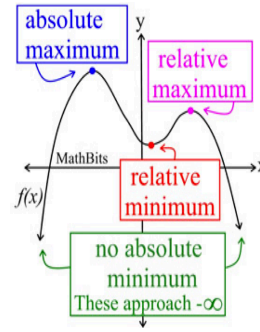
CP Algebra 2

Unit 3 – (Ch 6) Polynomials and Polynomial Functions



NOTES PACKET

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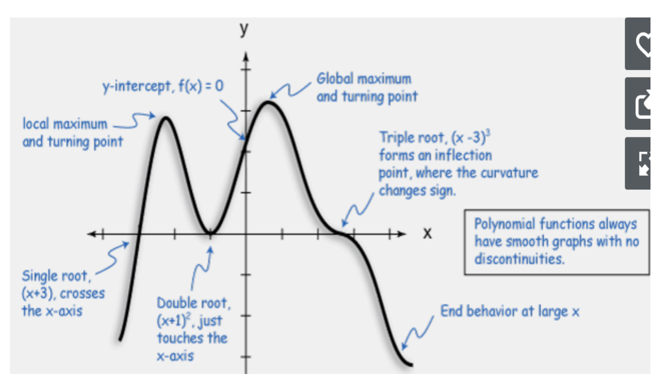
Learning Targets:

	PART 1
Polynomials: The Basics	1. I can classify polynomials by degree and number of terms. 2. I can use polynomial functions to model real life situations and make predictions 3. I can identify the characteristics of a polynomial function, such as the intervals of increase/decrease, intercepts, domain/range, relative minimum/maximum, and end behavior.
Factors and Zeros	4. I can write standard form polynomial equations in factored form and vice versa. 5. I can find the zeros (or x-intercepts or solutions) of a polynomial in factored form and identify the multiplicity of each zero. 6. I can write a polynomial function from its real roots.
Dividing Polynomials	7. I can use long division to divide polynomials. 8. I can use synthetic division to divide polynomials. 9. I can use synthetic division and the Remainder Theorem to evaluate polynomials.
	PART 2
Solving Polynomials	10. I can use the fundamental theorem of algebra to find the *expected* number of roots. 11. I can solve polynomials by graphing (with a calculator). 12. I can solve polynomials by factoring.
Finding and Using Roots	13. I can find all of the roots of a polynomial. 14. I can write a polynomial function from its complex roots.
Graphing	15. I can graph polynomials.

NAME ______________________________

PERIOD ________________



Polynomial: The Basics

After this lesson and practice, I will be able to …

LT1. classify polynomials by degree and number of terms.
LT2. use polynomial functions to model real life situations and make predictions
LT3. identify the characteristics of a polynomial function, such as the intervals of increase/decrease, intercepts, domain/range, relative minimum/maximum, and end behavior.

LT1. I can classify polynomials by degree and number of terms.

Let's start with some definitions:
Polynomial:
- a mathematical expression of 1 or more algebraic terms each of which consists of a constant multiplied by one or more variables raised to a nonnegative integral power (as $a + bxy + cy^2x^2$)
- a monomial or sum of monomials

Polynomial Function: A polynomial function is a function such as a quadratic, a cubic, a quartic, and so on, involving only **non-negative integer powers of x**. We can give a general definition of a polynomial, and define its degree**.**

Standard Form of a Polynomial:: $y = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ where $a_n, a_{n-1}, ..., a_1, a_0$ are the coefficients and n, n-1, n-2, ….0 are the powers of x, and all n's are a nonnegative integers.
- The exponents of the variables are given in descending order when written in general form.
- The term with the highest degrees first and place in the other terms in descending order.

Term - A of the monomial that is added in a polynomial.

Degree of a Term: the sum the exponents of each variable in each monomial.

Degree of a Polynomial: the greatest value of the sum of all exponents of each monomial.

There are special names we give to polynomials according to their degree and number of terms.

Degree	Name of Degree	Example		Number of Terms	Name	Example
0	Constant			1	Monomial	
1	Linear			2	Binomial	
2	Quadratic			3	Trinomial	
3	Cubic			4	Polynomial of 4 terms	
4	Quartic			n	Polynomial of n terms	
5	Quintic					
n	nth degree					

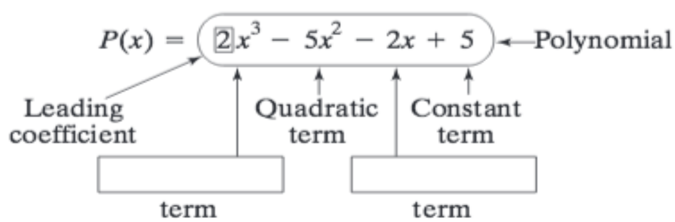
Complete the chart below using the information above.

Vocabulary and Key Concepts

Polynomial Function

$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ where n is a nonnegative integer and the coefficients $a_n, \ldots, a_0$ are ______ numbers.

Degree	Name Using Degree	Polynomial Example	Number of Terms	Name Using Number of Terms
0	constant	6	1	monomial
	linear	$x + 6$	2	binomial
2		$3x^2$	1	monomial
3	cubic	$2x^3 - 5x^2 - 2x$		trinomial
4	quartic	$x^4 + 3x^2$	2	
	quintic	$-2x^5 + 3x^2 - x + 4$	4	polynomial of 4 terms



1. Write each polynomial in standard form. Then classify each polynomial by its degree and number of terms. Finally, name the leading coefficient of each polynomial.

a. $9 + x^2$

b. $x^3 - 2x^2 - 3x^4$

More Examples:

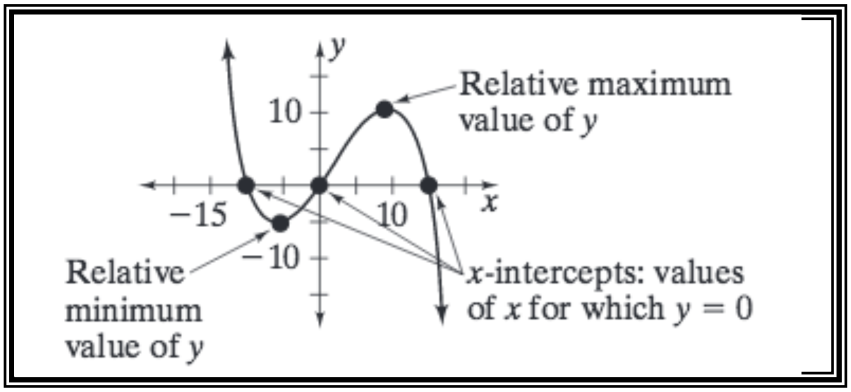
c. $-7x + 5x^4$

d. $x^2 - 4x + 3x^3 + 2x$

e. $4x - 6x + 5$

f. $6 - 3x^5$

LT3. I can identify the characteristics of a polynomial function, such as the intervals of increase/decrease, intercepts, domain/range, relative minimum/maximum, and end behavior.



Relative Maximum – the greatest y-value among the nearby points on the graph.
Relative Minimum – the smallest y-value among the nearby points on the graph.

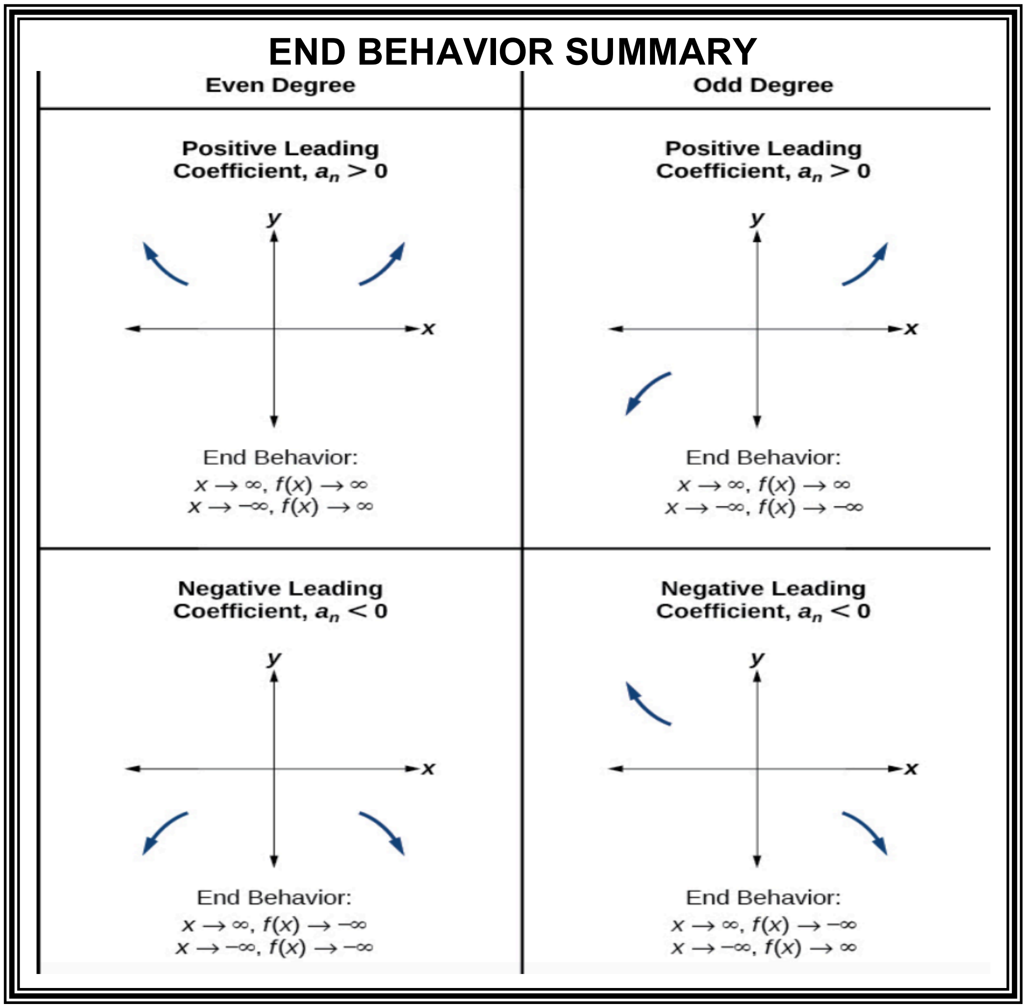
Multiple Zero – a zero of a linear factor that is repeated in the factored form of the polynomial

Multiplicity of a Zero – the number of times the related linear factor is repeated in the factored form of a polynomial.
-Impacts the behavior of the graph around the x-intercept (bounce, cross)

Domain: all possible x or input values
Range: all possible y or output values

Intervals of Increasing– the x values for which the y value are increasing
Intervals of Decreasing– the x values for which the y value are decreasing

	Quick Sketch of Function	Is the function always increasing, always decreasing, some of both, or neither?	What is the *largest* number of x-intercepts that the function can have?	What is the *smallest* number of x-intercepts that the function can have?	Domain
Constant Function					
1st Degree $f(x) = x$					
2nd Degree $f(x) = x^2$					
3rd Degree $f(x) = x^3$					
4th Degree $f(x) = x^4$					



Ex: 1. Describe the end behavior of the graph of each polynomial function by completing the statements and sketching arrows. Do this without looking at the graph.

a) $f(x) = -x^6 + 4x^2 + 2$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

b) $f(x) = 2x^3 + 2x^2 - 5x - 10$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

c) $f(x) = -2x^5 + x^2 - 1$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

d) $f(x) = x^4 - 5x + 10$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

Quick Check: Describe the end behavior of the graph of each polynomial function by completing the statements and sketching arrows. Do this without looking at the graph.

1) $f(x) = -5x^6 + 4x^2 + 2$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

2) $f(x) = 2x^5 + 2x^3 – 5x -6$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

3) $f(x) = 3x^4 + 4x^2 + 2$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

4) $f(x) = -2x^3 + 2x^2 – 5x -6$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

Summary of Minimums and Maximums

A relative minimum or maximum is a point that is the min. or max. relative to other nearby function values. (Note: Parabolas had an absolute min or max)

- Approximate the min or max *(First adjust your window as needed for your graph)*

1) Press 2nd TRACE, then press MIN or MAX (depending on the shape of your function).

2) Move your cursor just to the “left” of the relative min or relative max. Press ENTER.

3) Move your cursor just to the “right” of the relative min or relative max. Press ENTER.

4) The screen will show “Guess”. Press ENTER again.

5) The bottom of the screen will say X=____ Y =_______
The y value is the relative min or relative max. The x value is where the min or max is occurring.
The relative min or relative max in this example is _________ at _______.

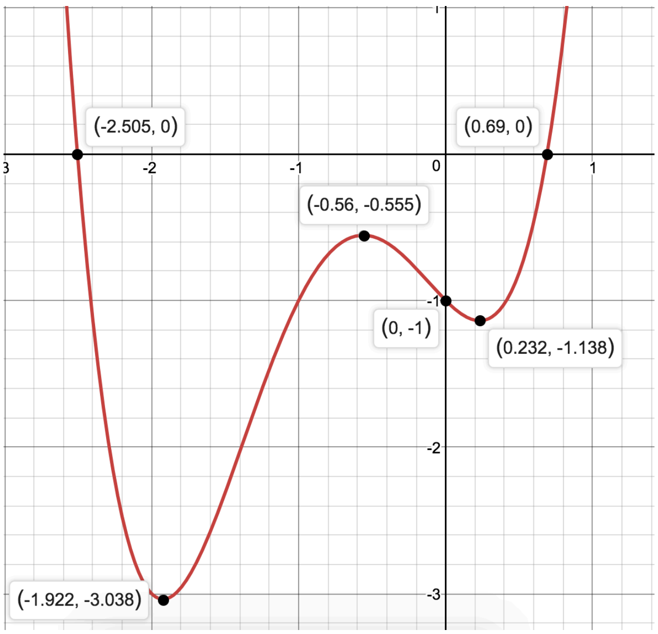
Ex 2: Graph the equation $y = 3x^3 - 5x + 5$ in your calculator. Then determine the coordinates of all relative minimums and maximums (rounded to 3 decimal places).

Quick Check Determine the coordinates of all relative minimums and maximums (rounded to 3 decimal places).

a. $y = 0.5x^4 - 3x^2 + 3$

b. $f(x) = -x^3 + 6x^2 - x - 1$

Ex 3: Determine the intervals of increase and decrease, the intercepts, the domain and range, and the coordinates of all relative minimums and maximums. Round all answers to three decimal places.



Intervals of increase: ___________________

Intervals of decrease: ______________________

y-Intercept: ___________________

x-Intercepts: __________________________

Domain: ___________

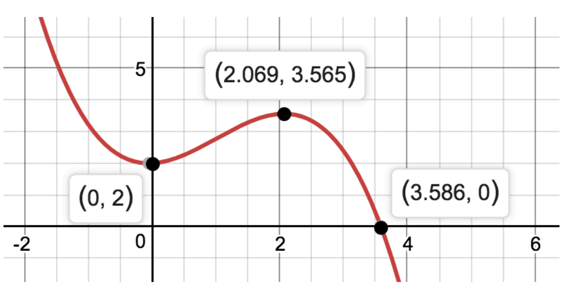
Range: _______________

Relative Minimum(s): ____________________

Relative Maximum(s): ___________________

Quick Check Determine the intervals of increase and decrease, the intercepts, the domain and range, and the coordinates of all relative minimums and maximums. Round all answers to three decimal places.

a)



Intervals of increase: __________________________

Intervals of decrease: __________________________

y-Intercept: ___________________

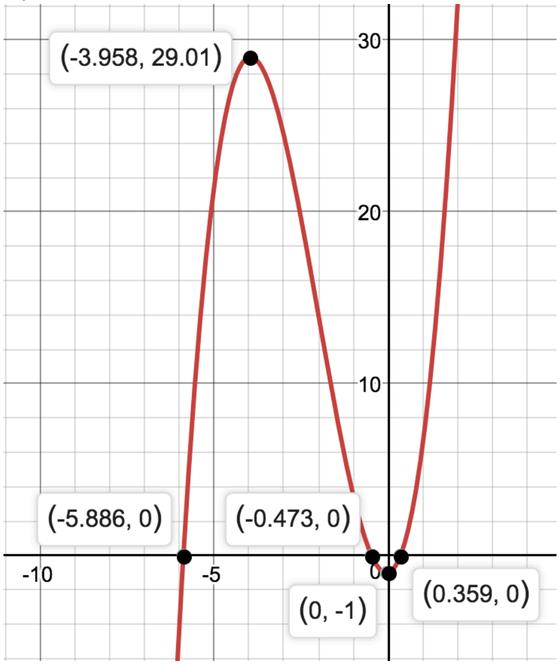
x-Intercepts: __________________________

Domain: ___________ Range: ________________

Relative Minimum(s): _________________________

Relative Maximum(s): _________________________

b)



Intervals of increase: ____________________________

Intervals of decrease: ____________________________

y-Intercept: ____________________

x-Intercepts: ____________________________

Domain: ____________ Range: ________________

Relative Minimum(s): ___________________________

Relative Maximum(s): ___________________________

More Practice.
Further analyze the graphs of the functions.

a. $y = 0.5x^4 - 3x^2 + 3$

Intervals of increase: ____________________________

Intervals of decrease: ____________________________

y-Intercept: ____________________ x-Intercepts: _____________________________________

Domain: ____________ Range: ________________

Relative Minimum(s): ___________________________

Relative Maximum(s): ___________________________

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b. $f(x) = -x^3 + 6x^2 - x - 1$

Intervals of increase: ____________________________

Intervals of decrease: ____________________________

y-Intercept: ____________________ x-Intercepts: ____________________________

Domain: ____________ Range: ________________

Relative Minimum(s): ___________________________

Relative Maximum(s): ___________________________

LT2. I can use polynomial functions to model real life situations and make predictions

Comparing Models Use a graphing calculator to find the best regression equation for the following data. Compare linear, quadratic and cubic regressions.

x	0	5	10	15	20
y	10.1	2.8	8.1	16.0	17.8

Recall finding a regression equation using STAT: (turn diagonstics on so r values are calculated)
- Enter your data (STAT, Edit…)
- Turn on Stat Plot 1 (2nd, STAT PLOT)
- Graph the data (ZOOM, 9)
- STAT, right, then choose your desired regression.
- Your command should look like ____Reg, Y_1 (VARS, right, ENTER, ENTER)

Which regression model is best? Why?

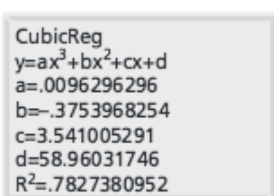
Predict the value of y when x = 12.

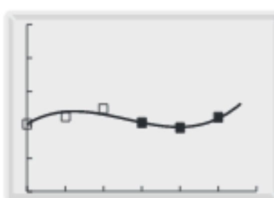
Example

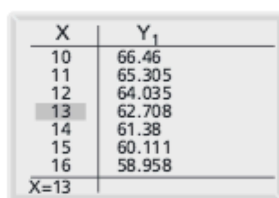
3 Using Cubic Functions The table shows data on the number of employees that a small company had from 1975 to 2000. Find a cubic function to model the data. Use it to estimate the number of employees in 1988.

Year	Number of Employees
1975	60
1980	65
1985	70
1990	60
1995	55
2000	64

Enter the data. Let 0 represent 1975. To find a cubic model, use the CubicReg option of a graphing calculator. Graph the model.







X	Y_1
10	66.46
11	65.305
12	64.035
13	62.708
14	61.38
15	60.111
16	58.958

X=13

The function $f(x) = 0.00963x^3 - 0.3754x^{\square} + 3.541x + 58.96$ is an approximate model for the ______ function.

To estimate the number of employees for 1988, you can use the Table option of a graphing calculator to find that $f(13) \approx 62.71$. According to the model, there were about ______ in 1988.

QUICK CHECK:

1. Use the cubic model in Example 3 to estimate the number of employees in 1999.

2. The table below shows world gold production for several years.

World Gold Production

Year	1975	1980	1985	1990	1995	2000
Production (millions of troy ounces)	38.5	39.2	49.3	70.2	71.8	82.6

SOURCES: *The World Almanac* and *World Gold*

a. Enter the data in your calculator. Let 0 represent 1975. Graph the data and sketch it here:

b. Find a quartic function to model the data: ____

c. Use your model to predict the gold production in 2018. __________________________

**NOTE: Some real life situations will not require a calculator to write an equation.*

3. Travel: Several popular models of carry-on luggage have a length of 10 inches greater than their depth. To comply with airline regulations, the sum of the length, width and depth may not exceed 40 inches.

a. Assume the sum of the length, width, and depth is 40 in. Write a function for the volume of the luggage.

b. What is the maximum possible volume of the luggage? What dimensions correspond to this volume?

Length: ____________ Width: ______________ Depth: _______________

Factors and Zeros

After this lesson and practice, I will be able to …

LT 4. write standard form polynomial equations in factored form and vice versa.
LT 5 find the zeros (or x-intercepts or solutions) of a polynomial in factored form and identify the multiplicity of each zero.
LT 6 write a polynomial function from its real roots.

LT 4. I can write standard form polynomial equations in factored form and vice versa.

Factor Theorem

The expression *x-a* is a linear factor of a polynomial if and only if the value of *a* is a __________________ of the related polynomial function.

Equivalent Statements about Polynomials

① -4 is a ____________ of $x^2 + 3x - 4 = 0$.

② -4 is an ____________ of the graph of $y = x^2 + 3x - 4$.

③ -4 is a ____________ of $y = x^2 + 3x - 4$.

④ $x + 4$ is a ____________ of $x^2 + 3x - 4$.

Writing a Polynomial in Standard Form.

Recall: **Standard Form of a Polynomial:** The term with the highest degrees first and place in the other terms in descending order.

1) Write (x - 1)(x + 3)(x + 4) as a polynomial in standard form.

QUICK CHECK:
2. Write each expression in standard form.

a. $(x-3)(x+2)(x-4)$

b. $(x+5)(x-1)(x+2)$

3. Write the expression (x + 1)(x – 1)(x + 2) as a polynomial in standard form.

Writing a Polynomial in Factored Form.

1) Write $3x^3 - 18x^2 + 24x$ as a polynomial in factored form. Check by multiplication or on the calculator.

QUICK CHECK:
2) Write each polynomial in factored form.

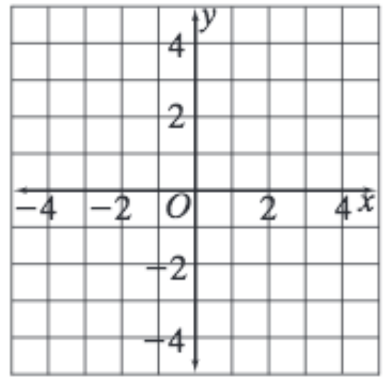
a) $3x^3 - 3x^2 - 36x$

b) $2x^3 + 10x^2 + 12x$

c) $6x^3 - 15x^2 - 36x$

LT 5 I can find the zeros (or x-intercepts or solutions) of a polynomial in factored form and identify the multiplicity of each zero

Finding the Zeros of a polynomial Function.



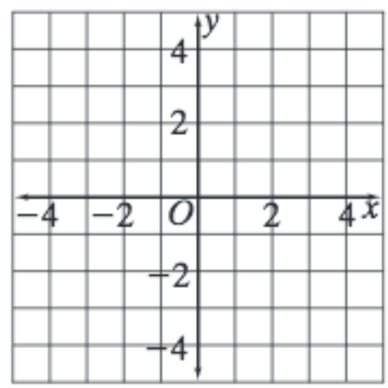
3) Find the zeros of the polynomial function

f(x) = (x + 1)(x - 1)(x + 3)

The Zeros: ___________, ___________ and ___________

Sketch the Function. Plot the x and y intercepts.
Hint: y intercept is when x = 0. (0,?)

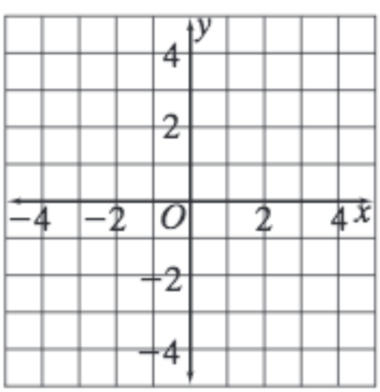
QUICK CHECK:
Find the zeros of each function. Then sketch a graph of the function, showing x and y intercepts.



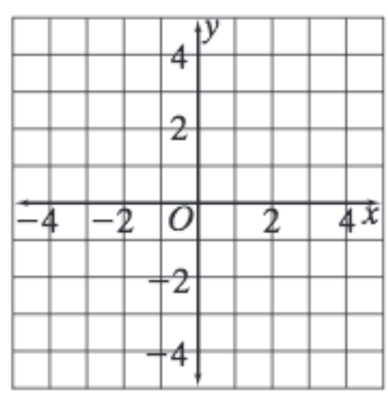
a. Find the zeros of the polynomial function
y = (x + 2)(x - 5)(x - 3)

The Zeros of the Function: ______, ______ and ______
Sketch the Function. Plot the x and y intercepts.

b. $y=(x-1)(x+2)(x+1)$



c. $y=(x-4)(x-1)(x+2)$



Finding the Multiplicity of a Zero

Recall:
Multiple Zero – a zero of a linear factor that is repeated in the factored form of the polynomial

Multiplicity of a Zero – the number of times the related linear factor is repeated in the factored form of a polynomial.
-Impacts the behavior of the graph around the x-intercept (bounce or cross)

Find the zeros of the function $\boldsymbol{y = (x-1)(x+2)(x+2)}$.

Notice that while the function has three total zeros, only __________ of then were __________ (______ is listed as a zero twice). When a zero (and thus its linear factor) is repeated, it is called a __________________ zero. In this example, -2 is said to have multiplicity _________ since it occurs as a zero _____________.

In general, the _______________________ *of a zero is equal to the number of times the zero occurs.*

QUICK CHECK:

1. Find all zeros of $f(x)=x^4+6x^3+8x^2$ and state the multiplicity of each zero.

2. Find the zeros and its multiplicity and the y-intercept for the function f(x)= $(x-5)(x-4)^3(x+1)^2$

3. Sketch: y= $5(x-1)(x+2)^3(x+5)^3$

State it's degree and leading coefficient.
Describe it's end behaviors.

LT 6. I can write a polynomial function from its real roots.

Writing a Polynomial Function From Its Zeros.

Recall: We did this with quadratic functions already.

1) Write a polynomial function in **standard form** with zeros at. 2, -3 and 0.

QUICK CHECK:

2) Write a polynomial function in standard form with zeros -4,-2,0.

3) Is there another polynomial function with zeros -4, -2, 0? Explain.

4) Write a polynomial function in standard form with zeros $\sqrt{5}, -\sqrt{5}$, -5, 1.
(Remember: Irrational zeros and complex zeros always come in conjugate pairs)

5) Write a polynomial equation in standard form with zeros at -2, 3, and 3.

6) Write a polynomial equation in standard form with zeros at 2, 1, and -3.

Dividing Polynomials

After this lesson and practice, I will be able to …
LT 7. use long division to divide polynomials.
LT 8. use synthetic division to divide polynomials.
LT 9. use synthetic division and the Remainder Theorem to evaluate polynomials.

LT 7. I can use long division to divide polynomials

Let's review long Division.

Consider $6\overline{)48}$ with quotient 8

-If the remainder is ______, then both the __________ and ________ are ________ of the ____________.
-If the remainder is not ______, then neither the __________ nor ________ are factors of the ____________.

This pattern holds true also for __________________ division.

Consider $x\overline{)2x^2}$

Long Division of Polynomials

Rules:

a) All terms must be put in ____ order.

b) Put in a _________________ to "hold the place" of any missing terms.

Ex. 1) $(2x^4 + x^3 – 2x^2 + 9x + 5) \div (2x + 1)$

$$2x + 1\overline{)\mathbf{2x^4 + x^3 - 2x^2 + 9x + 5}}$$

Ans.________________________________

Ex. 2) $(2x^3 + x^2 - 4x + 2) \div (x - 1)$

$$x - 1 \overline{\big) \mathbf{2x^3 + x^2 - 4x + 2}}$$

Ans.______________________________________

If there is a remainder, always add it to the end as the $\frac{remainder}{divisor}$

3) $x^2 + 2x - 6 \overline{\big) \mathbf{x^4 + 3x^3 - 5x^2 - 8x + 6}}$

Ans.______________________________

4) $(x^3 + 6x^2 - 5x + 20) \div (x^2 + 5)$ (hint: use place holder)

Ans.____________________________

QUICK CHECK:

5) $(x^2 + 9) \div (x + 3)$

6. Divide $x^2+3x-12$ by $x-3$ using long division.

__________________ __________________

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7. $\left(x^2+2x-30\right)\div\left(x-5\right)$

8. Divide $\left(12x^4-5x^2-3\right)\div\left(x-2\right)$

__________________ __________________

9. $\left(x^2+10x+16\right)\div\left(x+2\right)$

10. Divide $\left(x^4+3x^3-5x^2-8x+6\right)\div\left(x^2+2x-6\right)$

11. $(x^3+4x^2-x-1)\div(x+3)$

12. $\left(2x^3-14x^2+72\right)\div(x-6)$

13. $(9x^4-18x^3+10x-16)\div(x-2)$

14. $\left(9x^4+14x^3-14x^2-13x-2\right)\div(x+2)$

15. Divide $\left(x^4-x^3-2x^2-2x-8\right)\div\left(x^2+2\right)$

LT 8. I can use synthetic division to divide polynomials.

Synthetic Division

Review: Use Long Division to divide. Recall the need of a placeholder.

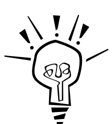
$3x^5 - 8x^3 + 2x^2 - x + 1$ divided by $x + 2$

1a) $x + 2 \overline{\big)\, 3x^5 + 0x^4 - 8x^3 + 2x^2 - x + 1}$

Ans: ____

Synthetic Division Now we'll do the same problem an easier way!!!

1b) $x + 2 = 0$ so $x =$ ____ | ___ ___ ___ ___ ___ ___



Drop down and
MAMA and MAMA!

The degree of the quotient is one less than the degree of the dividend.

Ans: ____

2) $(x^4 - 2x^3 - 31x - 4) \div (x - 4)$ (What's missing? ______)

___ | ____ ____ ____ ____ ____

The degree of the quotient is one less than the degree of the dividend.

Answer:____

3) $(x^3 - 2x^2 - 9) \div (x - 3)$

While synthetic division is very cool, there is one sad note. It only works for divisors of the form

________ or ________.
The coefficient of x must be 1!

It cannot be used with divisors like $2x - 3$ or $x^2 + 5$

Answer:__________________________________

Use division to determine whether $x+4$ is a factor of each polynomial.

a. x^2+6x+8

b. x^3+3x^2-6x-7

QUICK CHECK: Divide the polynomials using synthetic division.

5) $(x^2 + 9) \div (x + 3)$

6. Divide $x^2+3x-12$ by $x-3$

____

____

7. $\left(x^2+2x-30\right)\div\left(x-5\right)$

8. Divide $\left(12x^4-5x^2-3\right)\div\left(x-2\right)$

____

____

9. $\left(x^2+10x+16\right)\div\left(x+2\right)$

10. $\left(9x^4+14x^3-14x^2-13x-2\right)\div(x+2)$

____

____

11. $(x^3+4x^2-x-1)\div(x+3)$

12. $\left(2x^3-14x^2+72\right)\div(x-6)$

____

____

13. $(9x^4 - 18x^3 + 10x - 16) \div (x - 2)$

Recall, the purpose for learning to divide polynomials is to discover the _________ of the polynomial. Let's show how synthetic division can help you find all zeros of certain polynomials.

14: Find all zeros of each polynomial using synthetic division with the given factor.

a. $x^3 - 13x - 12$; one factor = $x + 1$

b. $x^3 + x^2 - 24x + 36$; one factor = $x - 3$

LT 9. I can use synthetic division and the Remainder Theorem to evaluate polynomials.

EVALUATING POLYNOMIALS BY THE REMAINDER THEOREM

Remainder Theorem – If a polynomial P(x) of degree of at least one is divided by _________, where ____ is a ________, then the remainder is ______.

****This theorem provides a fast way of evaluating some complex polynomials quickly and for determining if a given expression is a __________ of the polynomial.**

The remainder of the function has the same value as evaluating the function.

4) Evaluate f(-2) using the remainder theorem if $f(x) = 3x^3 + 8x^2 + 5x - 7$. Then determine if -2 is a zero of f(x).

Answer: f(–2) =

5) Evaluate f(-1) using the remainder theorem if $f(x) = 3x^4 - 18x^3 + 20x^2 - 24x$ Then determine if -1 is a zero of f(x).

Be careful on this one!

Answer: f(–1) =

6) Find $P(-4)$ for $P(x) = x^4 - 5x^2 + 4x + 12$. Then determine if -4 is a zero of P(x).

7) Find $P(-2)$ for $P(x) = 5x^3 + 12x^2 + x - 9$. Then determine if -2 is a zero of P(x).

USING SYNTHETIC DIVISION TO FIND FACTORS:

8) Given (x+3) is one factor of $2x^3 + 11x^2 + 18x + 9$. Find all of the factors & list all of the roots. Do not use fractions or decimals.

Factors: ______________________________

Roots, Zeros, x-intercepts, solutions: ________________
(depends if it is an equation = 0 or = y)

9) Given (x + 9) and x are factors of $x^4 + 14x^3 + 51x^2 + 54x$,
find the rest of the factors and list all of the roots.

Factors: _________________________________ *Roots:* _______________

Solving Polynomial Equations

After this lesson and practice, I will be able to …

LT 10. use the fundamental theorem of algebra to find the *expected* number of roots.
LT 11. solve polynomials by graphing (with a calculator).
LT 12. solve polynomials by factoring.

In the quadratics unit, you learned five strategies for solving quadratic equations. Let's see how many you can remember!

1)

2)

3)

4)

5)

To solve polynomials, use similar methods as to solving quadratics. There's one thing to learn first that will help solve polynomials …

LT 10. I can use the fundamental theorem of algebra to find the *expected* number of roots.

How does the degree of the polynomial relate to the number of x-intercepts?

The number of _____________ to a polynomial function is equal to the _____________ of the polynomial. This observation is a very important fact in algebra.

(Corollary to) The Fundamental Theorem of Algebra – Every polynomial in one variable of degree n>0 has exactly ________ zeros, including _____________ and ____________ zeros.

This theorem makes it possible to know the number and type of zeros in a given function, which can be helpful in finding all zeros of a polynomial.

Ex 1: Determine the number of zeros of the polynomial.

a. $f(x)=x^3-2x^2+4x-8$

b. $\boldsymbol{y=15x^{13}+3x^6-9}$

Recall: Only real zeros can be found using a calculator. Complex zeros must be found by hand. Additionally, sometimes it is required to fins the exact value of a real RATIONAL zeros. This must be found by hand also.

You can always use that trick to figure out how many zeros you should expect from a polynomial. Now let's solve!

LT 11. I can solve polynomials by graphing (with a calculator).

Solving polynomial equations by graphing is just like solving quadratic equations by graphing.

Solve $\boldsymbol{x^3 + 12x^2 + x - 1 = 0}$

Our graphing calculators will help us find zeros of a polynomial function.

$\boldsymbol{y = x^3 + 12x^2 + x - 1}$

1) Enter the equation in your calculator as Y_1 = Press GRAPH.

2) To make sure we can see the graph, click ZOOM and ZStandard.

You should see a skinny parabola that looks like it has two zeros. But let's use our Fundamental Theorem of Algebra trick to make sure there are only two zeros …

Based on the **Fundamental Theorem of Algebra**, how many zeros should this polynomial have?

Adjust the window until you can see all ______________ zeros. Then continue on with step 3.

3) Press 2nd TRACE, then press 2: ZERO.

4) Move your cursor just to the "left" of the first point of intersection. Press ENTER.

5) Move your cursor just to the "right" of the first point of intersection. Press ENTER.

6) The screen will show "Guess". Press ENTER again. The calculator will display the zero.

7) Repeat steps 3-6 to obtain the rest of the zeros.

Ex 2: Find the expected number of zeros, then use your graphing calculator to find the zeros of the function $\boldsymbol{y = x^3 + 12x^2 + x - 1}$ (Hint: You may need to zoom out!)

Ex 3: Find the expected number of zeros, then use your graphing calculator to find the zeros of the function $\boldsymbol{y = 2x^2 + x - 7}$.

Ex 4: Find the expected number of zeros, then use your graphing calculator to find the zeros of the function $\boldsymbol{y = x^4 + 2x^3 - 6x^2 - 2}$.

LT 12. I can solve polynomials by factoring.(6.4)

Recall our previous strategies for factoring quadratics:

1)

2)

3)

4)

5)

Here are two new factoring techniques.

SUM OF CUBES

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

DIFFERENCE OF CUBES

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Notice the signs are the same, the opposite, & then positive (SOP). SMILE!

What are the perfect cubes? *(look on table of calculator using $y_1 = x^3$)*

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$\mathbf{X^3}$									

Ex 1: Factor completely.

a. $x^3 + 64$

b. $16z^5 - 250z^2$

Ex 2: Factor completely.

a. $x^3 + 8$

b. $8x^3 - 1$

c. $512m^3 - 216$

Some polynomials of higher-degree can be solved using strategies you used when you factored quadratics. The key is in recognizing if the polynomial is in “quadratic form.”

Ex 2: Factor completely. *Hint: Make a temporary substitution of variables*

a. $x^4 - 2x^2 - 8$

b. $x^4 + 7x^2 + 6$

Ex 3: Factor completely.

a. $x^4 - x^2 - 2$

b. $x^4 + 8x^2 - 9$

Find the expected number of zeros, then solve each equation by factoring.
Solve. Find all complex roots.

1. $8x^3 + 125 = 0$

x = ________________________

Step 1: Factor
Step 2. Use Zero Property & Quadratic formula

2. $x^4 - 6x^2 - 27 = 0$

x = ________________________

Hint: Make a temporary substitution of variables

3. $x^3 + 8 = 0$

x = ________________________

4. $27x^3 = 1$

x = ________________________

5. $x^4 - 4x^2 - 45 = 0$

x = ________________________

6. $x^4 + 7x^2 + 6 = 0$

x = ________________________

7. $x^4 - 3x^2 - 10 = 0$ x = ________________________

8. $3x^3 + 2x^2 - 15x - 10 = 0$ x = ________________________

9. $x^4 + 11x^2 + 18 = 0$ x = ________________________

10. $x^4 - x^2 = 12$ x = ________________________

Finding and Using Roots

After this lesson and practice, I will be able to …
LT 13. find all of the roots of a polynomial.
LT 14. write a polynomial function from its complex roots.

LT 13. I can find all of the roots of a polynomial.

The Rational Root Theorem

If $\frac{p}{q}$ is in simplest form, and is a rational root of a polynomial equation having integer coefficients, then p must be a factor of the________________,

and q must be a factor of the ________________.

Theorems about Roots of Polynomial Equations (6.5)

Here's a simpler way of saying that…

All potential rational zeros are included in this list of numbers:

$$\frac{P = \text{Factors of constant term}}{Q = \text{Factors of leading coefficient}}$$

List all of the possible rational zeros

1) $y = x^3 - 6x^2 + 7x - 2$

$\frac{\text{factors of \_\_\_\_\_\_\_\_}}{\text{factors of}}$ = ____________ = { }

2) $y = 3x^4 + 2x^2 - 6x + 12$

$\frac{\text{factors of \_\_\_\_\_\_\_\_}}{\text{factors of}}$ = ________________________

{ }

3) $y = 2x^3 + 10x^2 + 2x + 10$ { }

Now graph each polynomial equation above to find out which potential rational zeros are actually zeros.

1) 2) 3)

Identify any **rational** roots by making a list or possible rational roots, graphing a polynomial in your calculator and using the zero function to find a root. Once you have a root, you can use synthetic division to get the polynomial down to a quadratic. See the box below for the steps.

Find all the roots of the function.

How many roots? _____

1) $f(x) = x^3 - 7x^2 + 2x + 40$

a) Find the possibilities by $\frac{\mathbf{p}}{\mathbf{q}}$ =

b) Put the function in your calculator and use the "value" feature in the CALC menu to determine if any potential rational zeros are actual zeros.

c) Use synthetic division to demonstrate what you see on the graph.

The zeros are ________________,
and the factored form of the function is

f(x) = ______________________________.

---

2) $f(x) = 2x^3 - 5x^2 - 14x + 8$ **(Follow the same steps you took in #1.)**

The zeros are ________________,
and the factored form of the function is

f(x) = ______________________________
or

f(x) = ______________________________
Check the a value and constant value . Do not use fractions in factored form

Quick Check:

3) Find all roots of each function and write each function in factored form with integer coefficients.

$f(x) = x^3 - 7x^2 + 2x + 40$

<u>Strategies for Finding All Roots of a Polynomial</u>
1) List all possible rational roots.
2) Use your calculator to verify one rational root.
3) Use synthetic division until the expression is quadratic and then use other algebraic techniques to find the remaining zeros.

4) Find all roots of the function $f(x) = 2x^3 + 3x^2 - 8x + 3$ and write it in factored form with integer coefficients

Now we'll take it to the next level & find irrational and complex (imaginary) zeros as well!

1) $f(x) = x^4 - 5x^3 - 11x^2 + 25x + 30$

The zeros are ______________________
(There should be 4 since a 4th degree)

2) $f(x) = 3x^3 + x^2 - x + 1$

The zeros are ____________________
(There should be 3 since a 3rd degree. At least one must be found on calc to get down to a 2nd degree)

If you need the product of factors would be:

3) $f(x) = x^3 - 2x^2 - 3x + 10$

The zeros are ________________________
(There should be 3 since a 3rd degree)

If you need the product of factors would be:

More Practice

Find all roots of each function and write each function in factored form.

*4. $f(x) = x^4 - 5x^3 - 11x^2 + 25x + 30$

Find all roots of each function and write each function in factored form.

*5. $f(x)=3x^3+x^2-x+1$

The results to these examples lead us to two additional polynomial theorems:

Irrational Root Theorem – If ___________ is a root of a polynomial equation with rational coefficients, then the _____________ __________ is also a root of the equation.

Imaginary Root Theorem – If ___________ is a root of a polynomial equation with real coefficients, then the _____________ __________ is also a root of the equation.

The Irrational Root Theorem

If $a+\sqrt{b}$ is a root of a polynomial equation with rational coefficients, then the conjugate $a-\sqrt{b}$ is also a root of the equation.

The Imaginary Root Theorem

If $a+bi$ is a root of a polynomial equation with real coefficients, then the conjugate $a-bi$ is also a root of the equation.

State the conjugate of the expression.

1. $3-\sqrt{7}$ 2. $1+2i$ 3. $-12-5i$ 4. $-\sqrt{15}$ 5. πi

2. A polynomial equation with rational coefficients has the following roots: $2-\sqrt{3}$ and $5+7i$. Find two additional roots.

3. Suppose a polynomial with rational coefficients has the following roots: $5+\sqrt{10}$ and $-4-\sqrt{2}$. Find two additional roots.

(MC) 4. A quartic polynomial with real coefficients has roots of -3 and $2-5i$. Which of the following cannot be another root of the polynomial?

A. 12 B. 0 C. $\sqrt{2}$ D. $2+5i$

(MC) 5. A fourth-degree polynomial with integer coefficients has zeros of –2, and $1+3i$. Which number cannot also be a zero of this polynomial?

a. 8 b. 0 c. $\sqrt{2}$ d. $1-3i$

More Examples:

Find ALL zeros and write as a product of factors.

1) $f(x) = 3x^4 - 8x^3 - 9x^2 + 16x + 6$

$\frac{\mathbf{p}}{\mathbf{q}} =$

2) $f(x) = 2x^4 - 7x^3 - 9x^2 + 22x - 8$

$\frac{\mathbf{p}}{\mathbf{q}} =$

*3) Find all roots of the function $f(x)=x^3-2x^2-3x+10$ and write it in factored form.

LT 14. I can write a polynomial function from its complex roots.

1) Find a polynomial function in standard form whose graph has x-intercepts 3, 5, -4, and y-intercept 180. Hint: Write in factored form then multiply by a factor to make the y-intercept that is needed

Recall from the previous lesson, that when polynomials have _____________ or ___________ zeros, they always appear as ________________ __________.

2) Write a polynomial function in standard form with real coefficients and zeros $x = 2, x = -5, x = 3 + 4i$.

More Practice:
Write a polynomials function of least degree with integral coefficients that has the given zeros.

1) 5, 3i

2) -4, 5, -5

3) -3 multiplicity of 2, -2+$\sqrt{7}$

4) -5, -1+$\sqrt{5}$

Graphing and Connections

After this lesson and practice, I will be able to ..
LT 15. graph polynomials.

Graphs of Polynomial Functions: End Behavior & Turning Points

	Odd degree		Even degree	
Sign of Leading Coefficient	Positive	Negative	Positive	Negative
End behavior	↙ ↗	↖ ↘	↖ ↗	↙ ↘
Number of turning points	n - 1 at most; each turning point is a local (or global) max. or min.			

LT 15. I can graph polynomials.
(sometimes without using a calculator**)**

The impact of multiplicity of a zero (factor) on the graph of the equation.

A zero or factor with an “ODD” multiplicity will cross or squiggle thru the x-axis.
If the multiplicity is ONE, it will cross. (look like y=x)
If the multiplicity is any other odd number it will squiggle (look like $y=x^3$).

A zero or factor with an “EVEN” multiplicity will bounce (or touch) the x-axis.
It will look like $y=x^2$.

When graphing is done **without** a calculator, the equation will be factorable, already factored, or you will be given factors to begin the analysis.

1) Find the zeros of f(x) = $\mathbf{x^3 + 3x^2 - x - 3}$ $\qquad \dfrac{\mathbf{p}}{\mathbf{q}} =$

a) Zeros (Roots): ____________________ b) Factors:______________________________
Do not use fractions in factored form

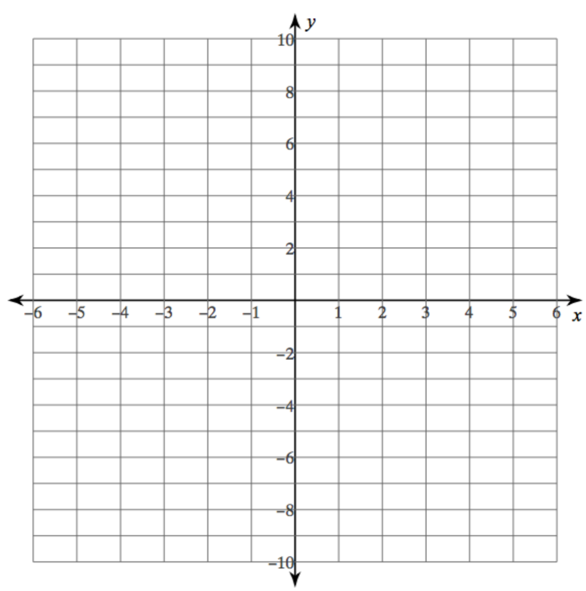
c) f( ) = , f( ) = , f( ) = because ___________

y-intercept = (,) **x-intercept(s) = (,) (,) (,)**

End behavior:

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->



2) Find the zeros of f(x) = $3x^3 - 10x^2 - 23x - 10$
One factor is (x+1) or use your calculator to begin.

$\frac{p}{q} =$

a) Zeros (Roots): ____________________

b) Factors:________________________________
Do not use fractions in factored form

c) f( ) = , f( ) = , f( ) = because ___________

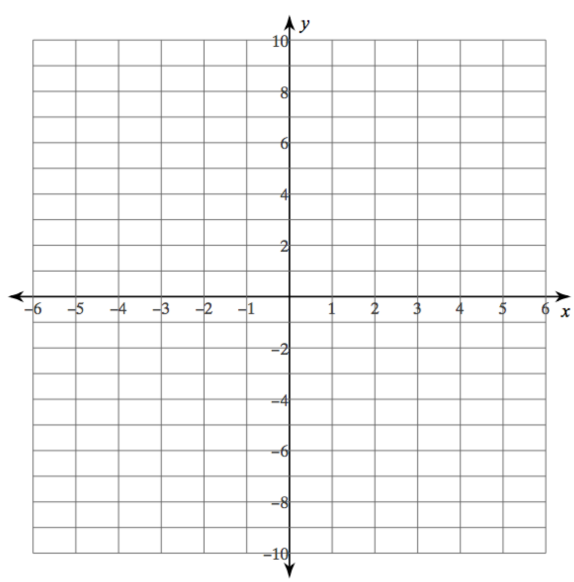
y-intercept = (,)

x-intercept(s) = (,) (,) (,)

End behavior:

as x-> $-\infty$ f(x) ->

as x-> $+\infty$ f(x) ->



More Practice

3) Find the zeros of f(x) = $x^4 - 2x^3 - 4x^2 + 2x + 3$
One factor is (x+1) and one factor is (x-1) or use your calculator to begin.

$\frac{p}{q} =$

a) Zeros (Roots): ____________ b) Factors:______________________________

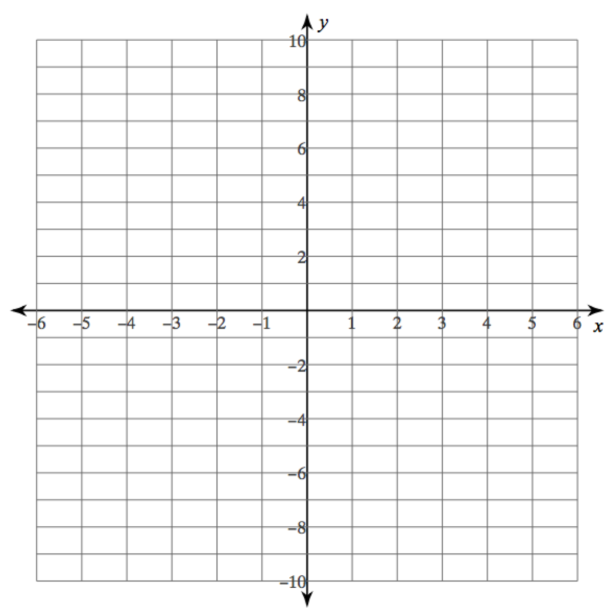
Do not use fractions in factored form

c) y-intercept = (,) x-intercept(s) = (,) (,) (,) (,)

End behavior:

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->



4) Decide whether the given value is a zero of the function.

$f(x) = \mathbf{2x^3 - 5x^2 - 14x + 8}$ $x = -2$; $x = 3$

$x = \mathbf{-2}$,

$x = 3$

5) Write a polynomial function that has zeros: 2, 4i, $\mathbf{-4}$ i, and $\mathbf{-1}$.
Write your answer in factored form then in standard form.

Factored form: ____________________________

Standard form: ____________________________

6) Write a polynomial function that has x-intercepts: (3,0) and (-½,0)
Write you answer if factored form. This is not the only function that has these intercepts. Please write at least 2 more.

Factored form: ____________________________

Another: ____________________________

Another: ____________________________

7) Write as the product of linear factors.
Use your calculator to begin.

$$x^4 - x^3 - 5x^2 - x - 6$$

Answer:____

Work:

The Fundamental Theorem of Algebra

Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $P(x) = 0$ has at least $\boxed{}$ complex root.

Corollary

Including imaginary roots and multiple roots, an $\boxed{}$ degree polynomial equation has exactly n roots; the related polynomial function has exactly $\boxed{}$ zeros.

Find the **ALL** of real and complex zeros WITHOUT using a calculator. Then sketch using x-intercepts, y-intercept, and end behaviors.

1. $y = x^3 - 2x^2 + 4x - 8$

____

2. $f(x) = x^5 + 3x^4 - x - 3$

____

*3. $f(x) = x^5 + x^3 + x^2 + 1$

____