UNIT 2 ERRORS IN MEASUREMENT

Structure

2.1 INTRODUCTION

The measurement of a quantity is based on some International fundamental standards. These fundamental standards are perfectly accurate, while others are derived from these. These derived standards are not perfectly accurate in spite of all precautions. In general, measurement of any quantity is done by comparing with derived standards which themselves are not perfectly accurate. So, the error in the measurement is not only due to error in methods but also due to standards (derived) not being perfectly accurate. Thus, the measurement with 100% accuracy is not possible with any method.

Error in the measurement of a physical quantity is its deviation from actual value. If an experimenter knew the error, he or she would correct it and it would no longer be an error. In other words, the real errors in experimental data are those factors that are always vague to some extent and carry some amount of uncertainty. A reasonable definition of experimental uncertainty may be taken as the possible value the error may have. The uncertainty may vary a great deal depending upon the circumstances of the experiment. Perhaps it is better to speak of experimental uncertainty instead of experimental error because the magnitude of an error is uncertain.

At this point, we may mention some of the types of errors that cause uncertainty is an experimental in measurement. First, there can always be those gross blunders in apparatus or instrument construction which may invalidate the data. Second, there may be certain fixed errors which will cause repeated readings to be in error by roughly some amount but for some unknown reasons. These are sometimes called systematic errors. Third, there are the random errors, which may be caused by personal fluctuation, random electronic fluctuation in apparatus or instruments, various influences of friction, etc.

Objectives

After studying this unit, you should be able to

- understand the nature of errors and their sources in the measurement,
- know accuracy and precision in the measurement, and
- explain the various methods of analysis of the errors.

2.2 CLASSIFICATION OF ERRORS

Errors will creep into all measurement regardless of the care which is exerted. But it is important for the person performing the experiment to take proper care so that the error can be minimized. Some of the errors are of random in nature, some will be due to gross blunder on the part of the experimenter and other will be due to the unknown reasons which are constant in nature.

Thus, we see that there are different sources of errors and generally errors are classified mainly into three categories as follows:

(a) Gross errors

(b) Systematic errors

(c) Random errors

2.2.1 Gross Errors

These errors are due to the gross blunder on the part of the experimenters or observers. These errors are caused by mistake in using instruments, recording data and calculating measurement results. For example: A person may read a pressure gage indicating 1.01 N/m^2 as 1.10 N/m^2. Someone may have a bad habit of memorizing data at a time of reading and writing a number of data together at later time. This may cause error in the data. Errors may be made in calculating the final results. Another gross error arises when an experimenter makes use (by mistake) of an ordinary flow meter having poor sensitivity to measure low pressure in a system.

2.2.2 Systematic Errors

These are inherent errors of apparatus or method. These errors always give a constant deviation. On the basis of the sources of errors, systematic errors may be divided into following sub-categories :

Constructional Error

None of the apparatus can be constructed to satisfy all specifications completely. This is the reason of giving guarantee within a limit. Therefore, a manufacturers always mention the minimum possible errors in the construction of the instruments.

Errors in Reading or Observation

Following are some of the reasons of errors in results of the indicating instruments :

(a) **Construction of the Scale :** There is a possibility of error due to the division of the scale not being uniform and clear.

(b) **Fitness and Straightness of the Pointer :** If the pointer is not fine and straight, then it always gives the error in the reading.

(c) **Parallax :** Without a mirror under the pointer there may be parallax error in reading.

(d) **Efficiency or Skillness of the Observer :** Error in the reading is largely dependent upon the skillness of the observer by which reading is noted accurately.

Determination Error

It is due to the indefiniteness in final adjustment of measuring apparatus. For example, Maxwell Bridge method of measuring inductances, it is difficult to find the differences in sound of head phones for small change in resistance at the time of final adjustment. The error varies from person to person.

Error due to Other Factors

Temperature Variation

Variation in temperature not only changes the values of the parameters but also brings changes in the reading of the instrument. For a consistent error, the temperature must be constant.

Effect of the Time on Instruments

There is a possibility of change in calibration error in the instrument with time. This may be called ageing of the instrument.

Effect of External Electrostatic and Magnetic Fields

These electrostatic and magnetic fields influence the readings of instruments. These effects can be minimized by proper shielding.

Mechanical Error

Friction between stationary and rotating parts and residual torsion in suspension wire cause errors in instruments. So, checking should be applied. Generally, these errors may be checked from time to time.

2.2.3 Random Errors

After corrections have been applied for all the parameters whose influences are known, there is left a residue of deviation. These are random error and their magnitudes are not constant. Persons performing the experiment have no control over the origin of these errors. These errors are due to so many reasons such as noise and fatigue in the working persons. These errors may be either positive or negative. To these errors the law of probability may be applied. Generally, these errors may be minimized by taking average of a large number of readings.

SAQ 1

(a) What is the difference between error and accuracy?

(b) What do you mean by uncertainty in measurement?

(c) What is the difference between fixed error and random error?

(d) Mention the role of the experimenter to minimize error in measurement.

(e) Identify the nature of error in the following cases :

- (i) The magnitude of a known voltage source of 100 V was measured with a voltmeter. Five readings were taken. The indicating readings were 101, 100, 102, 100 and 99.
- (ii) The temperature of a hot fluid is 200^{o}C. A glass bulb thermometer is used to measure the same for five times. The temperature indicated by the thermometer in each case is 180^{o}C.
- (iii) Five students were asked to take the readings of a pressure gage. The readings noted by them were 1.5 N/m^2, 1.51 N/m^2, 1.49 N/m^2, 1.48 N/m^2 and 1.5 N/m^2.
- (iv) Due to fluctuation of the voltage source, the pointer of the voltmeter indicates maximum and minimum readings of 230 and 220 volts respectively but the reading taken by the experimenter is 203 V.

2.3 ACCURACY AND PRECISION

Accuracy plays an important role in the measurement of any quantity. The word 'precision' is often used in place of accuracy as if they are interchangeable. The accuracy of measurement is defined as the deviation of measured value from the true value. On the other hand, the precision of measurement is defined as the deviation of different readings from the mean value. Thus, it is measure of consistency in measurement. An example will clarify this point. The value of a known voltage source of 100 V source is measured with a voltmeter. Five readings were taken. The indicated readings were 103 V, 105 V, 104 V, 103 V, 105 V. In this case, the accuracy of the instruments is better than 5%, because the maximum deviation from true value is 5 V. But the precision of the instrument is + 1 V because the deviation of the readings from mean value is + 1 V.

SAQ 2

(a) What is the difference between accuracy and precision?

(b) What do you mean by the 'accuracy of the instrument is better than 2%'?

2.4 CALIBRATION OF THE INSTRUMENT

The calibration of the instrument is done to find its accuracy. Before using an instrument, particularly a new one, in a measurement system, it is required to calibrate it to find the accuracy, precision or uncertainty of the instrument. It can be done by comparing its performance with

(a) a primary standard instrument,

(b) a secondary standard instrument having high accuracy, and

(c) a known input source.

For example, a flowmeter might be calibrated by

(a) comparing it with a standard flow measurement facility of the National Bureau of Standards,

(b) comparing it with another flowmeter of known accuracy, or

(c) direct calibration with a primary measurements such as weighing a certain amount of water in a tank and recording the time elapased for the quantity of flow through the meter.

SAQ 3

(a) What is the need of calibration of a measuring instrument?

(b) Mention the procedures of calibrating a pressure gage.

2.5 ANALYSIS OF THE ERRORS

When an experiment is performed and some data are obtained, then it is required to analyse these data to determine the error, precision and general validity of the experimental measurements. Bad data due to obvious blunder or reason may be discarded immediately. We cannot throw out the data because they do not conform with our hopes and expectations unless we see something obviously wrong. If such bad points fall outside the range of normally expected random deviations, they may be discarded on the basis of some consistent statistical analysis. The elimination of data point must be consistent and should not be dependent on human whims and biased based on what 'ought to be'. In many instances, it is very difficult for the individual to be consistent and unbiased. The presence of a deadline, disgust with previous experimental failures, and normal impatience all can influence rational thanking processes. However, the competent experimentalist will strive to maintain consistency in the primary data analysis.

2.5.1 Error Analysis on Common Sense Basis

Analysis of the data on common sense basis has many forms. One rule of thumb that could be used is that the error in the result is equal to maximum error in any parameter used to calculate the result. Another commonsense analysis would combine all the errors in the most detrimental way in order to determine the maximum error in the final result. Consider the calculation of electric power from

$$P = E \, . \, I \qquad \ldots (2.1)$$

where, E and I are measured as :

$$E = 100 \text{ V} \pm 2 \text{ V} \qquad \ldots (2.2)$$

$$I = 10 \text{ A} \pm 0.2 \text{ A} \qquad \ldots (2.3)$$

The nominal value of power is 100 × 10 = 1000 W. By taking worst possible variations in voltage and current, we could calculate

$$P_{max} = (100 + 2)\,(10 + 0.2)$$

$$= 1040.4 \text{ W}$$

$$P_{min} = (100 - 2)\,(10 - 0.2)$$

$$= 960.4 \text{ W}$$

Thus, using the method of calculation, the uncertainty in the power is + 4.04 percent or – 3.96 percent. It is quite unlikely that the power would be in error by these amounts because the voltmeter variation would probably not correspond with the ammeter variations. When voltmeter reads an extreme 'high', there is no reason why the ammeter must also read an extreme 'high' at that particular instant, indeed, this combination is most unlikely.

The simple calculation applied to the electric-power equation above is a useful way of inspecting experimental data to determine what error could result in a final calculations. However, the test is too severe and should be used only for rough inspection of data. It is significant to note, however, that if the results of the experiments appear to be in error by more than the amounts indicated by the above calculation, then the experimenter had better examine the data more closely. In particular, the experimenter should look for certain fixed errors in the instrumentation, which may be eliminated by applying either theoretical or empirical corrections.

2.5.2 Statistical Analysis of Experimental Data

It is important to define some pertinent terms before discussing some important methods of statistical analysis of experimental data.

Arithmetic Mean

When a set of readings of an instrument is taken, the individual readings will vary somewhat from each other, and the experimenter is usually concerned with the mean of all the readings. If each reading is denoted by x_i and there are n readings, the arithmetic mean is given by

$$x_m = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \ldots (2.4)$$

Deviation

The deviation, d, for each reading is given by

$$d_i = x_i - x_m \qquad \text{. . . (2.5)}$$

We may note that the average of the deviations of all readings is zero since

$$d_i = \frac{1}{n}\sum_{i=1}^{n} d_i = \frac{1}{n}\sum_{i=1}^{n} (x_i - x_m)$$

$$= x_m - \frac{1}{n}(nx_m)$$

$$= 0 \qquad \text{. . . (2.6)}$$

The average of the absolute value of the deviations is given by

$$|\bar{d}_i| = \frac{1}{n}\sum_{i=1}^{n} |d_i|$$

$$= \frac{1}{n}\sum_{i=1}^{n} [x_i - x_m] \qquad \text{. . . (2.7)}$$

Note that the quantity is not necessarily zero.

Standard Deviation

It is also called root mean-square deviation. It is defined as

$$\sigma = \left[\frac{1}{n}\sum_{i=1}^{n} (x_i - x_m)^2\right]^{1/2} \qquad \text{. . . (2.8)}$$

Variance

The square of standard deviation is called variance. This is sometimes called the population or biased standard deviation because it strictly applies only when a large number of samples is taken to describe the population.

Geometrical mean

It is appropriate to use a geometrical mean when studying phenomena which grow in proportion to their size. This would apply to certain biological processes and growth rate in financial resources. The geometrical mean is defined by

$$x_g = [x_1 \,.\, x_2 \,.\, x_3 \,.\,.\,.\, x_n]^{\frac{1}{n}} \qquad \text{. . . (2.9)}$$

Example 2.2

The following readings are taken of a certain physical length. Compute the mean reading, standard deviation, variance and average of the absolute value of the deviation using the biased bases.

Reading	1	2	3	4	5	6	7	8	9	10
x_i (cm)	5.30	5.73	6.77	5.26	4.33	5.45	6.09	5.64	5.81	5.75

Solution

$$x_m = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{10}(56.13)$$

$$= 5.613 \text{ cm}$$

The other quantities are computed with the aid of the following table.

Reading	$d_i = x_i - x_m$	$(x_i - x_m)^2$
1	– 0.313	0.09797
2	0.117	0.01369
3	1.157	1.33865
4	– 0.353	0.12461
5	– 1.283	16.4609
6	– 0.163	0.02657
7	0.477	0.22753
8	0.027	0.00729
9	0.197	0.03881
10	1.137	0.01877

$$\sigma = \left[\frac{1}{n} \sum_{i=1}^{n} (x_i - x_m)^2 \right]^{1/2}$$

$$= \left[\frac{1}{10} (3.533) \right]^{1/2} = 0.5944 \text{ cm}$$

$$\sigma^2 = 0.3533 \text{ cm}^2$$

$$|\bar{d}_i| = \frac{1}{n} \sum_{i=1}^{n} |d_i| = \frac{1}{n} \sum |x_i - x_m|$$

$$= \frac{1}{10} \times 4.224 = 0.4224 \text{ cm}$$

SAQ 4

(a) What is the need of analysis of an experimental data?

(b) What is the difference between error and uncertainty?

(c) What do you mean by limiting error?

(d) Following data points are expected to follow a functional variation between x and y in the form of

$$y = a\, e^{bx}$$

Find the best functional relation between x and y using the method of least squares.

x	1	2	3	4	5
y	8.0	7.2	6.5	4.2	2.5

(e) Three elements have following ratings :

$$R_1 = 40 \pm 5\%,\ R_2 = 80 \pm 5\%,\ R_3 = 50 \pm 5\%$$

where, $R_s = R_1 + R_2 + R_3$. Find the magnitude of R_s and the limiting errors in R_s and in percentage of three elements.

(f) The length and width of a rectangular plate are (0.163 ± 0.0005) m and (0.138 ± 0.0005) m respectively. Calculate the area of the plate and also uncertainity in the area.

(g) A physical quantity P is related to four parameters a, b, c and d as follows : $P = \frac{a^3 b^2}{c^{1/2}}$. The percentage error of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P?

2.6 SUMMARY

Error in the measurement of a physical quantity indicates the deviation from its actual value.

Errors can be classified as Gross error, Systematic error and Random error.

Accuracy and precision play important roles in the measurement of any physical quantity. Calibration of an instrument is done to find its accuracy. It can be done either by

(a) comparing with a standard instrument,

(b) comparing with an another instrument with known accuracy, or

(c) direct calibration with primary measurement.

When an experiment is performed and some data are obtained, then it is required to analyse these data to find error, precision and the general validity of the experimental measurements.

The error analysis of the experimental data can be done by various methods, such as common sense basis, uncertainty analysis, statistical analysis, probability error analysis, limiting error analysis etc.

2.7 KEY WORDS

Gross Errors	**:**	These are due to the gross blunder on the part of the experimenters or observers.
Systematic Errors	**:**	These are inherent errors of apparatus or method.
Random Errors	**:**	Their magnitudes are not constant. The law of probability may be applied to these errors.
Accuracy	**:**	Deviation of the measured value from true value.
Precision	**:**	Deviation of the different readings from mean reading.
Calibration	**:**	To make a comparison to find the accuracy.
The Method of Least Squares	:	Mean value that minimizes the sum of the squares of the deviations is the arithmetic mean.

2.8 ANSWERS TO SAQs

SAQ 1

(a) Error, in the measurement of a physical quantity, is the deviation from its actual value, whereas the precision is the deviation of some readings from their mean value.

(b) Experimental errors cause uncertainty in the results of the experimental measurements.

(c) Fixed error always gives constant deviation and it is one kind of systematic error. The random error is caused by personal fluctuations, random electronic fluctuation in the apparatus or instruments etc.

(d) The experimenter can minimise the error in the measurement by knowing the reasons of the error and adopting correct procedures for the experiments.

(e) (i) Random error

(ii) Systematic error

(iii) Random error

(iv) Gross error

SAQ 2

(a) The accuracy is the deviation of a measured value from its true value, whereas precision is the deviations of some readings from their mean value.

(b) The maximum possible error in the instrument is 2% and it is 98% accurate.

SAQ 3

(a) The calibration of an instrument is required to find its accuracy.

(b) A pressure gage can be calibrated

(i) By comparing with a standard pressure gage (ISI standard).

(ii) By comparing with another pressure gage with known accuracy.

(iii) By direct measurement of pressure against a known value.

SAQ 4

(a) When an experiment is performed and some data are obtained, then it is necessary to analyse these data to find error, precision and the general validity of the experimental measurements.

(b) The error is the deviation of the measured value from the true value, whereas uncertainty denotes the possible value the error may have, and it may vary in great deal depending upon the circumstances of the experiment.

(c) Most of the measuring instruments are guaranteed for their accuracy with a percentage deviation of full scale reading. This limiting deviation from the specified values are called limiting errors.

(d) $y = ae^{bx}$

$\Rightarrow \quad \ln y = \ln a + bx$

$\Rightarrow \quad Y = AX + B$

where, $Y = \ln y$

$X = x$

$A = b$

$B = \ln a$

x_i	y_i	X_i (= x_i)	Y_i (= ln y_i)	$X_i Y_i$	X_i^2	n
1	8.0	1	2.079	2.097	1	5
2	7.2	2	1.974	3.948	4	
3.	6.5	3	1.872	5.616	9	
4.	4.2	4	1.435	5.74	16	
5.	2.5	5	0.976	4.875	25	
		$\sum X_i = 15$	$\sum Y_i = 8.276$	$\sum X_i Y_i = 22.258$	$\sum X_i^2 = 55$	

Now, $$A = \frac{n \sum X_i Y_i - (\sum X_i)(\sum Y_i)}{n(\sum X_i^2) - (\sum X_i)^2}$$

$$= \frac{5 \times 22.258 - 15 \times 8.276}{5 \times 55 - 15^2}$$

$$= -0.257$$

$$= b$$

$$B = \frac{(\sum Y_i)(\sum X_i^2) - (\sum X_i Y_i)(\sum X_i)}{n(\sum X_i^2) - (\sum X_i)^2}$$

$$= \frac{8.276 \times 55 - 22.258 \times 15}{50}$$

$$= \frac{455.18 - 333.87}{50}$$

$$= 2.426$$

$$= \ln a$$

$\therefore$ $a = 11.313$

Hence, the best functional relation between y and x is

$$y = 11.313\, e^{-0.257x}$$

(e) $R_S = R_1 + R_2 + R_3$

$= 40 + 80 + 50$

$= 170$

$$\%\text{ error in } R_S = \frac{R_1}{R_S}\frac{\Delta R_1}{R_1} + \frac{R_2}{R_S}\frac{\Delta R_2}{R_2} + \frac{R_3}{R_S}\frac{\Delta R_3}{R_3}$$

$$= \frac{40}{170} \times 5\% + \frac{80}{170} \times 5\% + \frac{50}{170} \times 5\%$$

$$= \frac{5\%}{170}[40 + 80 + 50]$$

$$= 5\%$$

$$\%\text{ error in } R_S = \frac{5}{100} \times 170 = 8.5$$

(f) Length = (0.163 ± 0.0005) m; Δl = 0.0005 m

Width = (0.138 ± 0.0005) m; Δb = 0.0005 m

$\therefore$ $$\frac{\Delta l}{l} = \frac{0.0005}{0.163} = 0.003$$

$$\frac{\Delta b}{b} = \frac{0.0005}{0.138} = 0.0036$$

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} = 0.003 + 0.0036 = 0.0066$$

Relative % error = 0.0066 × 100 = 0.66%

Nominal area = 0.163 × 0.138 m^2 = 0.0225 m^2

Uncertainity in the area = (0.0066 × 0.0255) m^2

Area of the plate = (0.0225 ± 0.00015) m^2

(g) Maximum possible error in $\rho = \dfrac{\Delta\rho}{\rho} \times 100$

$$\frac{\Delta\rho}{\rho} \times 100 = 3\,\frac{\Delta a}{a} \times 100 + 2\,\frac{\Delta b}{b} \times 100$$

$$+ \frac{1}{2}\,\frac{\Delta c}{c} \times 100 + \frac{\Delta d}{d} \times 100$$

$$= 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 1 \times 2\%$$

$$= (3 + 6 + 2 + 23)\%$$

$$= 13\%$$