NAME ____________________

Date __________ Period__________

SYLLABUS

ALGEBRA 2 H

Unit 10: Quadratic Relations

DAY	TOPIC
1	Distance and Midpoint Formulas; Completing the Square
2	Parabolas Writing the Equation
3	Parabolas Graphs
4	Circles
5	Exploring Conic Sections –video **This will make everything crystal clear!**
6	**Review**
7	Ellipses Writing the Equation
8	Ellipses Graphs
9	Hyperbolas Writing the Equation
10	Hyperbolas Graphs; Foci
11	Translating Conics
12	Solving Quadratic Systems

Formulas for the Conic Sections Unit

DISTANCE FORMULA:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

MIDPOINT FORMULA:

$$M=\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)$$

PARABOLA:

$$(y-k)=\frac{1}{4p}(x-h)^2$$ **OR** $$(x-h)=\frac{1}{4p}(y-k)^2$$

CIRCLE:

$$(x-h)^2+(y-k)^2=r^2$$

ELLIPSE:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$ **or** $$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Width > Height
Horizontal Major Axis

Height > Width
Vertical Major Axis

HYPERBOLA:

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$ **or** $$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Unit 10: Day 1 Distance and Midpoint Formulas

For any two points on the coordinate axis (x_1, y_1) *and* (x_2, y_2) we can use the following formulas to find the distance and the midpoint between the points.

A. **The Distance Formula:**
Finds the distance between two points on the coordinate axis.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

B. **The Midpoint Formula:**
Finds the midpoint between two points on the coordinate axis.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

C. **Completing the Square:**

Examples:

1. $x^2 - 9y^2 + 36y - 45 = 0$

2. $3x^2 + 6x + 5y^2 - 20y - 13 = 0$

You try:

3. $x^2 + 2x + y^2 + 14y - 31 = 0$

Unit 10: Day 1 Worksheet

Find the length of each side of the right triangle.

1. A(4,1) B(10,1) C(10,8)

Find the distance between the given points. Express in simplest radical form.

2. P(0,-3) Q(5,-3)

3. P(2x,3y) Q(-3,y)

Find the values of x, if any, that make each of the following true for the distance from A to B.

4. A(x,4) B(-2,3) d=4

5. A(3,x) B(6,2) d=5

6. A(3,1) B(-x,4) d=3

7. A(-2,1) B(-4,x) d=6

Find the midpoint of each segment AB.

8. A(3,-7) B(2,0)

9. A(2,6) B(3.-5)

10. A(-5,1) B(4,7)

Complete the square for each variable in the equation.

11. $x^2 + y^2 - 10x + 8y + 5 = 0$

12. $x^2 + y^2 + 12x - 2y + 21 = 0$

13. $x^2 - y^2 + 4x - 18y + 69 = 0$

14. $x^2 + y^2 + 4x - 5 = 0$

15. $4x^2 - 9y^2 - 16x + 90y + 205 = 0$

Unit 10 (Quadratic Relations), Day 2: Conics Video

Answer the following from the video. Each item may not be the order of the video, so take a moment to glance at what you are looking for.

1. List the 4 conic sections.

2. Explain where the conic sections come from (in general.)

3. Circle

    Definition:

    Application:

    General Equation:

4. Ellipse

    Definition:

    Application:

    General Equation:

5. Parabola

    Definition:

    Application:

    General Equation:

6. Hyperbola

    Definition:

    Application:

    General Equation:

Unit 10 (Quadratic Relations), Day 3: The Parabola

Definition: A parabola is a set of points in the plane equidistant from fixed point called the focus and a fixed line called the directrix.

For each parabola the distance from the vertex to the focus and the vertex to the directrix (perpendicular) is c, where $|a| = \dfrac{1}{4c}$ (See below for a.)

Types of Parabolas:

$y = ax^2$; a > 0

$y = ax^2$; a < 0

$x = ay^2$; a > 0

$x = ay^2$; a < 0

Examples:

1. Write the equation of each parabola.

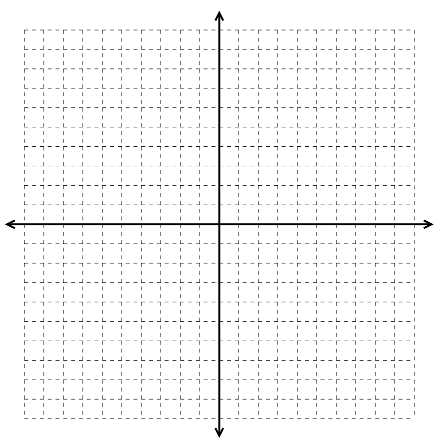
   a. Focus (2, 0) and directrix x = -2

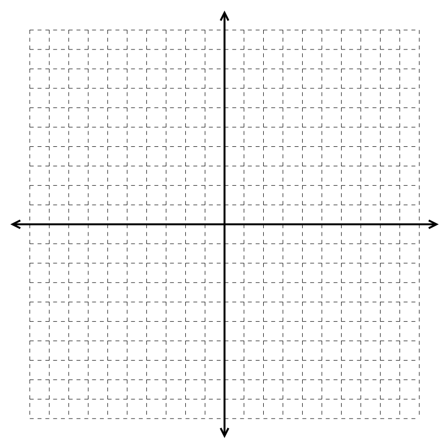
   b. Focus (0, -4) and directrix y = 4

2. Graph a sketch of the parabola, including the focus and directrix.

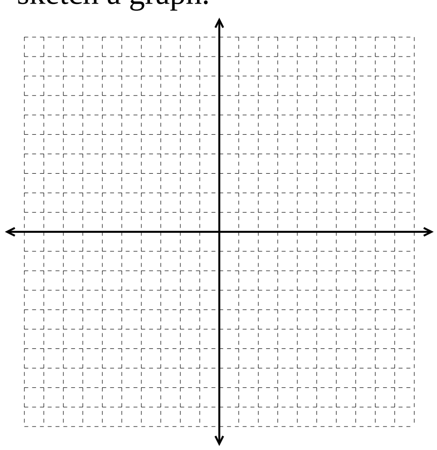
   a. $-20x = y^2$

   b. $y = \frac{1}{12}(x-2)^2$





3. Write the equation of the parabola in standard form, determine the focus and directrix, and sketch a graph. $x^2 + 6x + 4y + 5 = 0$



Unit 10 (Quadratic Relations), Day 4: The Parabola (continued)

Remember:

Standard form for parabolas:

$y = \pm a(x-h)^2 + k$ (when parabolas open up and down)

$x = \pm a(y-k)^2 + h$ (when parabolas open left and right)

Where (h,k) is the vertex of the parabola, $a = \frac{1}{4c}$, c being the directed distance from the vertex of the parabola to the focus of the parabola.

Warm Up

Write an equation whose graph is the set of all points in the plane equidistant from the given point and the given line.

a. F (0,8) and y=-8

b. F(3,0) and x=-3

Examples

1. Identify the focus and the directrix of the graph of the equation $y = \frac{-1}{16}x^2$

2. Identify the focus and the directrix of the graph of the equation $x = \frac{-1}{8}y^2$

3. Identify the focus and directrix of the graph of the equation $(x-2) = \frac{1}{12}(y+4)^2$

4. Write an equation of the parabola in standard form. Then identify the vertex, the focus, and the directrix and graph the equation.

$$y^2 - 4x - 4y + 16 = 0$$

5. Write the equation of the parabola in standard form. Then identify the vertex, the focus, and the directrix and graph the equation.

$$x^2 + 4x + 8y - 4 = 0$$

Practice

1. Identify the vertex, focus, and directrix of the graph of the equation. Then sketch the graph.

   a. $y + 1 = -\frac{1}{4}(x - 3)^2$

   b. $y^2 - 4x - 2y = 3$

2. Answer the last question on yesterday's notes as well ☺

Unit 10 (Quadratic Relations), Day 5: The Circle

Definition: The circle is a set of points in the plane that are equidistant from a fixed point, called the center. The radius is the distance from the center to each point on the circle.

Standard Equation: $(x-h)^2+(y-k)^2=r^2$, where (h, k) is the center of the circle and r is the radius.

A. Write the equation of the circle in standard form.

1. Center (0,0) with a radius of 6.

2. Center (-3, 0) and a radius of $2\sqrt{3}$.

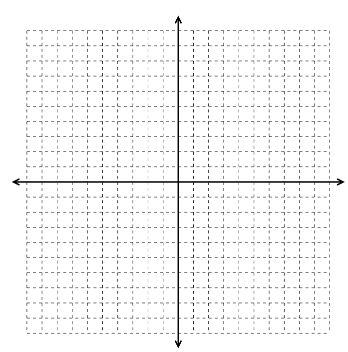
3. Center (-4, 2) and a diameter of 10.

4. Center (-5,6) and a point (2, 3) on the circle.

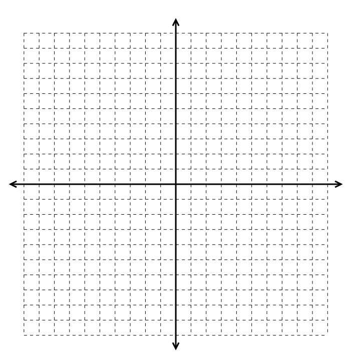
5. Endpoints of (0,8) and (-4, -2) are on the diameter of the circle.

B. Graph each circle.

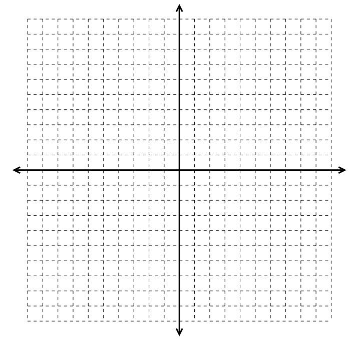
1. $x^2+(y-2)^2=9$



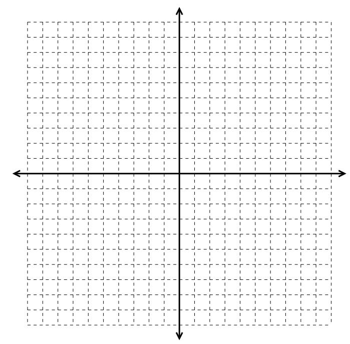
2. $(x+3)^2+(y+1)^2=25$



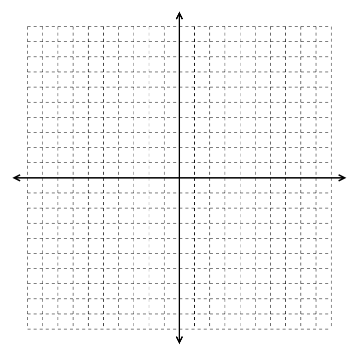
3. $8 - 2x^2 - 2y^2 = 0$



4. $x^2 + y^2 - 14x - 8y + 40 = 0$



5. $x^2 + y^2 + 4x + 8y - 2 = 0$



6. A circle has a radius of 5 and a point (3, 8) on the circle. If the center of the circle is (x, 4), find the value of x.

Unit 10 (Quadratic Relations): Review

Identify the directrix and the focus of each parabola.

1. $x - 5y^2 = 0$

2. $-8y = -x^2$

3. $-x - 3y^2 = 0$

Write an equation with the vertex at the origin.

4. focus at $(-2, 0)$

5. directrix at $y = -3$

6. focus at $(-5, 0)$

Write the equation whose graph is the set of all points in the plane equidistant from the given point and the given line.

7. $F(0, -4)$ and $y = 4$

8. $F(0, 1)$ and $y = -1$

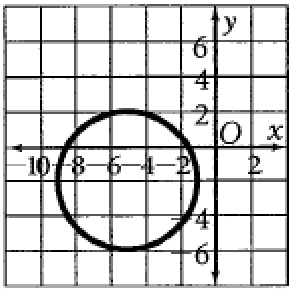
Identify the vertex, focus, and directrix of the graph of each equation. Then sketch the graph.

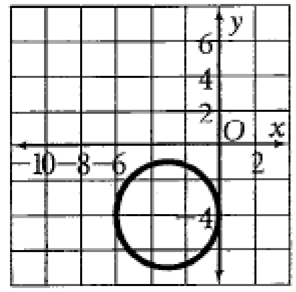
9. $x = 2y^2$

10. $y^2 - 4x - 2y = 3$

Answers! **1.** $\left(\frac{1}{20}, 0\right); x = \frac{-1}{20}$ **2.** (0,2); y=-2 **3.** $\left(-\frac{1}{12}, 0\right); x = \frac{1}{12}$ **4.** $x = -\frac{1}{8}y^2$ **5.** $y = \frac{1}{12}x^2$ **6.** $x = -\frac{1}{20}y^2$ 7. $y = -\frac{1}{16}x^2$ **8.** $y = \frac{1}{4}x^2$ **9.** $(0,0); (\frac{1}{8}, 0); x = -\frac{1}{8}$ **10.** $(-1,1); (0,1); x = -2$

Write an equation in standard form for each circle.

11. 

12. 

Write an equation of a circle with the given center and radius. Check your answers.

13. center (2, 0), radius 1

14. center (2, 3), diameter 1

Write an equation for each translation.

15. $x^2 + y^2 = 9$; right 4 and down 2

16. $x^2 + y^2 = 12$; left 2 and up 5

Find the center and radius of each circle.

17. $(x + 3)^2 + (y + 1)^2 = 2$

18. $x^2 + y^2 = 144$

Complete the square for the following conic sections. Identify the type of conic.

19. $x^2 + 6x - y + 7 = 0$

20. $x^2 + 2x + y^2 - 10y - 38 = 0$

Answers! **11**. $(x+5)^2 + (y+2)^2 = 16$ **12.** $(x-1)^2 + (y-4)^2 = 4$ **13.** $(x+2)^2 + (y-6)^2 = 16$ **14.** $(x-2)^2 + (y-3)^2 = \frac{1}{4}$ **15.** $(x-4)^2 + (y+2)^2 = 9$ **16.** $(x+2)^2 + (y-5)^2 = 1.21$ **17.** $(-3,-1); \sqrt{2}$ **18**. (0,0); 12 **19.** parabola **20.** circle

Unit 10 (Quadratic Relations): Day 7 : The Ellipse

Definition: The ellipse is a set of points in the plane such that the sum of the distances from two fixed points, called foci (plural of focus) to a point on the ellipse is constant.

	Type 1: Horizontal	**Type 2: Vertical**
General Equations:	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

Properties:

- (h, k) is the center
- a > b
- The vertices of the ellipse along the major axis are "a" units from the center.
- The co-vertices of the ellipse along the minor axis are "b" units from the center.
- The foci are located along the major axis.
- The distance from the center to each focus is "c", which is given by $c^2 = a^2 - b^2$

A. Write the equation of the ellipse in standard form.

1. Center (0,0) and intercepts at (0, $\pm$2) and ($\pm$5, 0).

2. Center (0,0) and intercepts at (0, $\pm$7) and ($\pm$1, 0).

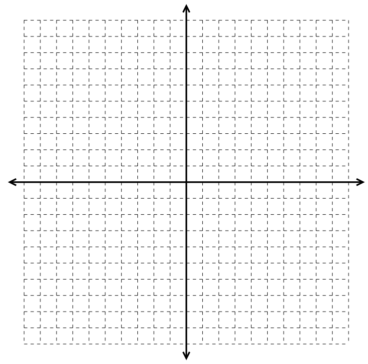
3. Center (1, -2), horizontal major axis of length 10 and vertical minor axis of length 4.

4. Vertices at (-2, 0), (-2, 2), (0, 1) and (-4, 1).

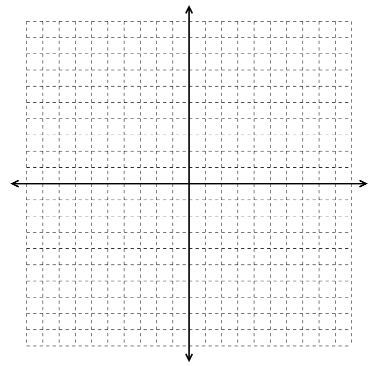
5. Minor axis of length 6 and foci at (0, 4) and (0, -4)

B. Graph each ellipse, including vertices and foci.

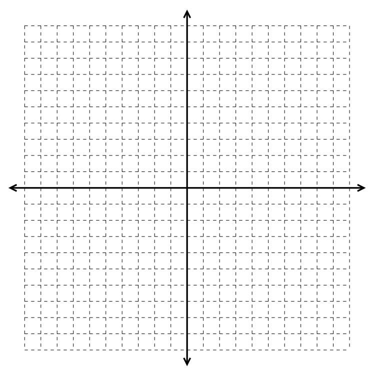
1. $\dfrac{x^2}{16}+\dfrac{(y+2)^2}{36}=1$



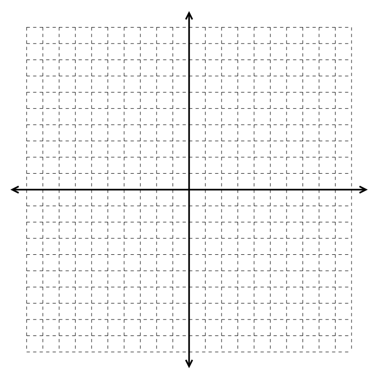
2. $x^2+4(y+1)^2=32$



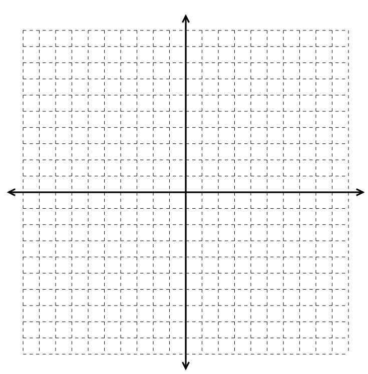
3. $9x^2+16y^2=144$



4. $9x^2+4y^2+54x-8y+49=0$



5. $16x^2+y^2-32x+2y-47=0$



Unit 10 (Quadratic Relations): Day 8 : The Ellipse (More practice)

Find the foci for each equation of an ellipse. Then graph the ellipse.

1. $\frac{x^2}{36} + \frac{y^2}{81} = 1$

2. $x^2 + \frac{y^2}{36} = 1$

3. $\frac{x^2}{9} + \frac{y^2}{100} = 1$

4. $3x^2 + 9y^2 = 9$

5. $4x^2 + 8y^2 = 16$

6. $12x^2 + 4y^2 = 48$

Write an equation of each ellipse in standard form with center at the origin and with the given characteristics.

7. foci $(\pm 5, 0)$; co-vertices $(0, \pm 2)$

8. height 10; width 8

9. vertex $(0, 10)$; co-vertex $(-7, 0)$

10. foci $(\pm 3, 0)$; co-vertices $(0, \pm 3)$

Unit 10 (Quadratic Relations): Day 9& 10 : The Hyperbola

Definition: The hyperbola is a set of points in the plane such that the difference of the distances from two fixed points, called foci (plural of focus) to a point on the hyperbola is constant.

	Type 1: Horizontal	Type 2: Vertical
General Equations:	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

Properties:

- (h, k) is the center of the hyperbola.
- "a" is under the positive term.
- When "a" is under the x terms, the hyperbola opens right/left.
- When "a" is under the y terms, the hyperbola opens up/down.
- The vertices of the hyperbola "a" units from the center, which are on the *transverse axis.*
- The *conjugate axis* is "b" units from the center in either direction.
- The ends of the hyperbola are guided by asymptotes which have the slopes: $\pm b/a$ for the horizontal and $\pm a/b$ for the vertical hyperbola.
- The distance from the center to each focus is "c", which is given by $c^2 = a^2 + b^2$

A. Write the equation of the hyperbola in standard form.

1. Center (0,0) , intercepts at (0, $\pm$5) and a conjugate axis of 6.

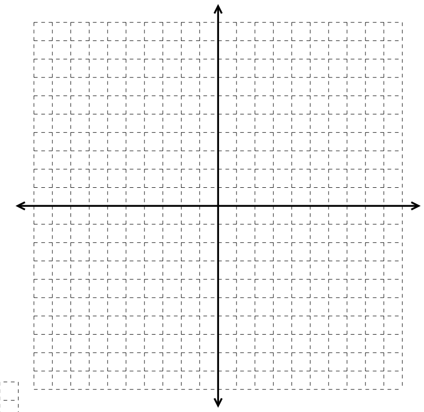
2. Center (0,0) , intercepts at ($\pm$4, 0) and foci at ($\pm$5, 0).

3. Vertices at (-2, 10) and (-2, -10) and foci at (-2, 14) and (-2, -14).

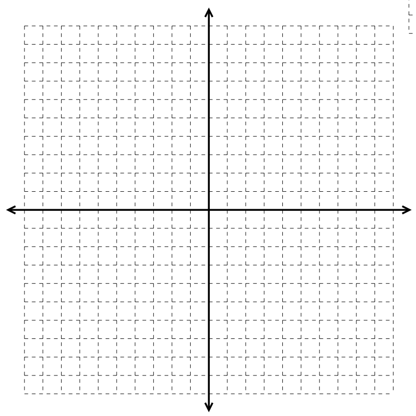
4. Center at (3, 1) with a horizontal transverse axis of 4 and a conjugate axis of 10.

B. Graph each hyperbola, including asymptotes and foci.

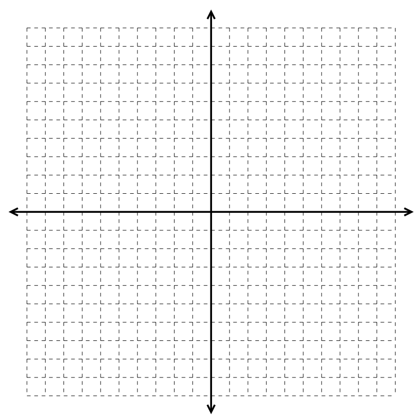
1. $\dfrac{y^2}{16} - \dfrac{x^2}{49} = 1$



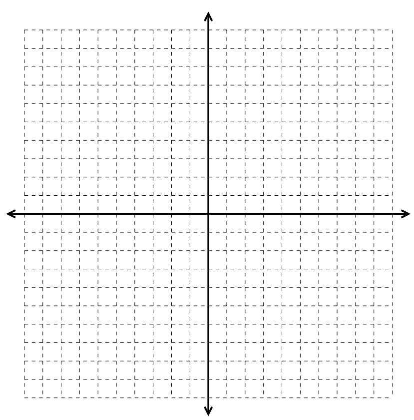
2. $x^2 - 4y^2 = 16$



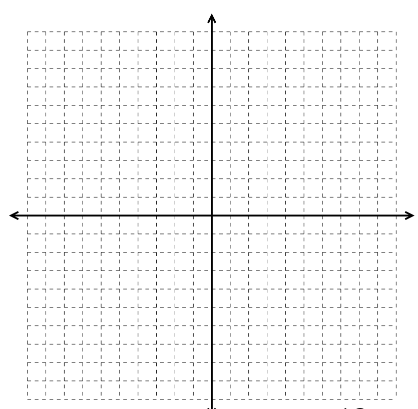
3. $\dfrac{(x+3)^2}{1} - \dfrac{(y+6)^2}{24} = 1$



4. $y^2 - x^2 + 10y + 6x + 12 = 0$



5. $16x^2 - 9y^2 - 36y - 180 = 0$



Unit 10 (Quadratic Relations): Day 9 & 10 : The Hyperbola

Write an equation for the hyperbola that meets each set of conditions.

1. The center at (4,-2) , a=2, b=3, and it has a vertical transverse axis.

2. The vertices are at (0,3) and (0,-3) and the focus is at (0,-9)

3. The length of the transverse axis is 6 units, and the foci are at (5,2) and (-5,2)

4. The center is at the origin and a=7 and c=9. Assume that the transverse axis is horizontal.

5. The center is at the origin and a=8 and c=10. Assume that the transverse axis is horizontal

Write an equation of a conic section with the given characteristics.

21. circle with center $(-4, 5)$, radius 6

22. hyperbola with center $(-4, 5)$, one vertex $(-4, 7)$, one focus $(-4, 8)$

23. Points on the hyperbola are 96 units closer to one focus than to the other. The foci are located at $(0, 0)$ and $(100, 0)$.

24. parabola with vertex $(1, -2)$, x-intercept 3, and opens to the right

25. ellipse with center $(0, 2)$, horizontal major axis of length 6, minor axis of length 4

26. ellipse with center $(-4, -5)$, endpoints of major and minor axes $(-4, -7)$, $(-4, -3)$, $(-1, -5)$, $(-7, -5)$

27. circle with center $(-1, 2)$, diameter 12

28. parabola with vertex $(-1, 5)$, y-intercept 4, and opens downward

29. hyperbola with vertices $(0, 2)$ and $(4, 2)$, foci $(-1, 2)$ and $(5, 2)$

30. ellipse with center $(2, -5)$, one end of each axis $(2, -9)$ and $(-3, -5)$

31. Points on the hyperbola are 12 units closer to one focus than to the other. The foci are located at $(0, 0)$ and $(250, 0)$.

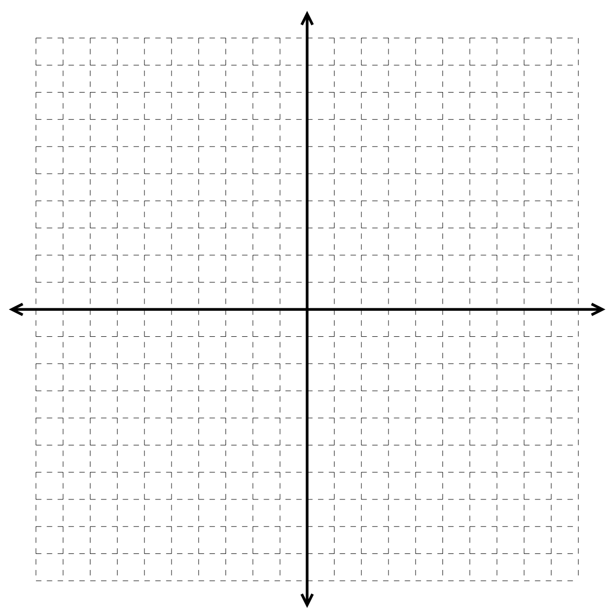
32. ellipse with center $(0, -2)$, vertical major axis of length 5, minor axis of length 3

Unit 10 (Quadratic Relations): Day 12 :

Show the algebraic and graphical methods of solving the system from page **577, #2.**

A. Algebraically:

B. Graphically:



Solutions:____
(Be sure you have all of them.)