
"First they build up your confidence with simple addition and subtraction, then they slam you with algebra and calculus. It's quite a clever scheme."

## Unit 3: Absolute Value

| Day 1 | Characteristics of Absolute Value |
| :---: | :---: |
| Day 2 | Transformations of Absolute Value |
| Day 5 | Absolute Value Equations |
| Day 6 | Absolute Value Inequalities |

## Schedule of Upcoming Classes

| Day 1 | A | Mon 9/21 | Introducing AV functions |
| :---: | :---: | :---: | :---: |
|  | B | Tues 9/22 |  |
| Day 2 | A | Wed 9/23 | Transformations \& Analyzing AV Graphs |
|  | B | Thurs 9/24 |  |
| Day 3 | A | Fri 9/25 | Review of Graphing AV functions |
|  | B | Mon 9/28 |  |
| Day 4 | A | Tues 9/29 | Quiz: Graphing \& Analyzing AV Functions |
|  | B | Wed 9/30 |  |
| Day 5 | A | Thurs 10/1 | Solving \& Graphing AV Equations |
|  | B | Fri 10/2 |  |
| Day 6 | A | Mon 10/5 | Solving \& Graphing AV Inequalities |
|  | B | Tues 10/6 |  |
| Day 7 | A | Wed 10/7 | Unit Review * |
|  | B | Thurs 10/8 |  |
| Day 8 | A | Fri 10/9 | Unit Test |
|  | B | Tues 10/13 |  |

* Skills Review due \& Skills Check


#### Abstract

Absent? See Ms. Huelsman AS SOON AS POSSIBLE to get work and any help you need. Notes are always posted online on the calendar. You may also email Ms. Huelsman at Kelsey.huelsman@lcps.org with any questions!


## Need Help?

Ms. Huelsman and Mu Alpha Theta are available to help
Monday, Tuesday, Thursday, and Friday mornings in L506 starting at 8:10.
Ms. Huelsman is also available after school until 4:30.

## Need to make up a test/ quiz?

Math Make Up Room is open Mon/Tues/Thurs/Fri mornings and Mon/Wed/Thurs afternoons.
Schedule is posted around the math hallway \& in Ms. Huelsman's classroom ©

## Day 1: I ntroducing... The Absolute Value Function

Let's take a look at $\mathrm{y}=\mathrm{x} .$. .

What happens if we change every negative $y$-value to a positive value?


Does this sound familiar? What takes negative values and makes them positive?

Introducing $\qquad$ the Absolute Value Function

$$
|x|= \begin{cases}x, & x \geq 0 \\ -x, & x<0\end{cases}
$$

We can analyze the parent function for special points and behavior -

$$
y=|x|
$$

Domain:

Range:
Vertex:
y-intercept:
zeros (roots, x-intercepts, solutions):


Increasing:
Decreasing:
End Behavior:
Slope of right branch:

We can also move the parent function to other places on the coordinate plane.


Are you noticing any patterns yet? Let's look at domain and range.

| $y=2\|x\|-5$ |  | $y=-\frac{1}{4}\|x-1\|+5$ |  |
| :--- | :--- | :--- | :--- |
| Domain: | Domain: |  |  |
| Range: | Range: |  |  |
| Vertex: | Vertex: |  |  |
| Y-intercept: | Y-intercept: |  |  |
| Zeros / X-intercepts: | Zeros / X-intercepts: |  |  |
| Increasing: | Increasing: |  |  |
| Decreasing: | Decreasing: |  |  |
| End Behavior: | End Behavior: |  |  |
| Slope of right branch: | Slope of right branch: |  |  |

Are you noticing any patterns yet? Let's look at slope of the right branch.

Graph the inverse of the Absolute Value Function
(start out with the original $y=|x|$ )
then, how do you graph an inverse?


## Is the inverse a function?

Graph an Absolute Value Function that has a removable discontinuity at $(3,4)$


## Day 2: Graphing Using TRANSFORMATI ONS

In these notes we will
Learn a new technique for graphing a function - shifting it up, down, left, right So we can

Eventually graph ANY function given its parent shape
First, let's graph the absolute value "parent function", $y=|x| \rightarrow$ Use your calculator to graph this function in $\mathrm{Y}_{1}$ What is the vertex of the graph?

## Exploration of Transformations - Vertical Shifts

1. Graph $y=|x|+2$ on your calculator in $Y_{2}$.
a) Sketch this graph and the "parent function".
b) How does the graph move? (up or down) $\qquad$
c) What is the vertex of the graph? $\qquad$

2. Graph $y=|x|-5$ on your calculator in $Y_{2}$.
a) Sketch this graph and the "parent function".
b) How does the graph move? (up or down) $\qquad$
c) What is the vertex of the graph? $\qquad$

3. Given that $\mathbf{y}=\mathbf{a} \mid \mathbf{x - h} \mathbf{~} \mathbf{+} \mathbf{k}$ is the symbolic form of the absolute value function, what does the parameter $k$ control?

If $\mathbf{k}$ is positive, what direction do we move? If $\mathbf{k}$ is negative, what direction do we move?

## Exploration of Transformations - Horizontal Shifts

1. Graph $\mathrm{y}=|\mathrm{x}|$ on your calculator in $\mathrm{Y}_{1}$.
a) Sketch a graph of the function.
b) What is the vertex of the graph? $\qquad$
2. Graph $y=|x-1|$ on your calculator in $Y_{2}$.
a) Sketch a graph of the function and the function in \#1.
b) How does the graph move? Left or Right?
c) What was the SIGN inside the absolute value?
d) What is the vertex of the graph? $\qquad$
3. Graph $y=|x-5|$ on your calculator in $Y_{2}$.
a) Sketch a graph of the function and the function in \#1.
b) How does the graph move? Left or Right?
c) What was the SIGN inside the absolute value?
d) What is the vertex of the graph? $\qquad$
4. Graph $y=|x+3|$ on your calculator in $Y_{2}$.
a) Sketch a graph of the function and the function in \#1.
b) How does the graph move? Left or Right?
c) What was the SI GN inside the absolute value?
d) What is the vertex of the graph? $\qquad$

5. Given that $\mathbf{y}=\mathbf{a} \mid \mathbf{x - h} \mathbf{~} \mathbf{+} \mathbf{k}$ is the symbolic from of the absolute value function, what does the parameter $h$ control?

When we have | $\mathbf{x}-\mathbf{h} \mid$, what direction does the graph move?
When we have | $\mathbf{x} \mathbf{+} \mathbf{h} \mid$, what direction does the graph move?
How is the motion related to the sign of $h$ ?

## Exploration of Transformations - Vertical Stretch or Shrink

1. Graph $\mathrm{y}=|\mathrm{x}|$ on your calculator in $\mathrm{Y}_{1}$.
a) What direction does the graph open? $\qquad$

| $x$ | $y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

2. Graph $y=2|x|$ on your calculator in $Y_{2}$.
a) What direction does the graph open? $\qquad$
b) What is the vertex of the graph?
c) Fill in the table. How do these $y$-coordinates compare with the $y$ coordinates in question 1 ? Is the graph fatter or skinnier?

| $x$ | $y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

3. Graph $\mathrm{y}=1 / 2|\mathrm{x}|$ on your calculator in $\mathrm{Y}_{2}$.
a) What direction does the graph open?
b) What is the vertex of the graph?
c) Fill in the table. How do these $y$-coordinates compare with the $y$ coordinates in question 1? Is the graph fatter or skinnier?

| $x$ | $y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

4. Graph $\mathrm{y}=-|\mathrm{x}|$ on your calculator in $\mathrm{Y}_{2}$.
a) Sketch a graph of the function and the function in \#1.
b) How did the graph change? $\qquad$
5. Graph $y=-2|x|$ on your calculator in $Y_{2}$.
a) Sketch a graph of the function and the function in \#1.
b) How did the graph change? $\qquad$

6. Given that $\mathbf{y}=\mathbf{a}|\mathbf{x}-\mathbf{h}|+\mathbf{k}$ is the symbolic from of the absolute value function, what does the parameter a control?
7. 

Examples 1 and 2
2.
Examples 3 and 4

## Exploration of ALL Transformations-

1. Graph $\mathrm{y}=|\mathrm{x}|$ on your calculator in $\mathrm{Y}_{1}$.
2. Graph $y=|x-3|-4$ on you calculator in $Y_{2}$.
a) What direction does the graph open?
b) How does the graph move? (left/right, up/down) $\qquad$
c) What is the vertex of the graph? $\qquad$
3. Graph $y=-|x-2|-3$ on you calculator in $Y_{2}$.
a) What direction does the graph open?
b) How does the graph move? (left/right, up/down)
c) What is the vertex of the graph? $\qquad$
4. Graph $y=2|x+6|-4$ on you calculator in $Y_{2}$.
a) What direction does the graph open? $\qquad$
b) How does the graph move? (left/right, up/down)
c) What is the vertex of the graph? $\qquad$
Given the absolute value function $\mathbf{y}=\mathbf{a}|\mathbf{x - h}| \mathbf{~} \mathbf{k}$
If $a>0$, does the graph open up or down? $\qquad$

If a $<0$, does the graph open up or down? $\qquad$
If $|a|>1$, does the graph have a vertical stretch or vertical shrink? $\qquad$

If $0<|a|<1$, does the graph have a vertical stretch or vertical shrink? $\qquad$

What does the parameter k control? $\qquad$
What does the parameter $h$ control? $\qquad$
What is the vertex? $\qquad$

Now generalize. . Fill in the table using your knowledge of transformations.

| Function | Direction/Opening <br> (up or down) | Vertex | Vertical Stretch or <br> shrink |
| :--- | :--- | :--- | :--- |
| 1. $y=1 / 4\|x+4\|-9$ |  |  |  |
| 2. $y=-2\|x+1\|+6$ |  |  |  |
| 3. $y=4\|x-3\|+5$ |  |  |  |
| 4. $y=-1 / 2\|x-7\|+3$ |  |  |  |
| 5. $y=2\|x+4\|-1$ |  |  |  |

Graph the following using your knowledge of transformations. Verify with your calculator.

1. $\mathrm{y}=1 / 4|\mathrm{x}+4|-9$

Domain: $\qquad$
Range: $\qquad$
2. $y=-2|x+1|+6$


Domain: $\qquad$
3. $y=4|x-3|+5$


Domain: $\qquad$

Range: $\qquad$

Given the absolute value equation graph, write the absolute value equation:

2.




## Quick Quiz Review

1. Which absolute value function(s) open up?
A. $y=-2|x-5|$
B. $y=|x+1|-7$
C. $y=-|x+4|+8$
D. $y=1 / 4|x-9|$
2. Which absolute value function(s) are vertically stretched?
A. $y=-2|x-5|$
B. $y=|x+1|-7$
C. $y=-|x+4|+8$
D. $y=1 / 4|x-9|$
3. Which absolute value function(s) have an absolute minimum at the vertex?
A. $y=-2|x-5|$
B. $y=|x+1|-7$
C. $y=-|x+4|+8$
D. $y=1 / 4|x-9|$
4. Given $f(x)=|x+9|$. The vertex of the function moves from $(0,0)$ nine units $\qquad$ :
A. left
B. right
C. up
D. down
5. Sketch $f(x)=|x-5|+1$

6. Sketch $f(x)=-\frac{4}{3}|x|+4 \quad$ What is the range?


What is the end behavior?

The vertex is... (circle all that apply)
A) a relative minimum
B) a relative maximum
C) an absolute minimum
D) an absolute maximum
7. Write the equation for the following absolute value functions





With a vertex at $(-8,2)$ that is vertically stretched by a scale of 2 :
$\qquad$
That opens downward and has a vertex at $(5,0)$ :

That is vertically shrunk and shifted to the right (you pick the details!)

## Day 5: Absolute Value Equations

Objective: understand the definition of absolute value and how to manipulate that symbol in solving equations.

Absolute Value means $\qquad$

## Absolute Value Equations

## What it MEANS: Graph an Absolute Value Equation on a Number Line

1. $|x|=4$


As distance: $\quad|x-0|=4$
"the set of points whose distance from 0 is 4 "
Another way - these are two FUNCTIONS. Where are they EQUAL? Graph $\mathrm{y}=|\mathrm{x}|$ and $\mathrm{y}=4 . .$.
3. $|x-4|=3$

As distance:

the set of points whose distance from $\qquad$ is equal to $\qquad$
As functions - What two functions are we looking at here?


How we DO it ALGEBRAI CALLY: Solve an Absolute Value Equation -

1. Isolate the absolute value symbol on one side of the equal sign
2. Break the equation into $\underline{\mathbf{2}}$ derived equations - the positive case and the negative case
3. Solve both equations
4. Check your solutions (WARNI NG: There may be extraneous solutions!)
5. $|x+3|=8$
6. $|3 x+1|-5=-3$

Isolate before writing the two cases!

Let's verify our answers graphically...

3. $|2 x+12|=4 x$
4. $|4 x+5|=2 x+4$

Let's verify our answers graphically...
$Y_{1}=$
$Y_{2}=$
$Y_{1}=$
$Y_{2}=$


5. $2|x+7|-5=15$
6. $-2|x+7|-5=15$

Discuss: What was different between \#5 and 6? What kind of absolute value equation would have no solution?

Let's Review solving linear inequalities (This is a lead-in to absolute value inequalities!)
A) Inequality Symbols: < $\qquad$ , $\leq$ $\qquad$
$>$ $\qquad$ , $\geq$ $\qquad$

Don't forget $\rightarrow$ switch the sign of the inequality when multiplying or dividing by a negative \#

| Switch | Don't switch |  |
| :---: | :---: | :---: |
| $-3 x<9$ | $3 x<-12$ | Original Problem(s) |
| $x>-3$ | $x<-4$ | Solution |

B) Graphing Linear Inequalities:

C) Solve the following linear inequalities, then graph each solution:
EX 1] $3 x+12<9$
EX 2] $4 x-3 \geq 6 x+15$

EX 3] $-4 x+16>4$ EX 4] $-3 x-6 \leq 3 x+6$

D) Graphing Compound I nequalities

What is different now?

$$
E X]-1<x<2
$$

$$
E X] x \leq-2 \text { or } x>1
$$

The solutions are all real numbers that are greater than -1 and less than 2.


The solutions are all real numbers that are less than or equal to -2 or greater than 1.


HOW DO I REMEMBER THESE???

Graph the following inequalities.

1. $-3<x \leq 2$
2. $x \leq 3$ or $x>7$


Solve the compound inequality, and then graph your solution.
3. $-10 \leq 2 x+4<14$
3. $3 x+2<-10$ or $2 x-4>-4$


## Day 6: Absolute Value I nequalities

Absolute value turns simple inequalities into compound inequalities because we have to consider the negative case.

| Less than: | Greater than: |
| :---: | :---: |
| $\|x\|<3 \text { means: }$ <br> set of points whose distance from is | $\|x\|>3$ means: <br> set of points whose distance from __ is |
| $-3<x<3$ | $x<-3$ or $x>3$ |

Practice: Write the absolute value inequalities that would correspond with these graphs:


AND or OR


AND or OR

Discuss: What was a key difference between solving linear equations and absolute value equations?

How we DO it: Solve an Absolute Value I nequalities -
6. Isolate the absolute value symbol on one side of the equal sign
7. Break the equation into derived equations - the positive case and the negative case (for the negative case - KEEP, CHANGE, CHANGE)
8. Solve both equations
9. Check your solutions (WARNI NG: There may be extraneous solutions!)

Solve and Graph the Absolute Value Inequality:
$|x+3| \geq 5$
AND or OR ?
Verify graphically

Graph your solution:



Try these:

1. $|3 x+1|+2<8$

2. $2|x-3|-2 \geq 8$

3. $|x-3|-2<-8$

THINK about this one! $)^{-}$


FSGPT: Change something about \#3 so that......

