Measures of Position - Quartiles and Percentiles

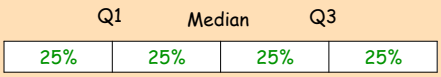
Position - Used to describe the position of a data value in relation to the rest of the data.

Types:
- Quartiles
- Percentiles
- Deciles

Quartiles: Values of the variables are divided into quarters - 4 equal parts.

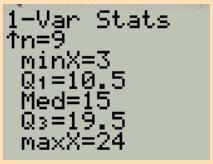
Named:

- Q1 - Lower Quartile
  - At most, 25% of data is smaller than Q1.
  - It divides the lower half of a data set in half.
- Q2 - Median
  - The median divides the data set in half.
  - 50% of the data values fall below the median, and 50% fall above.
- Q3 - Upper Quartile
  - At most, 25% of data is larger than Q3.
  - It divides the upper half of the data set in half.



Example:
Find Q1, median, and Q3

3 9 12 13 15 17 19 2024



Steps:
- Put data into L1.
- Run "Stat Calc One-variable stats (L1)."
- Scroll all the way down.

Percentiles: Values of the variable that divide a ranked set into 100 subsets

For example: P30 would be at 30%

Note:

- Q1 = P25
- Q2 = P50 = Median
- Q3 = P75

Percentile Example:
The 78th percentile means 78% are smaller than the given value. Does making the 80th percentile mean that you made an 80% on the test?

No, the 80th percentile would mean that a person did better than 80% of the students who took the same test.

Value that corresponds to the kth percentile: (this is a position locator formula).

$$c = \frac{np}{100}$$

If C is a decimal:
- round up to the next whole number
- this is the position where you will find your answer

If C is a whole number:
- you must average "c" and "c + 1"
- this gives you the position of the answer

Example:

2 3 5 6 8 10 12 15 18 20

Find P25 (the number in the 25th percentile position)

$$c = \frac{np}{100}$$
$$c = \frac{(10)(25)}{100} = 2.5$$

Round up to the 3rd position
2 3 5 6 8 10 12 15 18 20
The answer is 5

Example 2

2 3 5 6 8 10 12 15 18 20

Find p60

$$c = \frac{np}{100}$$
$$c = \frac{(10)(60)}{100} = 6$$

Average the numbers in the 6th and 7th positions together (10 & 12)

$$\frac{10+12}{2} = 11$$

The answer is 11.

Example 3:

2 3 5 6 8 10 12 15 18 20

Find P75 (this is the same as Q3)

$$c = \frac{np}{100}$$
$$c = \frac{(10)(75)}{100} = 7.5$$

Round up to the 8th position (15)
The answer is 15

Examples: You Try

3 4 4 6 8 10 10 12 12 12 13 15 15 15 16 17 20 22 25 27

1) Find P75

2) Find P30

3) Find P23

Midquartile: The number halfway between Q1 and Q3.

$$\text{Midquartile} = \frac{Q1+Q3}{2}$$

Example: If Q1 = 9 and Q3 = 16.5, find the midquartile

$$\frac{9+16.5}{2} = \frac{25.5}{2} = 12.75$$

Z-Scores: represent the number of standard deviations a data value falls above or below the mean.

It is used as a way to measure relative position.

Z-Score Formula:

$$\text{Z-Score} = \frac{value - mean}{st.deviation}$$

$$z = \frac{x - \bar{x}}{s}$$

Please round z-scores to 2 decimal places.

A positive z-score means that a score is above the mean.

A negative z-score means that a score is below the mean.

A z-score of 0 means that a score is exactly the same as the mean.

Example: A student scored a 65 on a math test that had a mean of 50 and a standard deviation of 10. She scored 30 on a history test with a mean of 25 and a standard deviation of 5. Compare her relative position on the two tests.

$$\text{Z-Score} = \frac{value - mean}{st.deviation}$$

Math: $z = \frac{65-50}{10} = \frac{15}{10} = 1.5$

History: $z = \frac{30-25}{5} = \frac{5}{5} = 1$

The student did better in math because the z-score was higher.

Example: Find the z-score for each test and state which test is better.

Test A $x = 38 \quad \bar{x} = 40 \quad s = 5$

Test B $x = 94 \quad \bar{x} = 100 \quad s = 10$

Example: A sample has a mean of 200 and a standard deviation of 25. Find the value of x that corresponds to a z-score of 2.35.

$$z = \frac{x - \bar{x}}{s}$$
$$2.35 = \frac{x - 200}{25}$$
$$x - 200 = (2.35)(25)$$
$$x - 200 = 58.75$$
$$x = 258.75$$

Box Plots (Box and Whiskers): A graph of a set of data obtained by drawing a horizontal line from the minimum to maximum values with quartiles labeled in between.

It is a graphical plot of 5 specific values called the 5-number summary.

5-number summary:

1. minimum
2. Q1
3. median
4. Q3
5. maximum

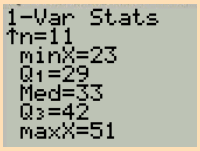
Steps:

1. Find the 5-number summary (use your calculator).
2. Draw and label a scale of equal intervals.
3. Place dots above the 5 numbers.
4. Put a box around Q1 and Q3.
5. Draw a vertical line through the median.
6. Draw "whiskers" from the minimum to Q1 and maximum to Q3.

Example: Draw a box plot of the following data.

33 38 43 30 29 40 51 27 42 23 31

Remember: You can put the numbers in L1 of your calculator and run "stat calc one-var stats L1" and then scroll down to the bottom to get the 5-number summary.



Let's draw the box plot by hand:

Min = 23

Q1 = 29

Median = 33

Q3 = 42

Max = 51

Example (you try): Draw a box plot for the following data.

3 8 15 20 22 23 23 24 28 29 29

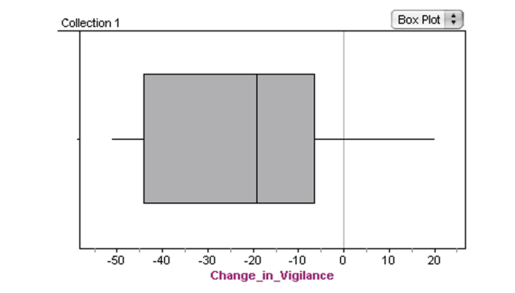
30 35 38

Skewness:

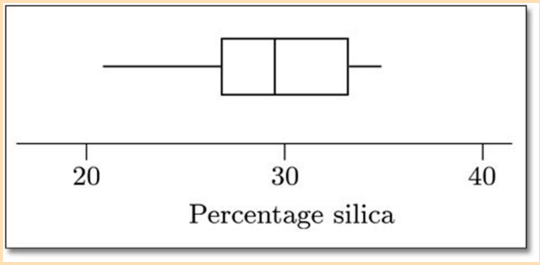
If the whisker to the right of the box is longer than the one to the left, there are more extreme values towards the positive end and so the distribution is positively skewed.

Similarly, if the whisker to the left is longer, the distribution is negatively skewed.

Skewed Left (Negative) or Right (Positive)?



Skewed Left (Negative) or Right (Positive)?



Can we figure out the numbers for the 5-number summary?