Day 7: The Quadratic Formula and Discriminants

Warm-Up:

9. Solve each of the quadratic functions by graphing and algebraic reasoning

   a. $x^2 - 3 = 0$

   b. $x^2 + 5x - 8 = 0$

   c. Explain why having alternative methods of solving is important.

Day 7: The Quadratic Formula and Discriminants

Standard form of a quadratic ***equation***: __________________________

The solutions of some quadratic equations are not rational, or are too messy to obtain by factoring. For such equations, the most common method of solution is the quadratic formula.

The quadratic formula: ________________________________ can be used to solve for the values of x.

Notice that there is a +/- sign in the formula. There are actually _____________________ for any quadratic formula.

Solve using the quadratic formula.

Example 1: $x^2 + 9x + 20 = 0$

Example 2: $x^2 - x = 5x - 9$

Example 3: $-x^2 + 2x = 2$

Example 4: $7x^2 - 12x + 3 = 0$

Example 5: $4x^2 + 12x + 9 = 0$

Example 6: $x^2 - 5x - 5 = 0$

Types of Zeroes

Given the following quadratic functions, use the quadratic formula to find the zeros:

1. $x^2 - x - 6 = 0$ 2. $x^2 + 16 = 0$ 3. $x^2 + 4x + 4 = 0$

Use your calculator to examine the graphs of each function. How does each graph relate to the number of solutions for that problem?

Which function's graph did not touch the x-axis? Which touched the x-axis once?

How was the quadratic formula different when you had two, one, or no zeroes?

1. When can you expect 2 solutions in a quadratic equation?
2. When can you expect 1 solution in a quadratic equation?
3. When can you expect a quadratic to have no solutions?

Recall the quadratic formula:

This part in the square root helps us to determine how many solutions a quadratic will have:

This is called the **Discriminant**. Calculate the discriminant for the 3 introductory problems.

1. $x^2 - x - 6 = 0$ 2. $x^2 + 16 = 0$ 3. $x^2 + 4x + 4 = 0$

<u>Types of Quadratic Solutions</u>

Quadratic solutions are either ________________ or __________________.

Real solutions are the solutions you get from factoring, the zeroes on the graph, and when you are able to do the square root in the quadratic formula.

Imaginary solutions do not show up on the graph or when factoring. In fact, quadratics with imaginary solutions cannot be factored.

- If the discriminant is **positive**, the quadratic has ___________________ solutions.
- If the discriminant is **zero**, the quadratic has ___________________ solution.
- If the discriminant is **negative**, the quadratic has ___________________ solutions

Real solutions can also be divided into two types:

- Rational solutions are when the discriminant evaluates to ___________________.
- Irrational solutions are when the discriminant evaluates to __________________.

Practice: Given the following quadratics, calculate the discriminant and determine how many and what types of solutions they will have. For real solutions, determine if they will be rational or irrational.

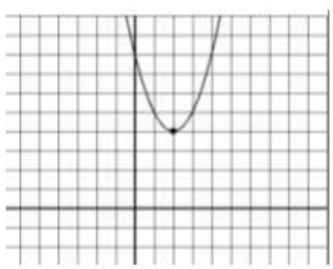
1. $x^2 - 6x + 11 = 2$___________________ 2. $3x^2 + 5x = 12$___________________

3. $3x^2 + 48 = 0$ __________________ 4. $x^2 - 27 = 0$__________________

5. $x^2 + x + 1 = 0$___________________ 6. $x^2 + 4x - 1 = 0$___________________

Given the following graphs of quadratic functions:

a) Determine the **sign** of the discriminant and **b)** whether the solutions are **real** or **non-real**.

1.


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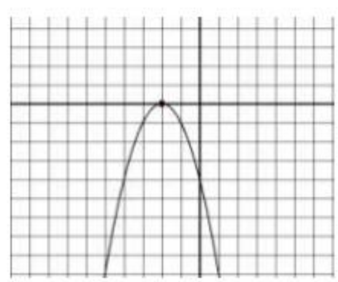

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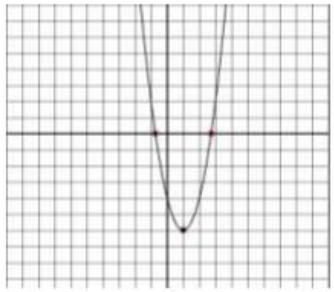
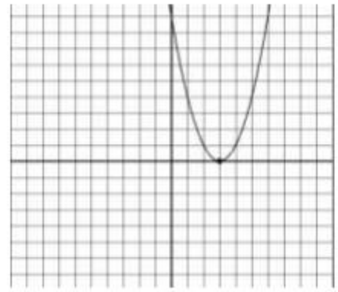


4.


Practice

Calculate the discriminant and determine the number and types of solutions.

Function	Discriminant	Number and Type of Solutions
1. $x^2 - 3x - 4 = 0$ $a = 1, b = -3, c = -4$ $(-3)^2 - 4(1)(-4)$ $9 - (-16)$ $9 + 16$	25	2 rational solutions
2. $x^2 - 6x + 9 = 0$		
3. $x^2 + 6x = -9$		
4. $x^2 - 6x - 16 = 0$		
5. $2x^2 - 6x - 13 = 0$		
6. $-x^2 + 2x - 1 = 0$		
7. $2x^2 + 3 = 2x$		
8. $x^2 + 2x + 1 = 0$		
9. $x^2 + 2x = -3$		
10. $x^2 - 6x + 9 = 0$		
11. $x^2 + 5x + 8 = 0$		
12. $2x^2 - 5x + 6 = 0$		
13. $x^2 - 5x = 10$		
14. $x^2 - 6x + 3x = 4 - 11$		

Day 8: Review and Quiz

Warm-Up:

10. Find the zeros of the following. Show all your work using the appropriate method.

a. $x^2 - 9x + 12 = 0$

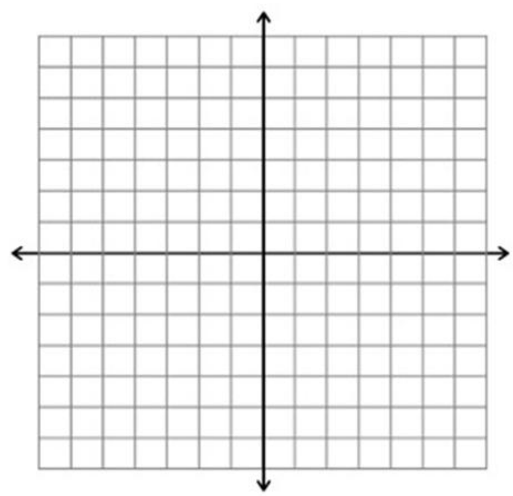
b. $x^2 - 16 = -4x$

c. $2x^2 + 8x = 13$

d. $x^2 + 3x = 28$

e. Show your work in the boxes to find the requested values of $y = 2x^2 - 5x - 3$ algebraically. Then graph.

<table>
<tr><td rowspan="3">Solve by factoring</td><td>x-intercepts</td><td>Vertex</td></tr>
<tr><td>y-intercepts</td><td>Maximum or minimum?</td></tr>
<tr><td colspan="2">Axis of symmetry</td></tr>
</table>



Day 9: FRED Functions

Warm-Up:

1. An electronics company has a new line of portable radios with CD players. Their research suggests that the daily sales, s, for the new product can be modeled by $s = -p^2 + 120p + 1400$, where p is the price of each unit.
   a. What is the maximum daily sales total for the new product?

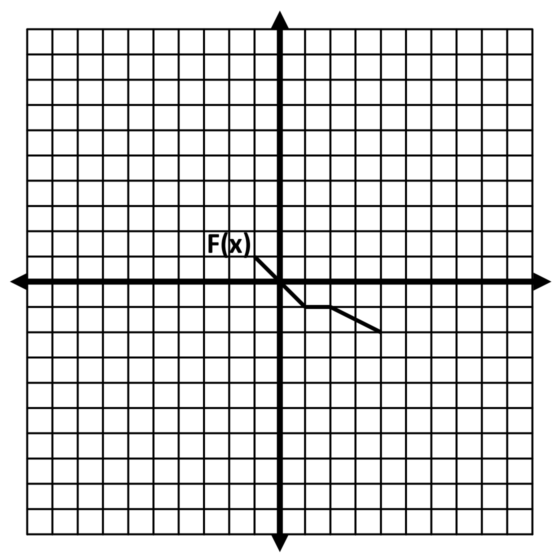
   b. What price should the company charge to make this profit?

2. The shape of the Gateway Arch in St. Louis is a catenary curve, which closely resembles a parabola. The function $y = -\frac{2}{315}x^2 + 4x$ closely models the shape of the arch, where y is the height in feet and x is the horizontal distance from the base of the left side of the arch in feet.
   a. What is the width of the arch at the base?

   b. What is the maximum height of the arch?

Day 9: FRED Functions Part 1

To the right is a graph of a **"Fred" function**. We can use Fred functions to explore transformations in the coordinate plane.



I. Let's review briefly.

1. a. Explain what a function is in your own words.

   b. Using the graph, how do we know that Fred is a function?

2. a. Explain what we mean by the term domain.

   b. Using the graph, what is the domain of Fred?

3. a. Explain what we mean by the term range.

   b. Using the graph, what is the range of Fred?

4. Let's explore the points on Fred.
   a. How many points lie on Fred? Can you list them all?

   b. What are the key points that would help us graph Fred?

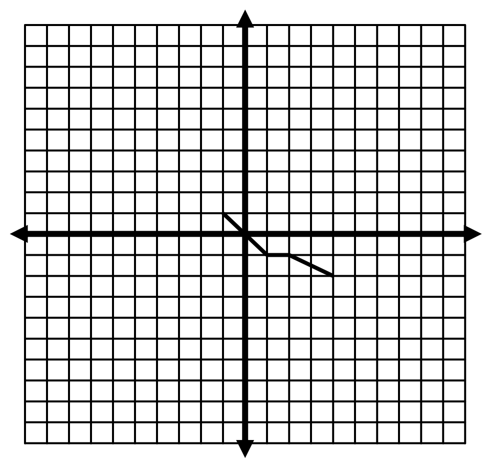
   We are going to call these key points **"characteristic" points**. It is important when graphing a function that you are able to identify these characteristic points.

   c. Use the graph of graph to evaluate the following.

F(1) = _____ F(–1) = _____ F(_____) = –2 F(5) = ______

II. Remember that F(x) is another name for the y-values.

Therefore the equation of Fred is **y = F(x)**.



x	F(x)
–1	
1	
2	
4	

1. Why did we choose those x-values to put in the table?

Now let's try graphing Freddie Jr.: **y = F(x) + 4**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) + 4

x	y
–1	
1	
2	
4	

2. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) + 4? (Be specific.)

3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

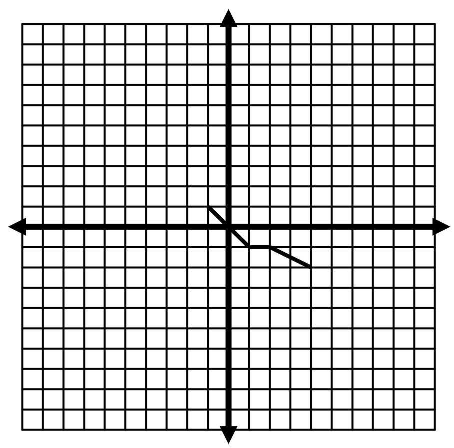
4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. In y = F(x) + 4, how did the "+4" affect the graph of Fred? Did it affect the domain or the range?

III. Suppose Freddie Jr's equation is: **y = F(x) – 3**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) – 3

x	y
–1	
1	
2	
4	



1. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) – 3? Be specific.

2. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

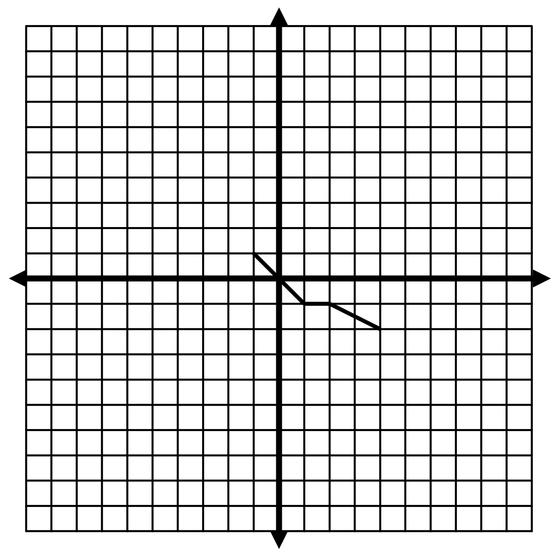
3. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

4. In y = F(x) – 3, how did the "– 3" affect the graph of Fred? Did it affect the domain or the range?

IV. Checkpoint: Using the understanding you have gained so far, describe the affect to Fred for the following functions.

Equation	Effect to Fred's graph
Example: y=F(x) + 18	Translate up 18 units
1. y = F(x) – 100	
2. y = F(x) + 73	
3. y = F(x) + 32	
4. y = F(x) – 521	

V. Suppose Freddie Jr's equation is: **y = F(x + 4)**.



1. Complete the table.

x	x + 4	y
–5	–1	1
	1	–1
	2	–1
	4	–2

*(**Hint**: Since, x + 4 = –1, subtract 4 from both sides of the equation, and x = –5. Use a similar method to find the missing x values.)*

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). The first point is (–5, 1).

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(x + 4)? (Be specific.)

4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

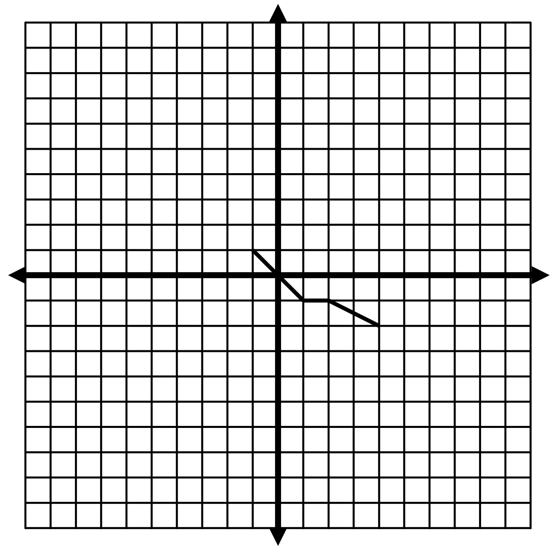
6. In y = F(x + 4), how did the "+4" affect the graph of Fred? Did it affect the domain or the range?

VI. Suppose Freddie Jr's equation is: **y = F(x – 3)**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

1. Complete the table.

y = F(x – 3)

x	**x – 3**	**y**
	–1	
	1	
	2	
	4	



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (2, 1).*]

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(x – 3)? (Be specific.)

4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

6. In y = F(x – 3), how did the " –3" affect the graph of Fred? Did it affect the domain or the range?

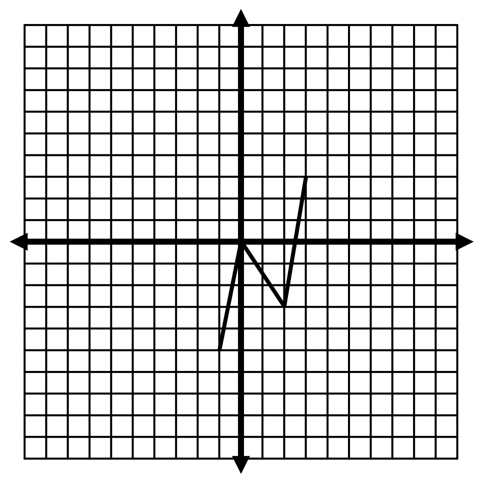
VII. Checkpoint: Using the understanding you have gained so far, describe the effect to Fred for the following functions.

Equation	**Effect to Fred's graph**
Example: y=F(x + 18)	Translate left 18 units
1. y = F(x – 10)	
2. y = F(x) + 7	
3. y = F(x + 48)	
4. y = F(x) – 22	
5. y = F(x + 30) + 18	

VIII. Checkpoint: Using the understanding you have gained so far, write the equation that would have the following effect on Fred's graph.

Equation	Effect to Fred's graph
Example: y=F(x + 8)	Translate left 8 units
1.	Translate up 29 units
2.	Translate right 7
3.	Translate left 45
4.	Translate left 5 and up 14
5.	Translate down 2 and right 6

IX. Now let's look at a new function.
Its notation is **H(x)**, and we will call it **Harry**.
Use Harry to demonstrate what you have learned so far about the transformations of functions.

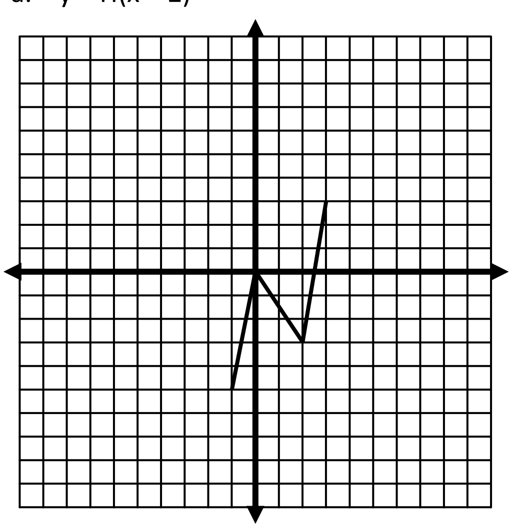


1. What are Harry's characteristic points?

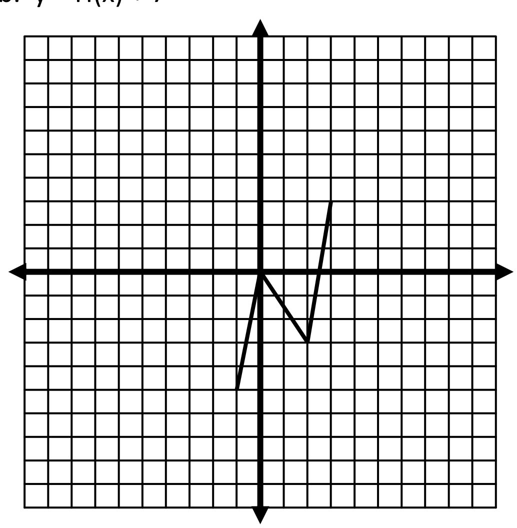
____

2. Describe the effect on Harry's graph for each of the following.
   a. H(x – 2) ____
   b. H(x) + 7 ____
   c. H(x+2) – 3 ____

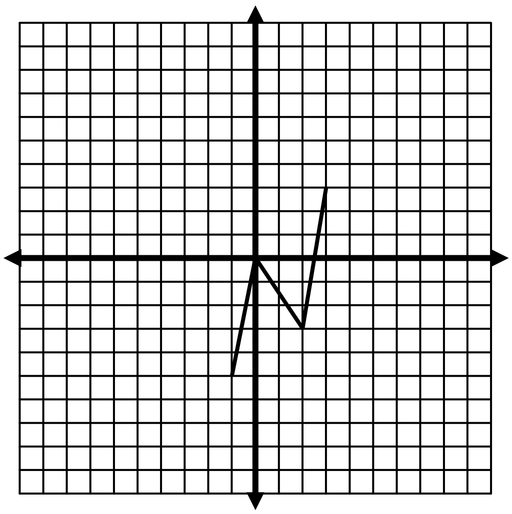
3. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.
   a. y = H(x – 2)



   b. y = H(x) + 7



c. $y = H(x+2) - 3$



Day 10: Transformations of Quadractics

Warm-Up:

11. Using the discriminant, determine the amount and type of solutions each equation will have. Then find the exact value of the solutions.

a. $x^2 + 4x + 5 = 0$

b. $x^2 - 2x + 1 = 0$

c. $2x^2 - 3x - 10 = 0$

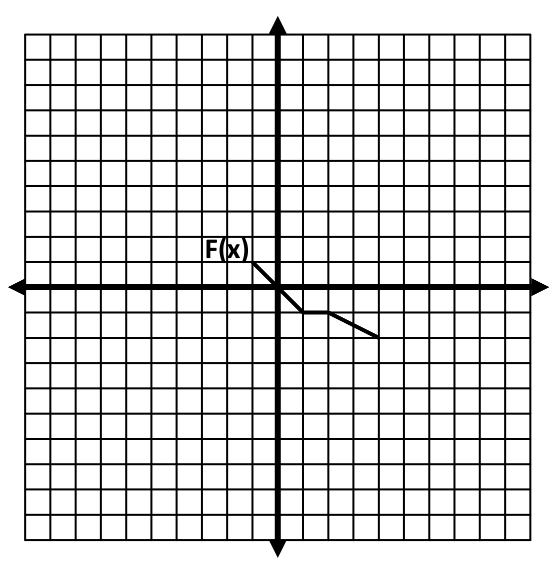
Day 10: FRED Functions Part 2

I. Let's suppose that Freddie Jr. is **y = – F(x)**

7. Complete the table.

y = – F(x)

x	F(x)	y
–1	1	–1
1		
2		
4		



8. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (–1, –1).*]

9. What type of transformation maps Fred, F(x), to Freddie Jr., –F(x)? (Be specific.)

10. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

11. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

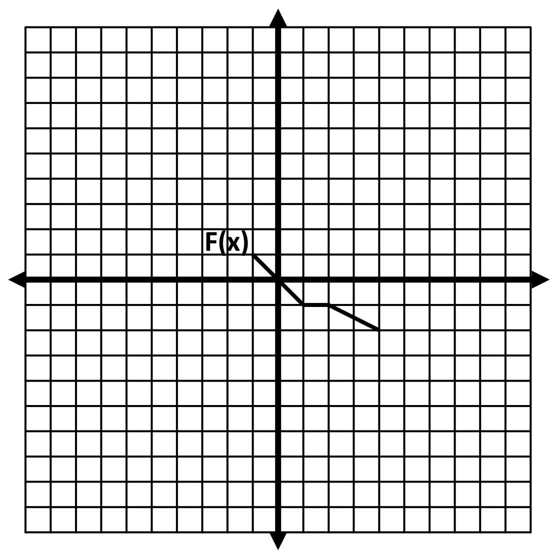
12. In y = – F(x), how did the negative coefficient of "F(x)" affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

II. Now let's suppose that Freddie Jr. is **y = F(–x)**

1. Complete the table.

y = F(–x)

x	**–x**	**y**
	–1	
	1	
	2	
	4	



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (1, 1).*]

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(–x)? (Be specific.)

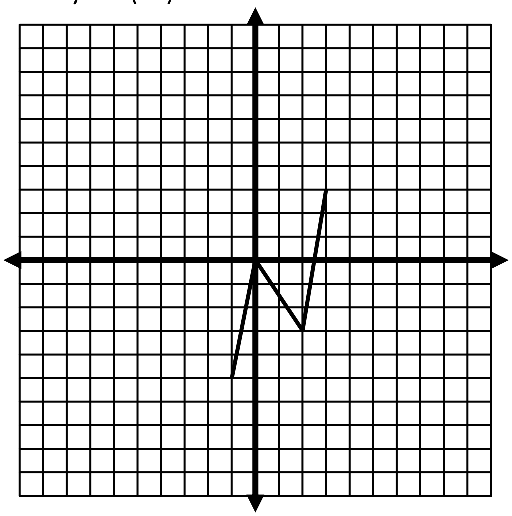
4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

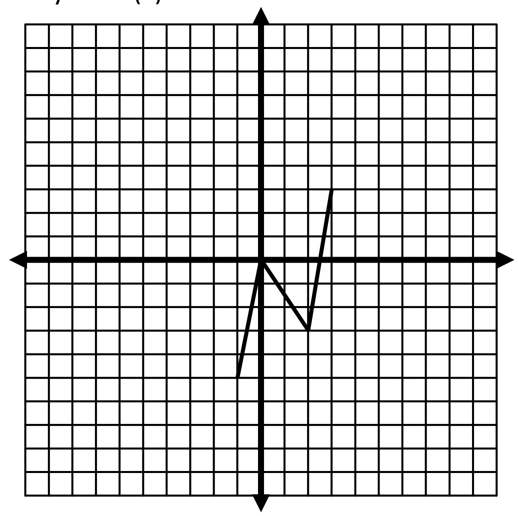
6. In y = F(–x), how did the negative coefficient of "x" affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

III. Checkpoint: Harry is H(x) and is shown on each grid. Use Harry's characteristic points to graph Harry's children without making a table.

1. y = H(–x)



2. y = – H(x)

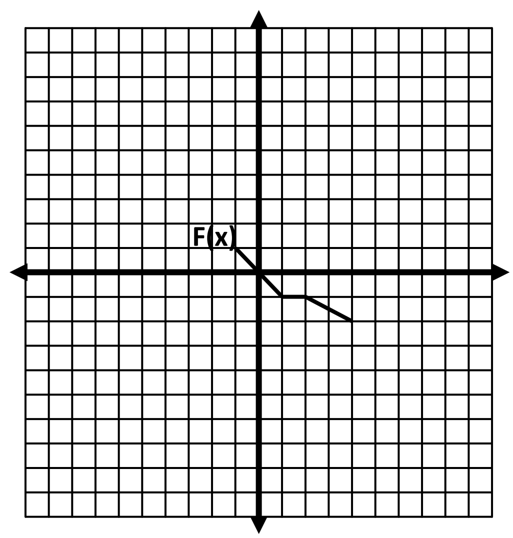


IV. Let's suppose that Freddie Jr. is **y = 4 F(x)**

1. Complete the table.

y = 4 F(x)

x	**F(x)**	**y**
–1		
1		
2		
4		



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, 4).*]

3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

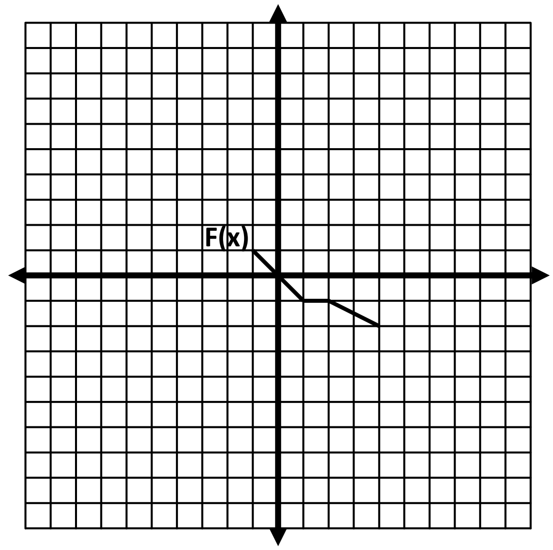
5. In y = 4 F(x), the coefficient of "F(x)" is 4. How did that affect the graph of Fred? Is this one of the transformations we studied? If so, which one? If not, explain.

V. Now let's suppose that Freddie Jr. is **y = ½ F(x)**.

1. Complete the table.

y = ½ F(x)

x	**F(x)**	**y**
–1		
1		
2		
4		



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, ½).*]

3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. In y = ½ F(x), the coefficient of "F(x)" is ½. How did that affect the graph of Fred? How is this different from the graph of y = 4 F(x) on the previous page?

VI. Checkpoint:

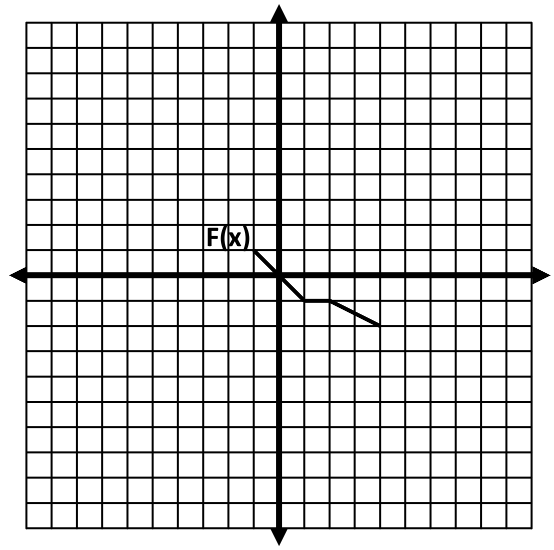
1. Complete each chart below. Each chart starts with the characteristic points of Fred.

x	**F(x)**	**3 F(x)**
–1	1	
1	–1	
2	–1	
4	–2	

x	**F(x)**	**¼ F(x)**
–1	1	
1	–1	
2	–1	
4	–2	

2. Compare the 2nd and 3rd columns of each chart above. The 2nd column is the y-value for Fred. Can you make a conjecture about how a coefficient changes the parent graph?

VII. Now let's suppose that Freddie Jr. is **y = –3 F(x)**.



1. Complete the table.

y = –3 F(x)

x	**F(x)**	**y**
–1		
1		
2		
4		

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, –3).*]

3. Reread the conjecture you made in #7 on the previous page. Does it hold true or do you need to refine it? If it does need some work, restate it more correctly here.

VIII. Checkpoint: Let's revisit Harry, H(x).

4. Describe the effect on Harry's graph for each of the following.

Example: –5H(x) <u>Each point is reflected in the x-axis and is 5 times as far from the x-axis.</u>

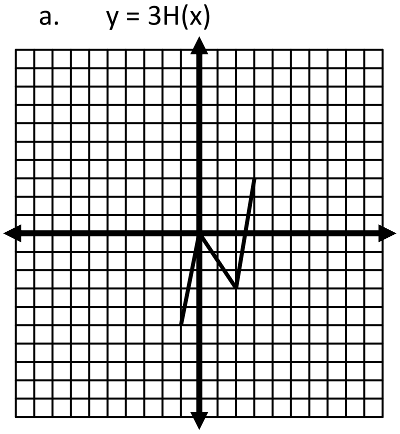
d. 3H(x) ____________________

e. –2H(x) ____________________

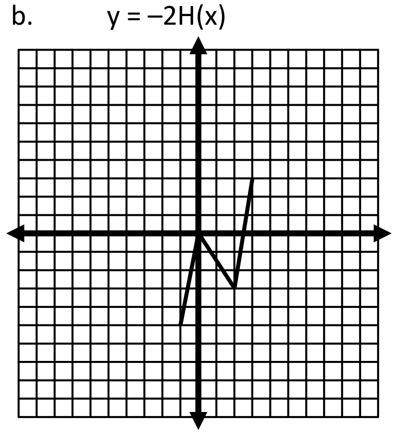
f. –H(x) ____________________

5. Use your answers to questions 1 and 2 to help you sketch each graph *<u>without using a table</u>*.

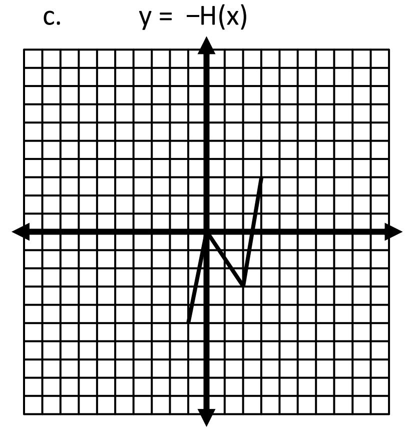
a. y = 3H(x)



b. y = –2H(x)



c. y = –H(x)



Quadratic Graphs

In your graphing calculator, graph the function y = x^2 (the quadratic parent function). Then graph each function below. Compare the new graph to the parent graph and write your observations about the location of the vertex, the overall shape, and the slope of the sides of the new graph in the blanks at the right.

Part A: The Effect of *a*

1. $y = 4x^2$

Vertex: _______________

Shape Change or Shift Change? : _________________

What was the change? __________________________

2. $y = \frac{1}{4}x^2$

Vertex: _____________

Shape Change or Shift Change? : __________________

What was the change? __________________________

3. $y = -4x^2$

Vertex: ___________________

Shape Change or Shift Change? : __________________

What was the change?____________________

4. $y = -\frac{1}{4}x^2$

Vertex: ________________________

Shape Change or Shift Change? : _________________

What was the change? _________________

Part B: The Effect of *h*

5. $y = (x+2)^2$

Vertex: _________________________

Shape Change or Shift Change? : _________________

What was the change? __________________________

6. $y = (x-4)^2$

Vertex: ______________________

Shape Change or Shift Change? : _________________

What was the change? ____________________

7. $y = -(x+5)^2$

Vertex: _______________________

Shape Change or Shift Change? : ________________

What was the change? __________________________

8. $y = -(x-6)^2$

Vertex: ______________________________

Shape Change or Shift Change? : _________________

What was the change? ___________________________

Part C: The Effect of *k*

9. $y = x^2 + 1$

Vertex: ___________________________

Shape Change or Shift Change? : ________________

What was the change? ________________________

10. $y = x^2 - 2$

Vertex: __________________________

Shape Change or Shift Change? : ______________

What was the change? ___________________

11. $y = -x^2 + 7$

Vertex: __________________________

Shape Change or Shift Change? : _________________

What was the change? ________________________

12. $y = -x^2 - 10$

Vertex: __________________________

Shape Change or Shift Change? : _________________

What was the change? ________________________

Day 11: Quadratic Systems

Warm-Up:

12. Given the following functions, specifically describe the transformation from the identity function $y = x^2$

a. $y = (x+3)^2 - 7$

b. $y = 5x^2 + 12$

c. $y = \frac{1}{2}(x - 2)^2 + 4$

Day 11: Solving and Graphing Quadratic Inequalities and Systems (Algebra 2 Text p. 269)

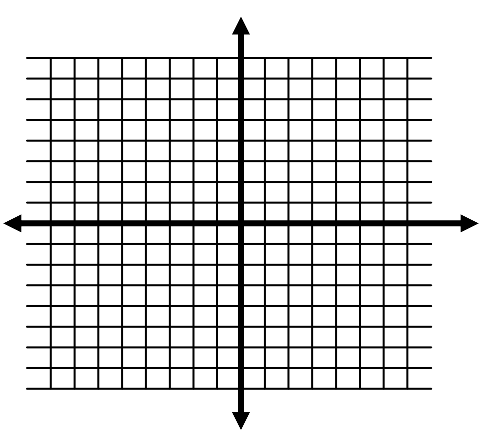
Graphing Quadratic Inequalities

STEPS: Graph the boundary. Determine if it should be solid (≤ , ≥) or dashed (> , <).

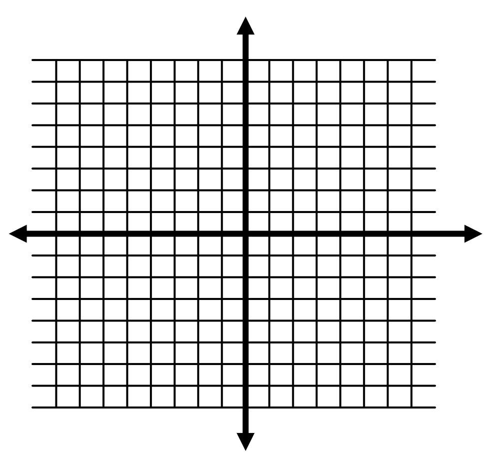
Test a point in each region.

Shade the region whose ordered pair results in a true inequality

EXAMPLE 1: Graph $y \le x^2 - 2x - 8$



EXAMPLE 2: Graph $y < -x^2 - 4x + 5$



Solving Quadratic Inequalities

Solve :

1) $0 > x^2 - 6x - 7$

2) $x^2 + 9x + 14 < 0$

3) $x^2 - x - 12 \geq 0$

4) $b^2 \geq 10b - 25$

5) $2x^2 + 5x < 12$

6) $n^2 \leq 3$

Interval Notation:

For < or > use (

For $\leq$ or $\geq$ use [

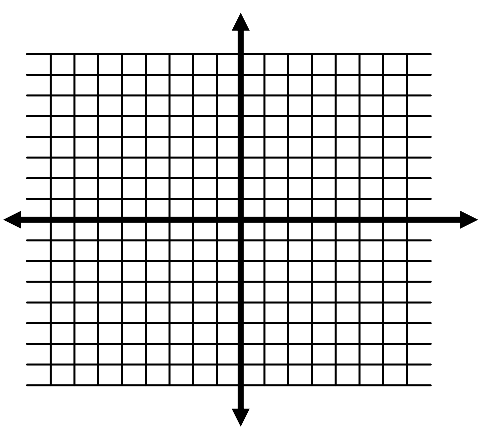
For $\leq$ or $\geq$ use [

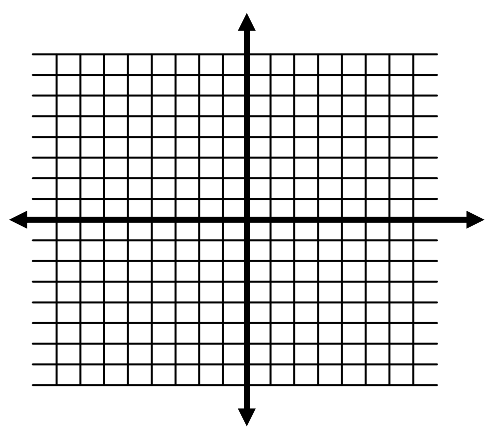
Practice:

Solve by graphing:

1. $y \geq 2x^2 - 2x - 4$

2. $y \leq -x^2 + 2x + 8$





Solve the inequality algebraically:

3. $x^2 - 3x - 10 < 0$

4. $x^2 + x \geq 8$

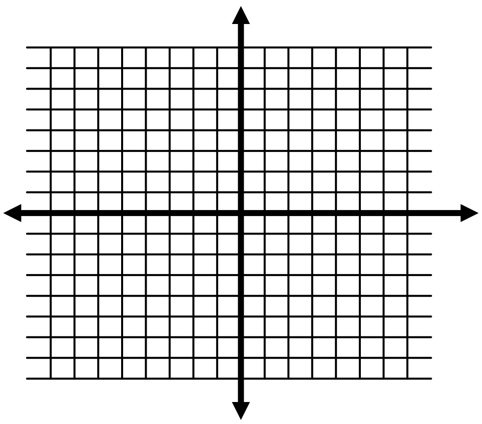
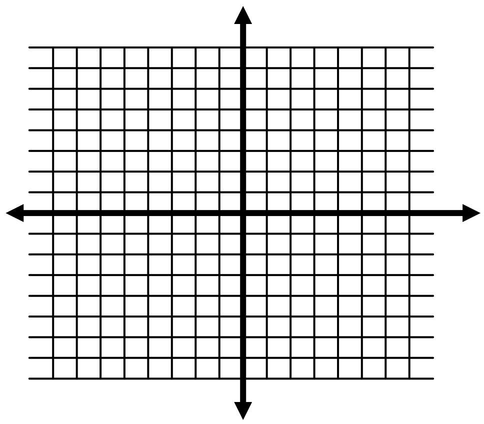
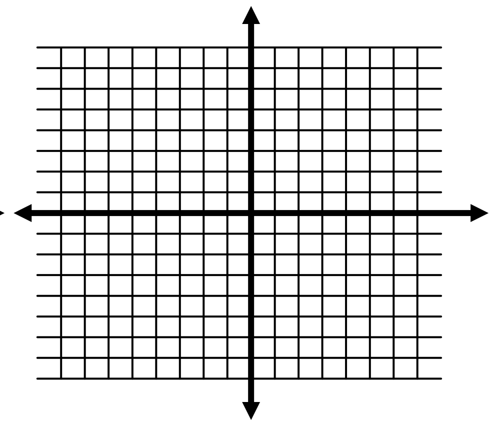
Solve a linear and Quadratic System by Graphing

Solve:

1) $y = x^2 + 5x + 6$
$y = x + 6$

2) $y \le -x^2 - x + 12$
$y \ge x^2 + 7x + 12$

3) $y < -x^2 + 4x - 3$
$y > x^2 + 6x + 8$

Practice: Musical Chairs activity

Day 12: Unit Review

Warm-Up:

13. Each year, a local school's Rock the Vote committee organizes a public rally. Based on previous years, the organizers decided that the Income from ticket sales, I(t) is related to ticket price *t* by the equation $I(t) = 400t - 40t^2$. *Cost* C(t) of operating the public event is also related to ticket price *t* by the equation $C(t) = 400 - 40t$.

a. What ticket price(s) would generate the greatest income? What is the greatest income possible? Explain how you obtained the value you got.

Ticket price(s) _______________ Income _________________

b. For what ticket price(s) would the operating costs be equal to the income from ticket sales? Explain how you obtained the answer.

c. Which of the following rules would give the predicted profit *P(t)* as a function of the ticket price?

i. $P(t) = -40t^2 + 440t - 400$
ii. $P(t) = -40t^2 - 440t - 400$
iii. $P(t) = -40t^2 - 360t + 400$
iv. $P(t) = -40t^2 - 360t - 400$
v. $P(t) = 40t^2 - 440t + 400$