

"First they build up your confidence with simple addition and subtraction, then they slam you with algebra and calculus. It's quite a clever scheme."

Unit 3: Absolute Value

Day 1	Characteristics of Absolute Value
Day 2	Transformations of Absolute Value
Day 5	Absolute Value Equations
Day 6	Absolute Value Inequalities

Schedule of Upcoming Classes

Day 1	A	Mon 9/21	Introducing AV/ functions
Day 1	В	Tues 9/22	Introducing AV functions
	Α	Wed 9/23	Transformations & Analyzing AV Cranha
Day 2	В	Thurs 9/24	Transformations & Analyzing AV Graphs
	А	Fri 9/25	Deview of Craphing AV functions
Day 3	В	Mon 9/28	Review of Graphing AV functions
Day 4	А	Tues 9/29	Quiz: Graphing & Analyzing
Day 4	В	Wed 9/30	AV Functions
Day F	А	Thurs 10/1	Solving & Craphing AV Equations
Day 5	В	Fri 10/2	Solving & Graphing AV Equations
Day 6	А	Mon 10/5	Solving & Graphing AV Inequalities
Day 0	В	Tues 10/6	Solving & Graphing AV mequalities
Day 7	А	Wed 10/7	
Day 7	В	Thurs 10/8	Unit Review *
	Α	Fri 10/9	Lipit Test
Day 8	В	Tues 10/13	Unit Test
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* Skills Review due & Skills Check

Absent?

See Ms. Huelsman AS SOON AS POSSIBLE to get work and any help you need.

Notes are <u>always</u> posted online on the calendar.

You may also email Ms. Huelsman at Kelsey.huelsman@lcps.org with any questions!

Need Help?

Ms. Huelsman and Mu Alpha Theta are available to help Monday, Tuesday, Thursday, and Friday **mornings** in L506 starting at 8:10.

Ms. Huelsman is also available after school until 4:30.

Need to make up a test/quiz?

Math Make Up Room is open Mon/Tues/Thurs/Fri mornings and Mon/Wed/Thurs afternoons.

Schedule is posted around the math hallway & in Ms. Huelsman's classroom $\ensuremath{\textcircled{\sc black}}$

Let's take a look at y = x...

What happens if we change every negative y-value to a positive value?

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Does this sound familiar? What takes negative values and makes them positive?

Introducing the Absolute Value Function

$$|x| = \begin{cases} x, & x \ge 0\\ -x, & x < 0 \end{cases}$$

We can analyze the parent function for special points and behavior -

y = |x|

Domain:

Range:

Vertex:

y-intercept:

zeros (roots, x-intercepts, solutions):

Increasing:

Decreasing:

End Behavior:

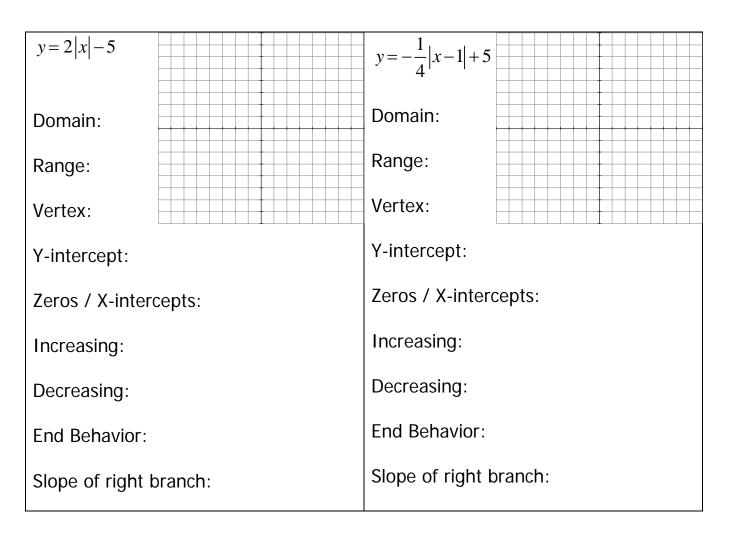
Slope of right branch:

 	 	 	 	-	 	 	 	
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We can also **move** the parent function to other places on the coordinate plane.

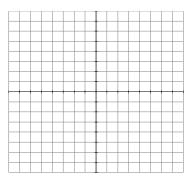
y = x - 4	y = x+3
Domain:	Domain:
Range:	Range:
Vertex:	Vertex:
Y-intercept:	Y-intercept:
Zeros / X-intercepts:	Zeros / X-intercepts:
Increasing:	Increasing:
Decreasing:	Decreasing:
End Behavior:	End Behavior:
Slope of right branch:	Slope of right branch:

Are you noticing any patterns yet? Let's look at domain and range.



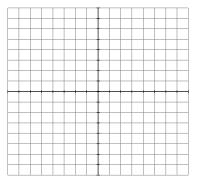
Are you noticing any patterns yet? Let's look at slope of the right branch.

Graph the **inverse** of the Absolute Value Function (start out with the original y = |x|) *then, how do you graph an inverse?*



Is the inverse a function?

Graph an Absolute Value Function that has a **removable discontinuity** at (3,4)



Day 2: Graphing Using TRANSFORMATIONS

In these notes we will

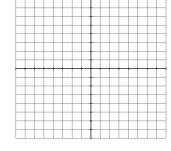
Learn a **new technique** for graphing a function – shifting it up, down, left, right So we can

Eventually graph **ANY** function given its parent shape

First, let's graph the absolute value "parent function", $y = |x| \rightarrow$ Use your calculator to graph this function in Y₁ What is the vertex of the graph?

Exploration of Transformations – Vertical Shifts

- 1. Graph y = |x| + 2 on your calculator in Y_2 .
 - a) Sketch this graph and the "parent function".
 - b) How does the graph move? (up or down) _____
 - c) What is the **vertex** of the graph? _____
- 2. Graph y = |x| 5 on your calculator in Y_2 .
 - a) Sketch this graph and the "parent function".
 - b) How does the graph move? (up or down) _____
 - c) What is the **vertex** of the graph? _____



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3. Given that $\mathbf{y} = \mathbf{a} |\mathbf{x} - \mathbf{h}| + \mathbf{k}$ is the symbolic form of the absolute value function, what does the **parameter** *k* **control**?

If k is positive, what direction do we move? If k is negative, what direction do we move?

Exploration of Transformations – Horizontal Shifts

- 1. Graph y = |x| on your calculator in Y_1 .
 - a) Sketch a graph of the function.
 - b) What is the vertex of the graph? _____
- 2. Graph y = |x 1| on your calculator in Y_2 .
 - a) Sketch a graph of the function and the function in #1.
 - b) How does the graph move? Left or Right?
 - c) What was the SIGN inside the absolute value?
 - d) What is the vertex of the graph? _____
- 3. Graph y = |x 5| on your calculator in Y_2 .
 - a) Sketch a graph of the function and the function in #1.
 - b) How does the graph move? Left or Right?
 - c) What was the **SIGN inside** the absolute value?
 - d) What is the vertex of the graph? _____
- 4. Graph y = |x + 3| on your calculator in Y_2 .
 - a) Sketch a graph of the function and the function in #1.
 - b) How does the graph move? Left or Right?
 - c) What was the **SIGN inside** the absolute value?
 - d) What is the vertex of the graph? _____
- 5. Given that $\mathbf{y} = \mathbf{a} |\mathbf{x} \mathbf{h}| + \mathbf{k}$ is the symbolic from of the absolute value function, what does the **parameter** *h* control?

When we have $|\mathbf{x} - \mathbf{h}|$, what direction does the graph move?

When we have $|\mathbf{x} + \mathbf{h}|$, what direction does the graph move?

How is the motion related to the sign of h?

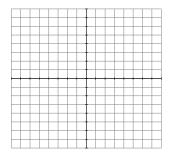
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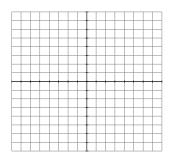
Exploration of Transformations – Vertical Stretch or Shrink

- 1. Graph y = |x| on your calculator in Y_1 .
 - a) What direction does the graph open? _____
 - b) What is the vertex of the graph? _____
 - c) Fill in the table to the right. These coordinates are the basic ordered pairs of the absolute value function.
- 2. Graph y = 2|x| on your calculator in Y_2 .
 - a) What direction does the graph open?
 - b) What is the vertex of the graph? _____
 - c) Fill in the table. How do these y-coordinates compare with the ycoordinates in question 1? Is the graph fatter or skinnier?
- 3. Graph $y = \frac{1}{2} |x|$ on your calculator in Y_2 .
 - a) What direction does the graph open? _____
 - b) What is the vertex of the graph?
 - c) Fill in the table. How do these y-coordinates compare with the ycoordinates in question 1? Is the graph fatter or skinnier?
- 4. Graph y = -|x| on your calculator in Y_2 .
 - a) Sketch a graph of the function and the function in #1.
 - b) How did the graph change? _____
- 5. Graph y = -2|x| on your calculator in Y_2 .
 - a) Sketch a graph of the function and the function in #1.
 - b) How did the graph change?

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1	
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	У
-2	У
-2 -1	У





6. Given that $\mathbf{y} = \mathbf{a} |\mathbf{x} - \mathbf{h}| + \mathbf{k}$ is the symbolic from of the absolute value function, what does the **parameter** *a* **control**?

1. _____ *Examples 1 and 2*

2.

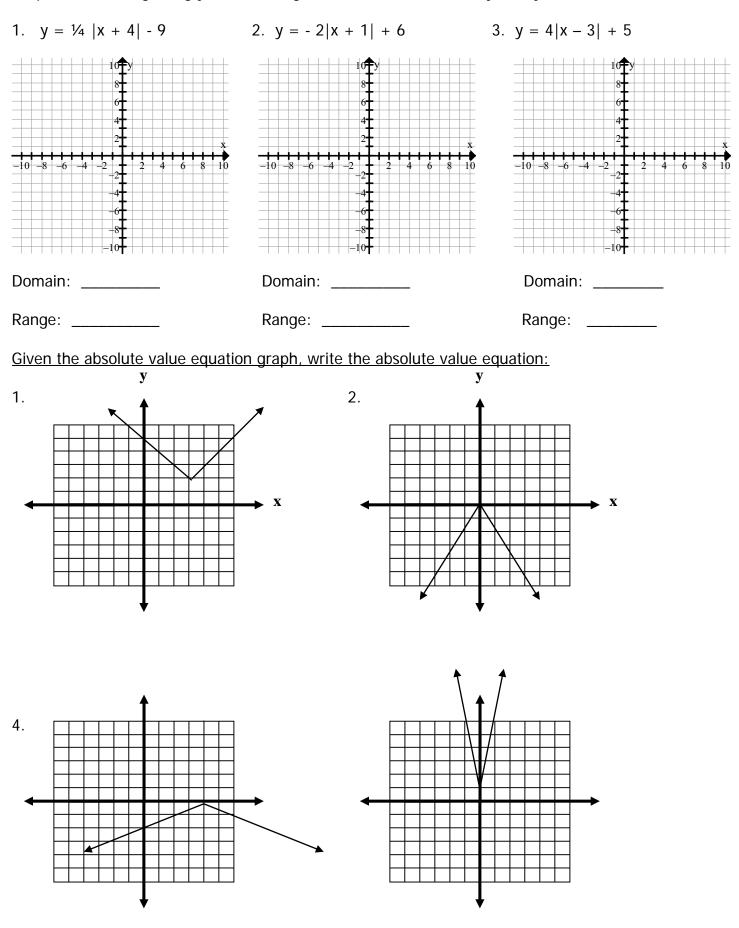
Examples 3 and 4

Exploration of ALL Transformations –

1.	Graph $y = x $ on your calculator in Y_1 .
2.	 Graph y = x - 3 - 4 on you calculator in Y₂. a) What direction does the graph open? b) How does the graph move? (left/right, up/down) c) What is the vertex of the graph?
3.	 Graph y = - x - 2 - 3 on you calculator in Y₂. a) What direction does the graph open? b) How does the graph move? (left/right, up/down) c) What is the vertex of the graph?
4.	 Graph y = 2 x + 6 - 4 on you calculator in Y₂. a) What direction does the graph open? b) How does the graph move? (left/right, up/down) c) What is the vertex of the graph?
	ven the absolute value function $\mathbf{y} = \mathbf{a} \mathbf{x} - \mathbf{h} + \mathbf{k}$ a > 0, does the graph open up or down?
lf a	a < 0, does the graph open up or down?
lf	a > 1, does the graph have a vertical stretch or vertical shrink?
lf (0 < a < 1, does the graph have a vertical stretch or vertical shrink?
Wł	nat does the parameter k control? nat does the parameter h control? nat is the vertex?
No	DW <i>generalize</i> Fill in the table using your knowledge of transformations.

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Function	Direction/Opening	Vertex	Vertical Stretch or
	(up or down)		shrink
1. $y = \frac{1}{4} x + 4 - 9$			
2. $y = -2 x + 1 + 6$			
3. $y = 4 x - 3 + 5$			
4. $y = -\frac{1}{2} x - 7 + 3$			
5. $y = 2 x + 4 - 1$			

Graph the following using your knowledge of transformations. Verify with your calculator.



- 1. Which absolute value function(s) open up?
 - B. y = |x + 1| 7A. y = -2|x - 5|C. y = -|x + 4| + 8D. $y = \frac{1}{4}|x - 9|$
- 2. Which absolute value function(s) are vertically stretched?
 - A. y = -2|x-5| B. y = |x+1| 7C. y = -|x + 4| + 8D. $y = \frac{1}{4}|x - 9|$
- 3. Which absolute value function(s) have an **absolute minimum** at the vertex?
 - A. y = -2|x-5| B. y = |x+1| 7C. y = -|x + 4| + 8D. $y = \frac{1}{4}|x - 9|$
- 4. Given f(x) = |x + 9|. The vertex of the function moves from (0, 0) nine units _____:
 - A. left B. right C. up D. down
- 5. Sketch f(x) = |x-5|+1

What is the range?

What is the end behavior?

The vertex is... (circle all that apply)

A) a relative minimum B) a relative maximum

C) an absolute minimum D) an absolute maximum

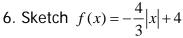
What is the range?

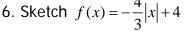
What is the end behavior?

The vertex is... (circle all that apply)

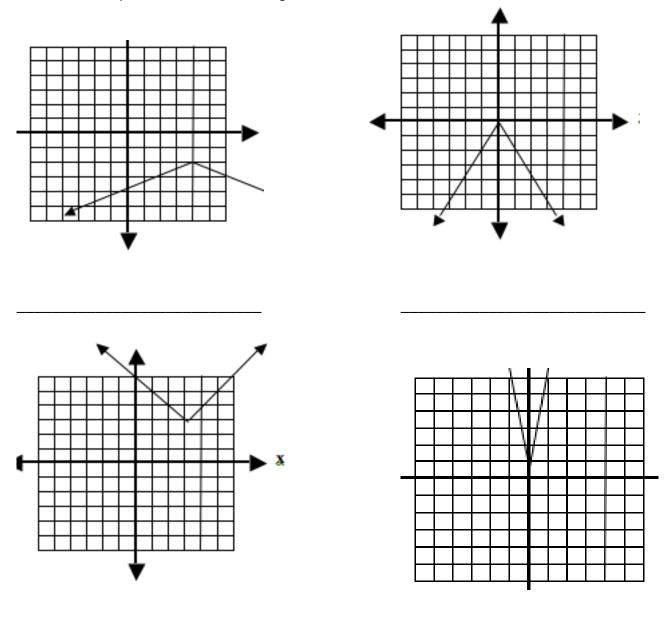
A) a relative minimum B) a relative maximum

C) an absolute minimum D) an absolute maximum





7. Write the equation for the following absolute value functions



With a vertex at (- 8, 2) that is vertically stretched by a scale of 2:

That opens downward and has a vertex at (5, 0):

That is vertically shrunk and shifted to the right (you pick the details!)

Day 5: Absolute Value Equations

Objective: understand the definition of absolute value and how to manipulate that symbol in solving equations.

Absolute Value means _____

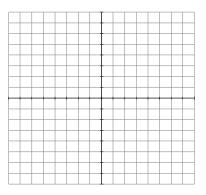
Absolute Value Equations

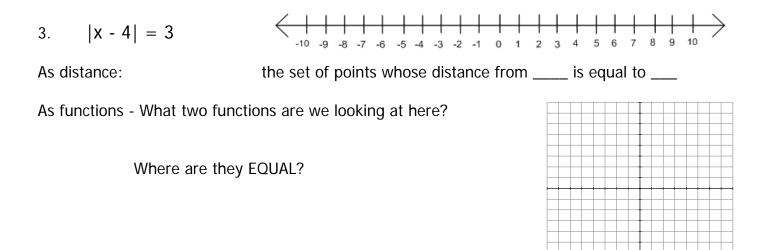
What it MEANS: Graph an Absolute Value Equation on a Number Line

1. $|\mathbf{X}| = 4$ (10.9 - 8.7 - 6.5 - 4.3 - 2.1 - 0.1 - 2.3 - 4.5 - 6.7 - 8.9 - 10)

As distance: |x - 0| = 4 "the set of points whose distance from 0 is 4"

Another way – these are two FUNCTIONS. Where are they EQUAL? Graph y=|x| and y=4...



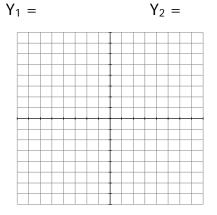


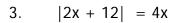
How we DO it ALGEBRAICALLY: Solve an Absolute Value Equation -

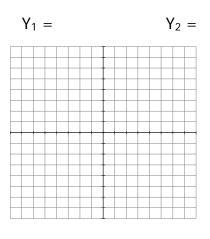
- 1. **Isolate** the absolute value symbol on one side of the equal sign
- 2. Break the equation into <u>2</u> derived equations the positive case and the negative case
- 3. Solve both equations
- 4. Check your solutions (WARNING: There may be extraneous solutions!)
- 1. |x+3| = 8

2. |3x + 1| - 5 = -3Isolate before writing the two cases!

Let's verify our answers graphically...

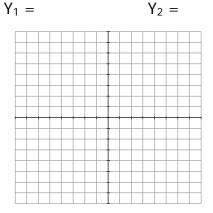


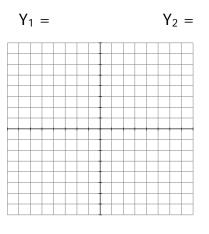




4. |4x + 5| = 2x + 4

Let's verify our answers graphically...



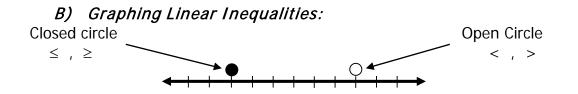


Discuss: What was different between #5 and 6? What kind of absolute value equation would have **no solution**?

Let's Review solving linear inequalities (This is a lead-in to absolute value inequalities!)

Don't forget \rightarrow switch the sign of the inequality when multiplying or dividing by a negative #

<u>Switch</u>	Don't switch	
-3x < 9	3x < -12	Original Problem(s)
x > -3	x < -4	Solution



C) Solve the following *linear inequalities*, then graph each solution:

EX 1] 3x + 12 < 9 EX 2] $4x - 3 \ge 6x + 15$





D) Graphing Compound Inequalities

What is different now?

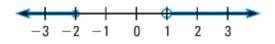
EX] -1 < x < 2

The solutions are all real numbers that are greater than -1 **and** less than 2.



EX] $x \le -2$ or x > 1

The solutions are all real numbers that are less than or equal to -2 or greater than 1.



HOW DO I REMEMBER THESE???

Graph the following inequalities.

1. $-3 < x \le 2$

2. $x \le 3$ or x > 7

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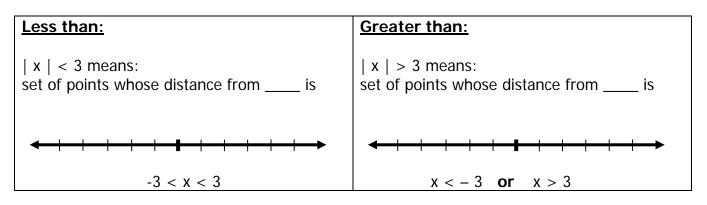


Solve the compound inequality, and then graph your solution.

3. $-10 \le 2x + 4 < 14$ 3. 3x + 2 < -10 or 2x - 4 > -4



Absolute value turns simple inequalities into <u>compound inequalities</u> because we have to <u>consider</u> <u>the negative case</u>.



Practice: Write the absolute value inequalities that would correspond with these graphs:



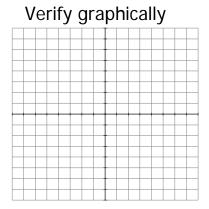
Discuss: What was a key difference between solving **linear** equations and **absolute value** equations?

How we DO it: Solve an Absolute Value Inequalities -

- 6. Isolate the absolute value symbol on one side of the equal sign
- 7. Break the equation into derived equations the positive case and the negative case (for the **negative** case **KEEP**, **CHANGE**, **CHANGE**)
- 8. Solve both equations
- 9. Check your solutions (WARNING: There may be extraneous solutions!)

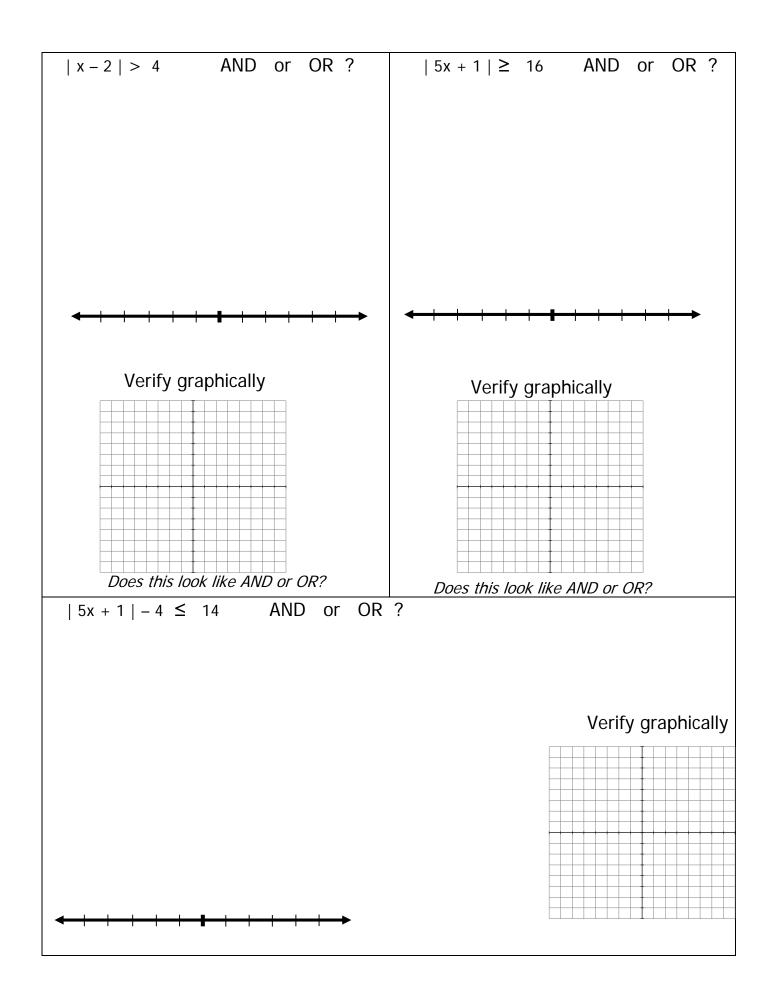
Solve and Graph the Absolute Value Inequality:

 $|x+3| \ge 5$ AND or OR ?

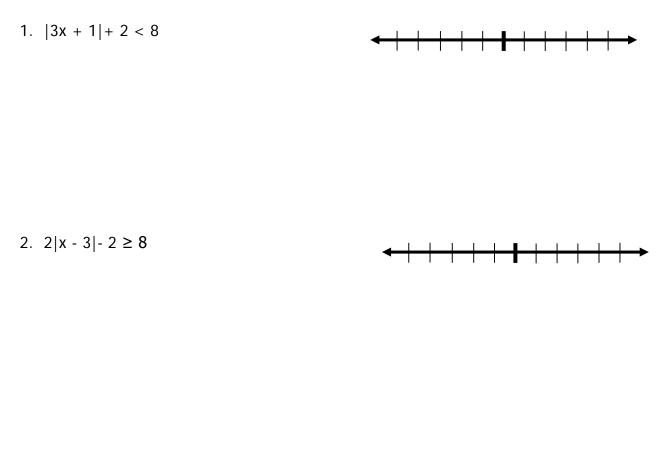


Graph your solution:

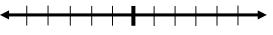
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Try these:



3. |x - 3|- 2 < -8 THINK about this one! ☺



FSGPT: Change something about #3 so that.....