-Plus Mathematics* curriculum, the term "assessment" is meant to include all instances of gathering information about students' levels of understanding and their disposition toward mathematics for purposes of making decisions about instruction. The dimensions of student performance that are assessed in this curriculum (see chart below) are consistent with the assessment recommendations of the National Council of Teachers of Mathematics in the *Assessment Standards for School Mathematics* (NCTM, 1995). They are more comprehensive than those of a typical testing program.

Assessment Dimensions		
Process	**Content**	**Disposition**
Problem Solving	Concepts	Beliefs
Reasoning	Applications	Perseverance
Communication	Representational Strategies	Confidence
Connections	Procedures	Enthusiasm

These unit resource masters contain the tools for formal assessment of the process and content dimensions of student performance. Calculators are assumed in most cases on these assessments. Teacher discretion should be used regarding student access to their textbooks and Math Toolkits for assessments. In general, if the goals to be assessed are problem solving and reasoning, while memory of facts and procedural skill are of less interest, resources should be allowed. However, if automaticity of procedures or unaided recall are being assessed, it is appropriate to prohibit resource materials.

You may want to consult the extended section on assessment in the front matter of the Course 3 *Core-Plus Mathematics Teacher's Guide* and *Implementing Core-Plus Mathematics*. Among the topics presented in these sources are curriculum-embedded assessment, student-generated assessment, and scoring assessments and assigning grades. Since the *Core-Plus Mathematics* approach and materials provide a wide variety of assessment information, the teacher will be in a good position to assign grades. With such a wide choice of assessment opportunities, a word of caution is appropriate: *It is easy to overassess students, and care must be taken to avoid doing so*. Since many rich opportunities for assessing students are embedded in the curriculum itself, you may choose not to use a quiz at the end of every lesson or to replace all or portions of an in-class test with take-home tasks or projects.

Name ______________________

Date ______________________

Evaluating My Collaborative Work

	Yes	Somewhat	No
1. I participated in this investigation by contributing ideas.	____	________	____
2. I was considerate of others, showed appreciation of ideas, and encouraged others to respond.	____	________	____
3. I paraphrased others' responses and asked others to explain their thinking and work.	____	________	____
4. I listened carefully and disagreed in an agreeable manner.	____	________	____
5. I checked others' understanding of the work.	____	________	____
6. I helped others in the group understand the solution(s) and strategies.	____	________	____
7. We all agreed on the solution(s).	____	________	____
8. I stayed on task and got the group back to work when necessary.	____	________	____
9. We asked the teacher for assistance only if everyone in the group had the same question.	____	________	____

10. What actions helped the group work productively?

11. What actions could make the group even more productive tomorrow?

Your signature: ______________________

Name ______________________________

Date ______________________________

Constructing a Math Toolkit

Name ______________________

Date ______________________

Sudoku

8				6				1
4	5	7	8					9
			2	5	9			4
2	3	9		8				
6	8						9	3
	7			9		1	6	8
7			9	2	8			
9			4		5	7	3	
1				7	3			2

Think About This Situation

Think about strategies you would use to solve the Sudoku puzzle shown below.

a. How would you decide where to begin?

b. Which square would you fill in first? Which one would you fill in next? Explain your reasoning.

c. Describe a strategy (or a combination of strategies) you would use to fill in the remaining squares.

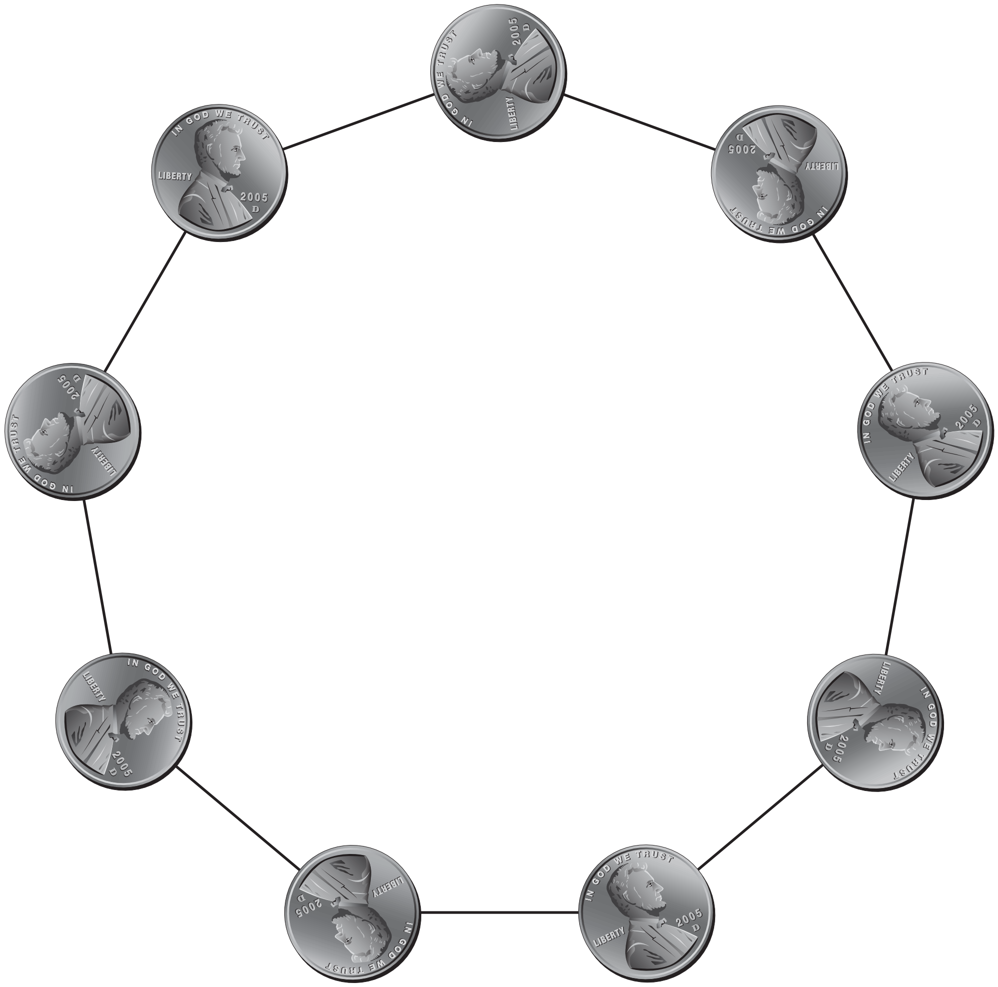
d. When the game is completed, what will be true about the sums of the row entries, the sums of the column entries, and the sums of the 3×3 block entries? Explain.

8				6				1
4	5	7	8					9
			2	5	9			4
2	3	9		8				
6	8						9	3
	7			9		1	6	8
7			9	2	8			
9			4		5	7	3	
1				7	3			2

Name ______________________________

Date ______________________________

Penny Game



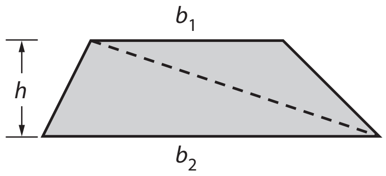
Name ______________________________

Date ______________________________

Reasoning about Areas

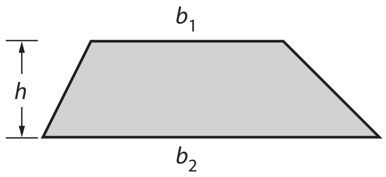
Problem 4

Angela: I can split the trapezoid into two triangles by drawing a diagonal. One triangle has area $\frac{1}{2}b_1h$. The other has area $\frac{1}{2}b_2h$. So, the area of the trapezoid is $\frac{1}{2}b_1h + \frac{1}{2}b_2h$, or $\frac{1}{2}(b_1 + b_2)h$.



Dylan: In a parallelogram, opposite sides are the same length. Any side can be used as the base. In the trapezoid shown,

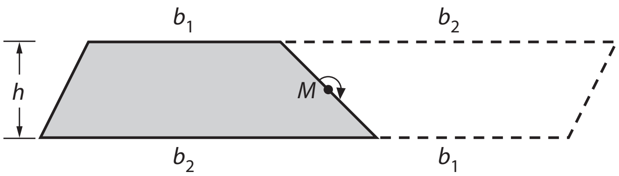
b_1h will underestimate the area.
b_2h will overestimate the area.



To find the correct area, you average the two estimates.

$$\frac{b_1h + b_2h}{2} = \frac{1}{2}(b_1h + b_2h)$$
$$= \frac{1}{2}(b_1 + b_2)h$$

Hsui: If I rotate the trapezoid 180° about the midpoint M, the trapezoid and its image form a parallelogram.



The length of the base of the parallelogram is $b_2 + b_1$ and the height is h. The area of the parallelogram is $(b_2 + b_1)h$. The area of the trapezoid is $\frac{1}{2}$ of this area, or $\frac{1}{2}(b_1 + b_2)h$.

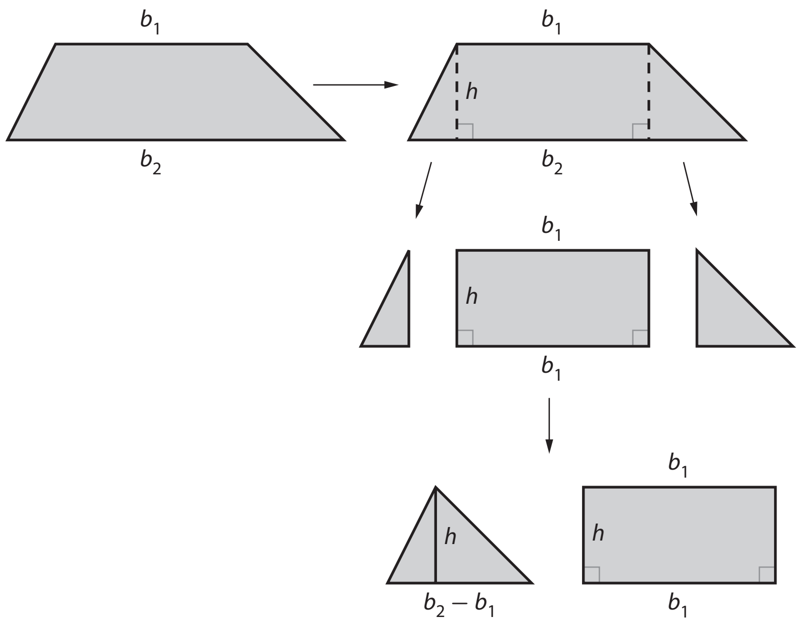
Name ________________________

Date ________________________

Reasoning about Areas

Problem 4

Barbara:



So, the area of the trapezoid is $\frac{1}{2}(b_2 - b_1)h + b_1h$, which equals $\frac{1}{2}(b_1 + b_2)h$.

Jorge: The area of the trapezoid is $\frac{1}{2}(b_1 + b_2)h$ because you can cut up the shape and find the areas of the individual pieces.

a. Which of the five arguments is closest to the argument you would have provided to justify the formula for the area of a trapezoid?

b. Which of these arguments show correct reasoning and which do not? Compare your responses with those of others and resolve any differences.

c. Select one of the arguments you think provides a correct proof of the area formula. Describe the features of the argument that you thought were good. What, if anything, would you add to that argument to make it easier to understand?

Summarize the Mathematics

In this investigation, you examined reasoning strategies and arguments in mathematical and nonmathematical contexts.

a Look back at the mathematical statements that the students were attempting to prove in Problems 3 and 4. In each case, answer the following questions.

i. What information was given?

ii. What conclusion was to be established?

iii. How was the given information used by Teresa to reason logically to the conclusion in Problem 3? By Angela to reason to the conclusion in Problem 4?

b How can you tell whether an argument provides a correct proof of a claim?

Be prepared to share your ideas with the rest of the class.

Summarize the Mathematics

Inductive reasoning and deductive reasoning are each important; they are complementary aspects of mathematical reasoning. Inductive reasoning often leads to conjectures of new relationships or properties that can be proven using deductive reasoning. Consider this conjecture.

The sum of any two consecutive odd numbers is divisible by 4.

a How could you arrive at this conjecture by using inductive reasoning?

b Write this conjecture in if-then form.

i. What is the hypothesis of your statement?

ii. What is the conclusion?

c How could you use deductive reasoning to prove this conjecture?

Be prepared to share your ideas and reasoning strategies with the entire class.

Name ______________________

Date ______________________

Lesson 1 Quiz

Form A

1. It is true that "all registered voters in the United States are at least 18 years old."
   a. Rewrite the statement in if-then form.

   b. Evan lives in Iowa and is 20 years old. What can you conclude?

   c. Brittany lives in Virginia and is registered to vote. What can you conclude?

2. A class of students at Jasper High School was asked to prove the following conjecture.

   If you double the length of a rectangle (and do not change the width), then the area of the rectangle doubles.

   Consider each argument below and decide if it is a correct proof. Explain your reasoning.

   a. Levi's Argument:

   A rectangle with length 5 cm and width 7 cm has an area of 35 cm^2. If you double the length to 10 cm and leave the width 7 cm, then the area is 70 cm^2. So, since $2(35) = 70$, when you double the length of a rectangle, you double its area.

b. Julian's Argument:

If the length of a rectangle is ℓ and the width of the rectangle is w, then the area of the rectangle is ℓw. If you double the length, the new length will be 2ℓ. If the new length is 2ℓ and the width is w, then the new area is $(2\ell)w$. Since $(2\ell)w$ is the same as $2(\ell w)$, the area of the new rectangle is twice the area of the original rectangle.

c. What might be added to the argument in Part b to make it easier for someone to follow?

3. If two angles are complementary, then the sum of their measures is 90°.

a. Suppose you know that $\angle A$ and $\angle B$ are complementary and that $m\angle B$ is 30°. What can you conclude?

b. Suppose that $m\angle C = 110°$. Is it possible that $\angle C$ and $\angle D$ are complementary? Provide a convincing argument to support your answer.

4. James made the following assertion: If a, b, and c are three consecutive whole numbers, then the sum $a + b + c$ is divisible by 3.

a. Check James' assertion using inductive reasoning.

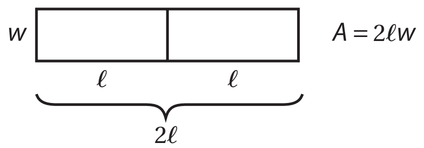
b. Give a convincing argument that James' assertion is true, or show that his assertion is false.

Lesson 1 Quiz

Form A
Suggested Solutions

1. **a.** If a person is registered to vote in the United States, then the person is at least 18 years old.
   b. We cannot conclude anything.
   c. Brittany is at least 18 years old.

2. **a.** Levi has not provided a correct proof. He has only shown that the conjecture is true for one example. A proof must be general.
   b. This is a correct proof. It uses the correct formula for the area of a rectangle, correctly applies the Associative Property of Equality, and uses general dimensions for the rectangle.
   c. A diagram such as the one below would improve the presentation of the proof in Part b.



3. **a.** Since the angles are complementary, you can conclude that $m\angle A = 60°$.
   b. It is not possible for $\angle C$ and $\angle D$ to be complementary since an angle measure is always positive, and it is impossible to add a positive angle measure to 110° and get 90°.

4. **a.** Students should find the sum of several sets of three consecutive whole numbers and note that each sum is divisible by 3.
   b. Let the three numbers be n, $n + 1$, and $n + 2$. Then the sum is $n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1)$. Thus, since 3 is a factor of the sum, the sum is divisible by 3.

Name ______________________

Date ______________________

Lesson 1 Quiz

Form B

1. Shane made the following assertion: All numbers that are divisible by 4 are even numbers.

   a. Write Shane's assertion in if-then form.

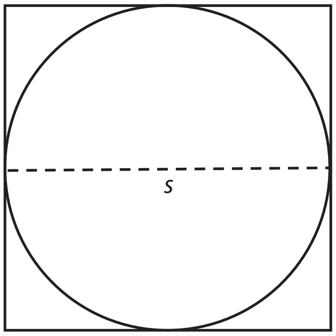
   b. Check Shane's assertion using three different numbers.

   c. Give a convincing argument that Shane's assertion is true or show that his assertion is false.

2. Is the following conjecture about the product of any two positive numbers true or false? Explain your reasoning.

   If x and y are positive numbers, then the product of x and y, xy, is greater than x.

3. Cheyenne and Patrick proved that the area of a square with side length s will always be greater than the area of a circle with diameter s by providing the following diagram.



Is this a complete proof? If so, explain why. If not, explain what else is needed.

4. Determine whether the argument below is correct. Explain your reasoning.

If Eva has a cast on her arm, then she will not be allowed to go swimming. Eva did not go swimming with her friends. Therefore, she must have a cast on her arm.

Lesson 1 Quiz

Form B
Suggested Solutions

1. **a.** If a number is divisible by 4, then it is even.

   b. Responses will vary. Students should choose three multiples of 4 and indicate that they are also even numbers.

   c. An even number is a number that is divisible by 2. If a number is divisible by 4, then it can be written as $4k$ for some k. But, $4k = 2(2k)$, so it is even.

2. This conjecture is false. If $0 < y \leq 1$ and x is any positive number, then $xy \leq x$. Students should give a specific counterexample.

3. This is not a complete proof. The diagram supports the conclusion, but reasoning from known facts should also be included. Students might indicate that there is area inside the square that is not inside the circle, and thus the area of the square is greater than the area of the circle. If students want to give a more algebraic proof, they can begin by finding the expressions for both areas.

   Square: $A_S = s^2$

   Circle: $A_C = \pi\left(\frac{s}{2}\right)^2$

   $= \pi\,\frac{s^2}{4}$

   $= \frac{\pi}{4}\,s^2$

   They then need to notice that $\frac{\pi}{4} < 1$, and thus $\frac{\pi}{4}\,s^2 < 1s^2$.

4. This is not a correct argument. Knowing that the conclusion of an if-then statement is satisfied does not allow you to infer that the hypothesis is also satisfied. Eva might not have gone swimming for some other reason.

Think About This Situation

Analyze the lift-bed truck mechanism shown below.

a How do you think the mechanism works?

b As the truck bed is raised or lowered, what elements change?

c As the truck bed is raised or lowered, what segment lengths and what angle measures remain unchanged?

d As the truck bed is raised or lowered, will it always remain parallel to the flat-bed frame of the truck? Explain your reasoning.

Name ______________________

Date ______________________

Selected Key Geometric Ideas From Courses 1 and 2

The geometric ideas listed are the ones that will be most useful as you complete Course 3. You may wish to add to this list or develop a different format to keep track of new geometric ideas.

TRIANGLES

Definition of isosceles triangle A triangle with at least two sides of equal length

Pythagorean Theorem If the lengths of the sides of a right triangle are a, b, c, with the side of length c opposite the right angle, then $a^2 + b^2 = c^2$.

Converse of the Pythagorean Theorem If the sum of the squares of the lengths of two sides of a triangle equals the square of the length of the third side, then the triangle is a right triangle.

Triangle Inequality The sum of the lengths of any two sides of a triangle is always greater than the length of the third side.

Triangle Angle Sum Property The sum of the measures of the angles in a triangle is 180°.

Base angles of isosceles triangle A triangle is an isosceles triangle if and only if it has two congruent angles.

Median of a triangle The line segment joining a vertex to the midpoint of the opposite side.

30°-60° right triangle relationship For a right triangle with acute angles of measures 30° and 60°, the length of the side opposite the 30° angle is half the length of the hypotenuse. The length of the side opposite the 60° angle is $\sqrt{3}$ times the length of the side opposite the 30° angle.

45°-45° right triangle relationship For a right triangle with acute angles of measures 45°, the length of the hypotenuse is $\sqrt{2}$ times the length of either of the equal legs of the right triangle.

Side-Side-Side (SSS) congruence condition If three sides of a triangle are congruent to the corresponding sides of another triangle, then the two triangles are congruent.

Side-Angle-Side (SAS) congruence condition If two sides and the angle between the sides of one triangle are congruent to the corresponding parts of another triangle, then the two triangles are congruent.

Angle-Side-Angle (ASA) congruence condition If two angles and the side between the angles of one triangle are congruent to the corresponding parts of another triangle, then the two triangles are congruent.

Name ______________________________

Date ______________________________

Selected Key Geometric Ideas From Courses 1 and 2

QUADRILATERALS AND POLYGONS

Definition of parallelogram A quadrilateral with opposite sides of equal length or a quadrilateral that has two pairs of sides parallel (See Course 3 Unit 3 p. 205 in the student text.)

Definition of rectangle A quadrilateral with four right angles

Definition of kite A convex quadrilateral with two distinct pairs of consecutive sides the same length

Definition of rhombus A quadrilateral with all four sides the same length

Quadrilateral Angle Sum Property The sum of the measures of the angles in a quadrilateral is 360°.

Polygon Angle Sum Property The sum of the measures of the interior angles of a polygon with n sides is $(n - 2)180°$.

Opposite Angles Property of Parallelograms Opposite angles in a parallelogram are congruent.

Condition ensuring a parallelogram If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Conditions ensuring a rectangle (1) If a parallelogram has one right angle, then it is a rectangle. (2) If the diagonals of a parallelogram are the same length, then the parallelogram is a rectangle.

RELATIONSHIPS AND TRANSFORMATIONS

Definition of congruent figures Figures that have the same shape and size, regardless of position or orientation

Definition of size transformation of magnitude *k* A size transformation of magnitude k is defined by the rule $(x, y) \rightarrow (kx, ky)$.

Definition of rotation A turning motion determined by a point called the center of rotation and a directed angle of rotation

Name ______________________________

Date ______________________________

Selected Key Geometric Ideas From Courses 1 and 2

ANGLES, SEGMENTS, AND CIRCLES

Definition of complementary angles Two angles whose measures sum to 90°

Definition of perpendicular bisector of a segment A line that is perpendicular to a segment and contains its midpoint

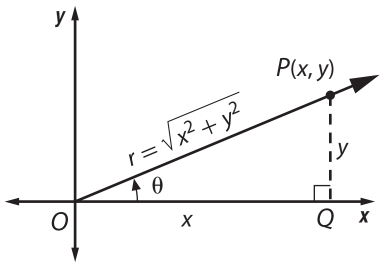
Definition of midpoint of a segment The point on the segment that is the same distance from each endpoint

Definition of slope of a segment The slope of a segment that contains two points with coordinates (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

Distance formula The distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Equation of a circle The equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$.

TRIGONOMETRY

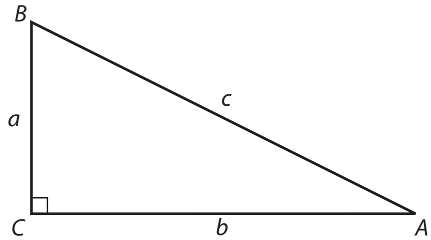


Definitions of trigonometric functions, $0° \leq \theta \leq 360°$

tangent of $\theta = \tan \theta = \frac{y}{x}$ $(x \neq 0)$

sine of $\theta = \sin \theta = \frac{y}{r}$

cosine of $\theta = \cos \theta = \frac{x}{r}$

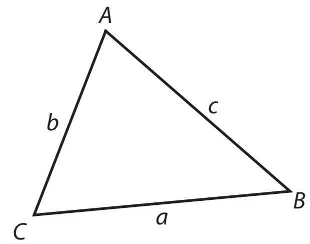


Definition of trigonometric ratios for right triangles

tangent of $\angle A = \tan A = \frac{a}{b} = \frac{\text{length of side } \textit{opposite } \angle A}{\text{lenght of side } \textit{adjacent to } \angle A}$

sine of $\angle A = \sin A = \frac{a}{c} = \frac{\text{length of side } \textit{opposite } \angle A}{\text{length of } \textit{hypotenuse}}$

cosine of $\angle A = \cos A = \frac{b}{c} = \frac{\text{length of side } \textit{adjacent to } \angle A}{\text{length of } \textit{hypotenuse}}$



Law of Sines In any triangle ABC with sides of lengths a, b, and c opposite $\angle A$, $\angle B$, and $\angle C$, respectively:
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Law of Cosines In any triangle ABC with sides of lengths a, b, and c opposite $\angle A$, $\angle B$, and $\angle C$, respectively:
$c^2 = a^2 + b^2 - 2ab \cos C$

Name ______________________________

Date ______________________________

Postulates and Theorems

Unit 1, Lesson 2

Names of Postulates and Theorem (if exist)	Postulates and Theorems	Labeled Illustration(s)

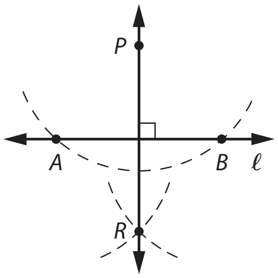
Name ______________________

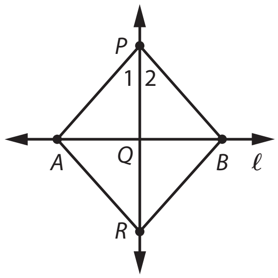
Date ______________________

Justifying the Construction of a Line Perpendicular to a Given Line through a Point not on the Line

Problem 4 Part cii

Refer to the construction procedure shown on page 34. Complete the argument below by supplying missing statements or reasons in the blanks provided.

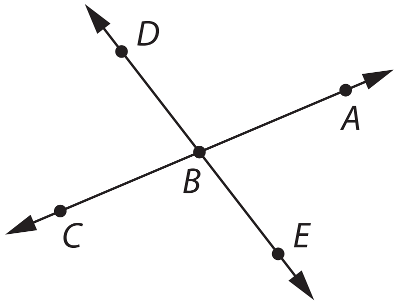




Label the point of intersection of ℓ and $\overleftrightarrow{PR}$, Q as shown. Draw $\overline{PA}$, $\overline{PB}$, $\overline{RA}$, and $\overline{RB}$. $PA = PB$ and $RA = RB$ because ______________________, and $PR = PR$ (common side in two triangles). It follows that ______________________ by the SSS congruence condition. So, $\angle 1 \cong \angle 2$ because ______________________. Since $\overline{PA} \cong \overline{PB}$, $\angle 1 \cong \angle 2$, and $\overline{PQ} \cong \overline{PQ}$, $\triangle APQ \cong \triangle BPQ$ by ______________. $\angle AQP \cong \angle BQP$ because ______________________. Since $\angle AQP$ and $\angle BQP$ are a linear pair, it follows that $m\angle AQR + m\angle BQP = 180°$ by ______________________. It follows that $m\angle AQP =$ __________. So, $\overleftrightarrow{PR} \perp \ell$ by ______________________.

Summarize the Mathematics

In this investigation, you used deductive reasoning to establish relationships between pairs of angles formed by two intersecting lines. In the diagram at the right, suppose the lines intersect so that m∠*DBA* = m∠*CBD*.



a What can you conclude about these two angles? Prepare an argument to prove your conjecture.

b What can you conclude about the other angles in the diagram? Write a proof of your conclusion.

c What mathematical facts did you use to help prove your statements in Parts a and b? Were these facts definitions, postulates, or theorems?

d Describe the relationship between $\overleftrightarrow{AC}$ and $\overleftrightarrow{DE}$.

Be prepared to share your conjectures and explain your proofs.

Name ______________________

Date ______________________

Justifying that If Two Lines are Cut by a Transversal and Interior Angles on Same Side of Transversal are Supplementary, then the Lines are Parallel

Problem 7 Part b

Make copies of this sheet. Cut these statement-reason strips along the dashed lines, mix and place the strips in envelopes for distribution and later storage.

Statements	Reasons
t a transversal cutting ℓ and m; $\angle 4$ and $\angle 5$ are supplementary	Given
$m\angle 5 + m\angle 4 = 180°$	Definition of supplementary angles
$m\angle 1 + m\angle 4 = 180°$	Linear Pair Postulate
$m\angle 1 + m\angle 4 = m\angle 5 + m\angle 4$	Substitution
$m\angle 1 = m\angle 5$	Subtraction Property of Equality
$\ell \parallel m$	Parallel Lines Postulate using corresponding angles $\angle 1$ and $\angle 5$

Summarize the Mathematics

In this investigation, you reasoned both inductively and deductively about angles formed by parallel lines and a transversal.

a What statements did you accept to be true without proof?

b What theorems and their converses were you able to prove about parallel lines and the angles they form with a transversal?

c Restate each theorem and its converse in Part b as a single if-and-only-if statement similar to the statement of the Parallel Lines Postulate.

Be prepared to compare your responses with those of others.

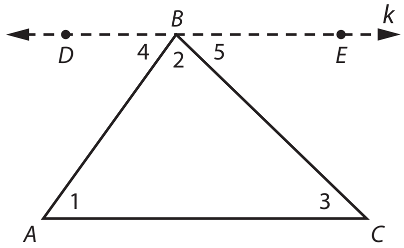
Name ______________________

Date ______________________

Justifying that the Sum of the Measures of the Angles of a Triangle is 180°

Connections Task 13 Part c

Prove: $m\angle 1 + m\angle 2\ m\angle 3 = 180°$



Statements	Reasons
(1) $k \parallel \overleftrightarrow{AC}$ with transversals $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$	(1)
(2) $m\angle 1 = m\angle 4$ $m\angle 3 = m\angle 5$	(2)
(3) $m\angle 4 + m\angle 2 = m\angle DBC$	(3)
(4) $m\angle 4 + m\angle 2 + m\angle 5 = m\angle DBE$	(4)
(5) $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	(5)
(6) $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	(6)

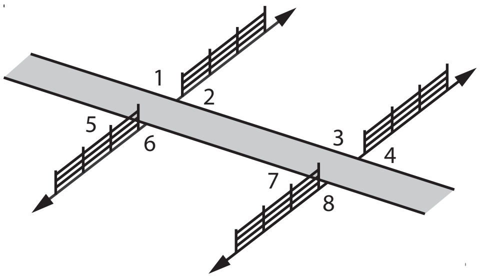
Name ______________________

Date ______________________

Lesson 2 Quiz

Form A

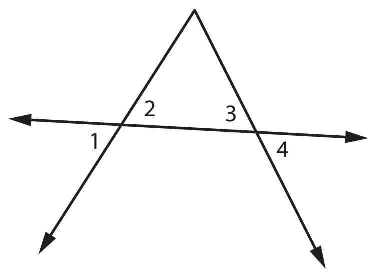
1. Emma wants to build two fence lines that are parallel to each other. They will intersect a straight private drive as shown below.



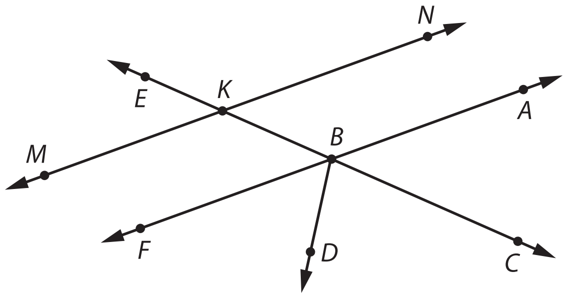
a. Identify three pairs of angles that she could make congruent in order to ensure that the fences are parallel.

b. If Emma found that $m\angle 6 + m\angle 4 = 180°$, could she conclude that the two fences are parallel? Explain your reasoning.

2. In the diagram at the right, if $m\angle 1 = m\angle 3$, prove that $m\angle 2 = m\angle 3$.



3. In the diagram below, $\overleftrightarrow{MN} \parallel \overleftrightarrow{AF}$, $m\angle ABC = 43°$, and $m\angle FBD = 58°$.



Find the measure of each indicated angle. Provide reasoning to support your answers.

a. $m\angle EBF =$ ____________

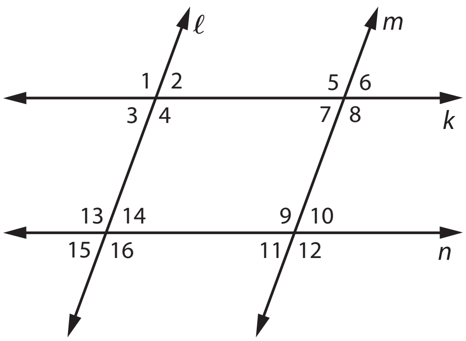
b. $m\angle EBA =$ ____________

c. $m\angle DBC =$ ____________

d. $m\angle EKN =$ ____________

e. $m\angle MKB =$ ____________

4. In the diagram below, $\ell \parallel m$ and $k \parallel n$. Prove that $m\angle 1 + m\angle 10 = 180°$.



Lesson 2 Quiz

Form A
Suggested Solutions

1. **a.** Responses will vary. Using the road as the transversal, any pair of corresponding angles, alternate interior angles, or alternate exterior angles being congruent is enough to ensure that the fences are parallel. Congruent pairs are L1 and L3, L5 and L7, L2 and L4, L6 and L8, L6 and L3, L2 and L7, L1 and L8, L4 and L5.

   b. Yes, this is enough to ensure that the fences are parallel. Note that $\angle 4$ and $\angle 7$ are vertical angles, so $m\angle 4 = m\angle 7$. If $m\angle 6 + m\angle 4 = 180°$, then $m\angle 6 + m\angle 7 = 180°$ because $m\angle 4 = m\angle 7$. Then the fences are parallel because $\angle 6$ and $\angle 7$ are interior angles on the same side of the transversal and are supplementary.

2. | | |
   |---|---|
   | $m\angle 1 = m\angle 2$ | Vertical angles are congruent. |
   | $m\angle 1 = m\angle 3$ | Given |
   | $m\angle 2 = m\angle 3$ | Substitution |

3. Students may have other correct reasoning for this item.

   a. $m\angle EBF = 43°$
   $\angle EBF$ and $\angle ABC$ are vertical angles, so they have equal measure.

   b. $m\angle EBA = 137°$
   $\angle EBA$ and $\angle ABC$ are a linear pair, so $m\angle EBA + m\angle ABC = 180°$.

   c. $m\angle DBC = 79°$
   $m\angle FBD + m\angle DBC + m\angle ABC = 180°$, so $58° + m\angle DBC + 43° = 180°$. Therefore, $101° + m\angle DBC = 180°$, or $m\angle DBC = 79°$.

   d. $m\angle EKN = 137°$; $\angle EKN$ and $\angle EBA$ are corresponding angles, so $m\angle EKN = m\angle EBA$. Alternatively, $\angle EKN$ and $\angle ABC$ are exterior angles on the same side of the transversal, so they are supplementary.

   e. $m\angle MKB = 137°$
   Students can justify this using vertical angles, alternate interior angles, or same side interior angles.

4. Proofs may vary. One possibility is provided here.

Statement	**Reason**
$\ell \parallel m$ and $k \parallel n$	Given
$m\angle 1 = m\angle 5$	Parallel Lines Postulate
$m\angle 5 = m\angle 9$	Parallel Lines Postulate
$m\angle 1 = m\angle 9$	Substitution
$m\angle 9 + m\angle 10 = 180°$	Linear Pair Postulate
$m\angle 1 + m\angle 10 = 180°$	Substitution

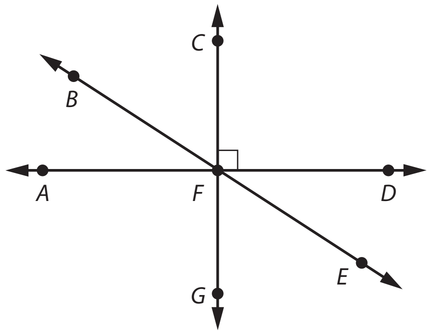
Name ______________________

Date ______________________

Lesson 2 Quiz

Form B

1. In the diagram below, m$\angle AFB = 32°$ and $\overline{CG} \perp \overline{AD}$.



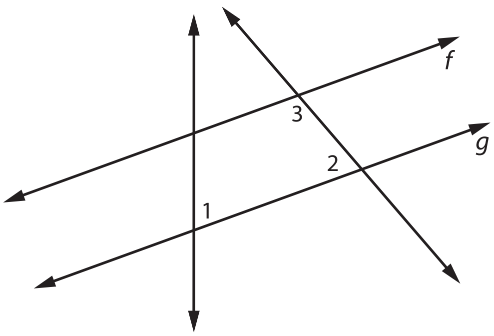
Find the measures of the indicated angles. Provide reasoning to support your answers.

a. m$\angle DFE$ = ____________

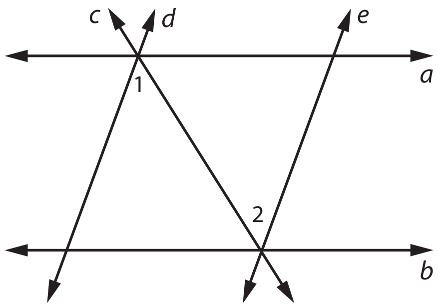
b. m$\angle BFC$ = ____________

c. m$\angle GFB$ = ____________

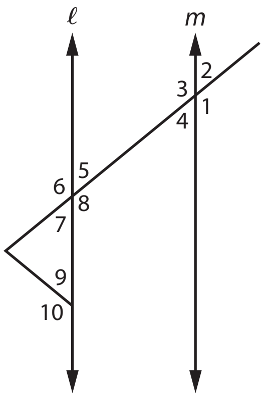
2. In the diagram below, if m$\angle 1$ = m$\angle 2$, and $f \parallel g$. Prove that m$\angle 1$ + m$\angle 3$ = 180°.



3. If m$\angle 1$ = m$\angle 2$, which lines must be parallel? Explain your reasoning.



4. In the diagram below, $\ell \parallel m$ and m$\angle 7$ = m$\angle 9$. Prove that m$\angle 2$ = m$\angle 9$.



Lesson 2 Quiz

Form B
Suggested Solutions

1. **a.** $m\angle DFE = 32°$
   $\angle AFB$ and $\angle DFE$ are vertical angles, so they have equal measures.

   b. $m\angle BFC = 58°$
   $m\angle BFC + m\angle AFB = 90°$ because $\overline{CG} \perp \overline{AD}$.

   c. $m\angle GFB = 122°$
   $m\angle AFG = 90°$ since $\overline{CG} \perp \overline{AD}$ and $m\angle GFB = m\angle AFB + m\angle AFG = 32° + 90° = 122°$.

2. Since $f \parallel g$ and $\angle 2$ and $\angle 3$ are interior angles on the same side of the transversal, $m\angle 2 + m\angle 3 = 180°$. We also know that $m\angle 1 = m\angle 2$; so by substitution, $m\angle 1 + m\angle 3 = 180°$.

3. $d \parallel e$
   $\angle 1$ and $\angle 2$ are alternate interior angles formed by line d, line e, and the transversal line c. Since $m\angle 1 = m\angle 2$, line d and line e must be parallel.

4. One possible proof follows:

Statement	Reason
$\ell \parallel m$	Given
$m\angle 7 = m\angle 9$	Given
$m\angle 2 = m\angle 4$	Vertical Angles Theorem
$m\angle 4 = m\angle 7$	Parallel Lines Postulate
$m\angle 2 = m\angle 7$	Substitution
$m\angle 2 = m\angle 9$	Substitution

Think About This Situation

The number magic is not so amazing if you think about it with algebraic reasoning.

a. What starting number do you think would lead to an end result of 39? Of 123? Of 513?

b. Can you explain how and why your teacher is able to find every starting number when told only the ending number?

Pick an integer between 0 and 20.
Add 5 to your number.
Multiply the result by 6.
Divide that result by 3.
Subtract 9 from that result.

Start Number	End Number
4	9
11	23
17	35
33	67
45	91

Name ______________________

Date ______________________

Proving Properties of Numbers and Functions

Problem 7 and 8

7. Recall that $y = \log x$ means $10y = x$. For example, $3 = \log 1{,}000$ because $10^3 = 1{,}000$. Use reasoning about powers of 10 or the **log** function on your calculator to complete the following table that compares $\log a$, $\log b$, and $\log ab$ for a sample of positive numbers a and b.

a	*b*	*ab*	log *a*	log *b*	log *ab*
100	1,000				
100	0.001				
0.1	0.001				
75	100				
75	0.01				
75	120				
50	20				

What pattern do you see that relates $\log a$, $\log b$, and $\log ab$? Describe the pattern in words and algebraically with symbols.

8. Rows in the table of Problem 7 illustrate a property of logarithms that connects multiplication and addition in a very useful way: For any a and $b > 0$, $\log ab = \log a + \log b$. Use what you know about logarithms and exponents to justify each step in the following algebraic proof of that property.

Statement		**Reason**
Since $a > 0$ and $b > 0$, there are numbers x and y so that $x = \log a$ and $y = \log b$.	(1)	
So, $a = 10^x$ and $b = 10^y$.	(2)	
$\log ab = \log (10^x 10^y)$	(3)	
$= \log 10^{x+y}$	(4)	
$= x + y$	(5)	
$= \log a = \log b$	(6)	
Therefore, $\log ab = \log a + \log b$.	(7)	

Summarize the Mathematics

In this investigation, you explored ways that algebraic reasoning explains interesting number patterns.

a What were the key steps in explaining the number tricks discovered in Problems 1 and 2?

b What overall strategy and algebraic properties were used to prove the generality of number patterns discovered in Problems 4 and 6?

c What overall strategy and algebraic properties were used to prove that $\log ab = \log a + \log b$ for all positive values of a and b?

Be prepared to explain your ideas to the class.

Name ______________________

Date ______________________

Proving the Law of Cosines

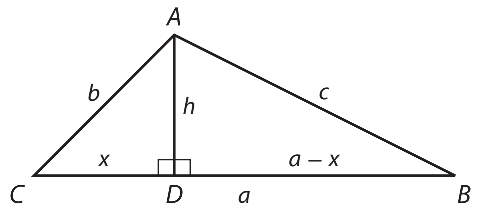
Problem 5

The acute angle case:

In a triangle …

$c^2 = a^2 + b^2 - 2ab \cos C$

Given: $\angle C$ is an acute angle; $\overline{AD}$ is an altitude to $\overline{BC}$ with point D between B and C.



Statements | **Reasons**

Step 1. In $\triangle ADC$: $\frac{x}{b} = \cos C$, so $x = b \cos C$.

Step 2. In $\triangle ABD$: $h^2 = c^2 - (a - x)^2$

$= c^2 - (a^2 - 2ax + x^2)$

$= c^2 - a^2 + 2ax - x^2$

Step 3. In $\triangle ADC$: $h^2 = b^2 - x^2$

Step 4. From Steps 2 and 3, we can conclude that:
$c^2 - a^2 + 2ax - x^2 = b^2 - x^2$.

Step 5. From the equation in Step 4, we can conclude that:
$c^2 = a^2 + b^2 - 2ax$.

Step 6. Finally, combining Steps 1 and 5, we conclude that:
$c^2 = a^2 + b^2 - 2ab \cos C$.

Name ______________________________

Date ______________________________

Proving the Law of Cosines

Problem 5

Make copies of this sheet. Cut these reason strips along the dashed lines, mix and place the strips in envelopes for distribution and later storage. (Note there are two sets of reasons below to save copying costs.)

Reasons Set I	**Reasons Set II**
The definition of cosine $\cos C = \frac{x}{b}$ and algebraic reasoning (multiplying both sides of the equation by b) gives the equivalent equation $x = b \cos C$.	The definition of cosine $\cos C = \frac{x}{b}$ and algebraic reasoning (multiplying both sides of the equation by b) gives the equivalent equation $x = b \cos C$.
Pythagorean Theorem, algebraic expansion of the square of a binomial, and the Distributive Property of Multiplication over Addition	Pythagorean Theorem, algebraic expansion of the square of a binomial, and the Distributive Property of Multiplication over Addition
Pythagorean Theorem	Pythagorean Theorem
Both sides of the equation are equal to h^2. So by substitution, they are equal to each other.	Both sides of the equation are equal to h^2. So by substitution, they are equal to each other.
Addition and Subtraction Properties of Equality	Addition and Subtraction Properties of Equality
Substitution Property of Equality	Substitution Property of Equality

Summarize the Mathematics

In this investigation, you explored ways that reasoning with equations can be used to prove important general principles in algebra, geometry, and trigonometry.

a What general properties of numbers, operations, and equations did you use to discover and prove a formula for solving equations in the form $ax + b = c$ with $a \neq 0$? In the form $ax + b = cx + d$ with $a \neq c$?

b What general properties of numbers, operations, and equations did you use to discover and prove formulas for slope and intercepts of graphs for linear equations in the form $ax + by + c$?

c What algebraic principles were used to justify steps in the proof of the Pythagorean Theorem?

d What is the main idea behind the proof of the Law of Cosines?

Be prepared to explain your ideas to the class.

Name ______________________________

Date ______________________________

Algebraic Properties and Properties of Equality

Algebraic Properties

Addition

For any numbers a, b, and c:

Commutative Property of Addition:	$a + b = b + a$
Associative Property of Addition:	$(a + b) + c = a + (b + c)$
Additive identity element (0):	$a + 0 = a$
Additive Inverse Property:	There is a number $-a$ such that $a + (-a) = 0$.

Multiplication

For any numbers a, b, and c:

Commutative Property of Multiplication:	$ab = ba$
Associative Property of Multiplication:	$(ab)c = a(bc)$
Multiplicative identity element (1):	$a1 = a$
Multiplicative Inverse Property:	For each $a \neq 0$, there is a number a^{-1} such that $a^{-1}a = 1$.

Distributive Property of Multiplication over Addition

For any numbers a, b, and c: $a(b + c) = (ab) + (ac)$

Properties of Equality

For any numbers a, b, and c:

Transitive Property of Equality:	If $a = b$ and $b = c$, then $a = c$.
Addition Property of Equality:	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality:	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality:	If $a = b$, then $ac = bc$.
Division Property of Equality:	If $a = b$, then $a \div c = b \div c$ (whenever $c \neq 0$).

Name ______________________

Date ______________________

Algebraic Properties and Properties of Equality

Applications Task 10

10. **a.** For any x, $3x + 5x = (3 + 5)x$ (1)

$= 8x$ (2)

b. For any x, $(3x + 5) + 7x = (5 + 3x) + 7x$ (1)

$= 5 + (3x + 7x)$ (2)

$= 5 + 10x$ (3)

c. If $7x + 5 = 5x + 14$, then $(7x + 5) + (-5) = (5x + 14) + (-5)$ (1)

$7x + (5 + (-5)) = 5x + (14 + (-5))$ (2)

$7x + 0 = 5x + 9$ (3)

$7x = 5x + 9$ (4)

$-5x + 7x = -5x + (5x + 9)$ (5)

$2x = (-5x + 5x) + 9$ (6)

$2x = 0x + 9$ (7)

$2x = 9$ (8)

$\left(\frac{1}{2}\right)(2)x = \left(\frac{1}{2}\right)9$ (9)

$1x = 4.5$ (10)

$x = 4.5$ (11)

Name ______________________________

Date ______________________________

Lesson 3 Quiz

Form A

1. Consider the following number trick.

Think of a number.
Add two to the number.
Triple the result.
Subtract the beginning number from that result.
Divide that result by 2.

a. Alexis starts with the number 3. What number does she have at the end?

b. Use algebraic reasoning to find a decoding procedure that can be used to find the start number when only the final result is known. Be sure to clearly indicate how you can use the ending number to obtain the beginning number. Show your work or explain your reasoning.

2. One interesting number pattern is 1, 9, 25, 49, 81, 121, 169, … . One expression for calculating the nth term in this sequence is $(2n - 1)^2$.

a. What is the 20th term in the sequence?

b. Write the sequence of numbers obtained by taking the differences of successive numbers in the original sequence up to the 7th term.

c. Write the shortest possible expression that shows how to calculate the difference between term n and term $(n + 1)$ in the original sequence.

d. Use what you know about simplifying algebraic expressions to prove that your answer to Part c is correct by showing that it is equivalent to $(2(n + 1) - 1)^2 - (2n - 1)^2$.

3. Consider linear equations of the form $a(x - b) = cx$.

a. Solve the equation $3(x - 5) = 7x$.

b. What formula shows how to calculate the solution to any linear equation of the form $a(x - b) = cx$, $a \neq c$? Show your work.

Lesson 3 Quiz

Form A
Suggested Solutions

1. **a.** 6; Students should be able to do this computation in their heads.

   b. The directions in the number trick can be represented by the expression $\frac{3(x+2)-x}{2}$. This simplifies to be $\frac{2x+6}{2} = x + 3$. Thus, the ending number is 3 greater than the beginning number. Subtracting 3 from the ending number will give the beginning number.

2. **a.** $(2(20) - 1)^2 = 39^2 = 1{,}521$

   b. 8, 16, 24, 32, 40, 48

   c. $8n$

   d. $$\begin{aligned}(2n+1)^2 - (2n-1)^2 &= (4n^2 + 4n + 1) - (4n^2 - 4n + 1)\\ &= 4n^2 + 4n + 1 - 4n^2 + 4n - 1\\ &= 8n\end{aligned}$$

3. **a.** $x = -\frac{15}{4}$

   b. $ax - ab = cx$

   $ax - cx = ab$

   $x(a - c) = ab$

   $x = \frac{ab}{a-c}$

Name ______________________

Date ______________________

Lesson 3 Quiz

Form B

1. Consider the patterns shown in the sequence of figures below.

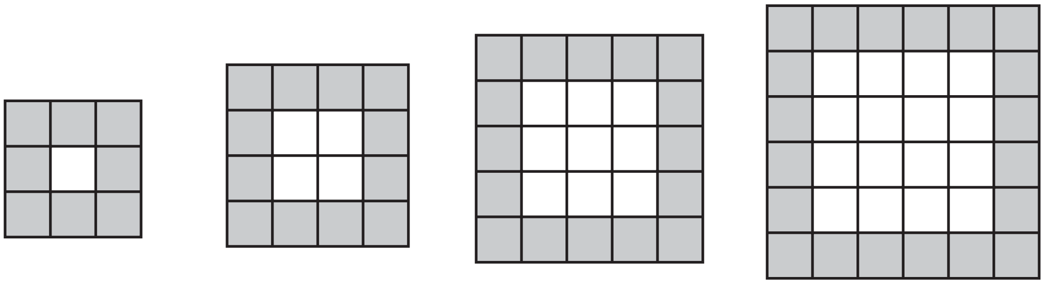


Figure 1 **Figure 2** **Figure 3** **Figure 4**

a. Complete the table below that indicates the number of shaded boxes in each figure.

Figure Number	1	2	3	4
Number of Shaded Boxes				

b. Dave thinks that a formula for the number of shaded squares in figure n is $4n + 4$. Does Dave's formula seem correct? Explain why or why not.

c. Olivia wrote the formula $2(n + 2) + 2n$. Use algebraic reasoning to determine if Olivia's formula is equivalent to Dave's.

d. Jaden wrote the formula $(n + 2)^2 - n^2$. Is Jaden's formula equivalent to Dave's? Use algebraic reasoning to justify your answer.

2. Suppose that the times, in seconds, for the top five finishers in the 400-meter race at a county preseason track meet were x_1, x_2, x_3, x_4, and x_5. The time for the first-place finisher was x_1, the time for the second-place finisher was x_2, and so on.

   a. What algebraic expression shows how to calculate the range (difference between the fastest time and the slowest time) for these times?

   b. Suppose that during the season, each of these runners decreased his or her time by 1.5 seconds. What expression would represent the new running time for the fastest runner?

   c. Use algebraic reasoning to show how decreasing each time by 1.5 seconds does or does not change the range for the running times.

3. Consider quadratic equations that occur in the form $ax^2 + b = c$, with $a > 0$ and $c > b$.

   a. Solve $2x^2 + 9 = 15$.

   b. Write a formula that will give solutions to any equation of the form $ax^2 + b = c$, with $a > 0$ and $c > b$, in terms of a, b, and c. Show the algebraic reasoning you used to derive your formula.

Lesson 3 Quiz

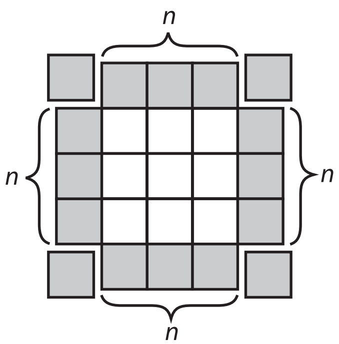
Form B
Suggested Solutions

1. a.

Figure Number	1	2	3	4
Number of Shaded Boxes	8	12	16	20

b. Yes, it seems correct. First of all, it gives the correct numbers. Also, it is clear from the numbers that this is a linear pattern with constant change of 4.

Visual thinking could also be used. In $4n + 4$, the $4n$ comes from the 4 sides of the figure without including the corners and the plus 4 comes from the four corners added in. For example, $n = 3$:



c. $2(n + 2) + 2n = 2n + 4 + 2n = 4n + 4$. Yes, Olivia's formula is equivalent.

d. $(n + 2)^2 - n^2 = n^2 + 4n + 4 - n^2$

$= 4n + 4$

Yes, Jaden's formula is equivalent to Dave's formula.

2. **a.** $x_5 - x_1$

b. $x_1 - 1.5$

c. $(x_5 - 1.5) - (x_1 - 1.5) = x_5 - 1.5 - x_1 + 1.5 = x_5 - x_1$. So, the range does not change.

3. **a.** $x = \pm\sqrt{3}$

b. $x = \pm\sqrt{\dfrac{c - b}{a}}$

Think About This Situation

Think about the design of this penny-stacking experiment and how you would interpret the results.

a Why is it important to agree on rules (*a protocol*) for how you must stack the pennies?

b Why is it important to divide your class into the two groups at random?

c Complete this experiment and then organize your data using plots and summary statistics. (Save the data, as you will need them in Investigation 2.)

d What can you conclude? Have you *proved* that, for students your age, one hand tends to be better than the other in stacking pennies? Why or why not?

Summarize the Mathematics

In this investigation, you examined characteristics of well-designed experiments.

a What are the three characteristics of a well-designed experiment? Why is each necessary?

b Why are subject blinding and evaluator blinding desirable in an experiment?

c What is the placebo effect? How can you account for it when designing an experiment?

Be prepared to share your deas with the class.

Name ______________________

Date ______________________

Test on Factoring into Primes

Circle the correct answer. There is no penalty for guessing, so if you do not know, guess.

1. The largest known prime is $2^{30,402,457} - 1$. How many digits does it have?

A 9,152,051 **B** 9,152,052

2. Twin primes are a pair of primes that differ by 2, such as 3 and 5 or 29 and 31. How many pairs of twin primes exist where both numbers are less than one million?

A 137 **B** 8,169

3. If you factor 17,422,457,186,352,049,329,324,779,900,065,324,265,471 into primes, how many different primes are there?

A 1 **B** 2

4. If you factor 5,439,042,183,600,204,290,159 into primes, how many different primes are there?

A 1 **B** 2

5. What is the 1,000th prime?

A 7,919 **B** 7,927

6. What do you get if you add up the reciprocals of all of the primes?

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \cdots$$

A about 10 **B** about 50 **C** neither of these

7. Suppose you factor 26! into primes. How many times does 5 appear as a factor?

A 5 **B** 6

8. How many perfect squares evenly divide $7{,}200 = 2^5 3^2 5^2$?

A 10 **B** 12

NUMBER CORRECT ______________________

Answers to Test on Factoring into Primes

1. B

2. B

3. B 17,422,457,186,352,049,329,324,779,900,065,324,265,471 =
32,032,215,596,496,435,569 • 5,439,042,183,600,204,290,159

4. A This number is prime.

5. A The other number also is prime.

6. C The sum of the reciprocals of the primes does not converge but grows larger and larger.

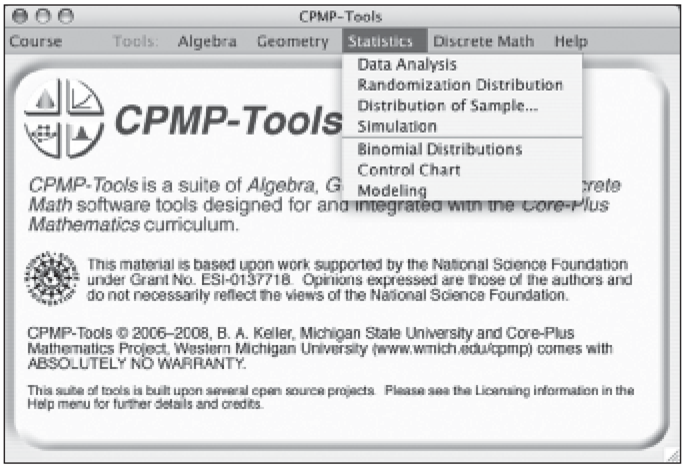
7. B The number $26! = 26 \cdot 25 \cdot 24 \cdot \cdots \cdot 3 \cdot 2 \cdot 1$ has one factor each of 5, 10, 15, 20, and 25. So, the prime factorization must contain 6 factors of 5.

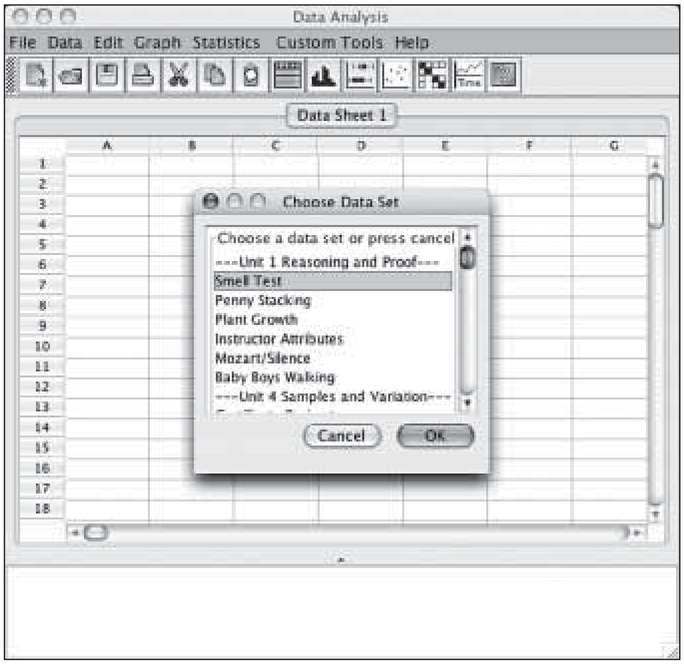
8. B A perfect square must have an even number of each factor. There can be zero, two, or four 2s, zero or two 3s, and zero or two 5s. So, there are $(3)(2)(2) = 12$ ways to build a square number from these factors.

Technology *Tip*
CPMP-Tools

Randomization Distribution

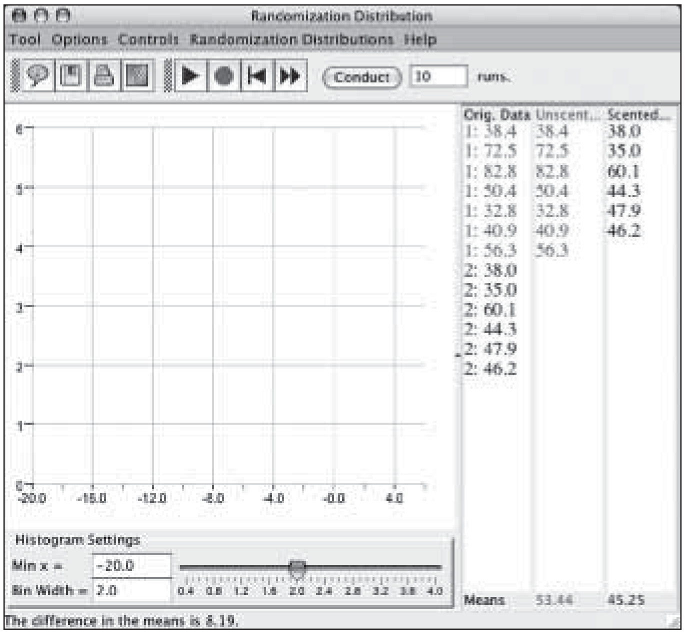
The instructions below are provided for the data analysis software in *CPMP-Tools* found at www.wmich.edu/cpmp/CPMP-Tools.

<table>
<tr><th></th><th></th></tr>
<tr><th>Instructions</th><th>Expected Display</th></tr>
<tr><td>To produce a random distribution for an experiment, select the Course 3 Statistics menu.</td><td>
</td></tr>
<tr><td>Select “Randomization Distribution” and the data set needed. Then choose OK two times.

If you enter your own data set, select “Randomization Distribution” from the Statistics menu.</td><td>
</td></tr>
</table>

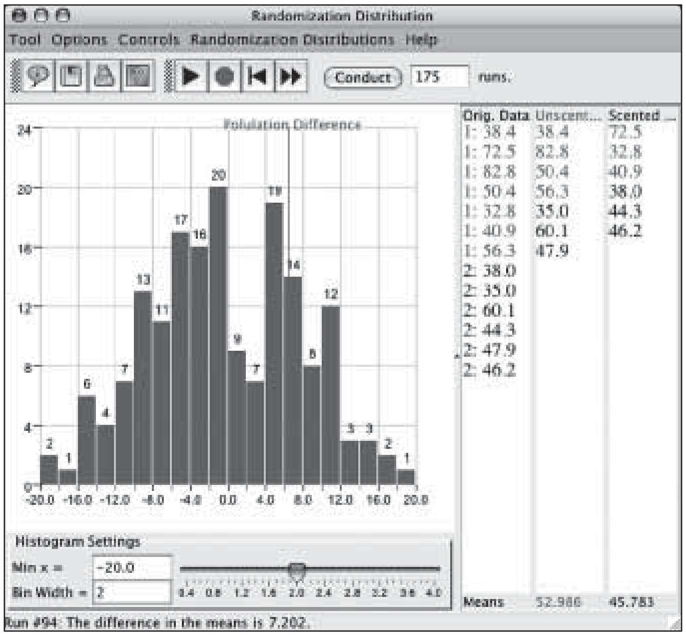
Notice that the responses from one treatment are printed in red and the responses from the other treatment are printed in blue. The difference in the means is displayed as a red vertical line and numerically below the histogram.

To adjust the histogram settings, choose options, Histogram Settings to display the Min x and Bin width (histogram width).



Then press Start to produce random assignments of the responses to treatments. Watch the red and blue responses being randomly assigned to the two treatments. For each random assignment, the difference of means will be calculated and added to the histogram.

Choose Label Bars under the Options menu to display the frequency of each bar.



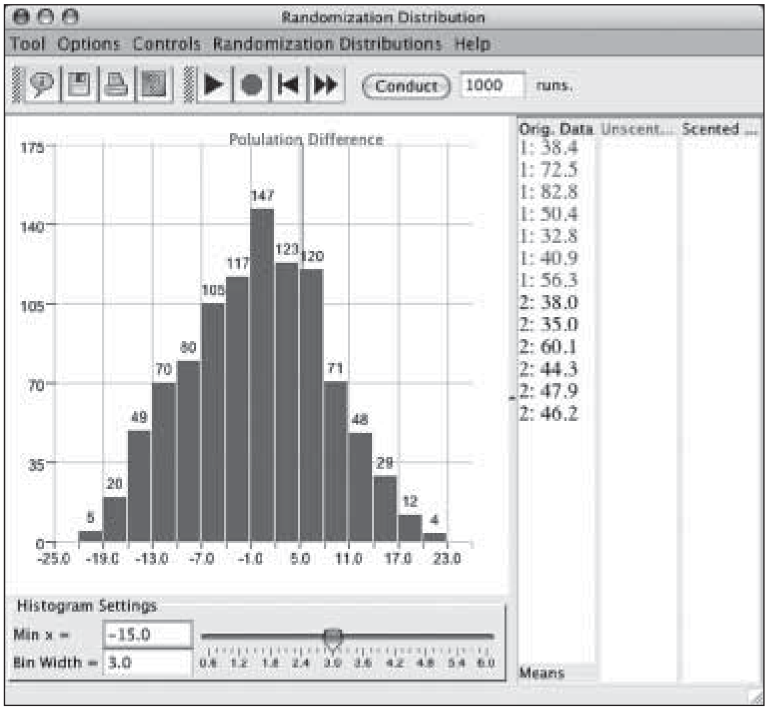
Name ______________________

Date ______________________

Scented and Unscented Masks

Problems 3 and 4

Unscented Mask (in seconds)	Scented Mask (in seconds)
38.4	38.0
72.5	35.0
82.8	60.1
50.4	44.3
32.8	47.9
40.9	46.2
56.3	



Name ______________________

Date ______________________

Randomization Test

Step 1. Assume that which treatment each subject gets makes absolutely no difference in his or her response. In other words, assume the subjects in the experiment would give the same response no matter which treatment they receive. In the next two steps, you will see if this assumption is plausible.

Step 2. Simulate the experiment.

- Write the name of each subject along with his or her response on a card.
- Randomly divide the cards into two treatment groups.
- Compute the mean for each treatment group.
- Find the difference of these means.
- Repeat this many times until you can see the shape of the distribution of differences.

Step 3. Locate the difference from the actual experiment on the distribution you generated in Step 2.

Step 4. If the difference from the actual experiment is in the outer 5% of the distribution, conclude that the results are **statistically significant**; that is, you have evidence that the treatments caused the difference in the mean response. If the difference is not in the outer 5% of the distribution, conclude that your original assumption was plausible. The difference can reasonably be attributed solely to the particular random assignment of treatments to subjects.

Summarize the Mathematics

In this investigation, you explored the randomization test. This test is one method of determining whether a difference between two treatment groups can be reasonably attributed to the random assignment of treatments to subjects or whether you should believe that the treatments caused the difference.

a Explain why this statement is true: Even if the response for each subject would be the same no matter which treatment he or she receives, there is almost always a nonzero difference in the means of the actual responses from the two treatments.

b What does it mean if the results of an experiment are called "statistically significant"?

c Explain the reasoning behind the steps of a randomization test.

d Explain how this statement applies to the reasoning in this unit: Statistical reasoning is different from mathematical proof because in statistics, you can never say you are certain.

Be prepared to share your responses and thinking with the class.

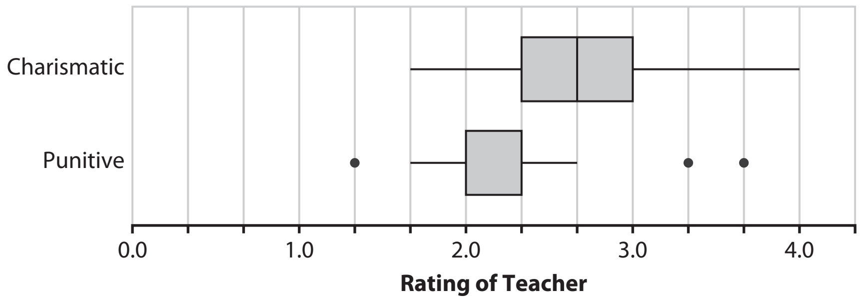
Name ______________________

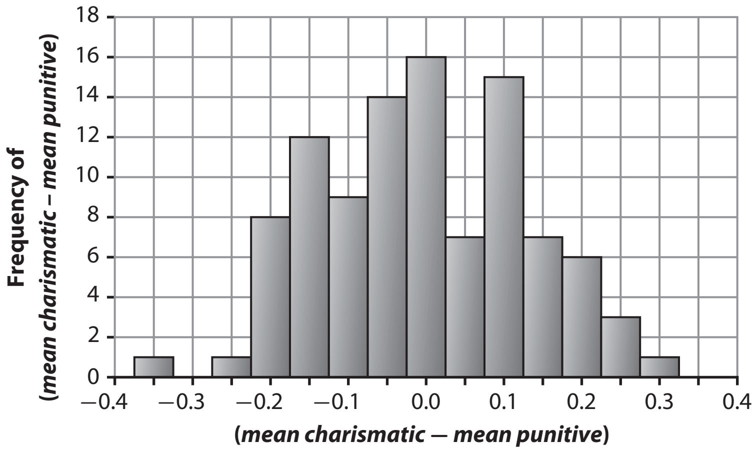
Date ______________________

Lecture Ratings

Check Your Understanding

Treatment	Mean	Standard Deviation	Number of Students
Charismatic	2.61	0.53	25
Punitive	2.24	0.54	24





Summarize the Mathematics

In this investigation, you examined the three main types of statistical studies.

a. What is a random sample?

b. How is randomization used in a sample survey? In an experiment? In an observational study?

c. What kind of conclusion can you draw from a sample survey? From an experiment? From an observational study?

Be prepared to share your responses with the class.

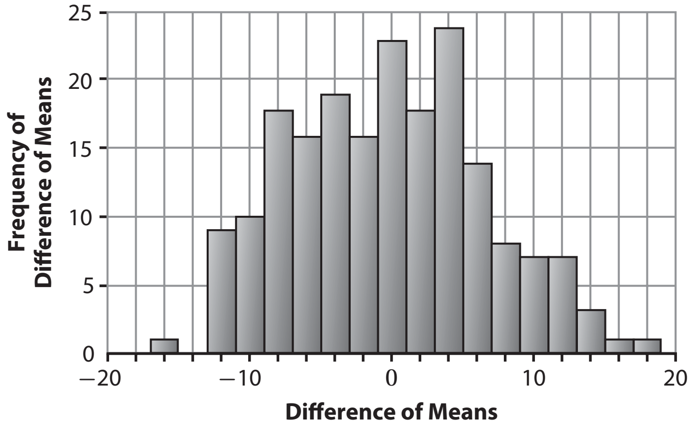
Name ____________________________

Date ____________________________

Mozart of Silence

Applications Task 4

The randomization distribution below shows 195 random assignments of Brian's results. Perform 5 runs for the randomization test by hand and add them to this distribution. Then answer Parts d and e.



d. Use the randomization distribution to estimate the probability that you could get a difference, just by the random assignment, that is as extreme as that from the real experiment.

e. Brian concluded that students who listen to Mozart during a test would have a better performance. Do you agree with this conclusion or do you think the difference can be reasonably attributed to the random assignment alone?

Name ______________________________

Date ______________________________

On Your Own

Connections Task 8

8. Fill in the missing two lines and they continue with Parts b, c, and d in your textbook.

Commercial Plastic Wrap	Mean of Plastic Wrap	Vacuum Wrap	Mean of Vacuum Wrap	Difference of Means
7.66, 6.98, 7.80	7.48	5.26, 5.44, 5.80	5.50	1.98
7.66, 6.98, 5.80	6.81	5.26, 5.44, 7.80	6.17	0.64
7.66, 6.98, 5.44	6.69	5.26, 5.80, 7.80	6.29	0.40
7.66, 6.98, 5.26	6.63	5.44, 5.80, 7.80	6.35	0.28
7.66, 7.80, 5.44	6.97	5.26, 5.80, 6.98	6.01	0.96
7.66, 7.80, 5.26	6.91	5.44, 5.80, 6.98	6.07	0.84
7.66, 5.26, 5.44	6.12	5.80, 6.98, 7.80	6.86	−0.74
7.66, 5.26, 5.80	6.24	5.44, 6.98, 7.80	6.74	−0.50
7.66, 5.44, 5.80	6.30	5.26, 6.98, 7.80	6.68	−0.38
5.26, 5.44, 5.80	5.50	7.66, 6.98, 7.80	7.48	−1.98
5.26, 5.44, 7.80	6.17	7.66, 6.98, 5.80	6.81	−0.64
5.26, 5.80, 7.80	6.29	7.66, 6.98, 5.44	6.69	−0.40
5.44, 5.80, 7.80	6.35	7.66, 6.98, 5.26	6.63	−0.28
5.26, 5.44, 6.98	5.89	7.66, 7.80, 5.80	7.09	−1.20
5.26, 5.80, 6.98	6.01	7.66, 7.80, 5.44	6.97	−0.96
5.44, 5.80, 6.98	6.07	7.66, 7.80, 5.26	6.91	−0.84
5.44, 6.98, 7.80	6.74	7.66, 5.26, 5.80	6.24	0.50
5.26, 6.98, 7.80	6.68	7.66, 5.44, 5.80	6.30	0.38

Name ____________________

Date ____________________

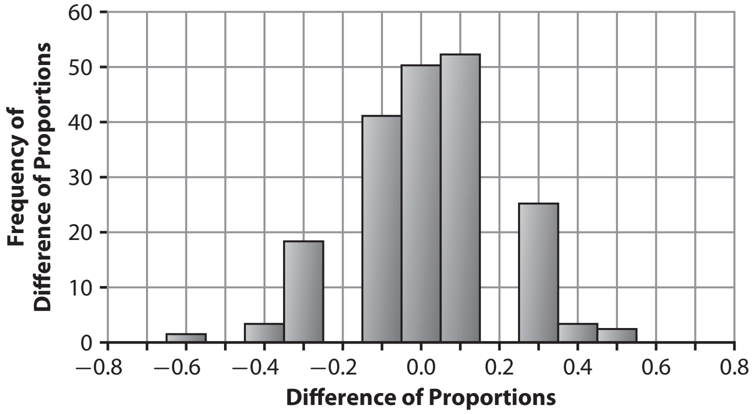
Psychology Experiment: Electric Shocks

Extensions Task 16

16. In a psychology experiment, a group of 17 female college students were told that they would be subjected to some painful electric shocks, and a group of 13 female college students were told that they would be subjected to some painless electric shocks. The subjects were given the choice of waiting with others or alone. (In fact, no one received any shocks.) Of the 17 students who were told that they would get painful shocks, 12 chose to wait with others. Of the 13 students told that they would get painless shocks, 4 chose to wait with others. (Source: Stanley Schachter. The *Psychology of Affiliation,* Stanford, CA: Stanford University Press, 1959, pp. 44–45.)

a. Describe how to use a randomization test to see if the difference in the proportions of students who choose to wait together is statistically significant. You can let 0 represent a response of waiting alone and 1 represent waiting with others.

b. Conduct 5 runs for your test and add them to a copy of the randomization distribution below, which shows 195 runs.



c. Is the difference in the proportions who choose to wait together statistically significant?

d. Why are there no differences between 0.15 and 0.25?

Name ______________________________

Date ______________________________

Shoe Experiment

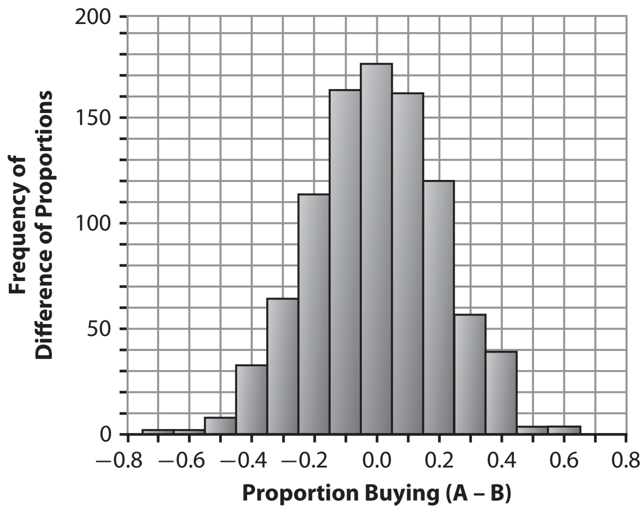
Extensions Task 17

17. A market research experiment was designed to determine how much a subtle color change in white tennis shoes mattered to people who wore them. Twenty people who volunteered to try a new brand of tennis shoe were randomly assigned to get one of two colors of the same shoe. They wore the shoes for a month and then were told they could buy the pair of tennis shoes at a greatly reduced price or return them. Of the 10 people who got color A, 9 decided to buy them. Of the 10 people who got color B, 4 decided to buy them.

a. What kind of a study was this?

b. Describe the logic of a randomization test to determine if the difference is statistically significant and how to do such a test. You can let 0 represent a response of not buying the tennis shoes and 1 represent a response of buying the shoes.

c. Perform five runs, adding them to the 1,000 runs in the randomization distribution shown below.



d. What is your conclusion?

Name ______________________

Date ______________________

Lesson 4 Quiz

Form A

1. Acupuncture is an important part of traditional Chinese medicine. This treatment is becoming more widely available throughout the United States.

   In England, a study was conducted to determine if acupuncture was an effective means of reducing chronic headache pain. In the study, 401 adults were randomly assigned to receive acupuncture treatments or to continue with their usual treatment for headaches. Acupuncture treatment was found to be more effective than usual care in decreasing headache severity. (Source: *American Family Physician,* Vol. 70, No. 3, August 1, 2004, pp. 574–5.)

   a. Does this study have the three characteristics of an experiment? Explain your reasoning.

   b. Was the experiment subject blind? Explain your reasoning.

   c. What were the treatments?

   d. What was the response?

2. Kayla conducted a study of how the color of water affects its evaporation rate. She filled each of ten identical containers to a height of 6.5 inches and then placed the containers on a windowsill. She then added red food coloring to the water in five randomly chosen containers. After one week, she measured the height of the water left in each of the containers. Her measurements are given below.

Height (in inches) of Water Left in Containers

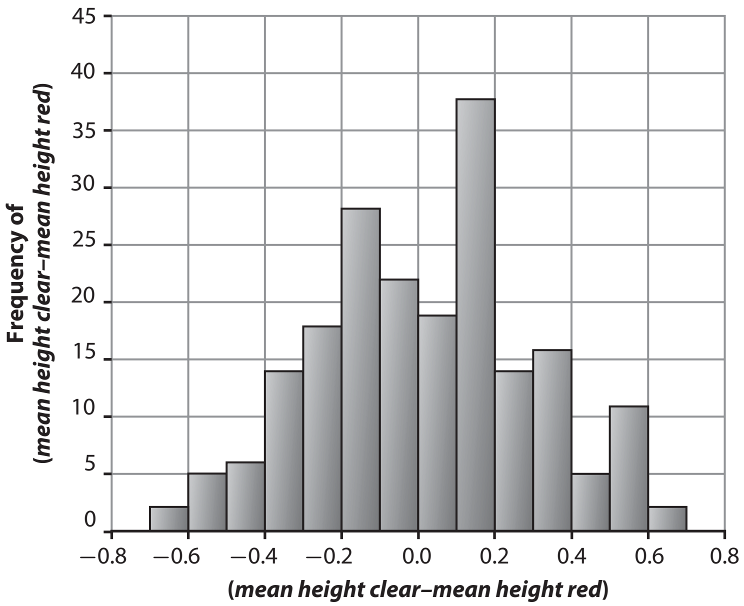
Clear Water	Red Water
5.4	5.2
5.6	5.5
5.8	5.4
6.0	4.8
6.3	5.1

   a. What are the treatments for this experiment?

b. Find the value of *mean height clear – mean height red*.

c. Describe how to conduct one run for a randomization test to decide whether the different colors cause a difference in the evaporation rate. Be as specific as you can.

d. The randomization distribution below provides the results of 200 random assignments. Each value plotted in the histogram is *mean height clear – mean height red* for one run.



Use this randomization distribution to estimate the probability that if the treatments made no difference, you could get a difference just by random assignment that is at least as extreme as what occurred in the real experiment.

e. Is the difference in evaporation rates statistically significant?

3. Alzheimer's disease is a disease that affects older people. One of the early symptoms of Alzheimer's disease is memory loss. A diet that is rich in fruits, vegetables, grains, olive oil, and fish is called a Mediterranean diet and may help prevent Alzheimer's disease.

Over a four-year period, researchers examined the health and diet of more than 2,000 people. The group of people had an average age of 76, and none of them had Alzheimer's disease at the beginning of the study. During the study, the researchers evaluated how closely each participant followed the Mediterranean diet. By the end of the study, 260 participants had been diagnosed with Alzheimer's disease. They found that participants who most closely followed the diet were less likely to develop Alzheimer's than were participants who did not follow the diet.
(Source: www.mayoclinic.com/health/mediterranean-diet/AN01475)

a. What type of study was this? Explain your reasoning.

b. Can you conclude from this study that eating a Mediterranean diet will cause a decrease in a person's chances of developing Alzheimer's disease? Explain your reasoning.

Lesson 4 Quiz

Form A
Suggested Solutions

1. **INSTRUCTIONAL NOTE If your students do not know what acupuncture is, you may need to provide a brief description before starting the quiz.**

   a. Yes, subjects were randomly assigned to the two treatments, there was a comparison group, and the number of participants was large.

   b. No, subjects knew if they were being given acupuncture or just continuing with their normal treatment.

   c. Acupuncture or usual treatment

   d. Headache severity

2. **a.** Red water or clear water

   b. *mean height clear* = 5.82 inches
   mean height red = 5.2 inches
   mean height clear − *mean height red* = 0.62 inches

   c. Write the ten heights on identical slips of paper. Mix up the slips and randomly draw five to be the heights for the red water. The remaining five will be the heights of the clear water. Find the mean of each group of five. Then find the difference *mean height clear* − *mean height red*.

   d. Since the difference in the real experiment is 0.62 inches and only 4 of the 200 trials had differences as extreme as 0.6 (0.6 or greater or −0.6 or less), the approximate probability that the difference will be 0.62 or larger is $\frac{4}{200}$, or 0.02, if the treatments made no difference.

   e. If the treatments made no difference, the probability that a difference is 0.62 or greater in absolute value is only 0.02. Thus, this experiment provides evidence that the evaporation rate for red water is different. The difference is statistically significant.

3. **a.** This study is an observational study. There is no randomization involved at all.

   b. No. You can only conclude that for these people, there is an association between how closely the person followed the Mediterranean diet and the likelihood of developing Alzheimer's disease. But, you cannot conclude a cause-and-effect relationship. One possible lurking variable is that people who had difficulty following a healthy diet were more likely to develop Alzheimer's disease.

Name ______________________

Date ______________________

Lesson 4 Quiz

Form B

1. Suppose you think that cows who are given a new weekly vitamin injection will produce more milk. You have a herd of 200 cows that you can use to test your hypothesis.

   a. Describe a randomized evaluator-blind experiment that could be used to test this hypothesis.

   b. Why might it be important for this experiment to be evaluator blind?

   c. What were the treatments in your experiment?

   d. What was the response variable in your experiment?

2. Andrew manages a comic book store and is trying to increase sales. He decides to run an experiment for the next 12 Mondays. He randomly picks 6 of the Mondays on which he will say hello to each person as they enter the store. On the other 6 Mondays, he will say hello to each person and offer them something to drink. For each day, he will determine the percentage of people who make a purchase. The percentage of people who made purchases each day and the treatment received are given below.

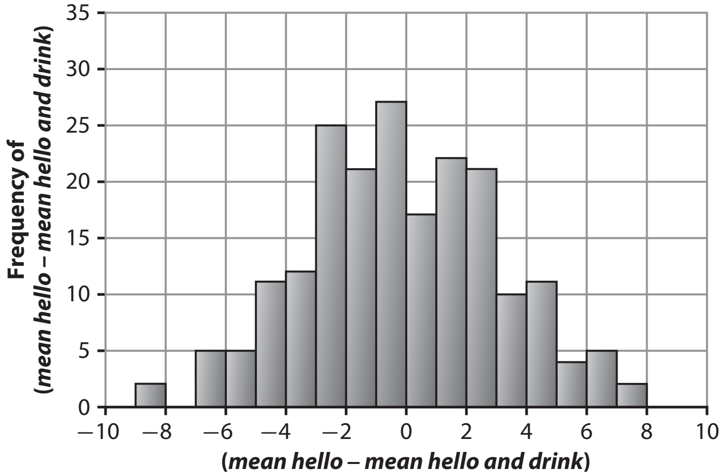
Hello Only	Hello and Drink Offer
19%	14%
20%	10%
25%	26%
12%	18%
18%	27%
23%	12%

   a. What are the treatments for this experiment?

b. Find the value of *mean hello – mean hello and drink.*

c. Describe how to conduct one run of a random assignment to decide whether the different treatments cause a difference in the percentage of people who make a purchase.

d. The histogram below provides the results of 200 random assignments. The value shown in the histogram is *mean hello – mean hello and drink.*



Use the histogram to estimate the probability that if the treatments made no difference, you could get a difference just by random assignment that is at least as extreme as what occurred in the real experiment.

e. Is the difference in the percentage of people making purchases statistically significant?

3. In 2003, a randomly selected group of 15-year-old students from several different countries completed the PISA test. Among other things, the test evaluated each student's mathematical problem-solving skills. The results showed that 15-year-old students who took the test in the United States scored significantly lower than 15-year-old students who took the test in Canada and significantly higher than 15-year-old students who took the test in Greece. (Source: nces.ed.gov/surveys/pisa/pisa2003highlightsfigures.asp?Quest=1&Figure=10)

a. What type of study was this? Explain your reasoning.

b. Is it correct to conclude that 15-year-old students from the United States have better mathematical problem-solving skills as measured by the PISA test than do 15-year-olds in Greece? Explain your reasoning.

Lesson 4 Quiz

Form B
Suggested Solutions

1. a. Randomly separate the cows into two groups of 100 cows each. Assign one of the groups to get the treatment of being given a new weekly vitamin injection. The other group will be given a placebo injection each week. Do this for several weeks or months and measure the amount of milk each cow produces. Be sure that the person doing the milking does not know which cows received the vitamin injection.

   b. If the evaluator knows which cows received the vitamin injection, he or she might try harder to get all the possible milk from those cows in order to show that they produced more milk.

   c. A weekly vitamin injection or a weekly placebo injection

   d. Amount of milk produced over the period of the experiment

2. a. Saying hello only or saying hello and being offered a drink

   b. *mean hello* = 19.5%
   mean hello and drink = 17.833%
   $19.5 - 17.833 \approx 1.67\%$

   c. Write the percentage of customers who made a purchase on 12 identical slips of paper. Then randomly separate them into two groups—one for each treatment. Find the mean of each group and then find the difference *mean hello – mean hello and drink*.

   d. To estimate this probability, you need to determine how many of the runs resulted in a difference greater than 1.67 or less than −1.67. Since 1.67 and −1.67 both fall in the middle of bars, students might choose to include half the trials in those bars or may choose to estimate the probability by looking at the differences of 2 or more. Either way, they will see that the probability is much greater than 0.05. Assuming that the treatments make no difference, $P($*greater than 2* or *less than* $-2) = \frac{53 + 60}{200} = 0.565$.

   e. Whichever method they use in Part e will lead to the conclusion that there is not enough evidence to show a difference. Since the probability of getting a difference of 1.67 or even more extreme if the treatments make no difference is 0.565, we do not have evidence that the treatments cause different percentages of people to make a purchase.

3. a. This study was a sample survey because the students who took the test were randomly selected from among all 15-year-olds in each country.

   b. This is a correct conclusion from this study. A sample survey can be generalized to the larger population.

Name ______________________

Date ______________________

In a Parallelogram, Opposite Angles Are Congruent

Task 2 Part c

Make copies of this sheet. Cut these statement-reason strips along the dashed lines, mix and place the strips in envelopes for distribution and later storage.

Statements	Reasons
S: ▱$ABCD$ is a parallelogram.	**R:** Given
S: $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{CD}$	**R:** Definition of parallelogram
S: $m\angle 1 = m\angle 3$, $m\angle 2 = m\angle 4$	**R:** If two parallel lines are cut by a transversal, then alternate interior angles have equal measure.
S: $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	**R:** Addition Property of Equality
S: $m\angle 1 + m\angle 2 = m\angle A$ $m\angle 3 + m\angle 4 = m\angle C$	**R:** Angle Addition Postulate
S: $m\angle A = m\angle C$	**R:** Substitution Property of Equality

Summarize the Mathematics

In this unit, you learned some basic reasoning principles and strategies that are useful in justifying claims involving concepts of geometry, algebra, and statistics.

a How is *deductive* reasoning different from *inductive* reasoning? Why are both types of reasoning important in mathematics?

b When trying to prove an if-then statement, with what facts do you begin? What do you try to deduce?

c What strategies and geometric properties can you use to prove that two lines are perpendicular? That two lines are parallel? Draw and label sketches to illustrate how those properties are used.

d How would you go about proving a statement like "The sum of the measures of the angles of a trapezoid is 360°"?

e What overall strategy and rules of algebra can you use to prove that an equation like $2ab + c^2 = (a + b)^2$ implies that $c^2 = a^2 + b^2$?

f If you want to see whether two algebraic expressions like $(n + 2)^2 - n^2$ and $4n + 4$ are equivalent, you could begin by comparing tables and graphs of the functions $y = (n + 2)^2 - n^2$ and $y = 4n + 4$. How would an algebraic proof give different evidence that the expressions are equivalent?

Summarize the Mathematics Cont.

g What are the mathematical conventions about *order of operations* when numerical patterns and relationships are represented with symbolic expressions and equations?

h What are the differences between sample surveys, experiments, and observational studies?

i How do you use a randomization test to determine if the result of an experiment to compare two treatments is statistically significant?

j How is statistical reasoning similar to algebraic and geometric reasoning and how is it different?

Be prepared to share your responses and reasoning with the class.

Name ______________________

Date ______________________

Unit Summary

In this unit, you examined and used reasoning principles and strategies to prove mathematical statements were always true. You also learned how to design experiments and use statistical reasoning to determine if one particular treatment was more effective than another treatment or no treatment at all.

What is inductive reasoning? Give an example.

What is deductive reasoning? Give an example.

How are inductive and deductive reasoning often used together in mathematics?

Definitions and assumed facts such as geometric postulates or algebraic properties are often used to support statements in mathematical arguments.

- Give an example of a definition:
  - in geometry ______________________
  - in algebra ______________________
- Give an example of an assumed fact:
  - in geometry ______________________
  - in algebra ______________________
- How are definitions and assumed facts different?

How is a theorem different from either a definition or a postulate?

Mathematical arguments often involve reasoning with if-then statements.

- Give an example of how if-then statements are used in reasoning.
- How do you go about proving a claim stated in if-then form?

In this unit, you used deductive reasoning to prove relations between angles formed by two intersecting lines or by two parallel lines cut by a transversal.

- If two lines intersect at a point, what relations exist between the measures of pairs of angles formed?
- If two parallel lines are intersected by a transversal, what relations exist between the measures of pairs of angles formed?
- What relations between pairs of angles formed when two lines are cut by a third line allow you to conclude that the lines are parallel?

Give an example of a geometric theorem and its converse that you proved in this unit.

The following algebraic expression represents the steps in a number trick. Use algebraic reasoning to prove that the resulting number $\frac{(2n + 2)(5)}{10}$ is always one greater than the starting number n.

What algebraic properties justify the following number principle that is often used in mental arithmetic: $(m + n) + p = (m + p) + n$?

What algebraic properties are used for solving equations of the form $a(x + c) = d$ for x?

How can you tell whether a mathematical argument provides a correct proof of a claim?

What are the characteristics of a well-designed experiment?

What are the differences between sample surveys, experiments, and observational studies?

Describe how to use a randomization test to decide whether a difference in the mean responses of two treatment groups is statistically significant.

Name ______________________

Date ______________________

Unit Test

Form A

1. Consider this interesting number pattern: 2, 6, 12, 20, 30, 42, 56, … .
One expression for calculating the nth term in this sequence is $n^2 + n$.

a. Verify that the expression gives the correct value for the 3rd term of the sequence.

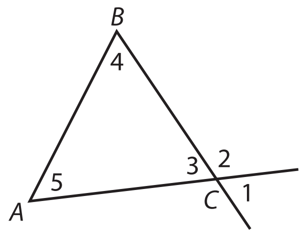
b. What is the 20th term in the sequence?

c. Write the sequence of numbers obtained by taking the differences of successive numbers in the original sequence up to the 7th term.

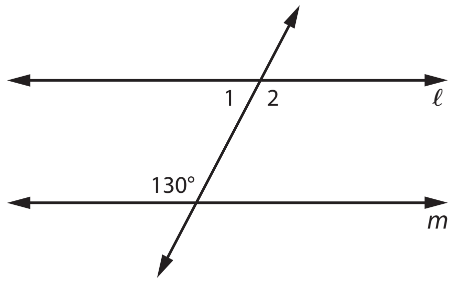
d. Use the pattern of differences in Part c to write an expression in the form $an + b$ for the difference between term n and term $(n + 1)$ in the original sequence.

e. Use what you know about simplifying algebraic expressions to prove that your answer to Part d is correct by showing that it is equivalent to $((n + 1)^2 + (n + 1)) - (n^2 + n)$.

2. Consider the diagram at the right. Write an argument that proves that m∠1 + m∠4 + m∠5 = 180°.



3. If $\ell \parallel m$, find the measures of $\angle 1$ and $\angle 2$. Explain your reasoning.



$m\angle 1 =$ ____________________

$m\angle 2 =$ ____________________

4. Consider the statement: Two lines are parallel if and only if their slopes are equal. Write two if-then statements that are implied by this statement.

5. Daniel recently discovered a new route to school. He wondered if the travel times were different for the two routes. He decided to do an experiment. He randomly assigned five days to the old route and five days to the new route. The travel times (in minutes) for each day are given below.

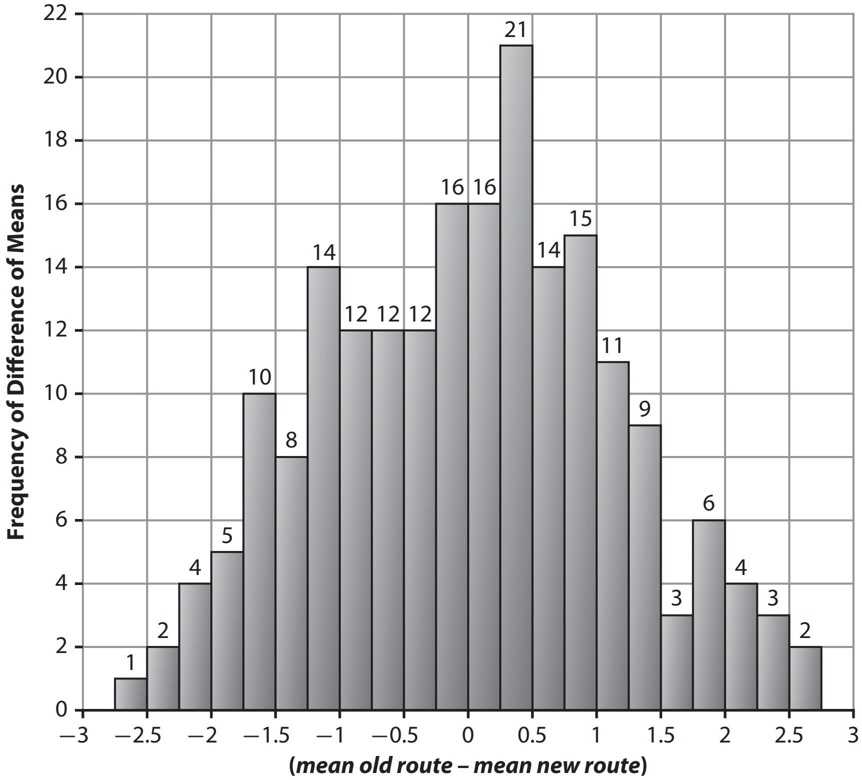
Old Route	New Route
8.5	8.5
11.3	12.4
10.4	7.9
12.7	9.0
9.5	8.8

The summary statistics for these two sets of data are given below.

	Median	Mean	Standard Deviation
Old Route	10.4	10.48	1.62
New Route	8.8	9.32	1.77

a. Describe how to use a randomization test to determine if the travel times were different.

b. Daniel performs 200 runs and creates the randomization distribution below. The values shown in the histogram are *mean old route – mean new route*. Use the randomization distribution to estimate the probability that, if the route made no difference, Daniel would get a difference just by the random assignment of days that is as extreme as what occurred.



c. What is your conclusion about whether there is statistically significant evidence that the travel times are different for the two routes?

Unit Test

Form A
Suggested Solutions

1. **a.** $3^2 + 3 = 12$

   b. $20^2 + 20 = 420$

   c. 4, 6, 8, 10, 12, 14

   d. $2n + 2$

   e. $((n + 1)^2 + (n + 1)) - (n^2 + n) = n^2 + 2n + 1 + n + 1 - n^2 - n = 2n + 2$

2.

Statement	**Reason**
$m\angle 1 = m\angle 3$	Vertical angles are congruent.
$m\angle 3 + m\angle 4 + m\angle 5 = 180°$	The sum of angles of a triangle is 180°.
$m\angle 1 + m\angle 4 + m\angle 5 = 180°$	Substitution

Alternatively, students might reason that $\angle 1$ and $\angle 2$ form a linear pair and that $\angle 2$ is an exterior angle of the triangle. Thus, $m\angle 1 + m\angle 2 = 180°$ and $m\angle 4 + m\angle 5 = m\angle 2$. So, by substitution, $m\angle 1 + m\angle 4 + m\angle 5 = 180°$.

3. $m\angle 1 = 50°$. Since $\angle 1$ and the angle with measure 130° are interior angles on the same side of the transversal, they must be supplementary. $m\angle 2 = 130°$. $\angle 2$ is an alternate interior angle to the 130° angle, so it will have the same measure.

4. If two lines are parallel, then their slopes are equal.
   If two lines have equal slopes, then the lines are parallel.

5. **a.** Write the travel times 12 times on identical slips of paper and randomly divide them into two groups. One group will be the old route times and one group will be the new route times. Find the difference between the means of the two groups (*mean old route − mean new route*). Repeat this process many times. Then using the distribution of differences, determine the probability that the difference between the means is as extreme as that observed. If the probability is small enough, then you can conclude that the travel times are different.

   b. The difference of the actual means is 1.16 seconds. The probability of getting an absolute difference greater than 1.25 is $\frac{30 + 27}{200} = \frac{57}{200} = 0.285$. So, the probability of getting an absolute difference of at least 1.16 is at least 0.285.

   c. If the route made no difference, you could expect a difference as extreme as 1.16 seconds at least 28.5% of the time. Thus, the experiment does not provide evidence to say the travel times are different.

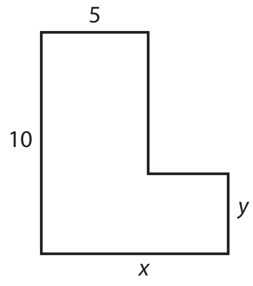
Name ______________________________

Date ______________________________

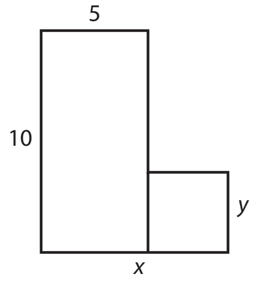
Unit Test

Form B

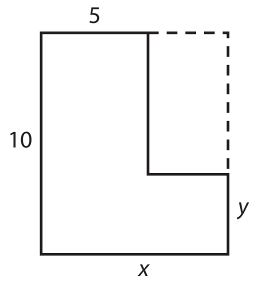
1. Consider the figure shown below.



a. Mary divided the diagram as shown below and calculated the area to be $50 + y(x - 5)$. Explain how Mary got her expression.

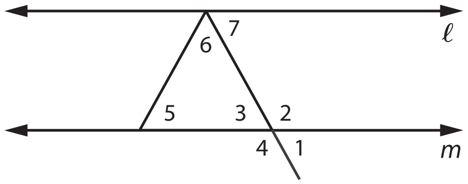


b. Jacob found the area using the diagram below and got the expression $10x - (10 - y)(x - 5)$. Explain how Jacob got his expression.

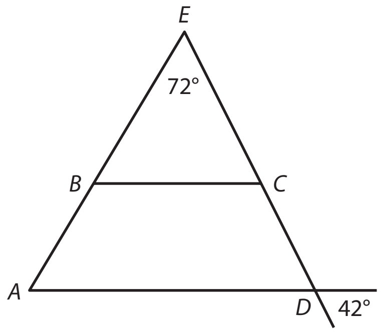


c. Use algebraic reasoning to prove that Jacob's and Mary's expressions will give the same area for any positive values of x and y.

2. Consider the figure below. If $\ell \parallel m$ and $m\angle 5 = m\angle 7$, prove that $m\angle 3 = m\angle 5$.



3. In the figure below, $\overline{BC} \parallel \overline{AD}$. Find the measures of $\angle A$ and $\angle BCD$. Show your work.



$m\angle A =$ ______________________________

$m\angle BCD =$ ____________________________

4. Consider the statement: If James has at least two quarters, then he has at least 50 cents.

a. Is this a true statement? Justify your reasoning.

b. Write the converse of this statement.

c. Is the converse of this statement true? Justify your reasoning.

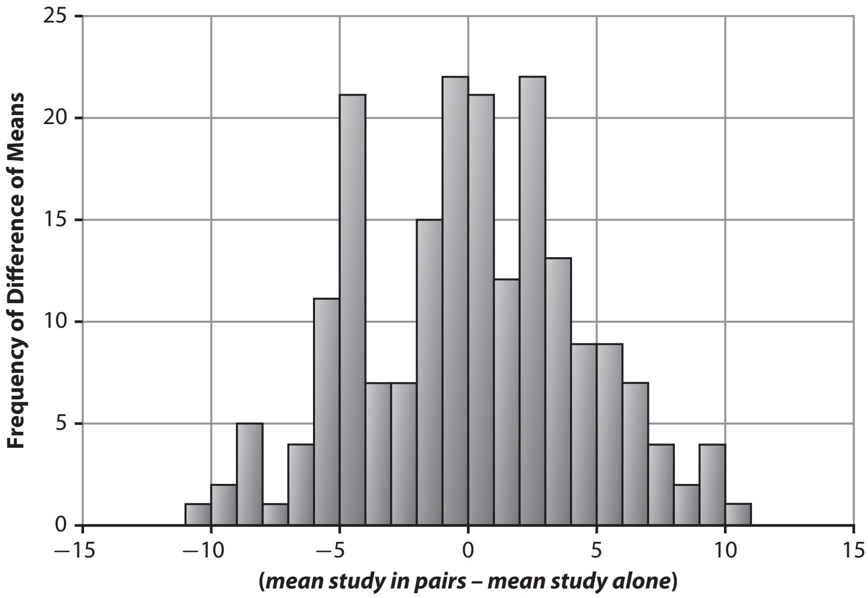
5. An experiment was designed to see whether studying Spanish vocabulary with another person would make a difference in how well students learn their vocabulary words. Approximately half of the students in the class were randomly chosen and given 5 minutes each day to work in pairs on their vocabulary, and the rest of the students were given 5 minutes each day to work by themselves on their vocabulary. Summary statistics for the students' scores on vocabulary tests are given below.

	Median	Mean	Standard Deviation	Number of Students
Study Alone	75	74.375	13.91	16
Study in Pairs	85	83	11.11	20

a. Explain why you would expect to have a difference in the mean scores for the two groups of students even if studying in pairs did not affect how well the students learn their vocabulary.

b. Describe how to use a randomization test to determine if students who studied in pairs performed differently than those who studied alone.

c. The histogram below shows a randomization distribution created from 200 runs. The values shown in the histogram are *mean study in pairs − mean study alone*. Does the randomization distribution provide evidence that students who studied in pairs did differently than those who did not study in pairs, or should the difference be attributed to chance?



Unit Test

Form B
Suggested Solutions

1. a. The area of the larger rectangle is $10(5) = 50$ square units and the area of the smaller rectangle is $y(x - 5)$ square units. So, the total area is $50 + y(x - 5)$ square units.

   b. The area of the whole region is $10x$ square units. Jacob then had to subtract the area of the small rectangle formed by the dotted lines. The area of that rectangle is $(10 - y)(x - 5)$ square units. Thus, the area of the original region is $10x - (10 - y)(x - 5)$ square units.

   c. Mary's expression can be rewritten as follows: $50 + y(x - 5) = 50 + xy - 5y$.
   Jacob's expression can be rewritten as follows: $10x - (10 - y)(x - 5) =$
   $10x - (10x - 50 - xy + 5y) = 10x - 10x + 50 + xy - 5y = 50 + xy - 5y$. Since the two expressions can be rewritten to be the same, they will always give the same values.

2.

Statement	**Reason**
$\ell \parallel m$	Given
$m\angle 3 = m\angle 7$	Alternate interior angles formed by parallel lines are congruent.
$m\angle 5 = m\angle 7$	Given
$m\angle 5 = m\angle 3$	Substitution

3. $m\angle A = 66°$
   $m\angle BCD = 138°$

   $m\angle ADE = 42°$ because $m\angle ADE$ and the 42° angle are vertical angles.
   Then $m\angle A + 42° + 72° = 180°$. So, $m\angle A = 66°$.

   $m\angle CDA = 42°$ by vertical angles. Also, $m\angle CDA + m\angle BCD = 180°$. Thus,
   $42° + m\angle BCD = 180°$ and $m\angle BCD = 138°$.

4. a. Yes, it is a true statement. Since each quarter is worth 25¢, having two of them gives him 50¢.

   b. If James has at least 50¢, then he has at least two quarters.

   c. No, the converse is not true. James could have 5 dimes and no quarters.

5. a. If studying in pairs made no difference, we would expect the scores to be similar but not exactly the same because there are different students in each group.

b. Write the score for each of the students on identical slips of paper. Then randomly divide the vocabulary test scores into two groups. One group will have 16 values and represent the study-alone scores, and one group will have 20 values and represent the study-in-pairs scores. Find the difference between the means of the two groups (*mean study in pairs − mean study alone*). Repeat this process many times. Then determine the probability that the difference between the means is as extreme as that observed. If the probability is small enough, then you can conclude that students who studied in pairs scored differently than those who studied alone.

c. The difference of means is 8.625. The probability of getting a difference greater than 9 or less than −9 is $\frac{8}{200} = 0.04$. Since this probability is less than 0.05, the data provides statistically significant evidence that students who studied in pairs scored differently on the vocabulary test.

Take-Home Assessments

1. The Nikhilam Sutra is a mathematical algorithm that is found in the *Atharva Veda,* an Indian writing dated between 1,000 and 500 B.C. The algorithm is a clever procedure for multiplying numbers that are relatively close to 100. For example, to multiply 94×87, the procedure works as follows:

    Step 1. Find the deficiency of each number from 100.
    (In this case, $100 - 94 = 6$ and $100 - 87 = 13$.)

    Step 2. Find the difference between each original number and the deficiency of the other number. Those differences will always be equal.
    (In this case, $94 - 13 = 81$ and $87 - 6 = 81$.)

    Step 3. Calculate the product of the deficiencies.
    (In this case, $13 \times 6 = 78$.)

    Step 4. Use results from Steps 2 and 3 to find the desired product by finding 100(*Step 2 result*) + (*Step 3 result*).
    (In this case, $100(81) + 78 = 8{,}178 = 94 \times 87$.)

    Steps in the Nikhilam Sutra algorithm were organized in a display like the following table.

Number	Deficiency
94	6
87	13
81	78

    a. Test your understanding of the procedure by calculating 91×93 and 85×92 in two ways—using the Nikhilam Sutra algorithm and with a modern algorithm or a calculator.

    b. Using the letters a and b to represent the factors of a product $a \times b$, write the formulas for each step of the Nikhilam Sutra calculation.

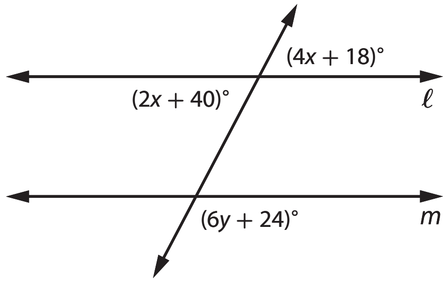
    Step 1. the deficiencies of a and b from 100

    Step 2. the common difference of each number and the deficiency of the other number

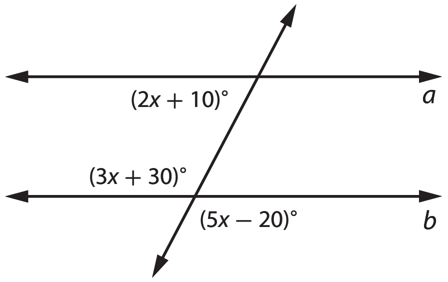
    Step 3. the product of the deficiencies

    Step 4. the final answer, which is 100(*Step 2 result*) + (*Step 3 result*)

2. a. If lines ℓ and m are parallel, find the values of x and y in the diagram below.



b. Are lines a and b parallel? Explain your reasoning?



3. In this unit, you learned about how people can use surveys, observational studies, and experiments to gather data to help answer questions.

   a. Identify a question that you think would best be answered using a survey. Describe why you think a survey would be the best method to gather the data.

   b. Identify a question that you think would be best answered using an observational study. Describe why you think an observational study would be the best way to gather this data.

   c. Identify a question that you think would be best answered using an experiment. Describe why you think an experiment would be the best way to gather this data.

Take-Home Assessments

Suggested Solutions

1. a. 91×93: $100 - 91 = 9$ and $100 - 93 = 7$; $91 - 7 = 84$ and $93 - 9 = 84$; $(9)(7) = 63$;
$91 \times 93 = 8{,}463$
Check: $(91)(93) = 8{,}463$

85×92: $100 - 85 = 15$ and $100 - 92 = 8$; $85 - 8 = 77$ and $92 - 15 = 77$; $(15)(8) = 120$;
$85 \times 92 = 7{,}820$
Check: $(85)(92) = 7{,}820$

b. **Step 1.** $100 - a$ and $100 - b$

Step 2. $a - (100 - b) = a + b - 100$ and $b - (100 - a) = b + a - 100$

Step 3. $(100 - a)(100 - b) = 10{,}000 - 100a - 100b + ab$

Step 4. $100(a + b - 100) + 10{,}000 - 100a - 100b + ab$

c. $a - (100 - b) = a + b - 100$ and
$b - (100 - a) = a + b - 100$

d. $100(a + b - 100) + 10{,}000 - 100a - 100b + ab$
$= 100a + 100b - 10{,}000 + 10{,}000 - 100a - 100b + ab$
$= ab$

2. a.

$2x + 40 = 4x + 18$	$6y + 24 + 4x + 18 = 180$
$22 = 2x$	$6y + 24 + 4(11) + 18 = 180$
$11 = x$	$6y = 94$
	$y = 15\frac{2}{3}$

b. $5x - 20 = 3x + 30$ because they are measures of vertical angles. This implies that $2x = 50$, or $x = 25$. If $x = 25$, then the value of $2x + 10 = 60$ and the value of $3x + 30 = 105$. So, $2x + 10 + 3x + 30 = 165$, but these two angles must be supplementary if lines a and b are parallel. Since they are not supplementary, the lines are not parallel. Alternatively, students might begin assuming $2x + 10 + 3x + 30 = 180$ and show that the two vertical angles would not be the same measure.

3. Student responses will vary for this task. Students' answers should reflect a clear understanding of the differences between the three types of studies and the conclusions that can be drawn from each.

a. Most students will probably identify a question related to people's opinions about an issue.

b. Questions that are best answered using observational studies are those for which you cannot randomly assign subjects to treatments for either practical or ethical reasons. Be sure that the question does not indicate that a cause-and-effect relationship is being explored.

c. Characteristics of questions for which an experiment are most appropriate are those in which the subjects can be randomly assigned to treatments and for which the experimenter wants to establish that one thing causes another.

Practicing for Standardized Tests

As you work closely with your classmates and teachers on a daily basis, they will have a good idea of what you know and what you are able to do with respect to the mathematics you are studying this year. However, your school district or state department of education may ask you to take tests that they design to measure the achievement of all students, classes, or schools in the district or state. Colleges also use external standardized tests, like the ACT and SAT, to compare the knowledge of different students applying for admission or scholarships and sometimes consider these results when placing students into mathematics courses.

External standardized tests usually present assessment tasks in formats that can easily be scored to produce simple percent-correct ratings of your knowledge. If you want to perform well on such standardized tests, it helps to have some practice with test items in multiple-choice formats. The following set of multiple-choice tasks has been designed to give you that kind of practice and to offer some strategic advice in working on such items. In addition, items on standardized tests are often written to assess knowledge in unique ways. Thus, frequently items on these Practice Sets will look different from those you have done in class. This is purposeful. You should use symbol sense and mathematical understanding to make a choice from those listed.

Name ______________________

Date ______________________

Practicing for Standardized Tests

Practice Set 1

Solve each problem. Then record the letter that corresponds to the correct answer.

1. If the length of a leg of a 45°-45° right triangle is 5 cm, how many centimeters long is the hypotenuse?

(a) 5 **(b)** $5\sqrt{2}$ **(c)** $5\sqrt{3}$ **(d)** 10 **(e)** 25

2. Which one of the following equations matches the graph below?

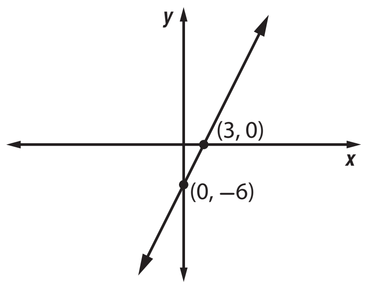
(a) $y = -2x - 6$

(b) $y = 3x - 6$

(c) $y = -6x - 2$

(d) $y = -2x - 3$

(e) $y = 2x - 6$



3. $|-9| - 3|-2| + |-6| =$

(a) -21 **(b)** -9 **(c)** -6 **(d)** 9 **(e)** 21

4. Given the lengths shown, in inches, find the length, in inches, of the third side of the triangle.

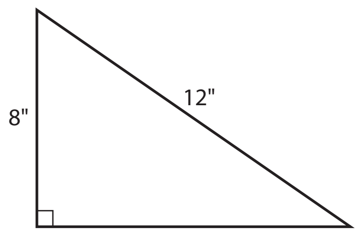
(a) $2\sqrt{5}$

(b) 4

(c) $4\sqrt{5}$

(d) $8\sqrt{5}$

(e) $4\sqrt{13}$



5. Multiply $x^2y^3z^5 \cdot 2xy^2z^5$.

(a) $x^4y^7z^{15}$

(b) $2x^2y^5z^{10}$

(c) $2x^3y^5z^{10}$

(d) $2x^3y^6z^{10}$

(e) $2x^2y^6z^{25}$

6. Which of the following expressions is *not* equivalent to $4(7 + x)$?

(a) $(7 + x)4$ **(b)** $4(x + 7)$ **(c)** $(x + 7)4$ **(d)** $7(4 + x)$ **(e)** $4x + 28$

7. If $x + y = 9$ and $y - x = 7$, then $x^2 + y^2 =$

(a) 1 **(b)** 2 **(c)** 8 **(d)** 63 **(e)** 65

8. If $\begin{bmatrix} 2 & 0 \\ x & 3 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ 6 & 9 \end{bmatrix}$, then $x =$

(a) $\frac{2}{3}$ **(b)** $\frac{3}{2}$ **(c)** 3 **(d)** 4 **(e)** 6

9. Which of the following is a good first step in solving $(x + 3)(x - 2) = 14$?

(a) Set the sum of the factors equal to 14.

(b) Set $(x + 3)$ equal to 2, and set $(x - 2)$ equal to 7.

(c) Set each factor equal to 0.

(d) Set each factor equal to 14.

(e) Multiply to remove the parentheses.

10. In the figure shown, determine the value of x.

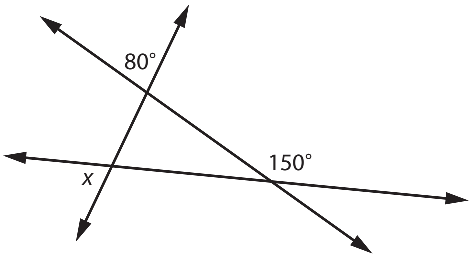
(a) 10°

(b) 30°

(c) 50°

(d) 70°

(e) 80°



Test-Taking Tip

Use Your Calculator to Evaluate Expressions.

When finding numerical answers, the form of your answer may not match any of the choices given. Use your calculator to find a decimal approximation of your answer. Then use your calculator to find the expression that has the same approximation from among the given choices.

Example Look back at Item 4. To use this strategy, first use the Pythagorean Theorem to determine that the length of the third side is $\sqrt{144 - 64} = \sqrt{80}$. The decimal approximation of this is 8.94427.

For choice (a): $2\sqrt{5} \approx 4.472136$.
For choice (c): $4\sqrt{5} \approx 8.94427$.

So, the answer is (c).

Find, if possible, another test item in the practice set for which this strategy might be helpful. Try it.