Unit 12
Solving Equations

Lesson Outline

	BIG PICTURE		
	Students will: • translate statements of mathematical relationship into equations; • solve and verify linear equations with one variable term using a variety of strategies.		
Day	**Lesson Title**	**Math Learning Goals**	**Expectations**
1	What Makes It True?	• Use inspection and estimation to 'guesstimate' integer solutions to equations having two or three terms, one of which contains a variable. • Justify estimations of solutions to simple equations. • Check estimation solutions by a formal verification process.	8m18, 8m23, 8m64 CGE 3c, 4f
2, 3	A Balancing Act	• Use 'inspection' and 'guess and check' to solve equations with integer solutions having one variable term and two constant terms. • Use a 'balance model' to solve equations with integer solutions having one variable term and two constant terms. • Virtual website http://matti.usu.edu/nlvm/nav/vlibrary.html Choose Index → Algebra Balance Scales → Algebra (9–12)	8m18, 8m23, 8m64 CGE 5b, 7b
4	Breaking the Code	• Find an expression for the general term of a given number pattern. • Determine, given the general term of a number pattern, which term has a particular value by solving an equation.	8m18, 8m56, 8m63, 8m64 CGE 3c, 4b
5, 6	Model Solutions	• Solve problems having integer solutions by first algebraically modelling the facts with an equation and then solving the equation.	8m22, 8m59, 8m61, 8m64 CGE 2b, 4f
7	Summative Assessment		

Unit 12: Day 1: What Makes It True?

Grade 8

	Math Learning Goals • Use inspection and estimation to 'guesstimate' integer solutions to equations having two or three terms, one of which contains a variable. • Justify estimations of solutions to simple equations. • Check estimation solutions by a formal verification process.	**Materials** • BLMs 12.1.1, 12.1.2, 12.1.3 • Chart paper and markers • White/blackboard • Tape • Whiteboard markers or chalk

Minds On…

Individual/Small Groups → Think/Share
Hand each student one card (BLM 12.1.1). Students solve their equation and find other students who have the same solution. In their groups, students discuss the strategies they used to solve their equations. How were these strategies the same/different?

On chart paper or white-/blackboard, create the headings "Inspection," "Systematic Substitution," and "Other." Have each student tape his/her equation under the title that best describes the method he/she used to solve the equation.

Key Words
Inspection
Systematic substitution
Guess-and-check

Assessment for learning – note where students place their equations and reasons

Action!

Whole Class → Discussion
Discuss with class the types of equations that they found they could solve through inspection and systematic substitution. Note in the discussion that systematic substitution requires thinking about the reasons behind selecting your numbers and using the information from each number to inform your choice of the next number to try. Briefly discuss the strategies they used and classified as "Other" noting any that used a balance model that you may reference on Day 2.

Partners → Open Question
An equation uses the variable "n." The equation equals 20 when n = 8. What might the equation be if you and your partner could solve it using inspection? What might the equation be if you and your partner needed to use systematic substitution to solve it?
Have pairs of students post their questions under the appropriate heading on the board. Discuss the differences between the equations produced in each category. How are they the same/different? When might you use inspection? Systematic substitution?

Flexible Partners → Investigation
Give each student either an expression or an answer (BLM 12.1.2). Have students with expressions pair up with students who have answers to make an equation. The paired students estimate what the solution to their equation might be, record their estimation and reasoning (BLM 12.1.3). They work together to solve for the unknown and record their actual answer. Students then verify their solutions by recording their equation on the white-/blackboard, substituting in their answer and solving to check. Create a spot on the white-/blackboard where students can post any equations that they felt were "not solvable" along with the reason why each equation did not seem to be solvable. Once a pair of students has solved their equation and verified it, they should find new partners and repeat the activity.

Assessment for learning – note types of equations student are able to solve by inspection versus systematic trial

Assessment for learning – note types of equations students indicate are not solvable and integrate into either Further Classroom Consolidation or next day's lesson

Consolidate Debrief

Whole Class → Discussion
Create a class list on chart paper of strategies used by students to support the reasonableness of their estimations. Include examples.

Assessment for/as learning – note strategies used and not used – keep chart to reference in future

Practice Exploration

Home Activity or Further Classroom Consolidation
Examine the list of equations that were "not solvable." Invite students to try to solve. Are they truly not solvable? Can we prove this?

TIPS4RM: Grade 8 – Unit 12: Solving Equations

Note: Cards are arranged in rows such that each row of equations has the same solution so that groups of 3 students will be formed. Cards should be cut apart and mixed up prior to distributing to students.

$n + 3 = 5$	$3n + 5 = 11$	$24 - n = 22$
$10 = n + 7$	$7 = 3n - 2$	$n - 3 = 0$
$5n + 1 = 21$	$n + 12 = 16$	$2 = n - 2$
$8 = 3n - 7$	$2 = n - 3$	$17 = n + 12$
$7 + n = 13$	$n - 3 = 3$	$4n - 10 = 14$

$11 = n + 4$	$70 + n = 77$	$93 - n = 86$
$9 = 2n - 7$	$n \times 2 = 16$	$n - 2 = 6$
$1 + n = 10$	$n \times n = 81$	$90/n = 10$
$90 + n = 100$	$5 = n/2$	$11n = 55$
$n - 15 = 10$	$4n = 100$	$n + n = 50$

Expressions

$n + 5$	$n - 5$	$5n$	$n/5$
$3n + 5$	$3n - 5$	$24 - n$	$24 + n$
$2n$	$n/2$	$2n + 4$	$2n - 4$
$4n + 8$	$4n - 10$	$n + 52$	$4/n$
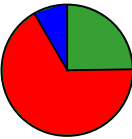
Answers

18	**20**	**2**	**10**
24	**100**	**1000**	**80**
32	**55**	**500**	**68**
6	**250**	**85**	**48**

12.1.3: What Makes It True?: Student Recording Sheet

Name:

Expression	Answer	Estimated Solution	Reason for My Estimation	Solution
Sample: $2n + 1 = 12$		**5**	**2 x 6 = 12 but it has to be less than 6 because you have to add 1 to get the 12**	**2 x 5 + 1 = 11 (5 is not large enough)** **2 x 6 + 1 = 13 (6 is too large; the number needs to be between 5 and 6** **2 x 5.5 + 1 = 12**
=				
=				
=				
=				

Unit 12: Days 2 & 3: A Balancing Act

		Materials
.	**Math Learning Goals** • Use 'inspection' and 'guess and check' to solve equations with integer solutions having one variable term and two constant terms. • Use a 'balance model' to solve equations with integer solutions having one variable term and two constant terms. • Virtual website http://matti.usu.edu/nlvm/nav/vlibrary.html Choose Index → Algebra Balance Scales → Algebra (9–12)	• BLM 12.2.1 • 2-pan balance scales/group • Linking cubes • Paper bags – 2/group with 3 linking cubes in each bag • Chart paper and markers • Tape • Post-it notes • Computers and internet • Coloured paper
Minds On…	**Pairs → Matching** Students will work with partner to match the equations with the statements (BLM 12.2.1) that explain a situation in words. Discussion questions: "How do you know that this equation goes with this statement?" "What does the letter x represent in this equation?" "What are the key words that helped you decide?" Students then solve the algebraic equations and state what their answer represents. Have students share their solutions and strategies. "Which equations were more challenging to solve? Why?" What does the "=" sign mean? **Pairs → Co-creating Exemplars** Have students create examples and non-examples of equations that are balanced or equal. Post as exemplars.	*Assessment for learning* - observe students' proficiency with matching and solving – plan to adjust program accordingly
Action!	**Small Groups → Investigation** Give each group a 2-pan balance, 2 paper bags each containing 3 linking cubes, 6 additional cubes. Have them place the 2 bags, without opening them, on one side of the scale. The other 6 cubes go on the other side of the scale. Students have to determine how many cubes are in each bag and record the steps they took to determine the answer, on chart paper. Post answers. Which answers keep the equation in balance in each step? Why or why not? Students could use post-it notes in a gallery walk to indicate places on other groups' answers where they have questions (?) or where the answer was particularly clear (*). Have students use the paper bags and additional linking cubes to create a new equation for another group to solve. Have students individually record their answers step-by-step. Have students in each group compare their answers with each other. Popcorn reflection: "How could I make my answer clearer for someone to read?" Repeat this activity. Have students practice balancing equations using the virtual website http://matti.usu.edu/nlvm/nav/vlibrary.html Choose Index → Algebra Balance Scales → Algebra (9–12) Give students examples of more challenging equations to try to solve using the balance model without the actual manipulatives. These may be drawn from Day 1's "Not Solvable" list or from the following: $12x + 7 = 79$; $50 - 3x = 65$; $18x + 7 = 151$. Students who need to continue to use the actual balance scale and cubes should be given equations with smaller numbers. Have students check their work and each other's asking "Is each line in balance?" Encourage students to verify their solutions by substituting in their answer and solving the equations.	*Assessment for learning* - observe students' proficiency in communicating each step – consider co-creating anchor charts about how to effectively communicate solutions *Assessment for learning* - number of repetitions depends on students' understanding, communication skills and the level of challenge required to engage them *Differentiation* of content based on readiness. *Assessment as learning* - students analyze their own work and each others

	Individual → Find the Error	
Consolidate Debrief	Give students an equation and solution that contains an error. Students must locate the error and correct it explaining why it is incorrect.	*Assessment **for** learning* - students' ability to find and correct the errors will provide information on their ability to solve equations
	Samples of incorrect solutions:	
	$\$55 + \$3n = \$203$ $\qquad\qquad$ $\$55 + \$3n = \$203$	
	$\$55 + \$55 + \$3n = \$202 + \$55$ $\qquad$ $\$55 - \$55 + \$3n = \$202 - \$55$	
	$\$3n = \$257$ $\qquad\qquad\qquad$ $\$3n = \$147$	
	$\$3n//\$3 = \$257/\$3$ $\qquad\qquad$ $\$3n//\$3 = \$147/\$3$	
	$n = 85.67$ $\qquad\qquad\qquad$ $n = 441$	
	Correct solution:	
	$\$55 + \$3n = \$203$	
	$\$55 - \$55 + \$3n = \$202 - \$55$	
	$\$3n = \$147$	
	$\$3n//\$3 = \$147/\$3$	
	$n = 49$	
	Note: Look for common errors or misconceptions in your students' work and use those to build your Find the Error question(s).	*Differentiation of content* based on student need.

Home Activity or Further Classroom Consolidation

Telephone Equation

Practice Exploration

In small groups, students create an equation on a coloured sheet of paper. Their equation is passed to the next group who solves the equation and copies the answer (i.e. just "$x = 10$") onto a new sheet of paper that is the same colour, titled Round 2. The Round 2 answer is passed to the next group who has to create an equation that has the same answer. They record their equation as Round 3 and pass it to the next group. Once the colour paper has returned to the original group, the game has ended. Post the equations and answers in sequence and discuss how the equations changed through the rounds. The answers should have stayed the same!

*Assessment **for** learning* - students' thinking and ability will be demonstrated through the equations they create and the solutions that they develop

*Assessment **as** learning* - students reflect on their work

Name:

2x + 7 = 29	**The kindergarten class had 32 shoes in the hallway. After a number of students put on their shoes and went outside, there were 20 shoes left in the hallway.**
2x – 9 = 53	**Four times a number, decreased by 7, is 21**
32 – 2x = 20	**Five times a number, increased by 7 is 27**
5x + 7 = 27	**Jackson's age is double Susan's age increased by 7. Jackson is 29.**
4x – 7 = 21	**Sam has a 53-meter blue ribbon that is 9 meters shorter than twice the length of his red ribbon.**

$2x + 7 = 29$ *x* = 11 *x* represents Susan's age	**Jackson's age is double Susan's age increased by 7. Jackson is 29.**
$2x - 9 = 53$ *x* = 31 *x* represents Susan's age	**Sam has a 53-meter blue ribbon that is 9 meters shorter than twice the length of his red ribbon.**
$32 - 2x = 20$ *x* = 6 *x* represents the number of students who went outside because they each put on 2 shoes	**The kindergarten class had 32 shoes in the hallway. After a number of students put on their shoes and went outside, there were 20 shoes left in the hallway.**
$5x + 7 = 27$ *x* = 4 *x* represents the unknown number	**Five times a number, increased by 7 is 27**
$4x - 7 = 21$ *x* = 7 *x* represents the unknown number	**Four times a number, decreased by 7, is 21**

Unit 12: Day 4: Breaking the Code

Grade 8

	Math Learning Goals • Find an expression for the general term of a given number pattern. • Determine, given the general term of a number pattern, which term has a particular value by solving an equation.	**Materials** • BLMs 12.4.1, 12.4.2
Minds On…	**Pairs → Open Question/Share** *Create the first 5 terms of a linear growing pattern. What is the general term for the pattern? What picture could you make of your pattern to help someone else see what the next terms might be? How does your picture show what the general term is?* Have each pair of students share their patterns and representations with another pair of students. Can the students solve each other's patterns for the next two terms and the general term? How does the picture help them do this?	*Differentiation* of content based on students' choice *Assessment **for** learning* - representations - strategies to solve for *n*th term
Action!	**Individuals/Small Groups → Stations Investigation** Divide the students into 8 groups. Station each group at centres (BLM 12.4.1) around the room – 2 copies of each centre. Have students move from centre to centre recording their answers on BLM 12.4.2. At each centre, students compare their patterns/strategies with others in their group and determine whose pattern is most like theirs and whose is different.	
Consolidate Debrief	**Whole Class → Discussion** Co-create an anchor chart of strategies students use to find the *n*th term. Use student work samples from the centres for examples. Discuss: How do you find the value of a particular term? How do you make an equation? How do you solve the equation? Use student work samples from the centre for examples.	*Assessment **for** learning* - note what strategies students are comfortable with and consider new strategies they may need exposure to
Practice	**Home Activity or Further Classroom Consolidation** Each student creates the first 5 terms of a linear growing pattern and a 3-digit number. Students exchange their patterns with another student who develops the next 3 terms, creates a representation of the pattern and finds the rule for the *n*th term. He/she then determines if the 3-digit number he/she has been given occurs in the pattern and explains why or why not. Each student shares his/her solution and reasoning with the pattern creator.	*Assessment **as** learning* (reflection)

12.4.1: Breaking the Code: Station Cards

(create 2 centres of each station so that there are 8 groups of students)

Station A

The 5th term of a linear pattern is 10.   What could the pattern be?

_____, _____, _____, _____,   10 , _____, _____, …

What is the general term for the pattern you created?

How do you know your pattern is linear?

Is your pattern increasing or decreasing?

If you extended your pattern would the number 205 be in it? How do you know?

Station B

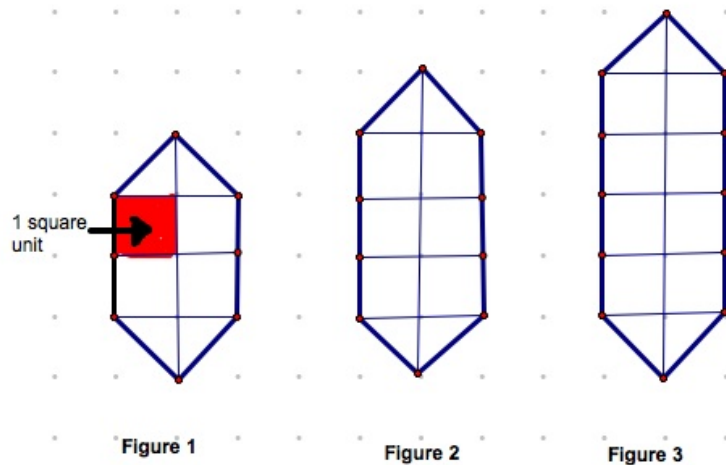The nth terms of two number patterns are:

Pattern I: 3n - 2

Pattern II: 7n + 6

  a) What is the 45th term of Pattern I?

  b) Which term in Pattern II has this same value? How do you know?

Station C

1 square unit

Figure 1   Figure 2   Figure 3

a) What is the area of each figure?

b) If this is a linear growing pattern, what would the area of Figure 10 be? Why do you think this?

c) What could the expression for the *n*th frame be?

d) What is the area of Figure 50? How do you know?

e) If the pattern is extended, would there be a figure with an area of 3,426 square units? How do you know?

Station D

Pick one of the patterns below:

Pattern A: $4n + 6$ Pattern B: $3n - 10$

a) If you extended the pattern, would the number 2,642 appear? How do you know?

b) Do you have to extend the pattern to find out if the number 2,642 appears? Why or why not?

c) What operations did you do to find out if the number 2,642 appears in the pattern? How do the operations you did relate to the operations in the pattern rule you chose?

Name:

Station A

Your pattern: _____, _____, _____, _____, _____ , _____, _____, …

General Term:

Is your pattern linear? How do you know?

Is your pattern increasing or decreasing?

Is the number 205 in your pattern? How do you know?

Of your group members, whose pattern is the same as /different than yours? Why**?**

SAME	DIFFERENT

Station B

a)   What is the 45th term of Pattern I?

b)   Which term in Pattern II has this same value? How do you know?

Of your group members, whose strategy is the same as /different than yours? Why?

SAME	DIFFERENT

Station C

a) What is the area of each figure?

Figure 1	Figure 2	Figure 3

b) What would the area of Figure 10 be? Why do you think this?

c) What could the expression for the *n*th frame be?

d) What is the area of Figure 50? How do you know?

e) If the pattern is extended, would there be a figure with an area of 3,426 square units? How do you know?

Of your group members, who solved it the same way you did? Differently than you?

SAME	DIFFERENT

Station D

a) If you extended the pattern, would the number 2,642 appear? How do you know?

b) Do you have to extend the pattern to find out if the number 2, 642 appears? Why or why not?

c) What operations did you do to find out if the number 2, 642 appears in the pattern? How do the operations you did relate to the operations in the pattern rule you chose?

Of your group members, who got the same answer to question (a)? Did they choose the same pattern that you did?

SAME	DIFFERENT
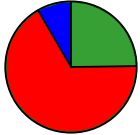
	Math Learning Goals • Solve problems having integer solutions by first algebraically modelling the facts with an equation and then solving the equation.	**Materials** • BLMs 12.6.1, 12.6.2 • Chart paper • Markers
Minds On…	**Pairs → Think, Pair, Share** *For the year-end graduation party, Mrs. Awesome is going to rent the local bowling alley. It will cost $95.00 for unlimited bowling for the class, but the shoe rental is $2 per student. Which of the following expressions could Mrs. Awesome use to determine the total cost of the bowling party for her class?* *A. 95s + 2 B. 2s + 95 C. 97s* *Explain your thinking.* Follow-up Questions: How could this expression be used to find out information about the party? (Possible answers: How much it costs for a certain number of students, the number of students going if given the total cost) If the total cost of the party was $151.00, what equation could you write to determine the number of students at the party? *(Answer: $151 = 2s + 95)* Why do the other two expressions not work for this situation? Choose one of those expressions and create a story that would work for it. Have some pairs share their stories with the whole class.	*Assessment for learning* - students' understanding of how parts of an equation are related to the problem - students' understanding of how an expression and equation are related
Action!	**Pairs → Gallery Walk** Have pairs of students work on the following problem, recording their work on chart paper with markers. *The average weight of a kitten at birth is 100 grams and can gain between 14-18 grams a day. If the average weight of an adult cat is 4990 grams, and kittens usually gain the same amount of weight each day, how long does it take the kitten to grow to full size? Create an algebraic equation that you can use to solve this problem. Solve the problem. Is your answer reasonable? Why or why not?* Note to teachers: Students can pick any number from 14 – 18 as the weight gained per day (student choice). Students share their solutions using a Gallery walk approach, looking at different approaches and how information was communicated (i.e., letters, pictures, symbols, words, etc.). How are the equations the same or different? What parts of students' solutions are clear? What parts of students' solutions leave the reader with a question about what they were thinking? Lead a class discussion of what they noticed during the gallery walk, comparing different approaches and communication of solutions. Consider co-creating an anchor chart with students of criteria for clearly communicating algebraic thinking.	*Assessment for learning* - students' understanding of how to create and use an algebraic equation *Assessment as learning* - students provide feedback to each other

	Pairs → Doing and Undoing "What's the Problem?" Students choose an equation card from the Equation Deck (BLM 12.6.1) and a card from the Theme Deck. (BLM 12.6.2) and with their partner they create a word problem using the theme that explains the equation. Students trade their story with another pair and try to write the equation from the problem. Students check in with the authors to confirm the correct equation and then solve the equation. If time, repeat process with another pair.	*Assessment **as** learning* - students provide feedback to each other
Consolidate Debrief	**Whole Class → Discussion** What equations/themes seemed to be the easiest to use to create a scenario? Which equations/themes did you find the most challenging to work with? Why? Are there any key parts that you found your scenario had to have? (Answers should lead to students' observing that something had to remain the same – constant – and something had to change – vary/variable) Did any pair have to change either their equation or their scenario? If so, why? Give students the following mismatched pair (equation and word problem): *Samir's cell phone plan charges him a flat rate of $35 per month plus $0.05 per text message that he sends. Incoming text messages are free. If "t" represents the number of text messages that Samir sends, the following equation can be used to determine the monthly cost of his cell phone:* $$\$35t + \$\,0.05$$ How could you change the equation to match the problem? How could you change the problem to match the equation?	*Assessment **as** learning* - students provide feedback to each other *Assessment **for** learning* - students' ability to relate equations and scenarios
Application Practice	**Home Activity or Further Classroom Consolidation** Pizza Palace problem – TIPS Continuum and Connections: Patterning to Algebraic Modelling, p. 24 Amusement Parks problem – TIPS Continuum and Connections: Solving Equations and Using Variables as Placeholders, p. 19	

$45 = 2x + 5$	$20 = 5x + 10$	$67 = 3x + 1$
$50 = 3x + 14$	$15 = 5x + 10$	$20 = 2x + 8$
$55 = 10x + 5$	$43 = 3x + 13$	$37 = 3x + 4$
$25 = 2x + 15$	$57 = 5x + 2$	$50 = 4x + 10$
$75 = 12x + 15$	$32 = 2x + 10$	$35 = 2x + 5$

SHOPPING	FOOD	ART
YOUR FAVORITE ATHLETE	AMUSEMENT PARK	BABY-SITTING
GYM	ONLINE GAMING	DOWN-LOADING MUSIC
PHONE APPS	YOUR FAVOURITE BAND	ANIMALS
SCHOOL	DANCE	YOUR CHOICE