Kinematics: Key Ideas

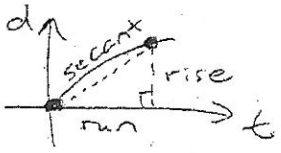
Scalar: a quantity that has magnitude (size) but no direction

Vector: a quantity that has both magnitude (size) and direction

length → arrowhead

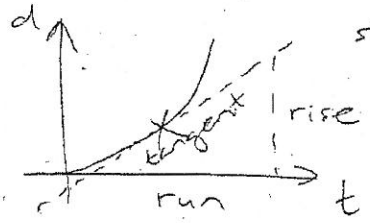
- Represented as arrows: length corresponds with magnitude (size); arrowhead shows direction

Displacement-Versus-Time Graphs (d vs. t)



slope = rise/run = average velocity

- Illustrates position at any given time
- Calculate slope to determine the velocity
  - Average velocity
    - Overall displacement travelled divided by time (slope of the secant)
  - Instantaneous velocity
    - Slope of the tangent



slope: rise/run = instantaneous velocity

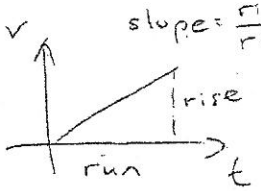
Equations: (assuming uniform acceleration)

1. $d = v_{avg}\, t$
2. $v_{avg} = \frac{1}{2}(v_i + v_f)$
3. $V_f = v_i + a\, t$
4. $d = v_i t + \frac{1}{2} a\, t^2$
5. $v_f^2 = v_i^2 + 2\, a\, d$

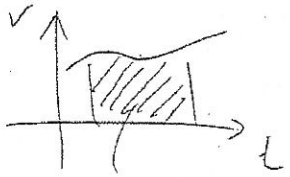
*where: d = displacement (m) v = velocity (m/s) v_{avg} = average velocity (m/s) t = time (s)
v_i = initial velocity v_f = final velocity a = acceleration (m/s^2)

Unit Conversion: 1m/s = 3.6 km/hr

Velocity-Versus-Time Graphs (v vs. t)



slope: rise/run = acceleration



area = displacement

- Illustrates velocity at any given time
- Calculate slope to determine the acceleration
- Compute area under the curve for displacement

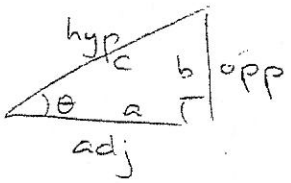
Vector Components

- Pythagorean Theorem: $a^2 + b^2 = c^2$
- Trigonometry:

$$\sin(\theta) = \frac{opp}{hyp}$$

$$\cos(\theta) = \frac{adj}{hyp}$$

$$\tan(\theta) = \frac{opp}{adj}$$



Acceleration Due To Gravity

- At the Earth's surface, $a_g = -9.80 m/s^2$
- Velocity is zero at the maximum height for objects thrown vertically

$a = -9.80 m/s^2$

Two-Dimensional Kinematics (Projectile Motion)

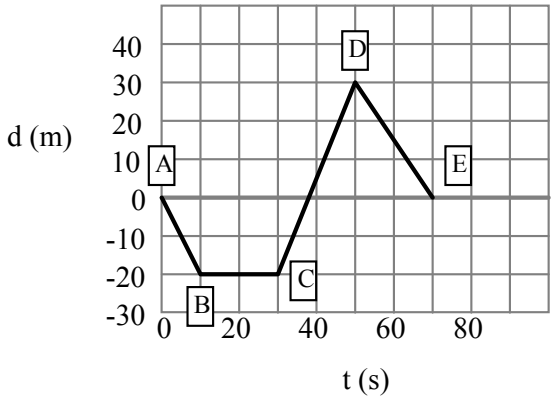
- The x part of motion occurs exactly as it would if the y part did not occur at all. Similarly, the y part of motion occurs exactly as it would if the x part of motion did not occur.

Unit 2: Kinematics in 1-D
Exam Preparation

1. 1. A bike first accelerates from 0.0 m/s to 5.0 m/s in 4.5 s, then continues at this constant speed for another 4.5 s. What is the total distance traveled by the bike?

2. A car traveling at 20 m/s when the driver sees a child standing in the road. He takes 0.80 s to react, then steps on the brakes and slows at 7.0 m/s^2. How far does the car go before it stops?

3. Answer the following questions about the car whose motion is graphed below:



   a. When was the car 20 m west of the origin?
   b. where was the car at 50 s?
   c. The car suddenly reversed direction. When and where did that occur?

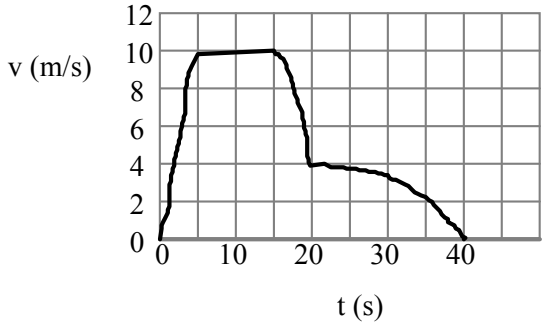
4. A car starts 200 m west of the town square and moves with a constant velocity of 15 m/s toward the east. Draw a graph that represents the motion of the car
   a. Where will the car be 10 minutes later?
   b. When will the car reach the town square?

5. At the same time the car in #4 left, a truck was 400 m east of the town square moving west at a constant velocity of 12 m/s.
   a. Add the truck's motion to the graph you drew for question #4.
   b. Find the time where the car passed the truck.

6. A car is coasting backwards downhill at a speed of 3.0 m/s when the driver gets the engine started. After 2.5 s, the car is moving uphill at 4.5 m/s. Assuming that uphill is positive direction, what is the car's average acceleration?

7. A car slows from 22 m/s to 3.0 m/s at a constant rate of 2.1 m/s^2. How many seconds are required before the car is traveling 3.0 m/s?

8. Look at the velocity-time graph given



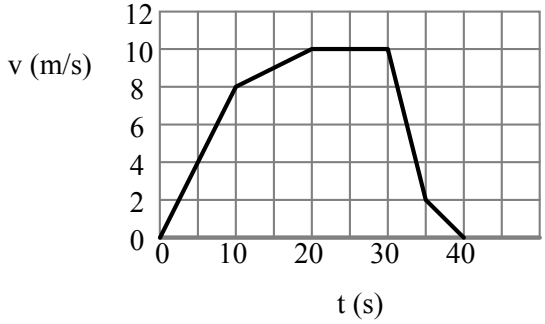
a. During which time interval or intervals is the speed constant?
b. During which interval or intervals is the train's acceleration positive?
c. During which time interval is its acceleration most negative?
d. Find the average acceleration during the following time intervals:
i. 0 to 5 s. ii. 15 to 20 s. iii 0 to 40 s.

9. An airplane starts from rest and accelerates at a constant rate of 3.00 m/s^2 for 30.0 s before leaving the ground.
a. How far did it move?
b. How fast was it going when it took off?

10. A brick is dropped from a high scaffold.
a. What is its velocity after 4.0 s?
b. How far does the brick fall during this time?

11. A tennis ball is thrown straight up with an initial speed of 22.5 m/s. It is caught at the same distance above the ground.
a. How high does the ball rise?
b. How long does the ball remain in the air?

12. Consider the following velocity-time graph.



Determine the displacement after t = ...
a. 10 s. b. 20 s. c. 30 s. d. 40 s.

13. A bag is dropped from a hovering helicopter. When the bag has fallen for 2.00 s,
a. what is the bag's velocity?
b. how far has the bag fallen?

1) 33.8 m 2) 44.6 m 3) a. Between B and C b. 30 m East c. D 4) a. 8800 m b. 13.3 s 5) b. 22.2 s 6) 3.0 m/s 7) 9.04 s 8) a. Between 5 and 15 s b. Between 0 and 5 s c. Between 15 and 20 s d. i. 2.0 m/s^2 ii. 1.2 m/s^2 iii 0 m/s^2 9) a. 1350 m b. 90 m/s 10) a. 39.2 s b. 78.4 m 11) 25.8 m b. 4.6 s 12) a. 40 m b. 130 m c. 230 m d. 265 m 13) a. 19.6 m/s b. 19.6 m

Vectors

A marathon runner runs 15 kilometers east then 10. kilometers south. What is displacement? **[18km 34° S of E]**

Find the x and y components of the displacement: 20.0m 52.0° W of N **[-15.8m, 12.3m]**

A student walks 45m south and turns to walks 35m 23° S of W. Find the displacement. **[67m 61° S of W]**

Add the following vectors: 5.00m E, 7.50m 60.0° S of W, 10.0m S, 3.50 m 35.0° N of E **[15.1m 74.1° S of E]**

Calculate the displacement and distance: 6.2km N, 3.5km E, 2.5km W and 8.5km S **[2.5km 67° S of E, 21km]**

A boat sails 100.km 35.0° north of east, then 20.0km west. What is the displacement? **[84.4km 42.8° N of E]**

A balloon drifts 30.km 75° west of south, then its engines are turned on and it flies 20.km 40.° east of south. What is the total displacement? **[28km 55° S of W]**

A ship leaves port and sails 26 kilometers at 72° south of east. Then the ship turns north and sails 4.0 kilometers. How far is the ship from port and what heading is the shortest way home? **[22km 69° S of E, 22km 69° N of W]**

A ship travels at 25.0km/h 64.0° east of north for 8.00h, then at 40.0km/h 20.0° south of east for 6.00h. What is the total displacement? **[406km 0.80° N of E]**

Kinematics practice problems:

1. Georgia is jogging with a velocity of 4 m/s when she accelerates at 2 m/s^2 for 3 seconds. How fast is Georgia running now?

2. In a football game, running back is at the 10 yard line and running up the field towards the 50 yard line, and runs for 3 seconds at 8 yd/s. What is his current position (in yards)?

3. A cat is moving at 18 m/s when it accelerates at 4 m/s^2 for 2 seconds. What is his new velocity?

4. A race car is traveling at +76 m/s when is slows down at -9 m/s^2 for 4 seconds. What is his new velocity?

5. An alien spaceship is 500 m above the ground and moving at a constant velocity of 150 m/s upwards. How high above the ground is the ship after 5 seconds?

6. A bicyclist is traveling at +25 m/s when he begins to decelerate at -4 m/s^2. How fast is he traveling after 5 seconds?

7. A squirrel is 5.0 m away from you while moving at a constant velocity of 3 m/s away from you. How far away is the squirrel after 5 seconds?

8. A ball is dropped off a very tall canyon ledge. Gravity accelerates the ball at 9.8 m/s^2. How fast is the ball traveling after 5 seconds?

9. During a race, a dragster is 200 m from the finish line when something goes wrong and it stops accelerating. It travels at a constant velocity of 45 m/s for 3 seconds to try to finish the race. How far from the finish line is the dragster after 3 seconds?

10. A dog is 60 m away while moving at a constant velocity of 10 m/s towards you. Where is the dog after 4 seconds?

11. Isaac throws an apple straight up (in the positive direction) from 1.0 m above the ground, reaching a maximum height of 35 meters. Neglecting air resistance, what is the ball's velocity when it hits the ground?

~~12. Two kittens are on opposite sides of a field, 250 m apart. Kitten A runs at a constant speed of 25 m/s due east on a collision course with kitten B, which is traveling west at 12 m/s. How much time elapses before the two kittens collide?~~

Worksheet 2.6 - Kinematic Equations

1. A ball rolling down a hill was displaced 19.6 m while uniformly accelerating from rest. If the final velocity was 5.00 m/s. what was the rate of acceleration?

2. A car starts from rest and accelerates uniformly to reach a speed of 21 m/s in 7.0 s. What was the speed of the object after 2.0 seconds?

3. A bike rider accelerates uniformly at 2.0 m/s² for 10.0 s. If the rider starts from rest, calculate the distance traveled in the **fourth** second.
(i.e. between t = 3 s and t = 4 s).

4. If a bullet leaves the muzzle of a rifle at 600.0 m/s, and the barrel is 0.90 m long, what was the acceleration of the bullet while in the barrel?

5. The Jamaican bobsled team hit the brakes on their sled so that it decelerates at a uniform rate of 0.43 m/s². How long does it take to stop if it travels 85 m before coming to rest?

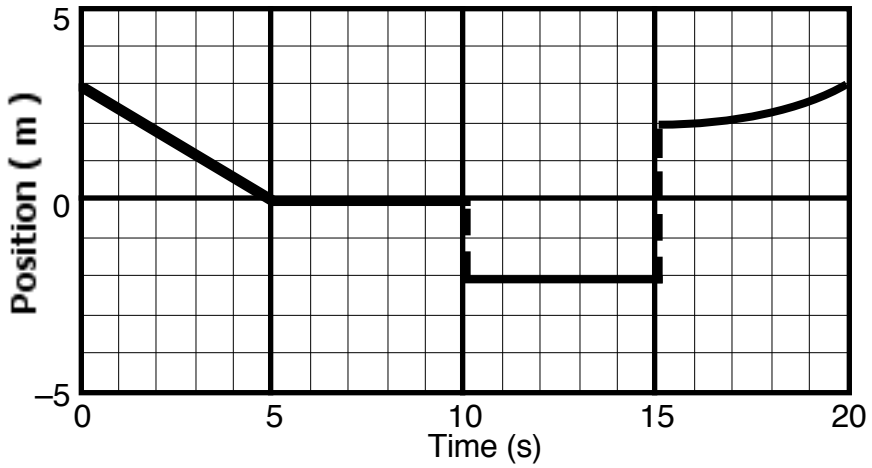
Bonus: A driver of a car going 90 km/h suddenly sees the lights of a barrier 40.0 m ahead. It takes the driver 0.75 s before he applies the brakes (this is known as reaction time). Once he does begin to brake, he decelerates at a rate of 10.0 m/s².

a) Does he hit the barrier?

b) SUPER-BONUS: What would be the maximum speed at which the car could travel and NOT hit the barrier 40.0 m ahead?

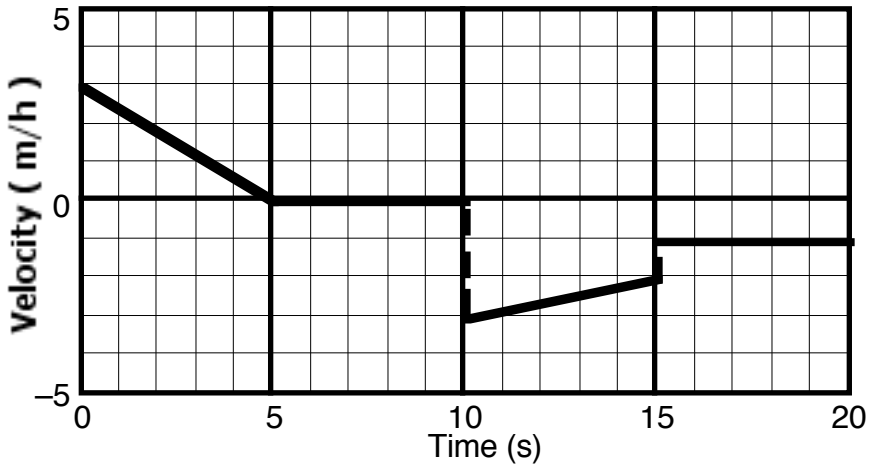
Graphing:

1.



a. Which 5 second interval(s) show a negative velocity?
b. Which 5 second interval(s) show a positive acceleration?
c. Which 5 second interval(s) show a velocity that is constant?
d. Which 5 second interval(s) show a velocity of zero?
e. What is the velocity at 6 seconds?
f. What is the velocity at 19 seconds?
g. What is the displacement from 5 to 15 seconds?
h. What are the units of slope from the graph above?

2.



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e. What is the velocity at 6 seconds?
f. What is the velocity at 19 seconds?
g. What are the units of slope from the graph above?

Free Falling

1) An object is thrown horizontally at a velocity of 10.m/s from the top of a 90.m building. Calculate the distance from the base of the building that the object will hit the ground. **[43m]**
2) A pop can is rolled off a 3.0m table top and lands 2.0m from the table top. How long was the pop can in the air? How fast was the pop can moving on the table top? **[0.78s, 2.6m/s]**
3) A person knows that he can roll a ball at exactly 3.2m/s off a 2.4m table top. If he wants the ball to land on an 'x' mark, how far from the table base should he place the 'x'? **[2.2m]**
4) If a ball pitched horizontally reaches a home plate 21.3m away falling 0.620m, how fast was the pitch? **[59.9m/s]**
5) During a thunderstorm, a tornado lifts a car to a height of 123m above the ground. The tornado flings the car horizontally with an initial speed of 80.0m/s horizontally. How long does the car take to reach the ground? How far does the car travel in the x direction before hitting the ground? **[5.00s, 401m]**
6) A person is going to try to roll a pop can off a 2.3m table top and have it land in a hole 2.3m from the base of the table. What should the velocity of the can be as it leaves the table? **[3.4m/s]**

Projectile Motion Worksheet

1. A ball rolls off a desk at a speed of 3.0 m/s and lands 0.40 seconds later.
   a) How far from the base of the desk does the ball land?
   b) How high is the desk?
   c) What is the speed and angle of impact?

2. A slingshot is used to launch a stone horizontally from the top of a 20.0 meter cliff. The stone lands 36.0 meters away.
   a) At what speed was the stone launched?
   b) What is the speed and angle of impact?

3. A ball rolls with a speed of 2.0 m/s across a level table that is 1.0 m above the floor. Upon reaching the edge of the table, it follows a parabolic path to the floor. How far along the floor is the landing spot from the table? [0.90 m]

4. A rescue pilot drops a survival kit while her plane is flying at an altitude of 2000.0 m with a forward velocity of 100.0 m/s. If air friction is disregarded, how far in advance of the starving explorer's drop zone should she release the package? [2020 m]

5. A rifle is fired horizontally and travels 200.0 m [E]. The rifle barrel is 1.90 m from the ground. What speed must the bullet have been travelling at? Ignore friction. [321 m/s]

6. A skier leaves the horizontal end of a ramp with a velocity of 25.0 m/s [E] and lands 70.0 m from the base of the ramp. How high is the end of the ramp from the ground? [38.5 m]

7. An astronaut stands on the edge of a lunar crater and throws a half-eaten Twinkie™ horizontally with a velocity of 5.00 m/s. The floor of the crater is 100.0 m below the astronaut. What

horizontal distance will the Twinkie™ travel before hitting the floor of the crater? (The acceleration of gravity on the moon is $1/6^{th}$ that of the Earth). [55.3 m]

8. A canon ball fired horizontally from a cliff has a velocity directed at 60^{o} below horizontal when it hits the ground 3.0 seconds later.
   a) How high is the cliff?
   b) How far from the base of the cliff does the canon ball land?

9. A baseball player leads off the game and hits a long home run. The ball leaves the bat at an angle of 30.0^{o} from the horizontal with a velocity of 40.0 m/s. How far will it travel in the air? [141 m]

10. A golfer is teeing off on a 170.0 m long par 3 hole. The ball leaves with a velocity of 40.0 m/s at 50.0^{o} to the horizontal. Assuming that she hits the ball on a direct path to the hole, how far from the hole will the ball land (no bounces or rolls)? [9.38 m]

11. A punter in a football game kicks a ball from the goal line at 60.0^{o} from the horizontal at 25.0 m/s.
    a) What is the hang time of the punt? [4.41 s]
    b) How far down field does the ball land? [55.2 m]

12. A cannon fires a cannonball 500.0 m downrange when set at a 45.0^{o} angle. At what velocity does the cannonball leave the cannon? [70.0 m/s at 45.0^{o}]

13. A lovesick lad wants to throw a bag of candy and love notes into the open window of his girlfriend's bedroom 10.0 m above. Assuming it just reaches the window, he throws the love gifts at 60.0^{o} to the ground:
    a) At what velocity should she throw the bag? [16.2 m/s at 60.0^{o} to the ground]
    b) How far from the house is he standing when he throws the bag? [11.5 m]

14. You are piloting a helicopter which is rising vertically at a uniform velocity of 14.70 m/s. When you reach 196.00 m, you see Barney (Uh-oh). A large object is projected with a horizontal velocity of 8.50 m/s from the rising helicopter.
    a) When does the ball reach Barney's head if he is standing in a hole with his head at ground level? [7.99 s]
    b) Where does Barney have to be horizontally relative to the helicopter's position? [68.0 m]
    c) What is the vertical velocity when it hits the ground? [- 63.7 m/s]

15. An object is punted at 25.0 m/s [40.0^{o} N of E] on G's home planet. What is the range of the object on level ground? (Use g = 18.0 m/s^2) [34.2 m]

16. An elastic loaded balloon launcher fires balloons at an angle of [38.0^{o} N of E] from the surface of the ground. If the initial velocity is 25.0 m/s, find how far away the balloons are from the launcher when they hit the level ground again. [61.8 m]

17. A movie stunt driver on a motorcycle speeds horizontally off a 50.0 m high cliff. How fast (in km/h) must the motorcycle leave the cliff-top if it's to land on the level ground below at a distance of 90.0 m from the base of the cliff? [101 km/h]

18. A football is kicked at 37.0^{o} to the horizontal at 20.0 m/s from the player's hand at 1.00 m from the ground. How far did the football travel before hitting the ground? [40.5 m]

19. The same football in #15 is kicked from the ground instead.
    a) Find the maximum height. [7.38 m]
    b) Find the time of travel. [2.45 s]
    c) How far away does it hit the ground? [39.2 m]
    d) Find the velocity vector at maximum height. [16.0 m/s which is horizontal]
    e) Find the acceleration vector at maximum height. [9.81 m/s^2 down]