Unit #2: Powers and Polynomials

Unit Outline:

Date	Lesson Title	Assignment Completed
	2.1 Introduction to Algebra	
	2.2 Discovering the Exponent Laws – Part 1	
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	2.5 Collecting Like Terms	
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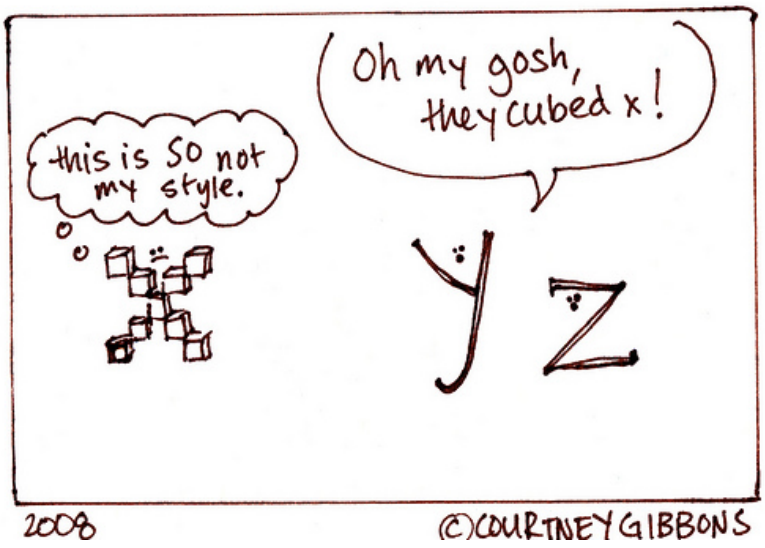
	2.7 Multiplying a Polynomial by a Monomial	
	Mid Unit Test → Lessons 2.5 to 2.7	
	2.8 Simplifying Algebraic Expressions – Part 1	
	2.9 Simplifying Algebraic Expressions – Part 2***	
	Final Algebra Test	



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2.1 Introduction to Algebra

Learning Goals: ***By the end of this lesson, you will be able to:***

What is algebra?

- Learning algebra is like learning another language. By learning algebra, mathematical models of real-world situations can be created and solved!

- In algebra, **letters are often used to represent numbers.**

Visualizing Algebra:

x x^2 x^3 $x+x$ $(x)(x)$

Vocabulary

→ **Term:**

→ **Variable:**

→ **Coefficient:**

→ **Constant:**

Draw the following algebraic terms:

$2x$ $2x^2$ 4

Vocabulary

→ **Like Terms:**

Come up with some examples!

Terms can be added and/or subtracted together to form **polynomials**.

Important: Polynomials are in simplest form <u>when they contain no like terms.</u>

Vocabulary

→ **Polynomial:**

→ **Trinomial:**

→ **Binomial:**

→ **Monomial:**

***Polynomials are in **standard form** when they are written with exponents **highest → lowest.**

Practice Problems:

1. State the coefficient and variable(s) in each term

a) $15x^2$
Coefficient:
Variable:

b) $-3y$
Coefficient:
Variable:

c) $-w^3$
Coefficient:
Variable:

2. Circle the **constant** term in each polynomial.

a) $3x^2 + 2x + 4$

b) $3y - 5$

c) $4a^2 + 5a$

3. Draw a model of each of the following.

a) $3x$ b) $-2x$ c) $x + 1$

d) $x - 2$ e) $3x^2$ f) $2x^3$

Writing Algebraic Expression

Writing algebraic expressions from word problems is a very important skill in mathematics.

1. Seven times a number plus three _____
2. Seven times the square of a number plus three _____
3. Three quarters of a number _____
4. A gym membership costs $35 upfront, then $10 per month _____
5. The sum of a number and seven _____
6. The difference between seven and a number _____
8. 2 times a number plus seven _____

Assignment 2.1: Introduction to Algebra

1. Sketch models to represent each of the following algebraic expressions.

a) x^2 b) x^3 c) $2y^2$ d) $5x^3$

2. For each term, identify the coefficient and the variable.

a) $4x$ b) $-5p^4$ c) $3m^2n$ d) g^3h^2

e) $-2y^5$ f) $-p^4q^5$ g) $\frac{3}{4}ab$ h) $0.6r^4s^2$

3. Classify each polynomial by type.

a) $2x + 1$ b) $3p^2 - p + 4$ c) $4b^2d^3$ d) $6 + gh^5$

e) $2 - y^5 - y^2 + 4y$ f) $x^2 - y^2 + 4$ g) $ab - b$ h) $6p^3q^3$

4. We read 4^2 as "4 squared" and 4^3 as "4 cubed". Sketch models that show why this makes sense.

5. Write an algebraic expression for each phrase.

a) double a number b) triple a number c) quadruple a number

d) one half of a number e) one third of a number f) one quarter of a number

g) 6 more than a number h) a number increased by 3 h) 2 increased by a number

j) 5 decreased by a number k) 7 less than a number l) a number decreased by 6

6. Write an algebraic expression for each phrase.

a) 4 more than triple a number b) half a number, less 5

c) quadruple a number decreased by 1 d) 2 less than double a number

7. Sate the problems that are in **<u>standard form</u>**. If it is not in standard form, re-write in standard form.

a. $x^3 - 11x^2$ b. $2 + 3x + 4x^2 + 3x^3$ c. $-3x + 17x^4 + 2x^2$ d. $-1 + 3x + 2x^2$

8. Given: $2x^3 - 5x^2 - 2x + 12$

a) How many terms are there?

b) What is the coefficient of the 3rd term?

c) What is the constant?

2.2 Discovering the Exponent Laws – Part 1

Learning Goals: ***By the end of this lesson, you will be able to:***

Investigation #1: Multiplication of Powers

Product	Expanded Form	Single Power
$x^4 \times x^3$	$= (x \times x \times x \times x) \times (x \times x \times x)$	$= x^7$
$y^5 \times y^6$	$= (y \times y \times y \times y \times y) \times (y \times y \times y \times y \times y \times y)$	$= y^{11}$
$m^3 \times m^2$	=	=
$a^4 \times a^5$	=	=
$t^6 \times t$	=	=
$w^7 \times w^2$	=	=
$h^2 \times h^4$	=	=
$p^8 \times p^5$	=	=
$x^a \times x^b$	=	=

Complete this statement: When multiplying powers with the **same base**….

Challenge:

$c^2d^3 \times c^4d^5$	$= (c \times c \times d \times d \times d) \times (c \times c \times c \times c \times d \times d \times d \times d \times d)$	$= c^6d^8$
$k^3j^4 \times k^5j^2$	=	=
$p^4q^2 \times p^2q^3$	=	=
$v^5z^3 \times v^3z^4$	=	=
$g^2t^5 \times g^4t^3$	=	=
$a^4b^2c^5 \times a^3b^4c^4$	=	=

Investigation #2: Division of Powers

Product	Expanded Form	Single Power
$x^7 \div x^3$	$= \frac{x \times x \times x \times x \times x \times x \times x}{x \times x \times x}$	$= x^4$
$y^5 \div y^2$	$= \frac{y \times y \times y \times y \times y}{y \times y}$	$= y^3$
$m^6 \div m^2$	=	=
$a^4 \div a^3$	=	=
$t^6 \div t$	=	=
$x^a \div x^b$	=	

Complete this statement: When **DIVIDING** powers with the **same base…**

Challenge:

$c^7d^3 \div c^4d^2$	$= \frac{c \times c \times c \times c \times c \times c \times c \times d \times d \times d}{c \times c \times c \times c \times d \times d}$	$= c^3d$
$k^8j^4 \div k^5j^2$	=	=
$p^4q^6 \div p^2q^3$	=	=
$v^5z^8 \div v^3z^4$	=	=
$g^6t^5 \div g^4t^3$	=	=
$a^4b^7c^6 \div a^3b^4c^4$	=	=

Let's Review:

$a^1 =$

$a^0 =$

$a^5 \bullet a^2 =$

$\frac{a^5}{a^3} =$

$x^4 \bullet y^3 =$

$\frac{x^4}{y^3} =$

$\frac{a^5 \bullet b^6}{a^4 \bullet b^3} =$

$\frac{b^5}{b^5} =$

Consolidation:

a) $\frac{\left(x^7\right)\left(x^3\right)}{x^6}$

b) $\left(a^5b^2\right)\left(a^4b^3\right)$

b) $\frac{\left(x^4y^3\right)\left(x^3y^5\right)}{x^5y^5}$

Confirm your answers by writing the expressions in standard form. (When it doubt, write it out)

Assignment 2.2 Discovering the Exponent Laws – Part 1

1. Write each expression as a single power.

   a) $7^2 \times 7^4$ b) $3^5 \times 3^3$ c) 5×5^2 d) $3^2 \times 3^4 \times 3^3$

   e) $(-2)^2 \times (-2)^3$ f) $(-1)^3 \times (-1)^2 \times (-1)$ g) $0.5^3 \times 0.5^2$ h) $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^3$

2. Evaluate each expression in question 1.

3. Write each expression as a single power.

   a) $8^6 \div 8^4$ b) $5^5 \div 5^3$ c) $7^7 \div 7^2$ d) $4^8 \div 4^5 \div 4$

   e) $(-9)^7 \div (-9)^6$ f) $0.1^6 \div 0.1^4$ g)$(-0.3)^4 \div (-0.3)$ h) $\left(\frac{2}{3}\right)^5 \div \left(\frac{2}{3}\right)^3$

4. Evaluate each expression in question 3.

5. Simplify.

   a) $b^5 \times b^3$ b) $p^4 \times p$ c) $w^5 \div w^2$ d) $x^8 \div x^4$

   e) $a^4b^5 \times ab^3$ f) $m^2n^4 \times m^3n^3$ g) $p^6q^5 \div p^3q^2$ h) $xy^2 \div y$

6. Simplify each of the following, write as a single power if possible.

   a) $10^{12} \bullet 10^{35}$ b) $a^7 \bullet a^{12}$ c) $c^3 \bullet c^8$

   d) $d^7 \bullet d^9$ e) $x^2 \bullet x^8$ f) $w^{103} \bullet w^{1030}$

   g) $a^6 \bullet b^5$ h) $10^a \bullet 10^b$ i) $g^{12} \bullet g^{19} \bullet g^{11}$

   j) $\frac{10^6}{10^2}$ k) $\frac{4^{17}}{4^{14}}$ l) $\frac{9^{210}}{9^{207}}$

7. Explain why it is necessary for the bases to be the same in order to apply the multiplication and division principles for exponents, use examples in your explanations.

Enrichment: Write $125 \times 625 \times 5$ as a single power

2.3 Discovering the Exponent Laws – Part 2

Warm Up:

What exponent goes in the box to make the following equation true?

$$\frac{x^{\square}x^6}{x^2} = x^{12}$$

a 9
b 8
c 4
d 3

Evaluate $\frac{5^6}{5^8}$ by writing the expanding form

Simplify $\frac{5^6}{5^8}$ using the exponent principles.

Investigation 3: Power of a Power

Product	*Expanded Form*	**Single Power**
$\left(x^4\right)^2$	$=\left(x\times x\times x\times x\right)\times\left(x\times x\times x\times x\right)$	$= x^8$
$\left(y^3\right)^4$	$=\left(y\times y\times y\right)\times\left(y\times y\times y\right)\times\left(y\times y\times y\right)\times\left(y\times y\times y\right)$	$= y^{12}$
$\left(m^3\right)^2$	=	=
$\left(g^2\right)^5$	=	=
$\left(t^4\right)^3$	=	=
$\left(x^a\right)^b$	=	=

	Do these using your SHORTCUT!
$\left(m^3\right)^5$	=
$\left(q^8\right)^6$	=
$\left(a^3b^4\right)^5$	=
$\left(x^2y^4z^3\right)^6$	
$\left(\frac{a^2}{b^3}\right)^4$	

When simplifying <u>POWERS OF POWERS</u>….

Let's Review:

There are still two more:

Example: $(xy)^3$ =

Example $\left(\frac{a}{b}\right)^3$ =

Practice Problems: Simplify

a) $(x^3y^2)^2$ **b)** $(5xy)^3$ c) $(2x^2y^4)^3$

d) $\left(\frac{2}{3}\right)^2$ **e**) $\left[\left(\frac{2}{5}\right)^2\right]^2$ **f)** $\left(\frac{4x}{3}\right)^5$

Exponent Rules can only be used when _____.

Practice: Simplify

a) $(n^4)^2(n^2)^3$

b) $\frac{\left(x^5y^2\right)^3}{xy}$

c) $\frac{\left(2x^2y^3\right)^3}{2xy^2}$

Extension: Determine the value that makes each statement true

a) $4^3 = 2^{?}$

b) $27^4 = 3^{?}$

c) $5^{?} = 64^0$

Assignment 2.3 Discovering the Exponent Laws – Part 2

1. Simplify each expression using exponent laws.

a) $\left(4^5 \div 4^3 \times 4^9\right)^2$ b) $\left(5^4\right)^2$ c) $\left(3^5\right)^7$

d) $\left(7^4\right)^5$ e) $7^{10} \div 7^4 \times 7^6$ f) $\left(3^2\right)^3 \times 3^5 \div 3^7$

2. Simplify each expression using exponent laws.

a) $\left(x^2\right)^5$ b) $\left(5^4\right)^2$ c) $\left(3^5\right)^7$

d) $\left(7^4\right)^5$ e) $\left(a^7\right)^3$ f) $3\left(x^5\right)^5$

g) $\left(2x^2\right)^3$ h) $\left(3y^4\right)^2$ i) $\left(2y^2\right)^4\left(x^2\right)^4$

j) $\left(x^2 \times x^3 \div x^5\right)^2$ k) $\left(s^3t^5\right)^5$ l) $\left(x^2y^5\right)^6$

m) $\left(\frac{a^2}{b^3}\right)^4$ n) $\left(\frac{x^{12}y^8}{y^6x^5}\right)^2$ o) $\left(\frac{a^5b^7}{a^3b^4}\right)^3$

3. Simplify each expression using exponent laws.

a) $(m^5)^2$ b) $(k^2)^3 \times k^2$ c) $g^5 \times g^5 \div g^7$ d) $(a^6)^3 \div (a^5)^2$

e) $(gh^4)^3$ f) $2k^2m^3 \times (2k^2)^2$ g) $(2g^5h^3)^2 \div 2gh^6$ h*) $\frac{6b^2d \times 3b^2d^2}{(3bd)^2}$

4. Show that 3^{10} is the same as 9^5 using your understanding of exponent.

5. Determine the value of the exponent that makes each expression true.

a) $27^4 = 3^{?}$ b) $(-125)^7 = (-5)^{?}$ c) $6^{?} = 216^2$

6. Evaluate for $a = 5$ and $b = 3$. Are the expressions equal? If not, which expression has the lesser value?

a) $a^2 + b^3$ or $a^3 + b^2$ b) a^2b^2 or $(ab)^2$

2.4 Multiplying and Dividing Monomials

Warm Up:

After marking a quiz, a teacher recorded the most common errors made by the students. In each case, **identify the error** made by the student and provide the correct simplification.

a) $2^3 \times 2^4 = 2^{12}$

b) $\left(4^3\right)^2 = 4^5$

c) $3^4 \times 3^5 = 9^9$

d) $\left(-5a^2b\right)^3 = -5a^6b^3$

Multiplying Monomials

$2x^2y \bullet 3xy^2$ "When in doubt, write it out".

MULTIPLYING MONOMIALS

$(6x^2y^6z)(2xy^4)$

Practice Problems: Multiply the following monomials.

a) $(2x)(3x)$

b) $(-5m^2n)(4m^3)$

c) $(-6z)(3x^2y)$

d) $(7mn^2)(2mn^4)$

e) $\left(\frac{2}{3}x^2\right)(15x^7)$

f) $(mnp^2)(8n)$

DIVIDING MONOMIALS

$$\frac{-81a^4b^5c^3}{18a^2b^5c}$$

Practice Problems: Divide the following monomials.

a) $\frac{10x^2}{5x}$

b) $\frac{-16m^2n}{-2mn}$

c) $\frac{48x^3y^5}{16xy^2}$

d) $(-15mp) \div (-5p)$

e) $\left(\frac{4}{5}x^4yz^2\right) \div \left(\frac{1}{5}xyz\right)$

f) $x^2 \div 4x^2$

Challenge Questions:

a) $\left(\frac{6x^5y^3}{8y^4}\right)^2$

b) $\frac{(-6x^{-2}y)(-9x^{-5}y^{-2})}{3x^2y^{-4}}$

Assignment 2.4 Multiplying and Dividing Monomials

1. Multiplying Monomials: Simplify each expression using exponent laws.

a) $(-2b)(4b)$ **b)** $3(6b)$ **c)** $(-2b)(3b^5)$

d) $3x(5x)$ **e)** $(-2b^2)^2(3b^5)$ **f)** $-(4x)(3x^2)$

g) $(-2b^2)(-3b^5)$ **h)** $5x(3y^2)(6x^2)(y^5)$ **i)** $5x^2(-3)$

j) $7a^2(4a^6)$ **k)** $(5x)(3x)(2x)$ **l)** $(-x^2)(3x)(-4x)$

m) $(2a^2b^3)(-3ab^2)$ **n)** $(-3xy)(-5y^2)$ **o)** $3a^4(2a)(5a^3)$

p) $-7m^3n^4(4m^5)(6n)$ **q)** $2x^3(-3x^3y^2)(-8y)$ **r)** $8x^3y^3(3x^4y^2)(-x^2y^4)$

2. Dividing Monomials: Simplify each expression using exponent laws.

- Fractions must be in lowest terms – no decimals!

a) $\frac{9x^2}{3}$ **b)** $\frac{28y^5}{4y}$ **c)** $\frac{12n^6}{18n^5}$

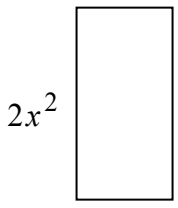
d) $\frac{45z^7}{-5z}$ **e)** $\frac{-27t^6}{-9t^4}$ **f)** $\frac{24x^7}{36x^5}$

g) $\frac{5x^3y^8}{-20x^7y^{12}}$ **h)** $\frac{14a^7b^9}{49a^{10}b^8}$ **i)** $\frac{9x^3(y^3)^3}{-81(x^{-2})^4y^{11}}$

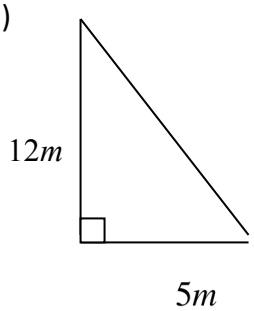
j) $\frac{72x^6y^3}{(6x^2y)(2x^2y)}$ **k)** $\frac{7m^5(5n^2)}{5m^3n^4}$ **l)** $\frac{(5x)^2(5y^3)}{(5x^3y^4)^3}$

3. Write an expression to represent the area of the following shapes.

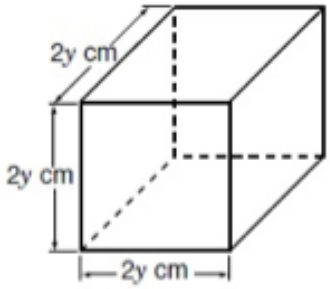
a)



b)

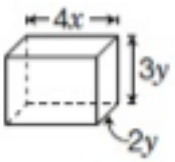


4. Determine the volume of the cube shown.

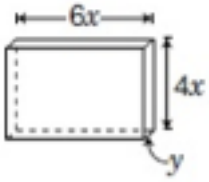


5. Determine the volume of the following shapes.

A



B



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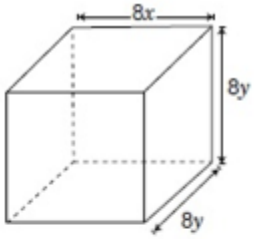

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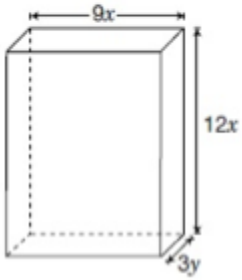
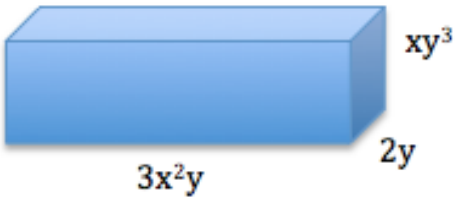


6. Write an expression to represent the volume of the following shape.



7. The volume of a box is $64a^5b^4$. The length of the box is $4a^3$ and the width of the box is $8b^4$. Determine the height of the box.

2. 5 Collecting Like Terms

Warm Up:

Draw a model to represent x, x^2, and x^3	Illustrate and explain the difference between a) 2x and x^2. b) $3x^2$ and $2x^3$

Start to fill out the FISHBONE diagram (next page) with the following definitions:

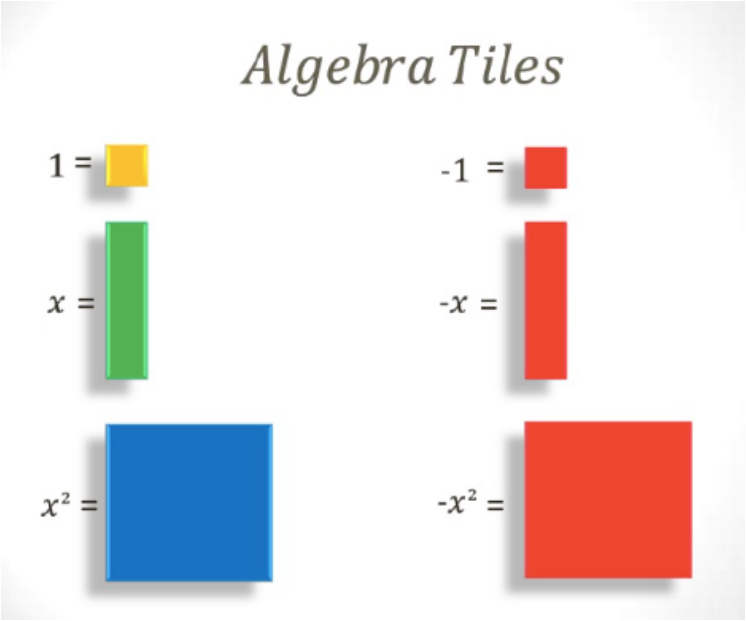
- ✓ **Variable**
- ✓ **Coefficient**
- ✓ **Constant**
- ✓ **Like Terms**
- ✓ **Polynomial**

Powers and Polynomials

20

Representing Polynomials

Algebra Tiles are often used to represent polynomials. Algebra tiles can help you visualize equivalent algebraic expressions and/or equations.



Represent the following polynomials **with algebra tiles**

a) $3x + 1$ b) $4x^2 - 3x$ c) $-2x^2 - 2x + 4$ d) $-x^2 - 5$

Like and Unlike Terms

Like terms have the same set of variable bases and corresponding exponents.

Examples: Represent each of the following sets with algebra tiles

a) $4x^2$ and $-5x^2$ b) -6x and 9x c) $3xy^2$ and $5xy^2$

Did you notice how algebra tiles with the same size and shape represent like terms?

Define and give an example of **Unlike Terms** -

Collecting Like Terms

Algebraic expressions that contain like terms can be simplified by combining each group of like terms.

Examples:

$3x + 4x$ $9x^2 – 6x^2$ $12x^3y^2 - 5x^3y^2$

Why **<u>can't</u>** you simplify?

$4x^2 + 4x$ $x^2 – 7$ $6x^3y + 5xy^3$

Practice Problems: **Simplify**

1) $7x + 5 – 3x$

2) $6w^2 + 11w + 8w^2 – 15w$

3) $6x + 4 – 5 – (-7x)$

4) $(-12x) – 5 – 7x – 11$

5) $2x^2 – (-3x) + 7 – (-3x^2) + 4x – 7$

6) $11a^2b – 12ab^2$

Assignment 2. 5: Collecting Like Terms

1. Are the terms in each pair like or unlike?

   a) $5a$ and $-2a$ b) $3x^2$ and x^3 c) $2p^3$ and $-p^3$ d) $4ab$ and $\frac{2}{3}ab$

   e) $-3b^4$ and $-4b^3$ f) $6a^2b$ and $3a^2b$ g) $9pq^3$ and $-p^3q$ h) $2x^2y$ and $3x^2y^2$

2. Simplify.

   a) $4 + v + 5v - 10$ b) $7a - 2b - a - 3b$ c) $8k + 1 + 3k - 5k + 4 + k$

   d) $2x^2 - 4x + 8x^2 + 5x$ e) $12 - 4m^2 - 8 - m^2 + 2m^2$ f) $-6y + 4y + 10 - 2y - 6 - y$

3. Simplify.

   a) $2a + 6b - 2 + b - 4 + a$ b) $4x + 3xy + y + 5x - 2xy - 3y$

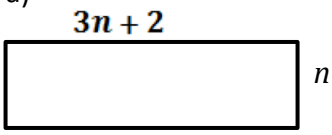
   c) $m^4 - m^2 + 1 + 3 - 2m^2 + m^4$ d) $x^2 + 3xy + 2y^2 - x^2 + 2xy - y^2$

4. Simplify:

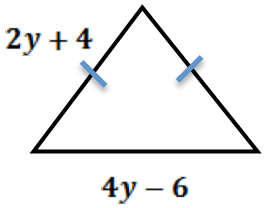
   a) $\frac{3}{4}w^2 - \frac{2}{3}w^2 + \frac{1}{4}w^2 - \frac{4}{3}w^2$ b) $\frac{3}{4}a - \frac{1}{5}b - \frac{2}{3}a + \frac{6}{8}b$

5. Find the perimeter of each figure below.

a)

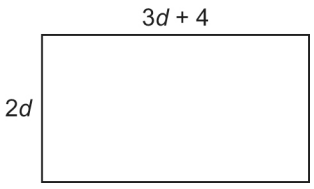


b)



2.6 Adding and Subtracting Polynomials

Warm Up:

Determine a simplified expression the perimeter of the following rectangle.  	Give an example of … • A monomial • A binomial • A trinomial • A constant term

Adding Polynomials

ACTIVITY:

Use tiles and two different colours to record your solution. Create zero pairs if you are adding positive & negative tiles. **Draw the tiles** under each polynomial & write answer in chart. Compare with neighbor. **compare with a neighbor.

Polynomial 1		Polynomial 2	Answer
$x^2 + 3x$	+	$2x^2+x+3$	
$-2x^2 - x$	+	$2x^2+x+3$	
$5x^2 + 3x - 4$	+	$-3x^2-2x+4$	
$-3x^2 - 7x + 5$	+	$-2x^2 + 6x - 4$	
$2x^2 - 3x + 6$	+	$2x^2 + 5x - 4$	

To separate one polynomial from another, often brackets are used:

Evaluate: $(3x^2 + 5x - 1) + (4x^2 - 2x)$

USING TILES: WITHOUT TILES:

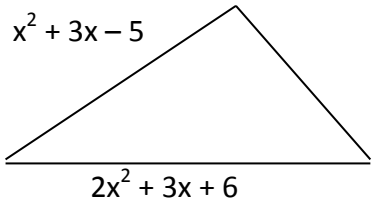
Practice Problems: Simplify. Choose your method!

a. $(4y - 7) + (-3y + 2)$ b. $(a^2 + 2a) + (6a^2 + 10a) + (5a + 1)$

PROBLEM SOLVING:

The measures of two sides of a triangle are given. P is the perimeter. Find the measure of the third side.

$P = 4x^2 + 5x + 5$



Enrichment:

Determine the polynomial that must be **added** to $3x^2 - 4xy + 6y^2$ to get: $6x^2 - 6xy + 8y^2$

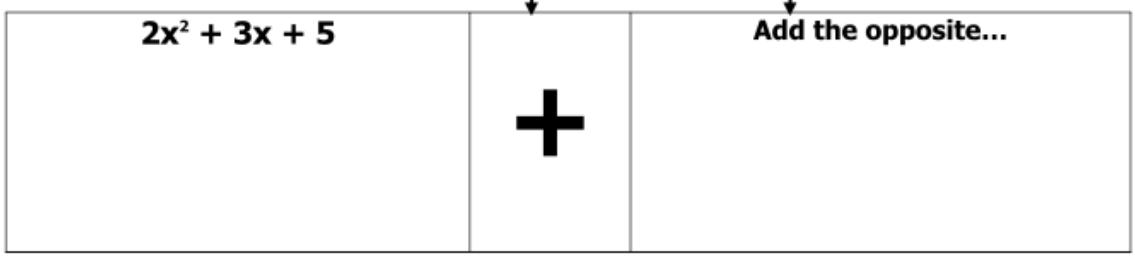
Create two three-term polynomials that when they are combined you get a 3-term polynomial

Subtracting Polynomials

Review: Subtracting is the same as _____ the _____.
This same idea can be used to **subtract polynomials**.

Ex. $(2x^2 + 3x + 5) - (x^2 + 2x + 4)$

Ex. $(2x^2 + 3x + 5)$ take away $(x^2 + 2x + 4)$



Answer: = _____

Ex. Try without tiles: $(7b^2 - 2b + 13) - (4b^2 - 5b - 6)$.

Practice Problems: Simplify (choose your method).

a. $(3x - 2) - (4x + 9)$

b. $(5z^2 - 12z) - (-4z - 8)$

c. $12 - (4x^2 + 5x - 1)$

d. $(8x^2 + 7x) - (4x^2 - 3) - (-5x + 9)$

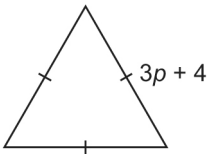
Assignment 2.6 Adding and Subtracting Polynomials

1. Add.
   a) $(y^2 + 6y - 5) + (-7y^2 + 2y - 2)$
   b) $(-2n + 2n^2 + 2) + (-1 - 7n^2 + n)$
   c) $(3m^2 + m) + (-10m^2 - m - 2)$
   d) $(-3d^2 + 2) + (-2 - 7d^2 + d)$

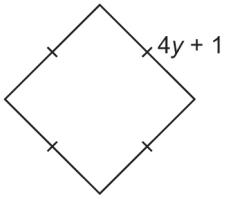
2. For each shape below, write the perimeter as a sum of polynomials and in simplest form.

   i)

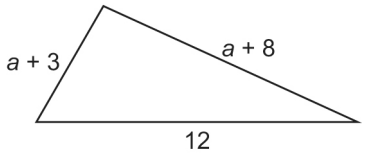
   ii)



   iii)



   iv)



3. Subtract.
   a) $(2x + 3) - (5x + 4)$
   b) $(4 - 8w) - (7w + 1)$
   c) $(x^2 + 2x - 4) - (4x^2 + 2x - 2)$
   d) $(-9z^2 - z - 2) - (3z^2 - z - 3)$

4. A student subtracted
   $(3y^2 + 5y + 2) - (4y^2 + 3y + 2)$ like this:
   $= 3y^2 - 5y - 2 - 4y^2 - 3y - 2$
   $= 3y^2 - 4y^2 - 5y - 3y - 2 - 2$
   $= -y^2 - 8y - 4$
   a) Explain why the student's solution is incorrect.
   b) What is the correct answer? Show your work.

5. The difference between two polynomials is $(5x + 3)$. One of the two polynomials is $(4x + 1 - 3x^2)$. What is the other polynomial? Explain how you found your answer.

6. Subtract.
   a) $(mn - 5m - 7) - (-6n + 2m + 1)$
   b) $(2a + 3b - 3a^2 + b^2) - (-a^2 + 8b^2 + 3a - b)$
   c) $(xy - x - 5y + 4y^2) - (6y^2 + 9y - xy)$

7. The sum of the perimeters of two shapes is represented by 13x + 4y. The perimeter of one shape is represented by 4x – 2y. Determine an expression for the perimeter of the other shape. Show your work.

8. A rectangular field has a perimeter of 10a – 6 meters. The width is 2a meters. Determine an expression for the length of this field.

2.7 Multiplying a Polynomial by a Monomial

Warm Up:

Simplify

a) $2(3b)$

b) $-2(6h)$

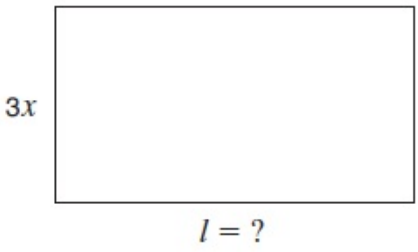
c) $4(2b^2)$

d) $-2(2x^2)$

e) $-2(-y^2)$

f) $-3(-2f)$

The area of the rectangle shown below is $6xy^2$ square units.



Hint: $A = lw$

If the width is $3x$ units, which expression represents the length of the rectangle?

a $2xy^2$ units

b $2y^2$ units

c $3xy^2$ units

d $3y^2$ units

Simplify.

a) $(6k - 4) + (2k + 4)$

b) $(2a + 1) - (4a + 2)$

c) $(b - 6) - (2 - 5b) + (b + 4)$

d) $(2m^2 + m + 12) - (3m^2 + 4m - 6)$

Simplify

a) $4a^5 \bullet 2a^3$

b) $6xy^3 \bullet 2x^2y^7$

c) $(-4xy^5)(3x^3y^4)$

d) $(9y^5)^2$

e) $\dfrac{8b^3d \times 4bd^2}{2(2bd)^2}$

The Distributive Property

The property known as the distributive property is also known as **expanding**.
We are making sure EVERY TERM in bracket gets multiplied.

Distributive Property: $a(b+c) = ab + ac$

With an integer….

a) $3(g + 4)$ b) $-7(q + 3)$ c) $-(2t - 1)$ d) $-4(-w - 5)$

With a variable…..

Example 1) Simplify: $x(2x+5)$

Practice Problems: Expand.

a) $b(b + 1)$ b) $3p(p + 4)$ c) $-r(-5r + 2)$ d) $-3w(2w - 1)$

With a variable <u>and</u> coefficient….

Example 2) Expand $3x(9x^2 - 4x)$.

Practice Problems: Simplify.

a) $3y(-4y^6 + 2y^3)$ b) $5z^2(3z - 1)$

c) $-2x^4(x^2 - 3x + 9)$ d) $(a - 1) \bullet 11a$

Example 3) Simplify $2(b^2 - 4b + 3) + 5b(b + 4)$

Practice Problems: Expand and simplify.

a) $-3(p - 2) + 6(p + 1)$ b) $3[-2(6 - t) + 5t]$ c) $-5m(m + 5) - 2(3m^2 - 4m - 7)$

Assignment 2.7 Multiplying a Polynomial by a monomial

1. Determine each product.

a) $4(3a + 2)$ **b)** $(d^2 + 2d)(-3)$ **c)** $2(4c^2 - 2c + 3)$

d) $(-2n^2 + n - 1)(6)$ **e)** $-3(-5m^2 + 6m + 7)$

2. Here is a student's solution for a multiplication question.

$(-5k^2 - k - 3)(-2)$

$= -2(5k^2) - 2(k) - 2(3)$

$= -10k^2 - 2k - 6$

a) Explain why the student's solution is incorrect.

b) What is the correct answer? Show your work.

3. Expand.

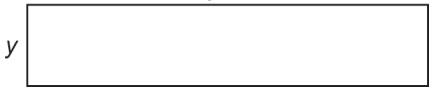
a) $3(x^2 + x - 4)$ **b)** $2(m^2 - 3m + 5)$ **c)** $-4(b^2 - 2b - 3)$ **d)** $5c(c^2 - 6c - 1)$

e) $-3h(4 - h^2)$ **f)** $(n^2 + 4n + 3)(-2)$ **g)** $(5t^2 - 2t)(-t)$ **h)** $(w^2 + 2w - 5)(4w)$

4. Write a simplified expression for the area of the following rectangles

a)

b)



5. Expand

a) $4x^2(3x + 2)$ b) $2n(2n - 3)$ c) $5m(4m^2 + 3m)$ d) $(y^4 - y^2)y^3$

e) $pq(3p + 2q)$ f) $3d(2d^2 - 4d + 1)$ g) $(t^2 + t - 4)(-2t)$ h) $3a^2b(3a^2 - b^2)$

6. Simplify

a) $2(x + 4) - 4(2x + 3)$ b) $3a(2a + 4b - 3) - 2b(3a + 2ab)$

c) $2p(p - 4) + 6(p^2 + 4p - 3)$ d) $4x(4x - 4y - 4) + 2y(6x + 3y)$

Enrichment: Do some research to learn how to multiply a binomial by a binomial (or a polynomial)

a) $(x + 1)(x - 3)$ b) $(3x - 5)(x - 2)$ c) $(4x - 3y)(5x + y)$

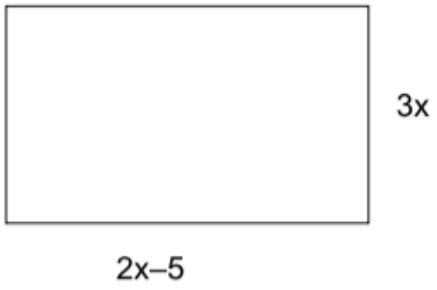
2.8 Simplifying Algebraic Expressions

Solve the following word problems. **Include your steps and a clear and thorough solution.**

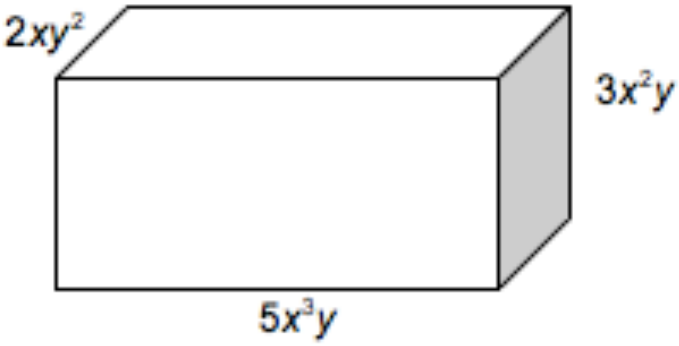
1. The area of a rectangle is represented by $36x^6y^4$. One side is represented by $6x^3y^2$. What is the length of the other side?

2.

Write a simplified algebraic expression for the perimeter, P, and area A, of the following figure.

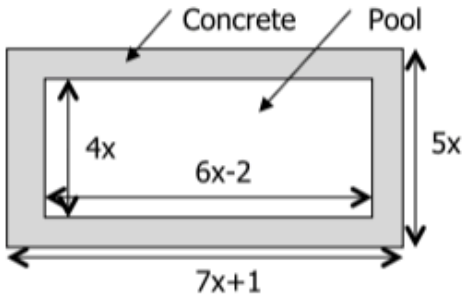


13. Find an expression to represent the volume of the rectangular prism below



4.

A concrete pad needs to be poured around a swimming pool with dimensions as shown on the diagram.



a) Determine an expression for the area of the concrete pad.

b) If x = 1.5 m, what is the area of the concrete pad?

Assignment 2.8 - Simplifying Algebraic Expressions

Simplify

1) $3p - 4q + 2p + 3 + 5q - 21$

2) $5x^2y + 12xy^2 - 8x^2 - 12xy^2$

3) $5 - 2y^2 + 3y - 8y - 9y^2 - 12$

4) $2x^2 + 3x - 7x - (-5x^2)$

5) $5(a - 3b) + 2(-b - 4a)$

6) $3(x + y) - 5(-2y + 3x)$

7) $-3b(5a - 3b) + 4(-3ab - 5b^2)$

8) $3x(x - 2y) - 4(-3x^2 - 2xy)$

9) $-3(x^2 + 3y) + 5(-6y - x^2)$

10) $-3(7xy - 11y^2) - 2y(-2x + 3y)$

11) $4(2 - x) - 3(-5 - 12x)$

12) $7(3 - x) - 6(8 - 13x)$

13) $\frac{1}{2}x^2 - 3y - \frac{1}{3}y + \frac{1}{4}x^2$

14) $\frac{1}{5}a^2 - 2b - \frac{1}{2}a^2 - 3b$

15) $\frac{1}{3}x - \frac{2}{3}y - \frac{2}{5}x + \frac{4}{7}y$

17) $\frac{2}{5}s - \frac{3}{8}t - \frac{4}{15}s - \frac{5}{12}t$

18) $3\left[6 - 2(x + y)\right]$

19) $2(x - 2y) - [3 - 2(x - y)]$

20) $-3\left\{7x - 2\left[x - (2x - 1)\right]\right\}$

21) $5x^2(x + 6) - 2\left[x - 2\left(1 + 2x^2\right)\right]$

22) $-4\left\{3a^2 - 2\left[4a^2 - \left(b + a^2\right)\right]\right\}$

23) $6b - \left\{5a - 2\left[a + (b - 2a)\right]\right\}$

24) $2\left[3x - (y + w)\right] - 3\left[2x + 2(3y - 2w)\right]$

25) $2x(x - 3) + 4(x^2 + 5)$

26) A triangle has sides of length $2a$ centimeters, $7b$ centimeters, and $5a + 3$ centimeters. What is the perimeter of the triangle?

27) A rectangle has sides of length $7x - 2$ meters and $3x + 4$ meters. What is the perimeter of the rectangle?

28) A square has a side of length $9x - 2$ inches. Each side is shortened by 3 inches. What is the perimeter of the new smaller square?

29) A triangle has sides of length $4a - 5$ feet, $3a + 8$ feet, and $9a + 2$ feet. Each side is doubled in length. What is the perimeter of the new enlarged triangle?

30) Subtract four times the quantity $2x + 5$ from three times the quantity $7x - 8$.

Possible Test Questions

1. If $x = 3$, what is the value of $2x^2 + 5x$?

a 21 **b** 27 **c** 33 **d** 51

2. Simplify the following expression: $3x(2x + 3) – 5x$

a $6x^2 – 5x + 3$ **b** $6x^2 – 6x$ **c** $15x^2 – 5x$ **d** $6x^2 + 4x$

3. Simplify the following algebraic expression: $\dfrac{a^6b^4}{a^2b}$

a $\dfrac{a^3}{b^3}$ **b** $\dfrac{a^4}{b^3}$ **c** a^3b^3 **d** a^4b^3

4. Tim shows the steps he took in simplifying the following algebraic expression: $\dfrac{(a^2)^3}{a^2a^3}$
In which step did Tim make an **error**?

a Step 1

b Step 2

c Step 3

d Step 4

$= \dfrac{a^5}{a^2a^3}$ Step 1

$= \dfrac{a^5}{a^{2+3}}$ Step 2

$= \dfrac{a^5}{a^5}$ Step 3

$= 1$ Step 4

5. Sabeeta expands and simplifies the expression below. $2(3x^2 – 5x) + 4x(7 + x)$ Which expression is equivalent to the one above?

a $6x^2 + 22x$ **b** $10x^2 + 18x$ **c** $10x^2 – 38x$ **d** $28x^2$

1) $4x + (-1) - (+5x) - (-6)$

2) $(-5a + 6) + (8a - 7)$

3) $(b - 4) - (3b - 8)$

4) $2y(3y - 5)$

5) $3(x + 6) - 4(x + 3)$

6) $\frac{1}{2}(4x - 3) + \frac{2}{3}(9x + 1)$

7) $2m(3m - 5) - 4(m^2 - 2m - 1)$

8) $-3[2(p + 2) - 3p]$

9) $3\left[6 - 2(x + y)\right]$

10) $2\left[3x - (y + w)\right] - 3\left[2x + 2(3y - 2w)\right]$