

Name _____
St.No. _____

Date(YY/MM/DD) ____/____/____
Section _____

UNIT 12: ROTATIONAL MOTION

Approximate Time: Three 100-minute sessions



*To every thing - turn, turn, turn
there is a season -- turn, turn, turn
and a time for every purpose under heaven.*

--Pete Seeger

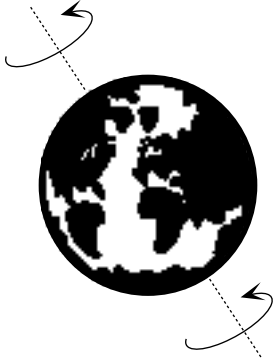
(With a little help from Ecclesiastes)

OBJECTIVES

1. To understand the definitions of angular velocity and angular acceleration.
2. To understand the kinematic equations for rotational motion on the basis of observations.
3. To discover the relationship between linear velocity and angular velocity and between linear acceleration and angular acceleration.
4. To develop definitions for rotational inertia as a measure of the resistance to rotational motion.
5. To understand torque and its relation to angular acceleration and rotational inertia on the basis of both observations and theory.

OVERVIEW

25 min



Earlier in the course, we spent a session on the study of centripetal force and acceleration, which characterize circular motion. In general, however, we have focused on studying motion along a straight line as well as the motion of projectiles. We have defined several measurable quantities to help us describe linear and parabolic motion, including position, velocity, acceleration, force, and mass. In the real world, many objects undergo circular motion and/or rotate while they move. The electron orbiting a proton in a hydrogen atom, an ice skater spinning, and a hammer which tumbles about while its centre-of-mass moves along a parabolic path are just three of many rotating objects.

Since many objects undergo rotational motion it is useful to be able to describe their motions mathematically. The study of rotational motion is also very useful in obtaining a deeper understanding of the nature of linear and parabolic motion.

We are going to try to define several new quantities and relationships to help us describe the rotational motion of rigid objects, i.e., objects which do not change shape. These quantities will include *angular velocity*, *angular acceleration*, *rotational inertia* and *torque*. We will then use these new concepts to develop an extension of Newton's second law to the description of rotational motion for masses more or less concentrated at a single point in space (e.g. a small marble) and for extended objects (like the tumbling hammer).

SESSION ONE: ROTATIONAL KINEMATICS

5 min

Rigid vs. Non-rigid Objects

We will begin our study of rotational motion with a consideration of some characteristics of the rotation of rigid objects about a fixed axis of rotation. The motions of objects, such as clouds, that change size and shape as time passes are hard to analyse mathematically. In this unit we will focus primarily on the study of the rotation of particles and *rigid objects* around an axis that is not moving. A *rigid object is defined as an object which can move along a line or can rotate without the relative distances between its parts changing.*



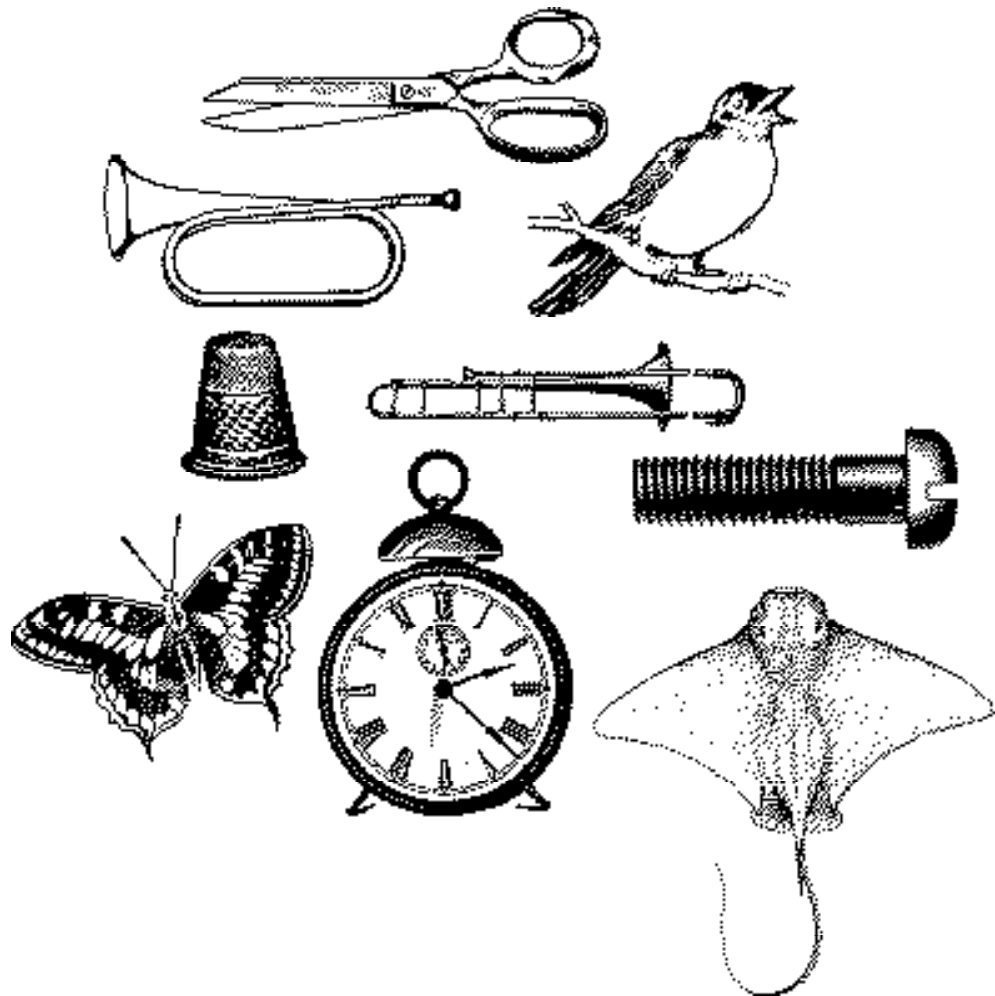
Figure 12-1: Examples of a non-rigid object in the form of a cloud which can change shape and of a rigid object in the form of an empty coffee cup which does not change shape.

The hammer we tossed end over end in our study of centre-of-mass and an empty coffee cup are examples of rigid objects. A ball of clay which deforms permanently in a collision and a cloud which grows are examples of non-rigid objects.

By using the definition of a *rigid* object just presented in the overview can you identify a rigid object?

Activity 12-1: Identifying Rigid Objects

A number of objects are pictured below. Circle the ones which are rigid and place an X through the ones which are not rigid.



10 min

A Puzzler

Use your imagination to solve the rotational puzzler outlined below. It's one that might stump someone who hasn't taken physics.

 **Activity 12-2: Horses of a Different Speed**

You are on a white horse, riding off at sunset with your partner on a chestnut mare riding at your side. Your horse has a speed of 4.0 m/s and your partner's horse has a speed of 3.5 m/s, yet he or she constantly remains at your side. Where are your horses? Make a sketch to explain your answer.

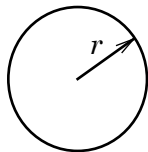


Review of the Geometry of Circles

Remember way back before you came to college when you studied equations for the circumference and the area of a circle? Let's review those equations now, since you'll need them a lot from here on in.

 **Activity 12-3: Circular Geometry**

(a) What is the equation for the circumference, C , of a circle of radius r ?



(b) What is the equation for the area, A , of a circle of radius r ?

(c) If someone told you that the area of a circle was $A = \pi r$, how could you refute them immediately? What's wrong with the idea of *area* being proportional to r ?

Distance from an Axis of Rotation and Speed

Let's begin our study by examining the rotation of objects about a common axis that is fixed. What happens to the speeds of different parts of a rigid object that rotates about a common axis? How does the speed of the object depend on its distance from an axis? You should be able to answer this question by observing the rotational speed of your own arms.

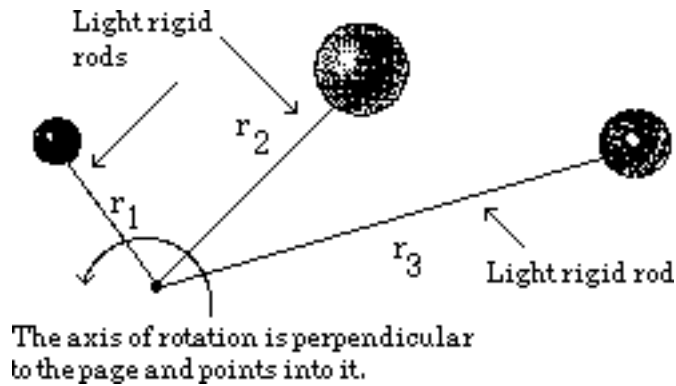


Figure 12-2: A rigid system of masses rotating about an axis

For this observation you will need:

- A partner
- A stopwatch
- A metre stick

Spread your arms and slowly rotate so that your fingertips move at a constant speed. Let your partner record the time as you turn.

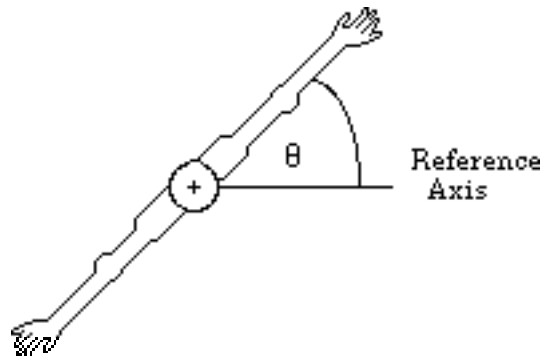


Figure 12-3: Rotating arms featuring elbows and hands

 **Activity 12-4: Twirling Your Arms – Speed vs. Radius**

(a) Measure how long it takes your arm to sweep through a known angle. Record the time and the angle in the space below.

(b) Calculate the distance of the paths traced out by your elbow and your fingers as you rotated through the angle you just recorded. (Note: What do you need to *measure* to perform this calculation?) Record your data below.

(c) Calculate the average *speed* of your elbow and the average *speed* of your fingers. How do they compare?

(d) Do the speeds seem to be related in any way to the distances of your elbow and of your finger tips from the axis of rotation? If so, describe the relationship mathematically.

(e) As you rotate, does the distance from the axis of rotation to your fingertips change?

(f) As you rotate, does the distance from the axis of rotation to your elbows change?

(g) At any given time during your rotation, is the angle between the reference axis and your elbow the same as the

angle between the axis and your fingertips, or do the angles differ?

(h) At any given time during your rotation, is the rate of change of the angle between the reference axis and your elbow the same as the rate of change of the angle between the axis and your fingertips, or do the rates differ?

(i) What happens to the linear *velocity*, \vec{v} , of your fingers as you rotate at a constant rate? **Hint:** What happens to the *magnitude* of the velocity, i.e., its speed? What happens to its *direction*?

(j) Are your finger tips accelerating? Why or why not?

20 min

Radians, Radii, and Arc Lengths

An understanding of the relationship between angles in radians, angles in degrees, and arc lengths is critical in the study of rotational motion. There are two common units used to measure angles—degrees and radians.

1. A *degree* is defined as 1/360th of a rotation in a complete circle.
2. A *radian* is defined as the angle for which the arc along the circle is equal to its radius as shown in Figure 12-4.

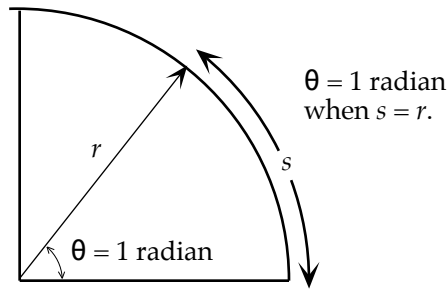
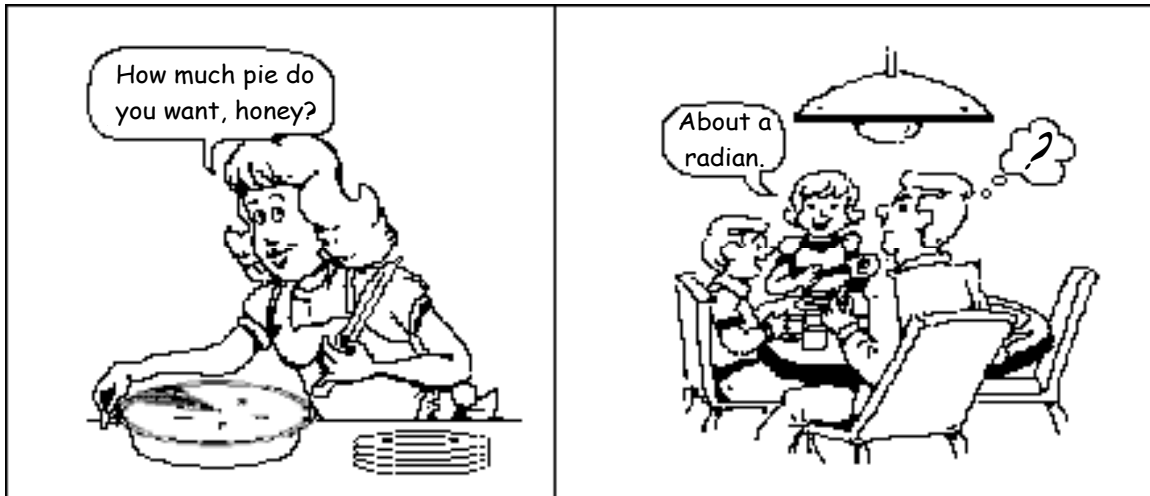


Figure 12-4: A diagram defining the radian



In the next series of activities you will be relating angles, arc lengths, and radii for a circle. To complete these activities you will need the following:

- A drawing compass
- A flexible ruler
- A protractor
- A pencil

Activity 12-5: Relating Arcs, Radii, and Angles

(a) Let's warm up with a review of some very basic mathematics. What should the constant of proportionality be between the circumference of a circle and its radius? How do you know?

(b) Now, test your prediction. You and your partners should draw four circles each with a different radius. Measure the radius and circumference of each circle. Enter your data into a spreadsheet and graphing routine

capable of doing simple fitting. Affix the plot in the space below.

(c) What is the slope of the line that you see (it should be straight)? Is that what you expected? What is the % discrepancy between the slope you obtained from your measurements and that which you predicted in part (a)?

(d) Approximately how many degrees are in one radian? Let's do this experimentally. Using the compass draw a circle and measure its radius. Then, use the flexible ruler to trace out a length of arc, s , that has the same length as the radius. Next measure the angle in degrees that is subtended by the arc.

(e) Theoretically, how many degrees are in one radian? *Please calculate your result to **three** significant figures.* Using the equation for the circumference of a circle as a function of its radius and the constant $\pi = 3.1415927\dots$

figure out a general equation to find degrees from radians.

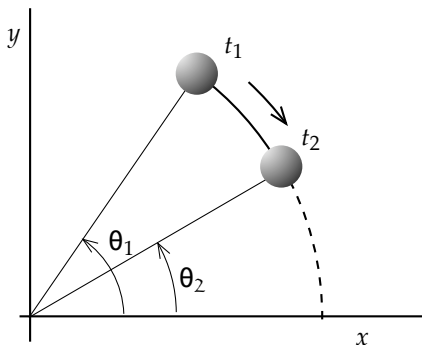
Hint: How many times does a radius fit onto the circumference of a circle? How many degrees fit in the circumference of a circle?

(e) If an object moves 30 degrees on the circumference of a circle of radius 1.5 m, what is the length of its path?

(f) If an object moves 0.42 radians on the circumference of a circle of radius 1.5 m, what is the length of its path?

(g) Remembering the relationship between the speed of your fingers and the distance, r , from the axis of your turn to your fingertips, what equations would you use to define the magnitude of the average "angular" velocity, $\langle \omega \rangle$?

Hint: In words, $\langle \omega \rangle$ is defined as the amount of angle swept out by the object per unit time. Note that the answer is not simply θ/t !



(h) How many radians are there in a full circle consisting of 360° ?

(i) When an object moves in a complete circle in a fixed amount of time, what quantity (other than time) remains unchanged for circles of several different radii?

10 min

Relating Linear and Angular Quantities

It's very useful to know the relationship between the variables s , v , and a , which describe linear motion and the corresponding variables θ , ω , and α , which describe rotational motion. You now know enough to define these relationships.

Activity 12-6: Linear and Angular Variables

(a) Using the definition of the radian, what is the general relationship between a length of arc, s , on a circle and the variables r and θ in radians.

(b) Assume that an object is moving in a circle of constant radius, r . Take the derivative of s with respect to time to find the velocity of the object. By using the relationship you found in part (a) above, show that the magnitude of the linear velocity, v , is related to the magnitude of the angular velocity, ω , by the equation $v = \omega r$.

(c) Assume that an object is accelerating in a circle of constant radius, r . Take the derivative of v with respect to time to find the acceleration of the object. By using the relationship you found in part (b) above, show that the linear acceleration, a_t , tangent to the circle is related to the angular acceleration, α , by the equation $a_t = \alpha r$.

10 min

The Rotational Kinematic Equations for Constant α

The set of definitions of angular variables are the basis of the physicist's description of rotational motion. We can use them to derive a set of kinematic equations for rotational motion with constant angular acceleration that are similar to the equations for linear motion.

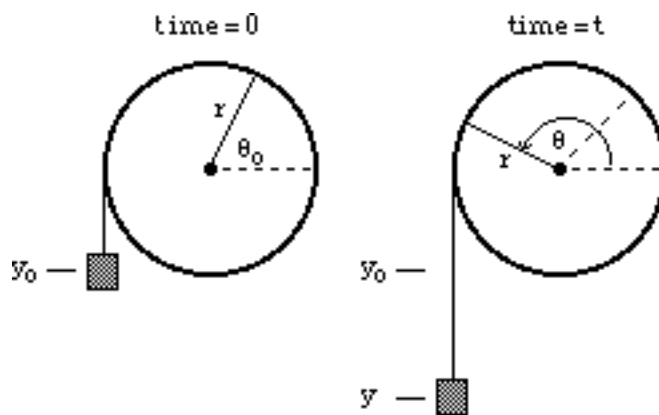


Figure 12-5: A massless string is wound around a spool of radius r . The mass falls with a constant acceleration, a

Activity 12-7: The Rotational Kinematic Equations

Refer to Figure 12-5 and answer the following questions.

- What is the equation for θ in terms of y and r ?
- What is the equation for ω in terms of v and r ?
- What is the equation for α in terms of a and r ?

(d) Consider the falling mass in Figure 12-5 above. Suppose you are standing on your head so that the positive y -axis is pointing down. Using the relationships between the linear and angular variables in parts (a), (b), and (c), *derive* the rotational kinematic equations for constant accelerations for each to the linear kinematic equations listed below. **Warning:** *Don't just write the analogous equations! Show the substitutions needed to derive the equations on the right from those on the left.*

(a) $v = v_0 + at$ $\omega =$

(b) $y = y_0 + v_0t + \frac{1}{2}at^2$ $\theta =$

(c) $v^2 = v_0^2 + 2ay$ $\omega^2 =$

SESSION TWO: TORQUE, ROTATIONAL INERTIA, & NEWTON'S LAWS

25 min

Causing and Preventing Rotation

Up to now we have been considering rotational motion without considering its cause. Of course, this is also the way we proceeded for linear motion. Linear motions are attributed to forces acting on objects. We need to define the rotational analogue to force.

Recall that an object tends to rotate when a force is applied to it along a line that does not pass through its centre-of-mass. Let's apply some forces to a rigid bar. What happens when the applied forces don't act along a line passing through the centre-of-mass of the bar?

The Rotational Analogue of Force – What Should It Be?

If linear equilibrium results when the vector sum of the forces on an object is zero (i.e., there is no *change* in the motion of the centre of mass of the object), we would like to demand that the sum of some new set of rotational quantities on a stationary non-rotating object also be zero. By making some careful observations you should be able to figure out how to define a new quantity which is analogous to force when it comes to causing or preventing rotation. For this set of observations you will need:

- A vertical pivot
- A clamp stand to hold the pivot
- An aluminum rod with holes drilled in it
- Two identical spring scales
- A ruler

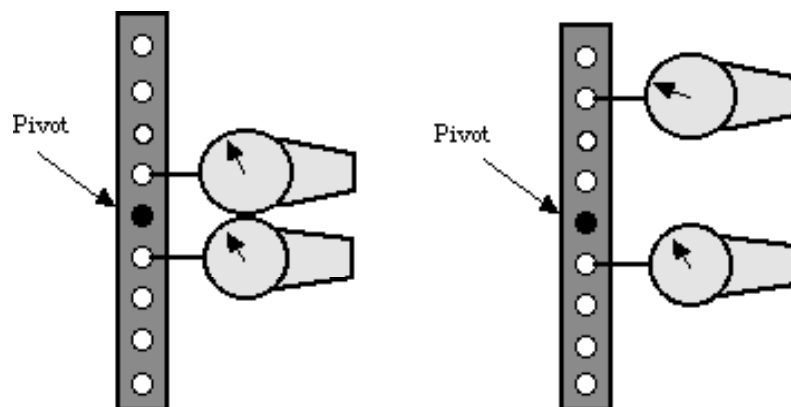


Figure 12-6: Aluminum Rod with Holes and Spring Balances

 **Activity 12-8: Force and Lever Arm Combinations**

(a) Set the aluminum rod horizontally on the vertical pivot. Try pulling horizontally with each scale when they are hooked on holes that are the same distance from the pivot as shown in diagram (a) above. What ratio of forces is needed to keep the rod in equilibrium?

(b) Try moving one of the spring scales to some other hole as shown in Figure 12-6. Now what ratio of forces is needed to keep the rod from rotating? How do these ratios relate to the distances? Try this for several different situations and record your results in the table below.

| | Original Force (N) | Original Distance (cm) | Balancing Force (N) | Balancing Distance (cm) |
|---|--------------------|------------------------|---------------------|-------------------------|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |

(c) What mathematical relationship between the original force and distance and the balancing force and distance give a constant for both cases? *How would you define the rotational factor mathematically?* Cite evidence for your conclusion.

(d) Show quantitatively that your original and final rotational factors are the same within the limits of experimental uncertainty for *all four* of the situations you set up.

The rotational factor that you just discovered is officially known as *torque* and is usually denoted by the Greek letter τ ("tau", which rhymes with "cow"). The distance from the

pivot to the point of application of a force you applied with the spring scale is defined as the *lever arm* for that force.

Seeking a "Second Law" of Rotational Motion

Consider an object of mass m moving along a straight line. According to Newton's second law an object will undergo a linear acceleration a when it is subjected to a linear force where $\vec{F} = m\vec{a}$. Let's postulate that a similar law can be formulated for rotational motion in which a torque τ is proportional to an angular acceleration α . If we define the constant of proportionality as the *rotational inertia*, I , then the rotational second law can be expressed by the equation

$$\tau = I\alpha$$

(I is also commonly called the *moment of inertia*. Actually, τ and α are vector quantities. For now we will not worry about including vector signs as the vector nature of τ and α will be treated in the next unit.)

We need to know how to determine the rotational inertia, I , mathematically. You can predict, on the basis of direct observation, what properties of a rotating object influence the rotational inertia. For these observations you will need the following equipment.

- A vertical pivot
- A clamp stand to hold the pivot
- An aluminum rod with holes drilled in it
- Two aluminum "point masses" that mount over holes in the rod
- A metre stick

This observation relates a fixed torque applied by you to the resulting angular velocity of a spinning rod with masses on it. When the resulting angular acceleration is small for a given effort, we say that the rotational inertia is large. Conversely, a small rotational inertia will lead to a large rotational acceleration. In this observation you can place masses at different distances from an axis of rotation to determine what factors cause rotational inertia to increase.

Centre a light aluminum rod on the almost frictionless pivot that is fixed at your table. With your finger, push the rod at a point about halfway between the pivot point and one end of the rod. Spin the rod gently with different mass configurations as shown in the diagram below.

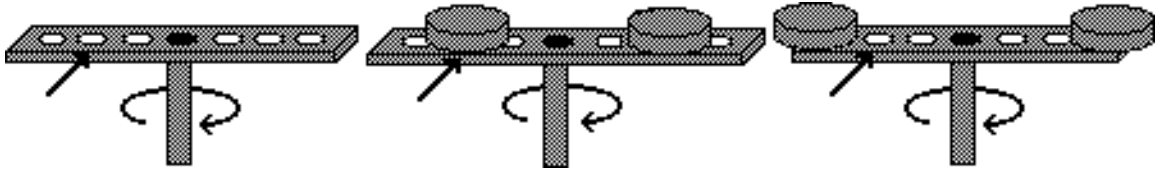


Figure 12-7: Causing a rod to rotate under the influence of a constant applied torque for three different mass configurations.

Activity 12-9: Rotational Inertia Factors

(a) What do you predict will happen if you exert a constant torque on the rotating rod using a uniform pressure applied by your finger at a fixed lever arm? Will it undergo an angular acceleration, move at a constant angular velocity, or what?

(b) What do you expect to happen differently if you use the same torque on a rod with two masses added to the rod as shown in the middle of Figure 12-7?

(c) Will the motion be different if you relocate the masses farther from the axis of rotation as shown in Figure 12-7 on the right?

(d) While applying a constant torque, observe the rotation of:

(1) the rod, (2) the rod with masses placed close to the axis of rotation, and (3) the rod with the same masses placed far from the axis of rotation. Look carefully at the motions. Does the rod appear to undergo angular acceleration or does it move at a constant angular velocity?

(e) How did your predictions pan out? What factors does the rotational inertia, I , depend on?

10 min

The Equation for the Rotational Inertia of a Point Mass

Now that you have a feel for the factors on which I depends, let's derive the mathematical expression for the rotational inertia of an ideal point mass, m , which is rotating at a known distance, r , from an axis of rotation. To do this, recall the following equations for a point mass that is rotating:

$$a = \alpha r \qquad \tau = rF$$

Activity 12-10: Defining I Using the Law of Rotation

Show that if $F=ma$ and $\tau = I\alpha$, then I for a point mass that is rotating on an ultra-light rod at a distance r from an axis is given by

$$I = mr^2$$

45 min

Rotational Inertia for Rigid Extended Masses

Because very few rotating objects are point masses at the end of light rods, we need to consider the physics of rotation for objects in which the mass is distributed over a volume, like heavy rods, hoops, disks, jagged rocks, human bodies, and so on. We begin our discussion with the concept of rotational inertia for the simplest possible ideal case, namely that of one point mass at the end of a light rigid rod as in the previous activity. Then we will present the general mathematical expression for the rotational inertia for rigid bodies. In our first rigid body example you will show how the rotational inertia for one point mass can easily be extended to that of two point masses, a hoop, and finally a cylinder or disk.

The Rotational Inertia of Point Masses and a Hoop

Let's start by considering the rotational inertia at a distance r from a blob of clay that approximates a point mass where the clay blob is a distance r from the axis of

rotation. Now, suppose the blob of clay is split into two point masses still at a distance r from the axis of rotation. Then consider the blob of clay split into eight point masses, and, finally, the same blob of clay fashioned into a hoop as shown in the diagram below.

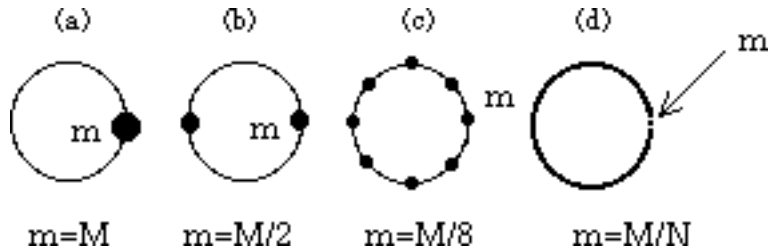


Figure 12-8: Masses rotating at a constant radius

Activity 12-11: The Rotational Inertia of a Hoop

(a) Write the equation for the rotational inertia, I , of the point mass shown in diagram (a) of Figure 12-8 above in terms of its total mass, M , and the radius of rotation of the mass, r .

(b) Write the equation for the rotational inertia, I , of the two "point" masses shown in diagram (b) of Figure 12-8 above in terms of its individual masses m and their common radius of rotation r . By replacing m with $M/2$ in the equation, express I as a function of the total mass M of the two particle system and the common radius of rotation r of the mass elements.

(c) Write the equation for the rotational inertia, I , of the eight "point" masses shown in diagram (c) of Figure 12-8 above in terms of its individual masses m and their common radius of rotation r . By replacing m with $M/8$ in the equation, express I as a function of the total mass M of the eight particle system and the common radius of rotation r of the mass elements.

(d) Write the equation for the rotational inertia, I , of the N "point" masses shown in diagram (d) of Figure 12-8 above in terms of its individual masses m and their common radius of rotation r . By replacing m with M/N in the equation, express I as a function of the total mass M of the

N particle system and the common radius of rotation r of the mass elements.

(e) What is the equation for the rotational inertia, I , of a hoop of radius r and mass M rotating about its centre?

The Rotational Inertia of a Disk

The basic equation for the moment of inertia of a point mass is mr^2 . Note that as r increases I increases, rather dramatically, as the square of r . Let's apply this fact to the consideration of the motion of a matched hoop and disk down an inclined plane. To make the observation of rotational motion your class will need one set-up of the following demonstration apparatus:

- A hoop and cylinder with the same mass and radius as the cylinder
- An inclined plane

Activity 12-12: Which Rotational Inertia is Larger?

(a) If a hoop and a disk both have the same outer radius and mass, which one will have the largest rotational inertia (i.e., which object has its mass distributed *farther* away from an axis of rotation through its centre)? Why?

(b) Which object should be more resistant to rotation – the hoop or the disk? Explain. **Hint:** You may want to use the results of your observation in Activity 12-9(d).

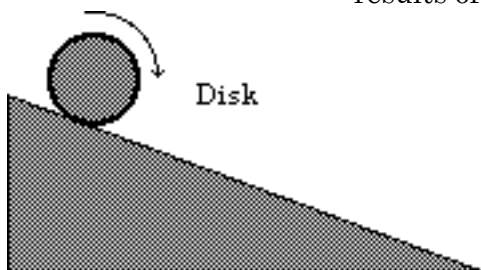


Figure 12-9: A disk rolling down an incline

(c) What will happen if a hoop and disk each having the same mass and outer radius are rolled down an incline? Which will roll faster? Why?

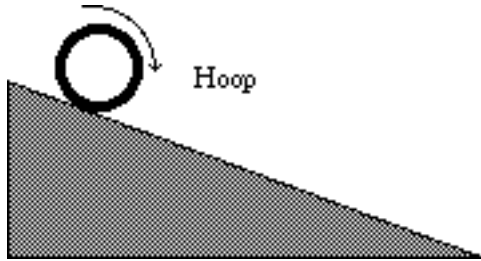


Figure 12-10: A hoop rolling down an incline

(d) What did you actually observe, and how valid was your prediction?

It can be shown experimentally that the rotational inertia of any rotating body is the sum of the rotational inertias of each tiny mass element, dm , of the rotating body. If an infinitesimal element of mass, dm , is located at a distance r from an axis of rotation then its contribution to the rotational inertia of the body is given by $r^2 dm$.

Mathematical theory tells us that since the total rotational inertia of the system is the sum of the rotational inertias of each of its mass elements, the rotational inertia I is the integral of $r^2 dm$ over all m . This is shown in the equation below.

$$I = \int r^2 dm$$

When this integration is performed for a disk or cylinder rotating about its axis, the rotational inertia turns out to be

$$I = \frac{1}{2} Mr^2$$

where M is the total mass of the cylinder and r is its radius. See almost any standard introductory physics textbook for details of how to do this integral.

It is often convenient to define a distance from the pivot point where the total mass could be located to give the same rotational inertia. This distance is called the *radius of gyration* and is symbolized by k .

$$\text{radius of gyration: } k = \sqrt{\frac{I}{M}}$$

A disk or cylinder can be thought of as a series of nested, concentric hoops. This is shown in the figure below.

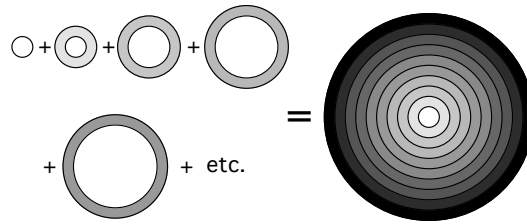


Figure 12-11: A disk or cylinder as a set of concentric hoops

It is instructive to compare the theoretical rotational inertia of a disk, calculated using an integral, with a spreadsheet calculation of the rotational inertia approximated as a series of concentric hoops.

Suppose the disk pictured above is a life-sized drawing of a disk that has a total mass, M , of 2.0 kg. Assume that the disk has a uniform density and a constant thickness so that the piece of mass represented by each hoop is proportional to its cross sectional area.

Activity 12-13: The Rotational Inertia of a Disk (6 points)

(a) Measure the radius of the disk shown in Figure 12-11. Use the equation shown below (obtained from integration) to calculate the theoretical value of the rotational inertia of the disk pictured in Figure 12-11.

$$I = \frac{1}{2} Mr^2 =$$

(b) Develop a set of equations to calculate the numerical value for I by considering I to be the sum of the ΔI_i s for each hoop of mass $m = (\Delta A_i/A)M$ (where A is the total cross sectional area of the disk and ΔA_i represents the area of

the i -th hoop). Hints: (1) Start by making the measurements needed to calculate the ΔA of the inner hoop and move out from there; (2) The area of the face of a disk of radius r is given by πr^2 .

$$\Delta A_i =$$

$$\Delta I_i =$$

$$I =$$

(Express in terms of ΔI .)

(c) Create a spreadsheet with the values of ΔA and m for each hoop along with the value of rotational inertia contributed by each of the hoops. Upload the spreadsheet using WebCT.

(d) How closely does your numerical spreadsheet calculation for the total rotational inertia of the disk compare with the value you calculated theoretically? Write down the general equation for the % discrepancy and display the steps in your calculation of it below.

(e) How could you change your procedures to make the % discrepancy smaller?

(f) What is the *radius of gyration* of the disk?

SESSION THREE: VERIFYING NEWTON'S 2ND LAW FOR ROTATION

100 min

Experimental Verification that $\tau = I\alpha$ for a Rotating Disk

In the last session, you used the definition of rotational inertia, I , and spreadsheet calculations to determine a *theoretical* equation for the rotational inertia of a disk. This equation was given by

$$I = \frac{1}{2} Mr^2$$

Does this equation adequately describe the rotational inertia of a rotating disk system? If so, then we should find that, if we apply a known torque, τ , to the disk system, its resulting angular acceleration, α , is actually related to the system's rotational inertia, I , by the equation

$$\tau = I\alpha$$

or

$$\alpha = \frac{\tau}{I}$$

The purpose of this experiment is to determine if, within the limits of experimental uncertainty, the measured angular acceleration of a rotating disk system is the same as its theoretical value. The theoretical value of angular acceleration can be calculated using theoretically determined values for the torque on the system and its rotational inertia.

The following apparatus will be available to you:

- A Rotating Cylinder System
- A 20 g or 50 g hanging mass (for applying torque)
- A clamp stand to mount the system on
- String
- A metre stick and a ruler
- A Motion Detection System
- A scale for determining mass

Theoretical Calculations

You'll need to take some basic measurements on the rotating cylinder system to determine theoretical values for I and τ . Values of rotational inertia calculated from the dimensions of a rotating object are theoretical because they purport to describe the resistance of an object to rotation. An experimental value is obtained by applying a known

torque to the object and measuring the resultant angular acceleration.

Activity 12-14: Theoretical Calculations

(a) Calculate the theoretical value of the rotational inertia of the stack of CDs using basic measurements of its radius and mass. Be sure to state units! If there's a significant size hole in the centre, try to estimate the error involved in ignoring it.

For the CDs, ignoring the hole:

$$r_d = \qquad M_d =$$

$$I_d =$$

For the hole in the centre:

$$r_h = \qquad M_h =$$

$$I_h =$$

$$I_{CD} = I_d - I_h =$$

(b) Calculate the theoretical value of the rotational inertia of the spool using basic measurements of its radius and mass. **Note:** You'll have to do a bit of estimation here. Be sure to state units.

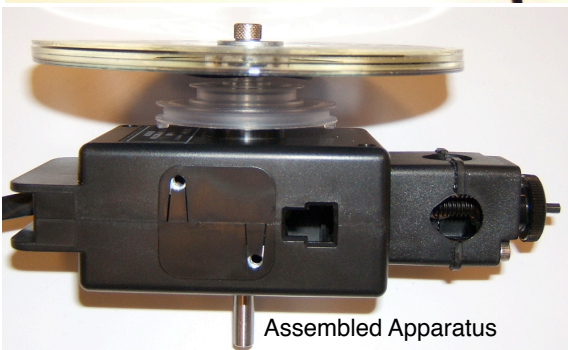
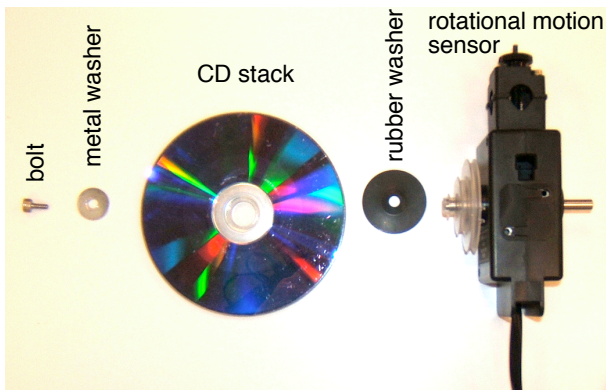
$$r_s = \qquad M_s =$$

$$I_s =$$

(c) Calculate the theoretical value of the rotational inertia, I , of the *whole* system. Don't forget to include the units. **Note:** By noting how small the rotational inertia of the spool is compared to that of the disk, you should be able to convince yourself that you can neglect the rotational inertia of the rotating axle in your calculations.

$$I =$$

(c) In preparation for calculating the torque on your system, summarize the measurements for the falling mass, m , and the radius of the spool in the space below. Don't forget the units!



$m =$

$r_s =$

(d) Use the equation you derived in parts (b) and (c) of problem SP12-4 to calculate the theoretical value for the torque on the rotating system as a function of the magnitude of the hanging mass and the radius, r_s , of the spool.

$\tau =$

(e) Based on the values of torque and rotational inertia of the system, what is the theoretical value of the angular acceleration of the disk? What are the units?

$\alpha_{th} =$

Experimental Measurement of Angular Acceleration

Devise a good way to *measure* the linear acceleration, a , of the hanging mass with a minimum of uncertainty and then use that value to determine α . You will need to take enough measurements to find a standard deviation for your measurement of a and eventually α . Can you see why it is desirable to make several runs for this experiment? Should you use a spread sheet?

Note: The value of a is not the same as that of the gravitational acceleration, g .

If you choose to use a graphical technique to find the acceleration be sure to include a copy of your graph and the equation that best fits the graph. Also show all the equations and data used in your calculations. Discuss the sources of uncertainties and errors and ways to reduce them.

 **Activity 12-15: Experimental Write-up for Finding α (10 pts)**

Describe your experiment in detail in the space below. Show your data *and* your calculations.

Compare your experimental results for α to your theoretical calculation of α for the rotating system. Present this comparison with a neat summary of your data and calculated results.

 **Activity 12-16: Comparing Theory with Experiment**

(a) Summarize the theoretical and experimental values of angular acceleration along with the standard deviation for the experimental value.

$$\alpha_{\text{th}} =$$

$$\alpha_{\text{exp}} =$$

$$\sigma_{\text{exp}} =$$

(b) Do theory and experiment agree within the limits of experimental uncertainty?