Unit 2 Guided Notes

Quadratic Functions

Standards: **A.CED.1, A.REI.4a, A.REI.4b, A.SSE.1a, A.SSE.2, A.SSE.3b, F.BF.1, F.BF.3, F.IF.5, F.IF.6, F.IF.7a, F.IF.8, F.IF.8a, F.IF.9, G.GPE.1, G.GPE.2, N.CN.1, N.CN.2, N.CN.7**

Clio High School – Algebra 2A

Name: ______________________________ Period: ______________

Need help? Support is available!

- Miss Seitz's tutoring: See schedule in classroom
- Website with all videos and resources

www.msseitz.weebly.com

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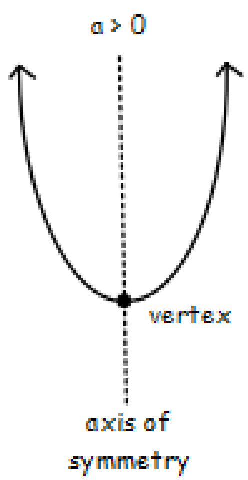
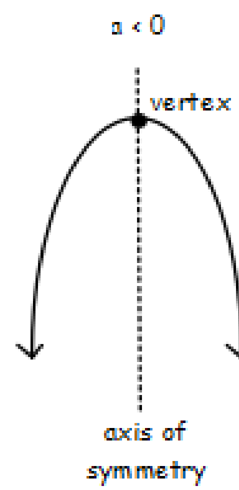
Concept #	What we will be learning…	Text
#1	**Vertex Form and Transformations** ☐ Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx) and f(x + k) for specific values of k (both positive and negative) ☐ Find the value of k given the graph ☐ Graph quadratic functions and show intercepts, maxima and minima	4.1
#2	**Standard Form of a Quadratic Function** ☐ Write an equation that describes how two things are related based on a real world context	4.2
#3	**Factoring Quadratics** ☐ Use the structure of an expression to identify ways to rewrite it	4.4
#4	**Solve by Factoring** ☐ Solve quadratic equations by factoring	4.5
#5	**Completing the Square** ☐ Use the method of completing the square to transform any quadratic equation into the form $(x-p)^2=q$	4.6
#6	**Quadratic Formula** ☐ Explain how to derive the quadratic formula from $(x-p)^2 = q$. ☐ Solve quadratic equations using the quadratic formula	4.7
#7	**Complex Numbers** ☐ Use the commutative, associative, and distributive properties to add and subtract complex numbers. ☐ Use the relation $i^2 = -1$ to multiply two imaginary numbers to get a real number ☐ Multiply two complex numbers	4.8
#8	**Parabolas in a Different Light** ☐ Derive the equation of a parabola given the focus and directrix	10.2
#9	**Circles** ☐ Identify the center and radius from the equation of a circle ☐ Use completing the square to write the equation of a circle ☐ Explain how to derive the equation of a circle given the center and radius using the Pythagorean Theorem	10.3

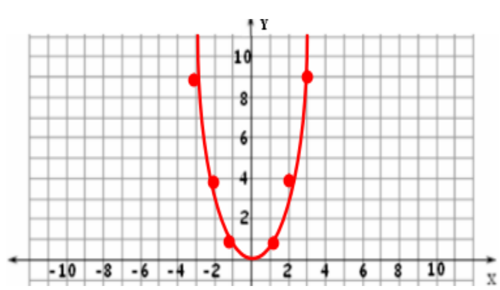
#1

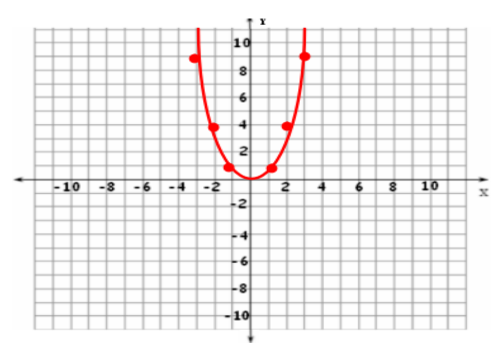
Vertex Form and Transformations **Text: 4.1**

- ☐ Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx) and f(x + k) for specific values of k (both positive and negative)
- ☐ Find the value of k given the graph
- ☐ Graph quadratic functions and show intercepts, maxima and minima

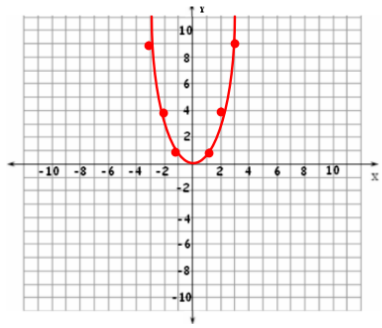
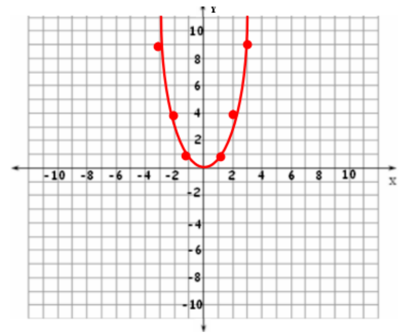
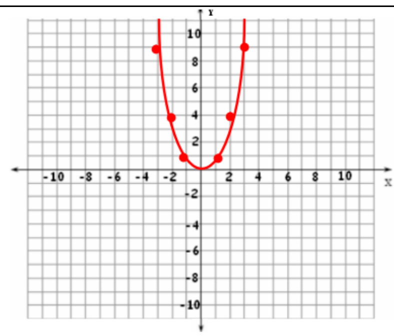
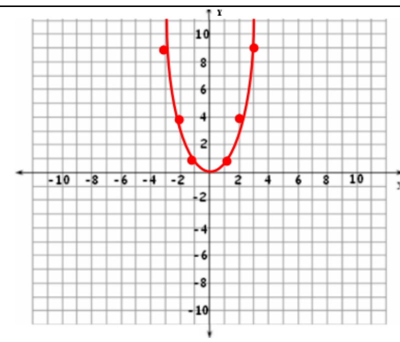
Vocabulary: parabola, vertex form, maximum, minimum, vertex

<table>
<tr><th colspan="2">Vertex Form of a Parabola
$y = A(x - h)^2 + k$ (h, k) is the vertex</th></tr>
<tr><td>The V_________ of a parabola is the highest or lowest point on the graph.</td><td>The A_______ of S________________ is the vertical line that passes through h</td></tr>
<tr><td>A parabola has a M_____________
when the graph opens __________.
This is because A is P________________.</td><td>A parabola has a M_____________
when the graph opens __________.
This is because A is N_______________.</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">The domain of a quadratic function is A_____ R_________ N_____________</td></tr>
<tr><td>The range of a quadratic function that opens up is y ≥ k</td><td>The range of a quadratic function that opens down is y ≤ k</td></tr>
</table>

<table>
<tr><th colspan="2">Identifying Transformations
Hint: It's just like Unit 1 Concept 7!</th></tr>
<tr><td>A p_______________ is the graph of a quadratic function.


Parent Function: $y = x^2$</td><td>

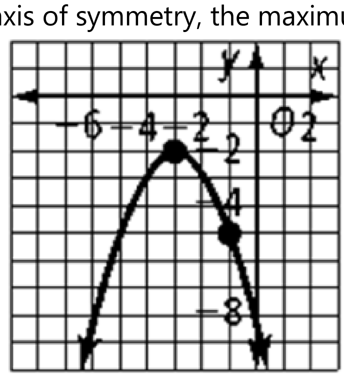
$y = -x^2$
What does the negative do?</td></tr>
</table>

 $y = 2x^2$ What does the 2 do?	 $y = \frac{1}{2}x^2$ What does the ½ do?
 $y = (x-3)^2 + 5$ What does the 3 do? What does the 5 do?	 $y = (x+1)^2 - 6$ What does the 1 do? What does the - 6 do?

You Try It! Identify the transformations

1.) $y = 3(x+2)^2$

2.) $y = -(x+5)^2 + 1$

Writing the Equation in Vertex Form from a Graph (or when given the vertex and a point)
Example 1: Write the equation of the parabola in vertex form. Identify the vertex, axis of symmetry, the maximum/minimum value, and the domain and range. 

#2	**Standard Form of a Quadratic Function** ☐ Write an equation that describes how two things are related based on a real world context Vocabulary: standard form

Definitions
The Standard Form of a Quadratic Equation is $\boldsymbol{y = Ax^2 + Bx + C}$ where A is not zero.

Finding the Vertex	
Vertex: $\left(\frac{B}{-2A},\ f\left(\frac{B}{-2A}\right)\right)$	**<u>Example 1:</u>** Identify the vertex of $y = x^2 - 4x + 1$
<u>Steps:</u> 1. Find $x = \frac{B}{-2A}$	
2. Plug that value into the original equation to find y	

You Try It! Find the vertex, axis of symmetry, maximum/minimum value, and range of the parabola

1.) $y = -x^2 + 2x + 3$

2.) $y = 2x^2 + 3x - 5$

Standard Form to Vertex Form	
HINT: **A** is the same in both forms!	**<u>Example 2:</u>** Write the function in vertex form $y = x^2 - 8x + 19$
<u>Steps:</u> 1. Find the vertex	
2. Plug **A**, **h**, and **k** into vertex form $\mathbf{y = A(x - h)^2 + k}$	

You Try It! Write each equation in vertex form

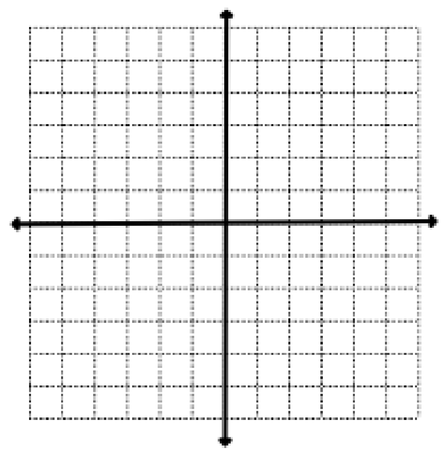
3.) $y = x^2 + 3x$

4.) $y = x^2 - 2x - 6$

<table>
<tr><th colspan="2">Graphing Standard Form</th></tr>
<tr><td>The y – intercept is the point (0, C)

<u>Steps:</u>
1. Find the vertex

2. Identify the following:
y-intercept:
axis of symmetry:
direction of opening:

3. Sketch the graph</td><td><u>Example 3:</u> Graph $y = x^2 + 2x - 5$
</td></tr>
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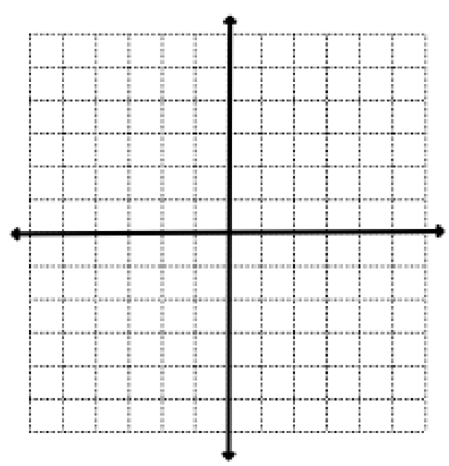
You Try It! Graph $y = 2x^2 + 4x - 4$

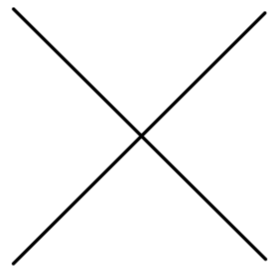
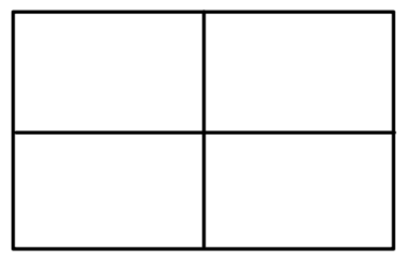


#3 | **Factoring Quadratics** | **Text: 4.4**

☐ Use the structure of an expression to identify ways to rewrite it

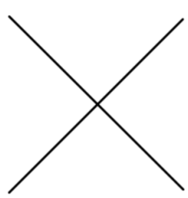
Vocabulary: X-Box, Box Method, Factor, Difference of Squares

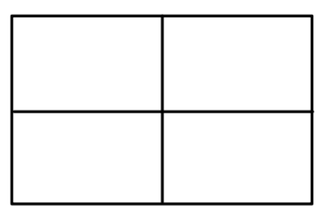
Factoring Using the X-Box Method

Steps:	**Example 1:** Factor $12x^3 + 10x^2 - 12x$
1. Factor out any common factors	
2. Put ***A********C*** in top and ***B*** in bottom 3. Find two numbers that multiply to make the top number that also add to make the bottom number	
4. Put $\mathbf{Ax^2}$ in the top left box and **C** in the bottom right box. 5. Put sides of your X in leftover boxes 6. Factor out what is common to each row and column	
7. Write out all the factors (Including step 1)	

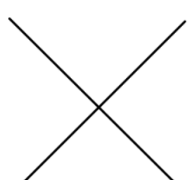
You Try It! Factor

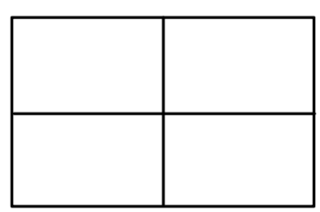
1.) $3x^2 + 8x - 3$





2.) $4x^2 + 12x + 9$





Difference of Squares $\boldsymbol{a^2 - b^2 = (a+b)(a-b)}$	
Example 3: Factor $4x^2 - 9$	
Using the D.o.S.	**Using the Box Method**

You Try It! Factor

3.) $x^2 - 36$

4.) $9x^4 - 81$

#4	**Solving by Factoring** **Text: 4.5** ☐ Solve quadratic equations by factoring Vocabulary: Factored form, zero product property, roots, zeros

Definitions	
The R________ or Z__________ of a Quadratic Function are any values of *x* for which f(*x*) = 0.	The Z_______ P_____________ P_____________ says ***If a·b = 0, then a = 0 or b = 0.***

Using the Zero Product Property	
<u>Example 1:</u> Find the solutions of $(x + 4)(x - 9) = 0$	**<u>Example 2:</u>** Find the solutions of $(x + 5)(x + 8) = 0$

Solving by Factoring	
<u>Example 3:</u> Solve $x^2 - x - 30 = 0$	
<u>Steps:</u> 1. Factor using X-Box	
2. Use the Zero Product Property	

You Try It! Solve each by factoring

1.) $2x^2 + 8x - 10 = 0$

2.) $x^2 + 6x = 40$

#5	**Completing the Square** **Text: 4.6** ☐ Use the method of completing the square to transform any quadratic equation into the form $(x - p)^2 = q$ Vocabulary: completing the square, perfect square trinomial

Solve Using Square Roots		
1. $3x^2 = 75$	**2.** $5x^2 = 45$	**3.** $(x + 4)^2 = 25$

Writing Equations in Standard Form		
4. $(x - 2)^2 =$	**5.** $(x + 3)^2 =$	**6.** $(x - 5)^2 =$

What do you notice about the number in the parentheses and the middle term in standard form?	What do you notice about the number in the parentheses and the last term in standard form?

A **P**____________ **S**__________ **T** ____________ has these special relationships.

If we can write a quadratic equation in this way then we can take the square root of each side to solve.

Solving Using Square Roots	
7. $x^2 + 12x + 36 = 25$	**8.** $x^2 - 10x + 25 = 144$

Completing the Square	
You can form a perfect square trinomial from x^2 + Bx by **adding** $\left(\frac{B}{2}\right)^2$. $$x^2 + Bx + \left(\frac{B}{2}\right)^2 = \left(x + \frac{B}{2}\right)^2$$	
<u>Example 1:</u> Complete the square $x^2 + 22x + \square$	
<u>Steps:</u> 1. Identify ***B***	
2. Divide ***B*** by 2	
3. Square $\frac{B}{2}$	

You Try It! Complete the square

1.) $x^2 + 2x$

2.) $x^2 - 6x$

Solving by Completing the Square	
<u>Example 2:</u> Solve $x^2 + 10x - 1 = 0$ by Completing the Square.	
<u>Steps:</u> 1. Rewrite so all terms with *x* are on the same side	
2. Find $\left(\frac{B}{2}\right)^2$	
3. Add $\left(\frac{B}{2}\right)^2$ to both sides of the equation	
4. Factor the trinomial **THINK:** $(x + \frac{B}{2})^2$	
5. Take the square root of both sides	
6. Solve for x	

You Try It! Solve by completing the square

3.) $x^2 + 2x = 7$

4.) $x^2 - 6x = 10$

#6	**The Quadratic Formula** **Text: 4.7** ☐ Explain how to derive the quadratic formula from $(x - p)^2 = q$. ☐ Solve quadratic equations using the quadratic formula Vocabulary: quadratic formula, discriminant

Identifying A, B, and C

Example 1: Identify A, B, and C in each equation

A. $4x^2 + 3x - 5$

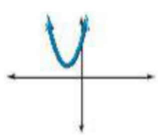
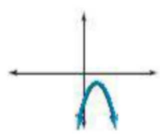
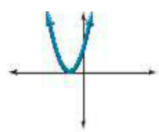
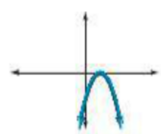
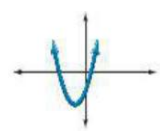
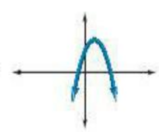
B. $-2x^2 - 4x + 5$

The Discriminant

The D________________ of a quadratic equation in the form $Ax^2 + Bx + C = 0$ is

$$(B)^2 - 4(A)(C).$$

It tells us how many ***real solutions*** there are to a quadratic equation.

If $B^2 - 4AC$ is N_______________, there are ______ real solutions.	If $B^2 - 4AC = 0$, there is ________ real solution.	If $B^2 - 4AC$ is P_______________, there are ______ real solutions.
 	 	 

Example 2: Evaluate the discriminant and determine how many real solutions for $x^2 - 4x = -4$

A =

B =

C =

You Try It! Evaluate the discriminant and determine how many real solutions

1.) $x^2 - x + 6 = 0$

2.) $2x - 5 = x^2$

<table>
<tr><th colspan="2">The Quadratic Formula</th></tr>
<tr><td colspan="2">$$\boldsymbol{x = \frac{-(B) \pm \sqrt{(B)^2 - 4(A)(C)}}{2(A)}}$$</td></tr>
<tr><td>A =
B =
C =</td><td rowspan="2"><u>Example 3:</u> Solve using the quadratic formula:
$x^2 - 3x - 10 = 0$</td></tr>
<tr><td>Find the discriminant
$B^2 - 4AC$</td></tr>
</table>

You Try It! Use the Quadratic Formula to solve each equation

3.) $x^2 + 6x + 9 = 0$

4.) $4x^2 + x = 1$

#7	**Complex Numbers** **Text: 4.8** ☐ Use the commutative, associative, and distributive properties to add and subtract complex numbers. ☐ Use the relation $i^2 = -1$ to multiply two imaginary numbers to get a real number ☐ Multiply two complex numbers Vocabulary: imaginary number, complex number

Imaginary Numbers	
You can take the square root of a negative number by using the I__________________ number ***i***. $i = \sqrt{-1}$	**Example 1:** Write $\sqrt{-18}$ using the imaginary number i. Simplify the radical as much as possible.

You Try It! Simplify each number by using the imaginary number i

1.) $\sqrt{-8}$

2.) $\sqrt{-144}$

Complex Numbers	
A C_______________ N______________ has two parts; a real part and an imaginary part (it has "i "). It is written in the form $a + bi$ where a and b are real numbers and $b \neq 0$. **Example:** $5 + 6i$	
Adding & Subtracting Complex Numbers *When adding or subtracting complex numbers, combine the real parts, and then combine the imaginary parts (just like combining like terms!!!).*	
Example 2: Find the sum $(3+ i) + (2+ 3i)$	**Example 3:** Find the sum $\left(6-\sqrt{-16}\right)+\left(-4+\sqrt{-25}\right)$
Example 4: Find the difference $(4 + 2i) - (6 - 3i)$	

You Try It! Find the sum or difference

3.) $(5 + 6i) + (- 2 + 4i)$

4.) $(12 + 5i) - (2 - i)$

Multiplying Complex Numbers

When multiplying complex numbers, use the Distributive Property or the Box Method.

<u>Example 5:</u> Find the product $(7 - 3i)(-4 + 9i)$

You Try It! Find each product

5.) $3i(1 - 2i)$

6.) $(3 + i)(2 + i)$

Finding Complex Solutions

Use the Quadratic Formula: A = B = C =	**<u>Example 5:</u>** What are the solutions of $2x^2 - 3x + 5 = 0$?

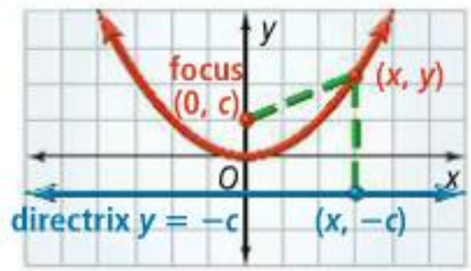
You Try It! Find the solutions to the quadratic equation

7.) $3x^2 - x + 2 = 0$

#8	**Parabolas in a Different Light** **Text: 10.2** ☐ Derive the equation of a parabola given the focus and directrix Vocabulary: focus, directrix, parabola

Definitions

A P_______________ is the set of all points in a plane that are the same distance from a fixed line and a fixed point not on the line.



The fixed point is called the F___________.

The fixed line is called the D__________________.

Vertex Form: y = A(x – h)^2 + k (h, k) is the vertex

Transformations of a Parabola

	Vertex (0, 0)	Vertex (h, k)
Equation	$y = \dfrac{1}{4c}x^2$	$y = \dfrac{1}{4c}(x-h)^2 + k$
Focus	**(0, c)**	**(h, k + c)**
Directrix	**y = –c**	**y = k – c**

Vertex at the Origin

When given ***focus*** **(0, c)**

Steps: 1. Identify **c** 2. Find $a = \frac{1}{4c}$ 3. Write equation	**Example 1:** Vertex at origin, Focus: $\left(0, \dfrac{1}{28}\right)$

When given ***directrix*** **y = – c**

Steps: 1. Identify **c** 2. Find $a = \frac{1}{4c}$ 3. Write equation	**Example 2:** Vertex at origin, Directrix: $y = -\dfrac{1}{8}$

You Try It! Use the information provided to write the vertex form of the parabola

1.) Vertex at origin, Focus: $\left(0, \frac{1}{44}\right)$

2.) Vertex at origin, Directrix: $y = -\frac{1}{4}$

Vertex at the (h, k)	
When given ***focus*** **(h, k + c)**	
<u>Steps:</u> 1. Identify **c** 2. Take **c** and **subtract k** 3. Find $\boldsymbol{a} = \frac{1}{4c}$ 4. Write equation	<u>**Example 3**</u>**:** Vertex: $(-8, -2)$, Focus: $\left(-8, -\frac{11}{4}\right)$
When given ***directrix*** **y = k – c**	
<u>Steps:</u> 1. Identify **c** 2. Take **–c** and **add k** 3. Find $\boldsymbol{a} = \frac{1}{4\boldsymbol{c}}$ 4. Write equation	<u>**Example 4**</u>**:** Vertex: $(-9, -5)$, Directrix: $y = -\frac{19}{4}$

You Try It! Use the information provided to write the vertex form of the parabola

3.) Vertex: $(4, -4)$, Focus: $\left(4, -\frac{49}{12}\right)$

4.) Vertex: $(-6, -9)$, Directrix: $y = -\frac{71}{8}$

Finding the Focus and Directrix	
Steps: 1. Identify the vertex 2. Use $\boldsymbol{a} = \frac{1}{4\boldsymbol{c}}$ to find c	**Example 5**: What are the vertex, focus, and directrix of the parabola with equation $y = \frac{1}{12}x^2$?

#9	**Circles** **Text: 10.3** ☐ Identify the center and radius from the equation of a circle ☐ Use completing the square to write the equation of a circle ☐ Explain how to derive the equation of a circle given the center and radius using the Pythagorean Theorem Vocabulary: circle, radius

Definitions	
A C__________ is the set of all points in a plane that are a distance *r* from a given point, the center of the circle.	The distance *r* is called the R_________.
Standard Form of an Equation of a Circle $(x - h)^2 + (y - k)^2 = r^2$ Center: **(h, k)** Radius: **r**	

Derive the Standard Form of an Equation of a Circle.	
Start with the Distance Formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
The radius is the distance from the center (h, k) to any point (x, y) on the circle.	
Square both sides.	

Writing the Equation of a Circle	
Example 1: Write the equation in standard form of a circle with center (–1, 3) and radius 10	
Steps: 1. Write the standard form of an equation of a circle.	
2. Plug in **h**, **k**, and **r**	
3. Simplify	

You Try It! Write the equation in standard form

1.) center (2, 3) radius 4.5

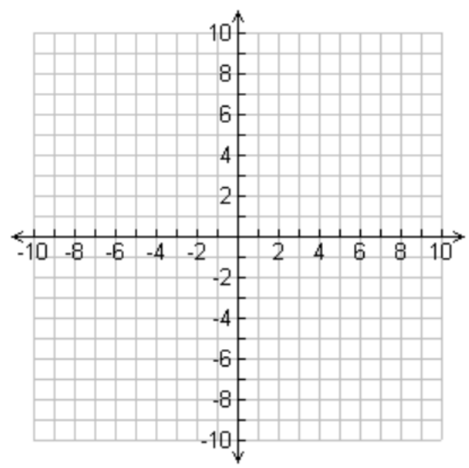
2.) center (0, 0) radius 10

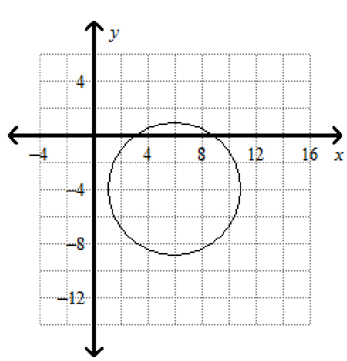
Finding the Center and Radius	
Example 2: Find the center and radius of the circle with equation $(x + 1)^2 + (y - 3)^2 = 16$	
Identify h and k	
Take the square root of the right side	

You Try It! Find the center and radius of each circle

3.) $x^2 + (y + 1)^2 = 25$

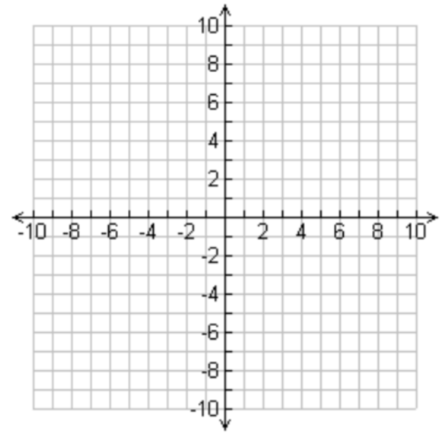
4.) $x^2 + y^2 = 64$

Graphing Circles	
Example 3: Use the center and radius to graph the circle with equation $(x + 3)^2 + (y + 2)^2 = 4$	
Center: (____, ____) Radius: ______	
Plot the center	
Go out your radius number of spaces in four directions	
Draw a circle between your four points	

Example 4: Identify the center and radius and write the equation of the graph.	
	

You Try It!

5.) Use the center and radius to graph the circle. $(x + 4)^2 + (y - 1)^2 = 1$



6.) Identify the center and radius and write the equation of the graph.