Identify Pairs of Lines and Angles

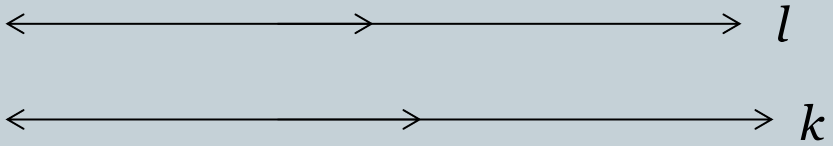
"LIFE IS WHAT YOU MAKE IT." –MR. H'S DAD

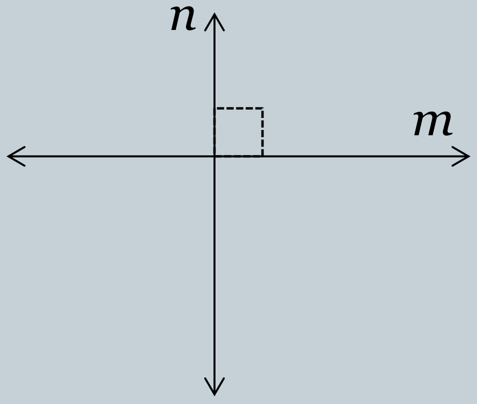
Goal

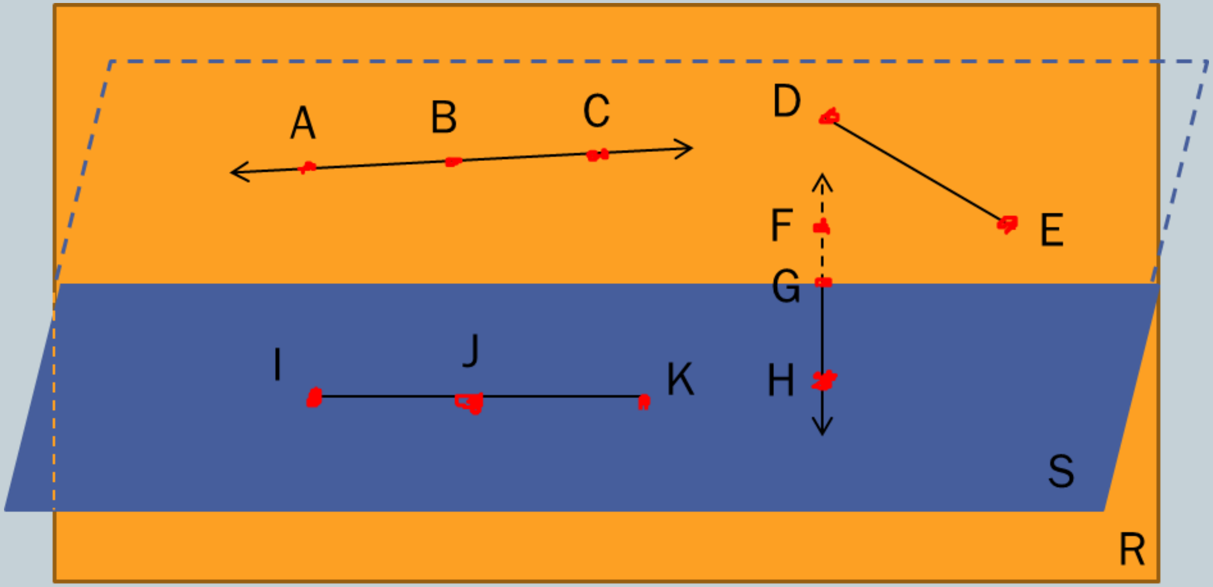
- Students will learn the different ways lines can interact with each other. This included coplanar and non-coplanar lines.
- Students will learn what a transversal is and the different types of angles formed by a transversal.

Parallel and Skew

- Parallel lines-never intersect and are coplanar.
- Skew lines-never intersect and are not coplanar.
- Parallel planes-planes that never intersect and are always the same distance apart.
- Perpendicular lines-lines that intersect at a right angle.
- $l \parallel k$ and $n \perp m$






A
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Postulates

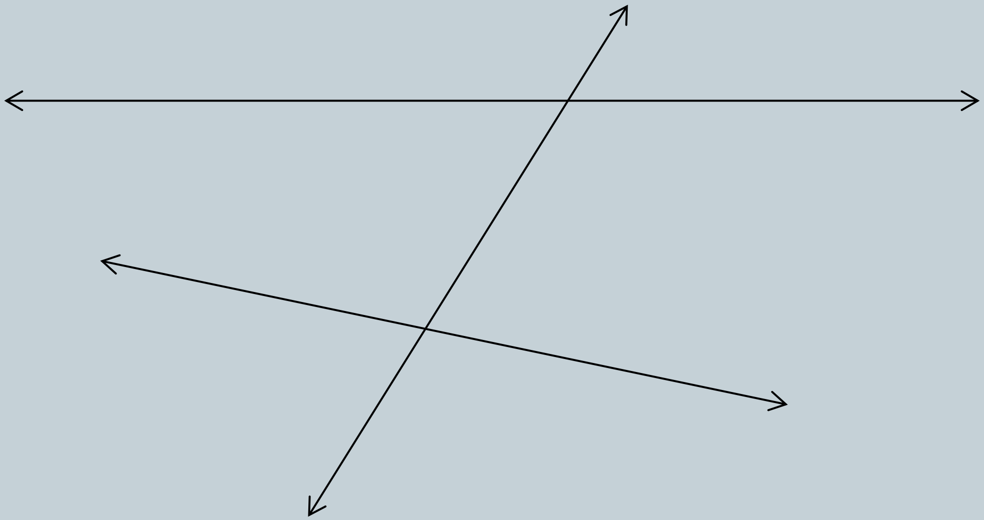
- Parallel Postulate
  - If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.
- Perpendicular Postulate
  - If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

Questions

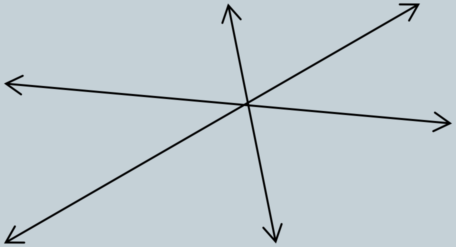
- What are the three different names for the interaction of lines?
  - Parallel, intersection, or skew.
- If I have a point and a line, how many lines could I draw through the point that would be parallel to my given line?
  - Exactly one
- T/F Two lines in a plane either intersect or are parallel.

Transversals

- A transversal is a line that intersects two or more coplanar lines at different points.



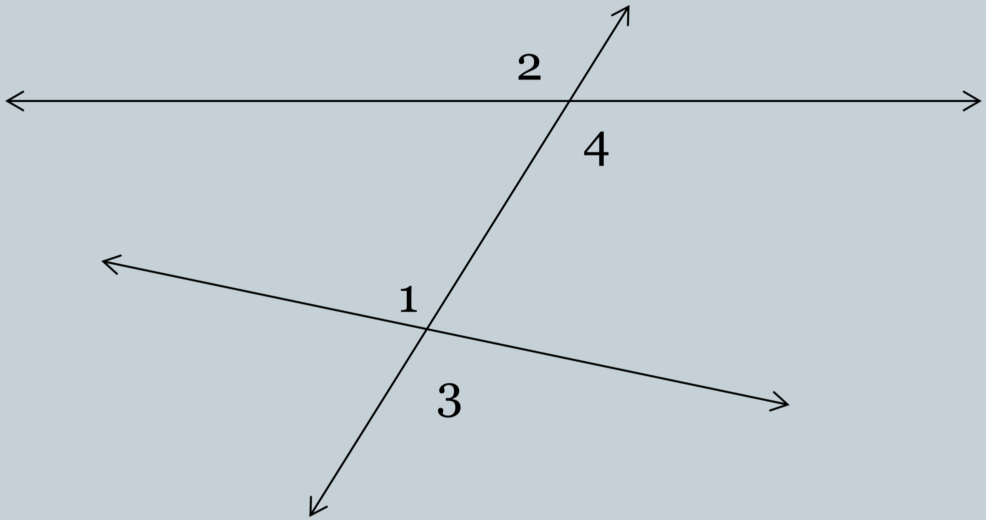
transversal



No transversal

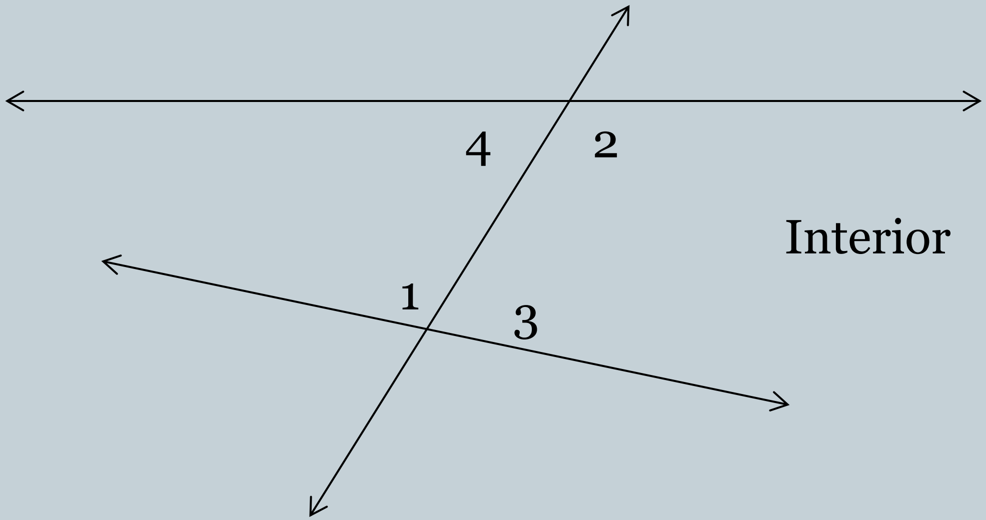
Corresponding Angles

- Angles that are on the same side of the transversal and in the same corresponding positions.



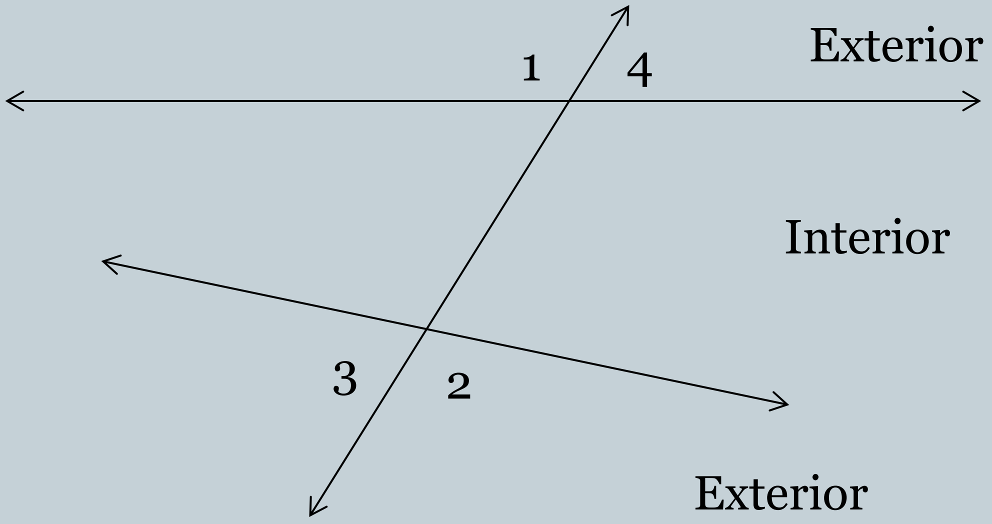
Alternate Interior Angles

- Angles that are on the opposite sides of the transversal and between the two lines.



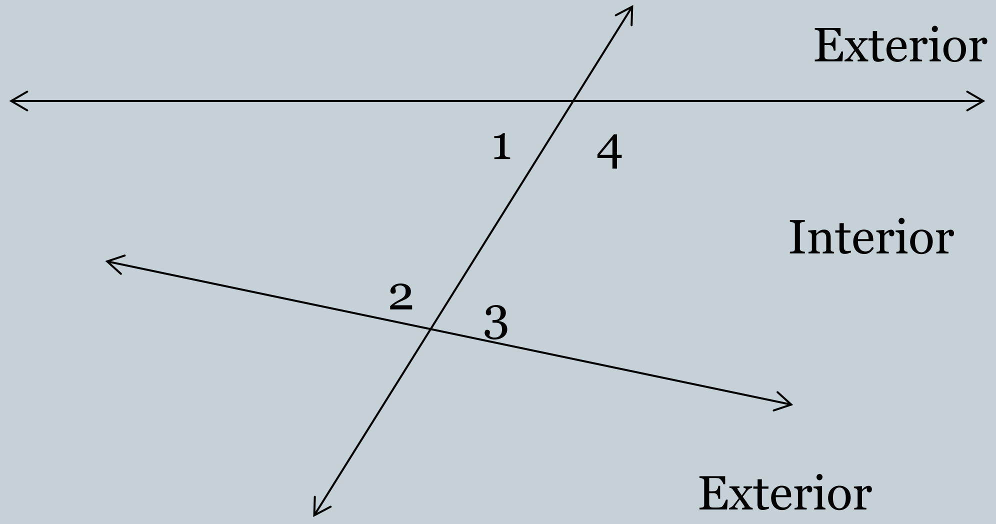
Alternate Exterior Angles

- Angles that are on the opposite sides of the transversal and outside the two lines.



Consecutive interior Angles

- Angles that are on the same side of the transversal and between the two lines.

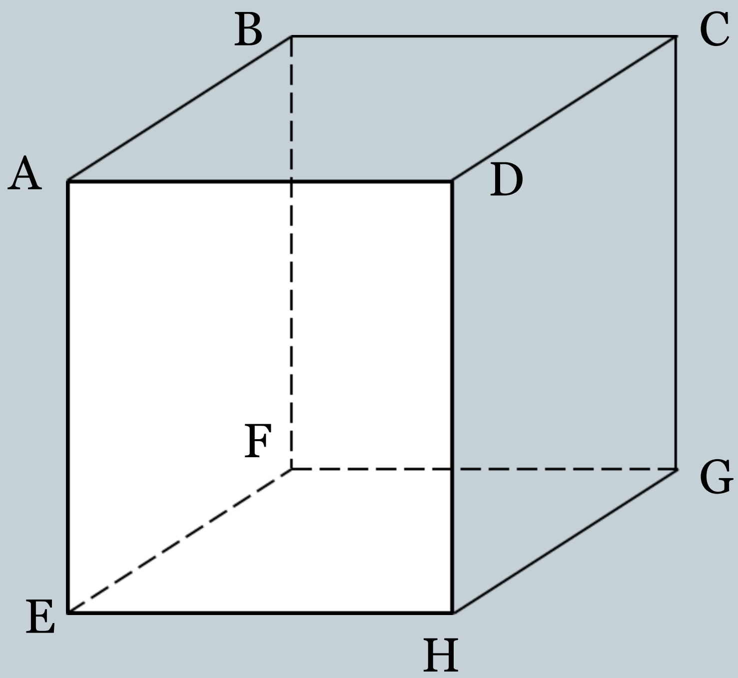


Summary

- Students will learn the different ways lines can interact with each other. This included coplanar and non-coplanar lines.
- Students will learn what a transversal is and the different types of angles formed by a transversal.

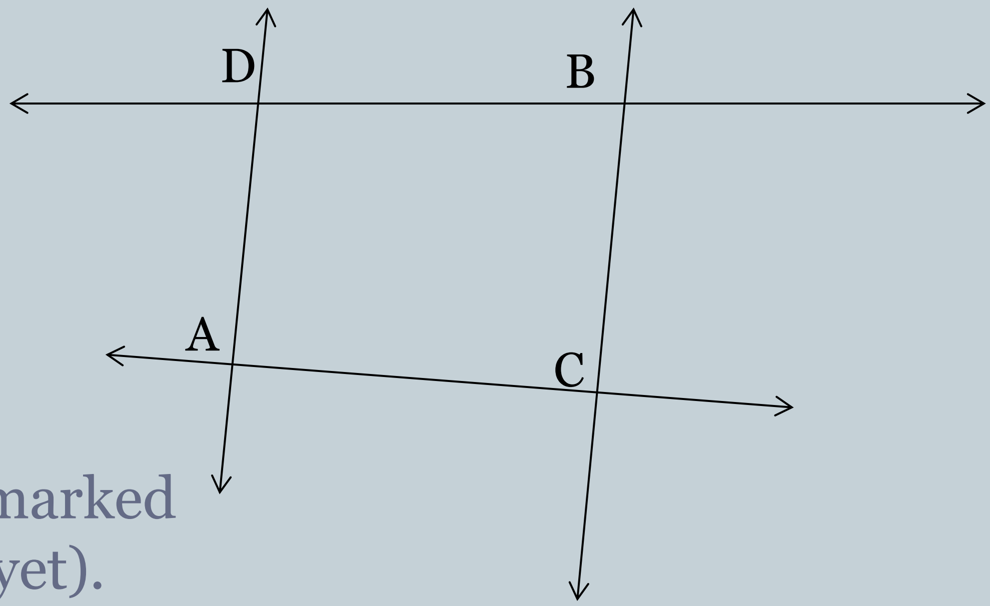
Example 1

- Which lines or planes match the given description.
  - Line(s) parallel to line AD and containing point C
    - $\overleftrightarrow{BC}$
  - Line(s) perpendicular to line HD and containing point G
    - $\overleftrightarrow{HG}$
  - Line(s) skew to line EF and containing point H
    - $\overleftrightarrow{HD}$



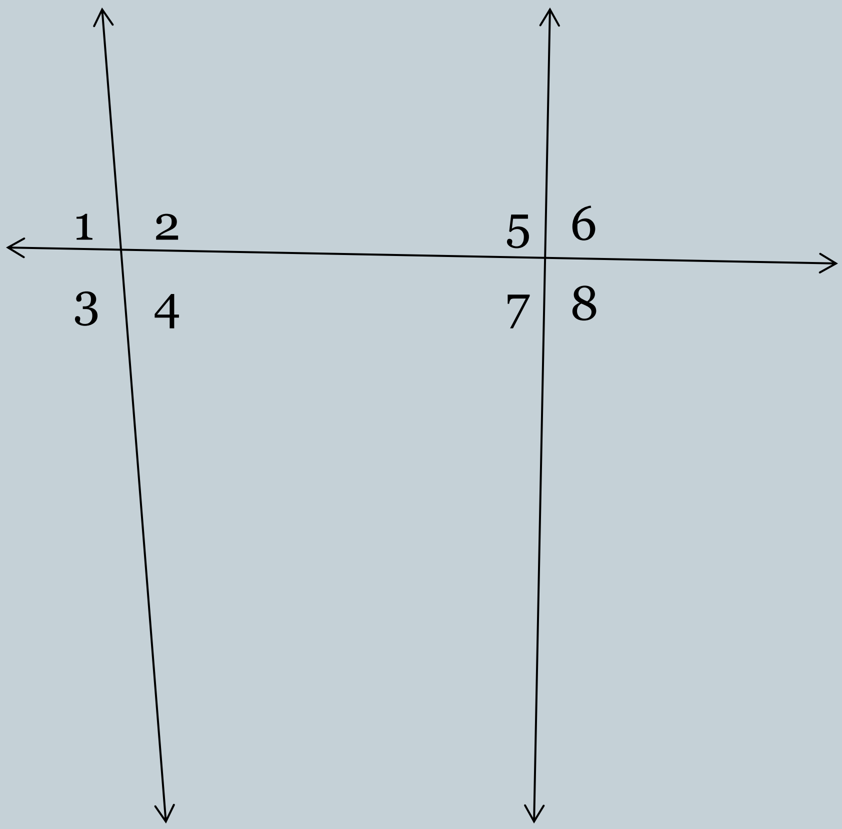
Example 2

- Name a pair of parallel lines.
  - Line AC ll Line BD
- Name a pair of perpendicular lines.
  - Line AC ⊥ Line BC
- Is Line AC ⊥ Line AD? Explain.
  - Not enough info. It is not marked and no way to determine (yet).
- Please look at Ex. 2 part c in book.



Example 3

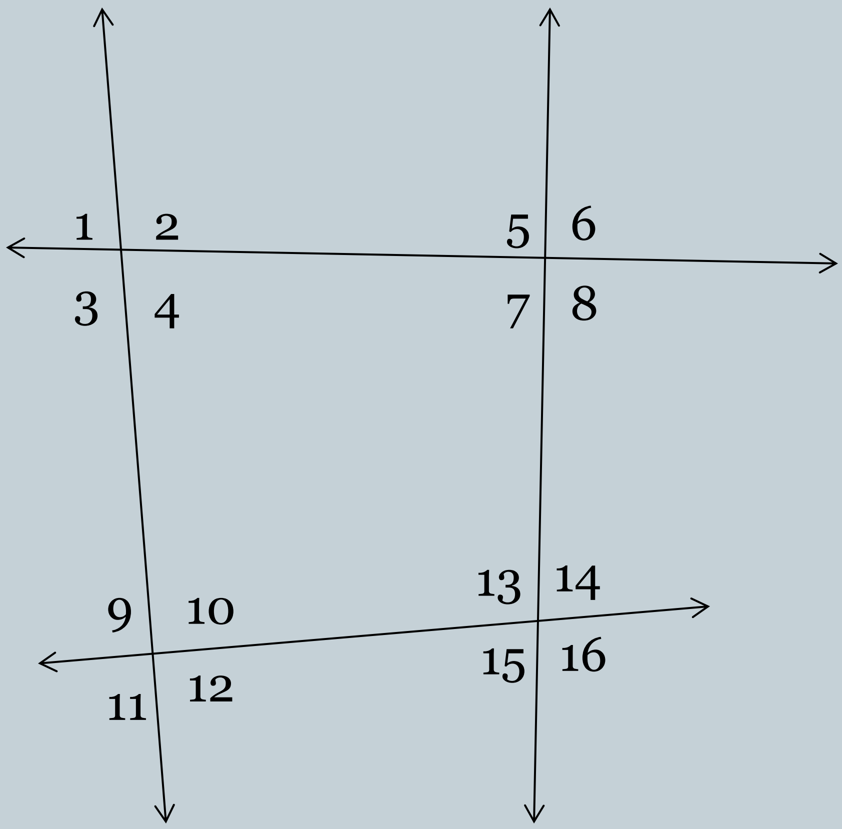
- Identify all corresponding angles, alternate interior angles, alternate exterior angles, and consecutive interior angles.



Example 4

Determine what each angle pair is called.

- Angle 1 and angle 2
- Angle 9 and angle 13
- Angle 11 and angle 2
- Angle 9 and angle 4
- Angle 9 and angle 16
- Angle 8 and angle 14
- Angle 6 and angle 7
- Angle 5 and angle 1



Use Parallel Lines and Transversals

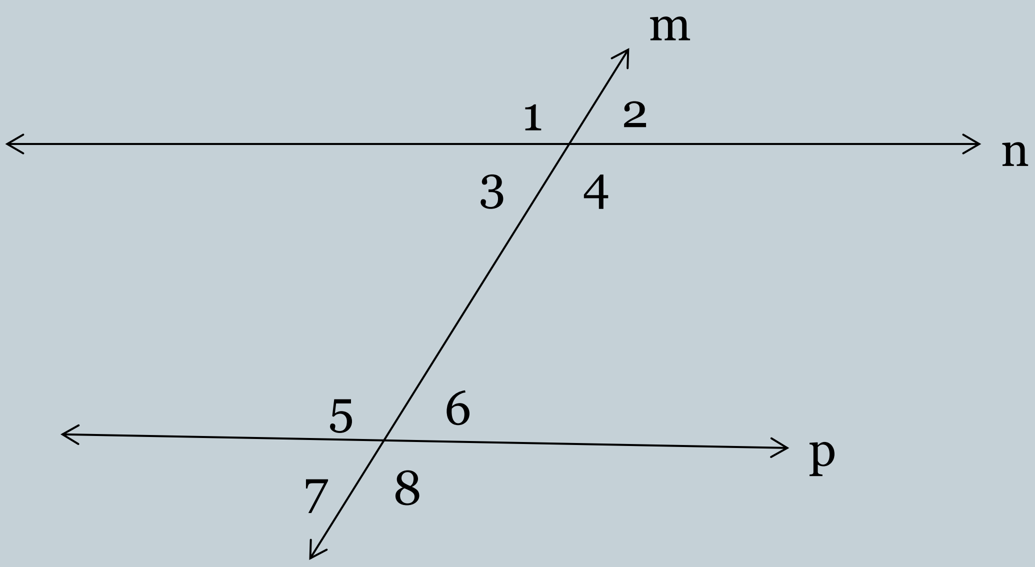
"THE GREAT USE OF LIFE IS TO SPEND IT FOR SOMETHING THAT WILL OUTLAST IT." – WILLIAM JAMES

Goal

- Students will learn how lines being parallel affects the angles formed by a transversal.
- Students will be able to justify why angles are congruent or supplementary based on their position in a diagram.

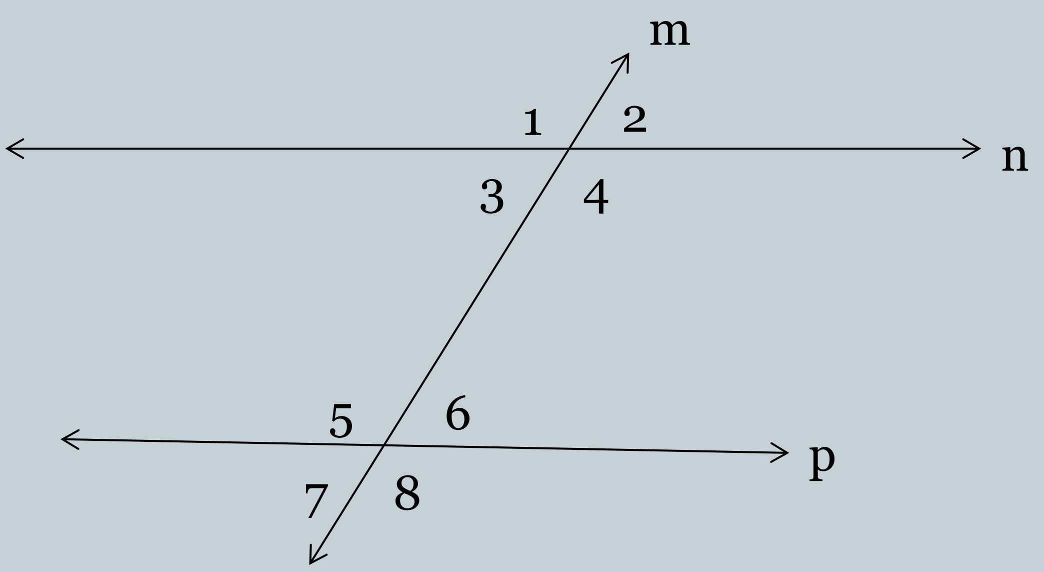
Corresponding Angles Postulate

- If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.
- $\angle 1 \cong \angle 5$
- $\angle 2 \cong \angle 6$
- $\angle 3 \cong \angle 7$
- $\angle 4 \cong \angle 8$



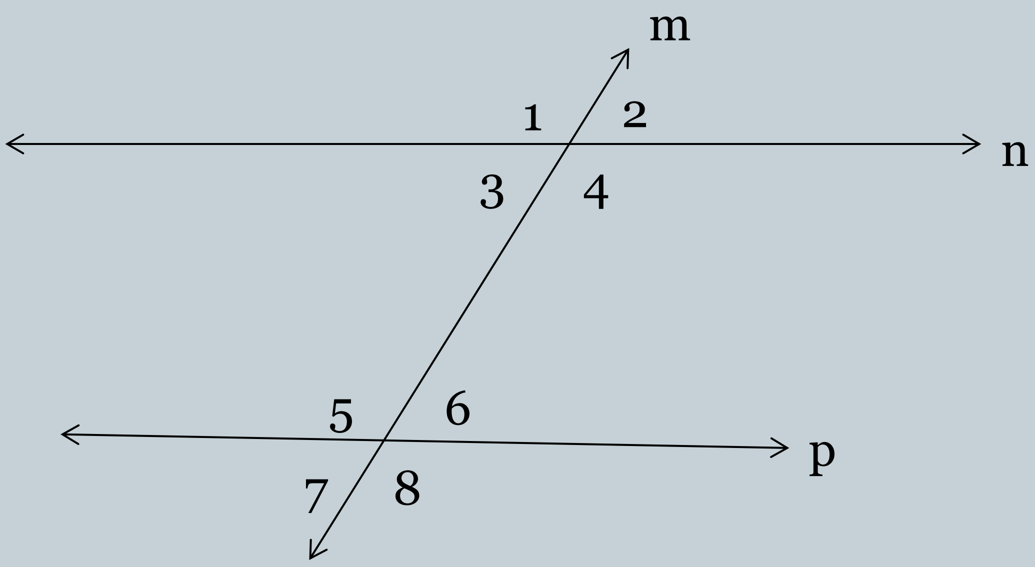
Alternate Interior Angles Theorem

- If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.
- $\angle 3 \cong \angle 6$
- $\angle 4 \cong \angle 5$



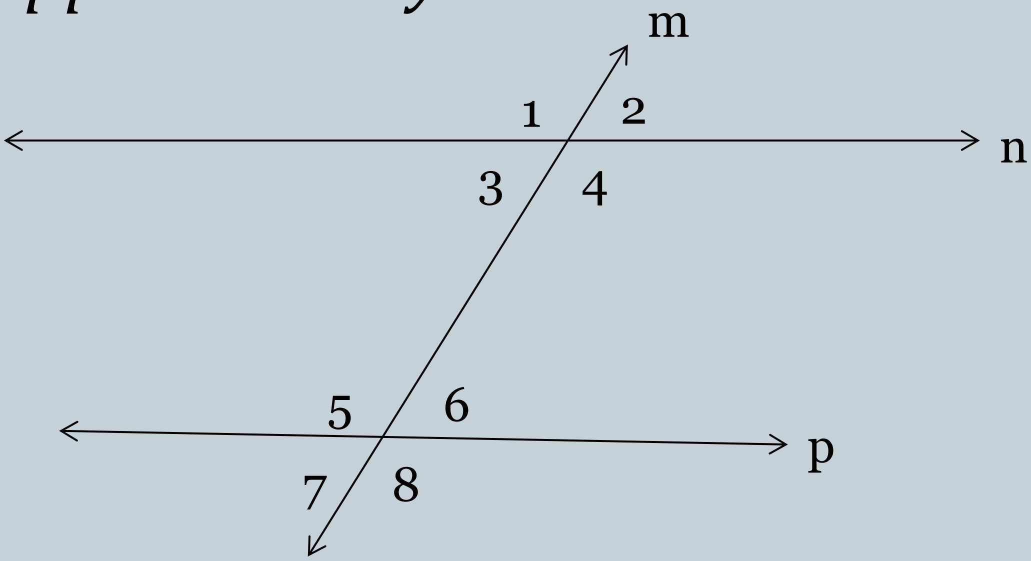
Alternate Exterior Angles Theorem

- If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.
- $\angle 1 \cong \angle 8$
- $\angle 2 \cong \angle 7$



Consecutive interior Angles Theorem

- If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.
- $\angle 3 \text{ and } \angle 5 \text{ are supplementary}$
- $\angle 4 \text{ and } \angle 6 \text{ are supplementary}$

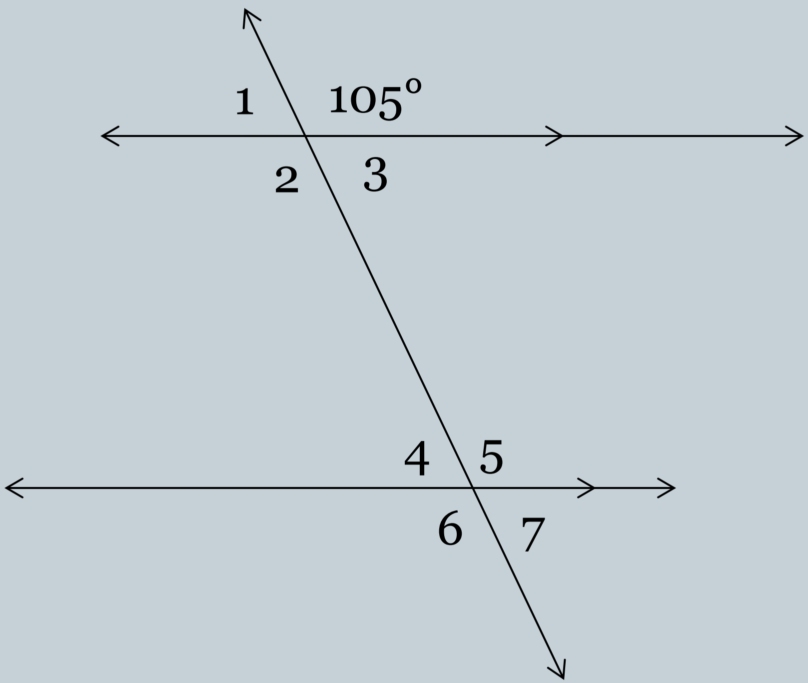


Summary

- At this point, you should be able to:
  - Identify special properties of the four new angle pairs when two parallel lines are cut by a transversal.

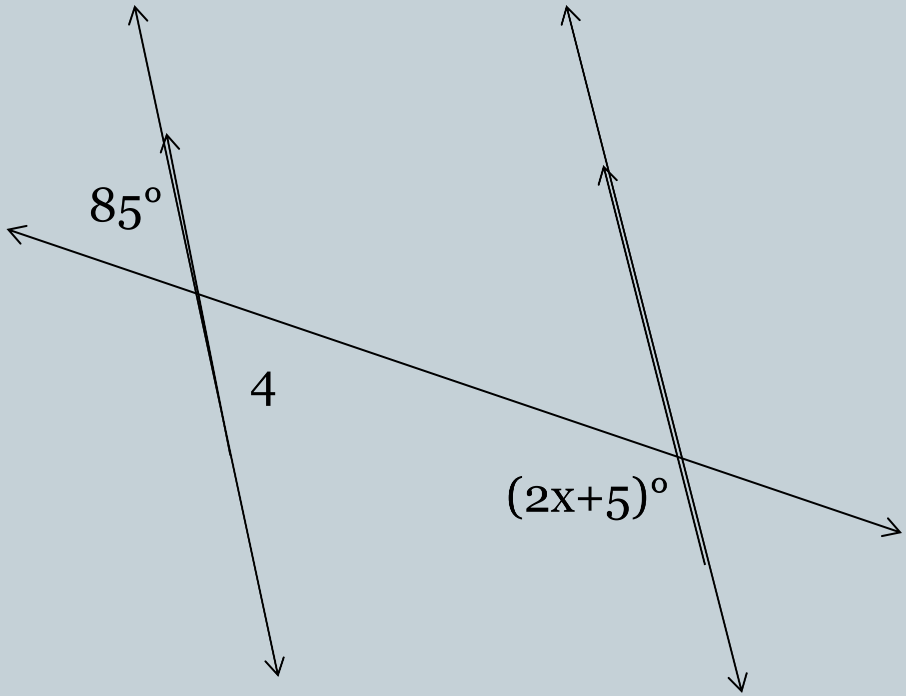
Example 1

- Find the unknown angle measures.
- $m\angle 2=105^{o}$
- $m\angle 5=105^{o}$
- $m\angle 6=105^{o}$
- $m\angle 3+m\angle 5=180^{o}$
- $m\angle 3=75^{o}$
- $m\angle 1=75^{o}$
- $m\angle 4=75^{o}$
- $m\angle 7=75^{o}$



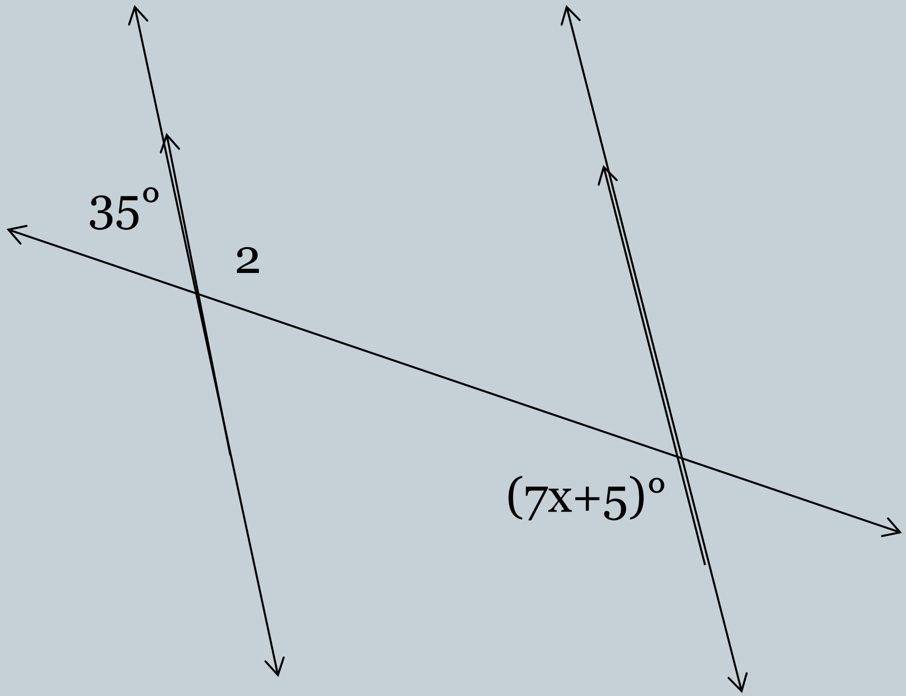
Example 2

- Find the value of x.
- $m\angle 4 = 85°$
- $m\angle 4 + 2x + 5 = 180$
- $85 + 2x + 5 = 180$
- $2x + 90 = 180$
- $2x = 90$
- $x = 45$



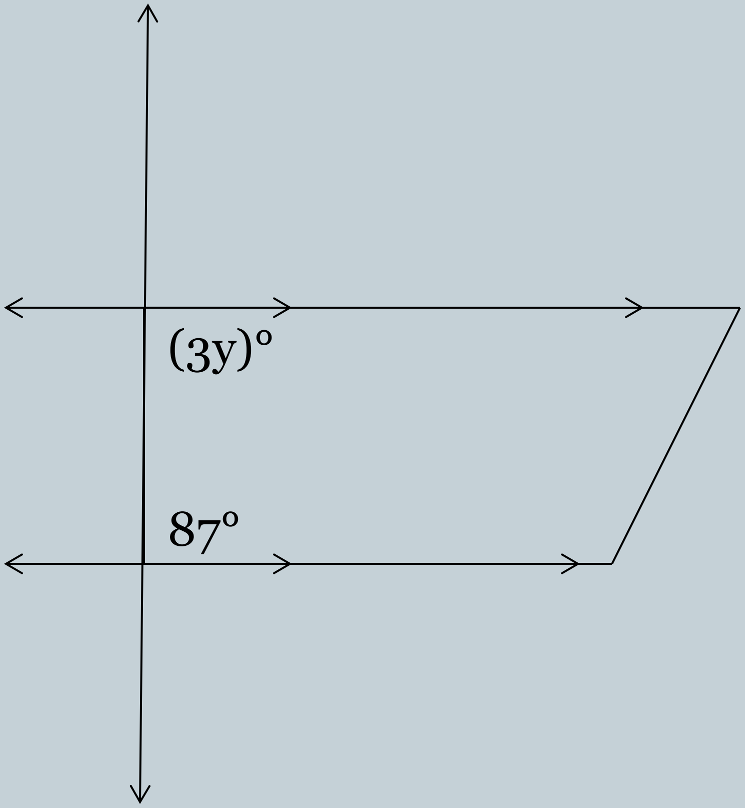
Example 3

- Find the value of x.
- $35° + m\angle 2 = 180°$
- $m\angle 2 = 145°$
- $m\angle 2 = (7x+5)°$
- $145 = 7x + 5$
- $140 = 7x$
- $20 = x$



Example 4

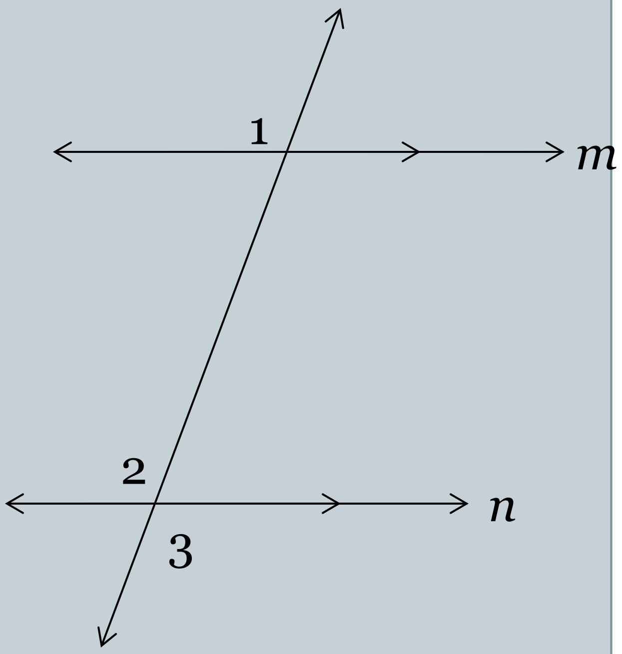
- Find the value of y.
- $3y+87=180$
- $3y=93$
- $y=31$



Example 5

- Prove the Alternate Exterior Angles Theorem.
- Given: $m \| n$
- Prove: $\angle 1 \cong \angle 3$

Statements	Reasons
1. $m \| n$	1. Given
2. $\angle 1 \cong \angle 2$	2. Corresponding angles Postulate
3. $\angle 3 \cong \angle 2$	3. Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 3$	4. Transitive Property



Prove Lines are Parallel

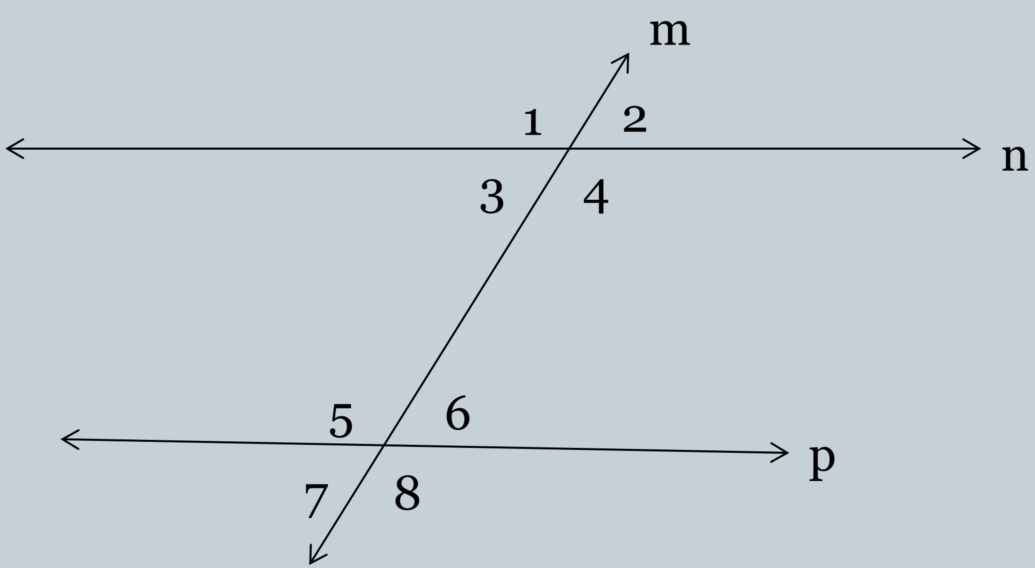
"ONLY THE PERSON WHO HAS FAITH IN HIMSELF IS ABLE TO BE FAITHFUL TO OTHERS."–ERICH FROMM

Goal

- Students will learn how the angles formed by a transversal can be used to determine that lines are parallel.
- Students will be able to justify why lines are parallel.

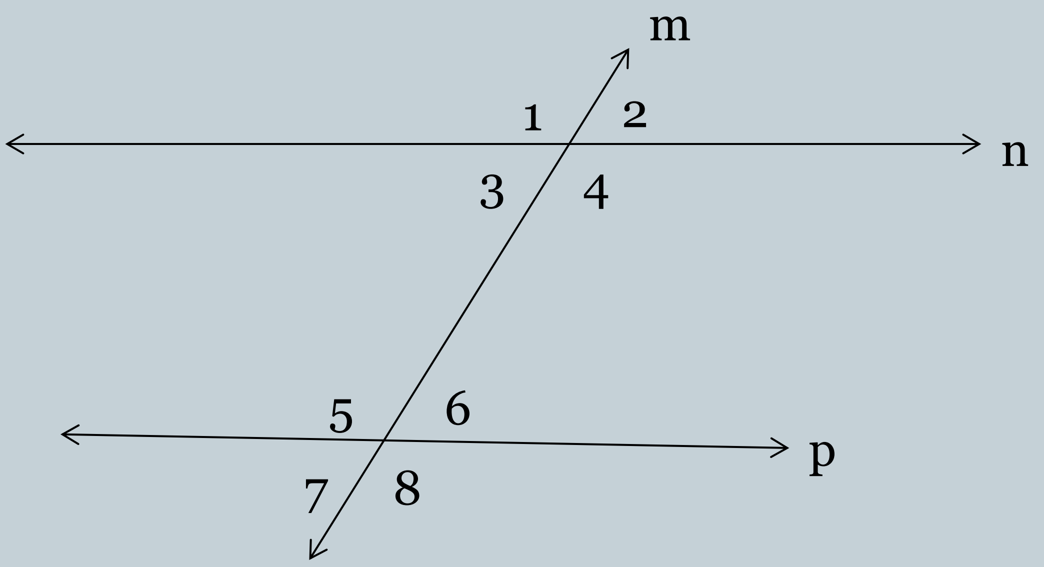
Corresponding Angles ConversePostulate

- If two lines are cut by a transversal and the pairs of corresponding angles are congruent, then the lines are parallel.
- $\angle 1 \cong \angle 5$
- $\angle 2 \cong \angle 6$
- $\angle 3 \cong \angle 7$
- $\angle 4 \cong \angle 8$



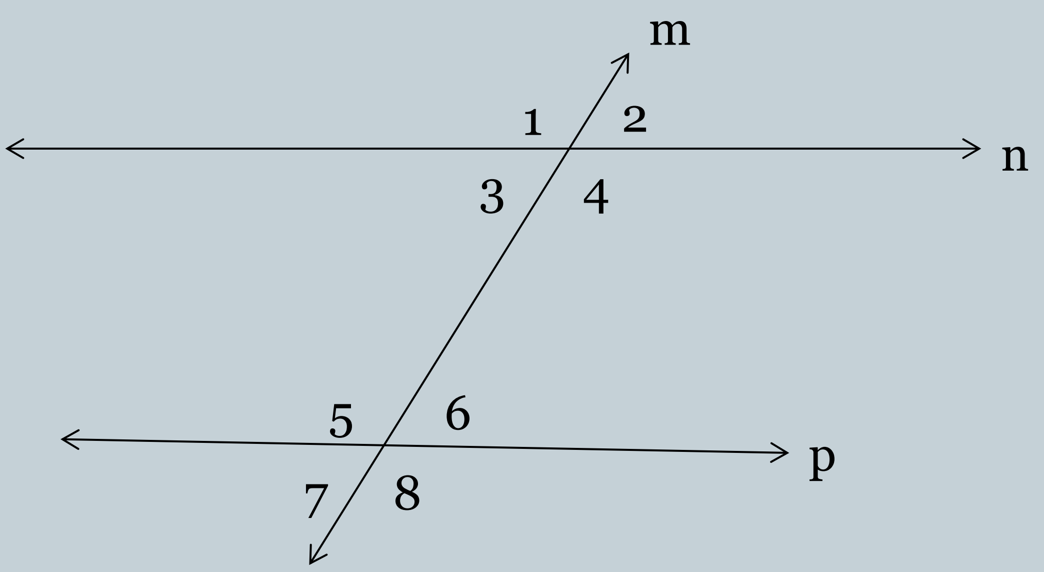
Alternate Interior Angles Converse Theorem

- If two lines are cut by a transversal and the pairs of alternate interior angles are congruent, then the lines are parallel.
- $\angle 3 \cong \angle 6$
- $\angle 4 \cong \angle 5$



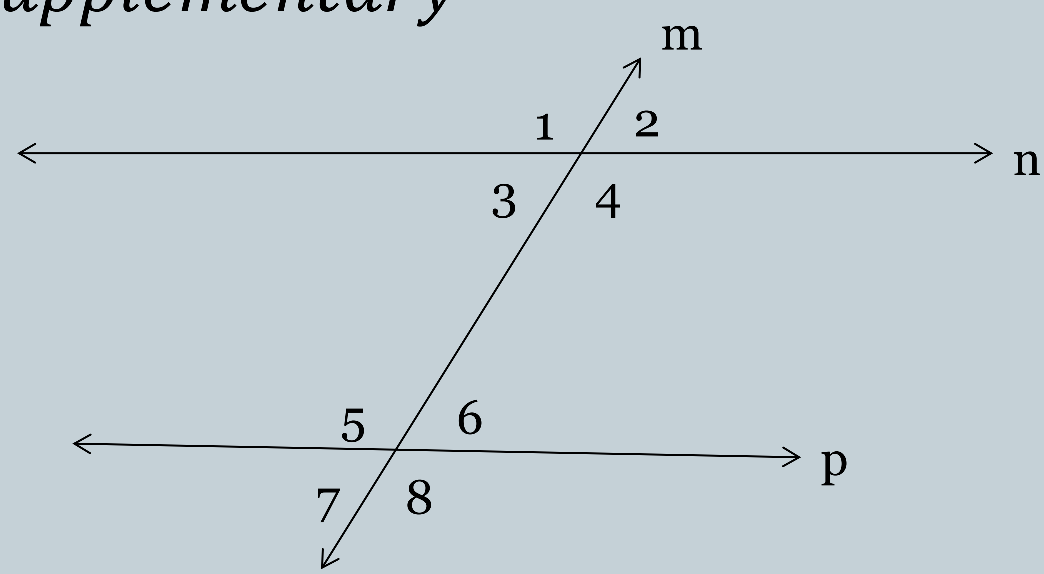
Alternate Exterior Angles Converse Theorem

- If two lines are cut by a transversal and the pairs of Alternate Exterior Angles are congruent, then the lines are parallel.
- $\angle 1 \cong \angle 8$
- $\angle 2 \cong \angle 7$



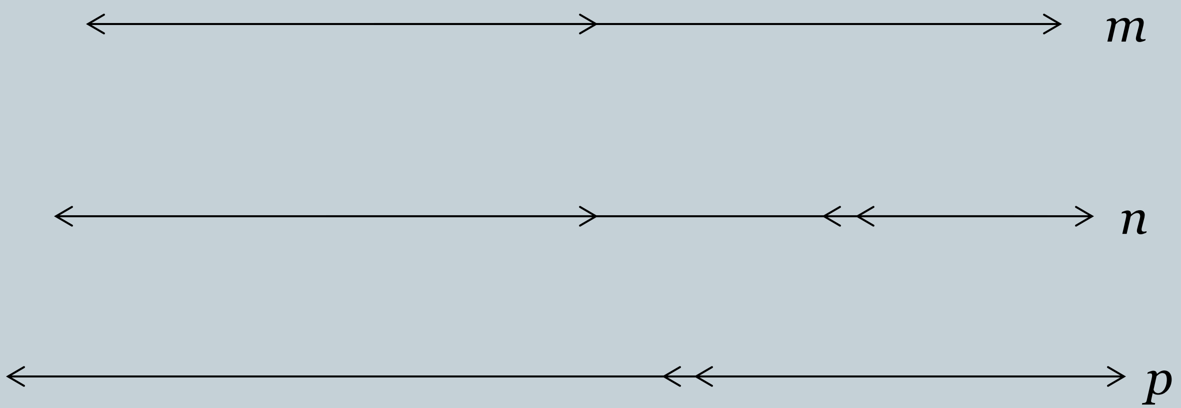
Consecutive Interior Angles Converse Theorem

- If two lines are cut by a transversal and the pairs of consecutive interior angles are supplementary, then the lines are parallel.
- $\angle 3 \text{ } and \text{ } \angle 5 \text{ } are \text{ } supplementary$
- $\angle 4 \text{ } and \text{ } \angle 6 \text{ } are \text{ } supplementary$



Transitive Property of Parallel Lines

- If two lines are parallel to the same line, then they are parallel to each other.
- If *m*ll*n* and *n*ll*p*, then *m*ll*p*.

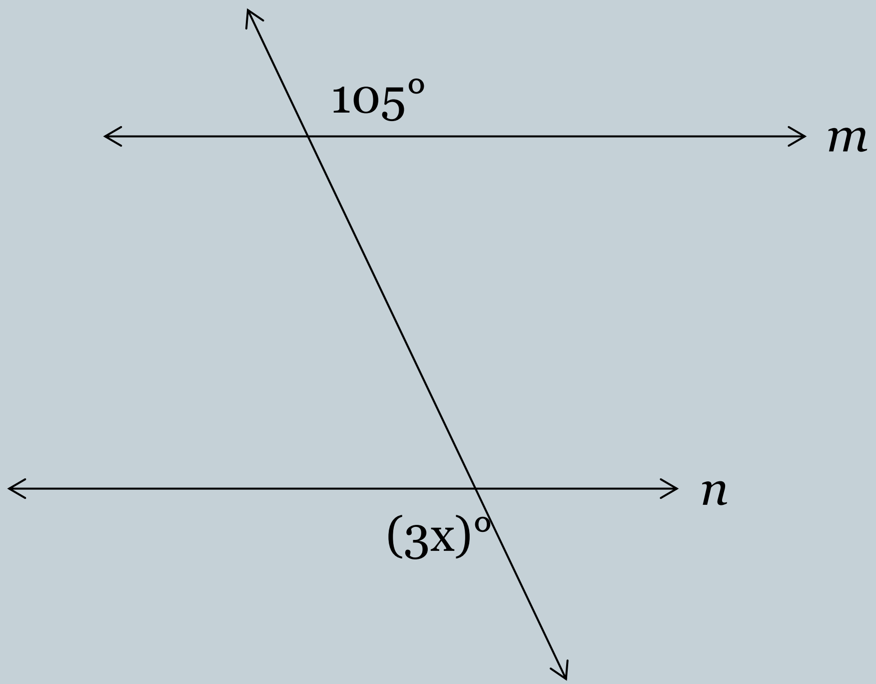


Summary

- At this point, you should be able to:
  - Identify special properties of the four new angle pairs formed when two lines are cut by a transversal that cause lines to be parallel.

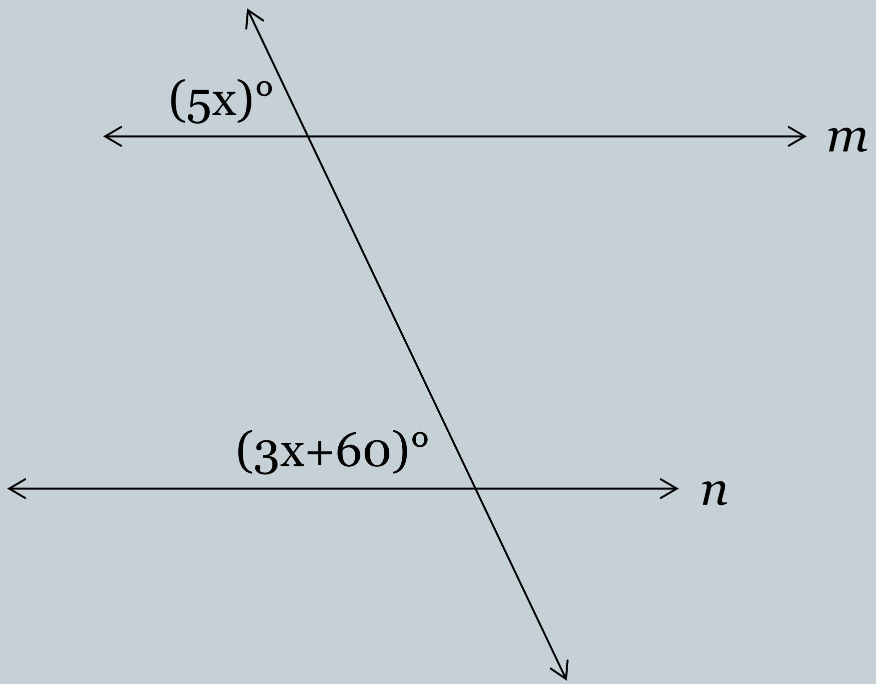
Example 1

- Find the value of x that makes *m*ll*n*.
- 105=3x
- 35=x



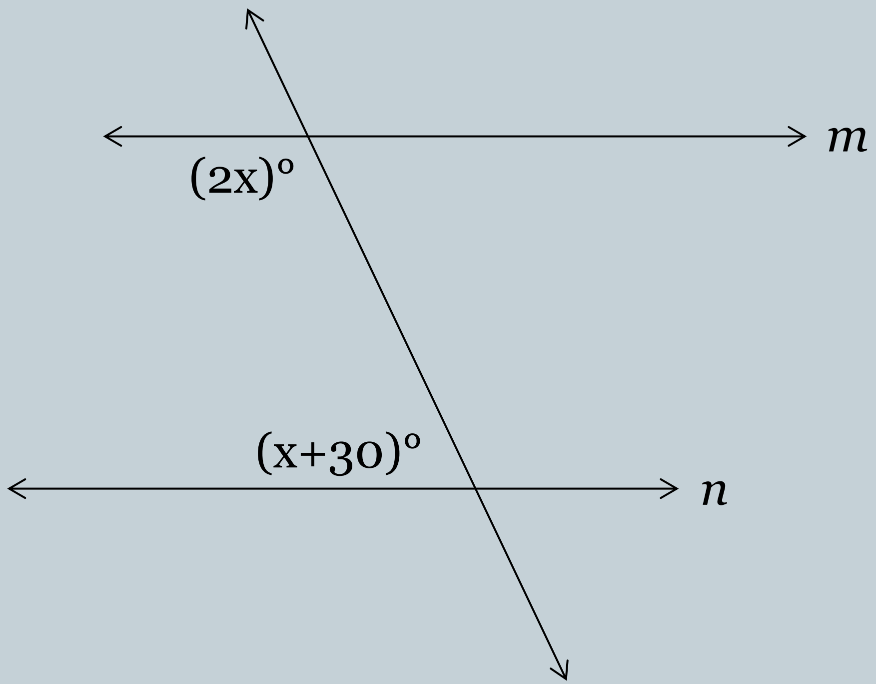
Example 2

- Find the value of x that makes *m*ll*n*.
- 5x=3x+60
- 2x=60
- x=30



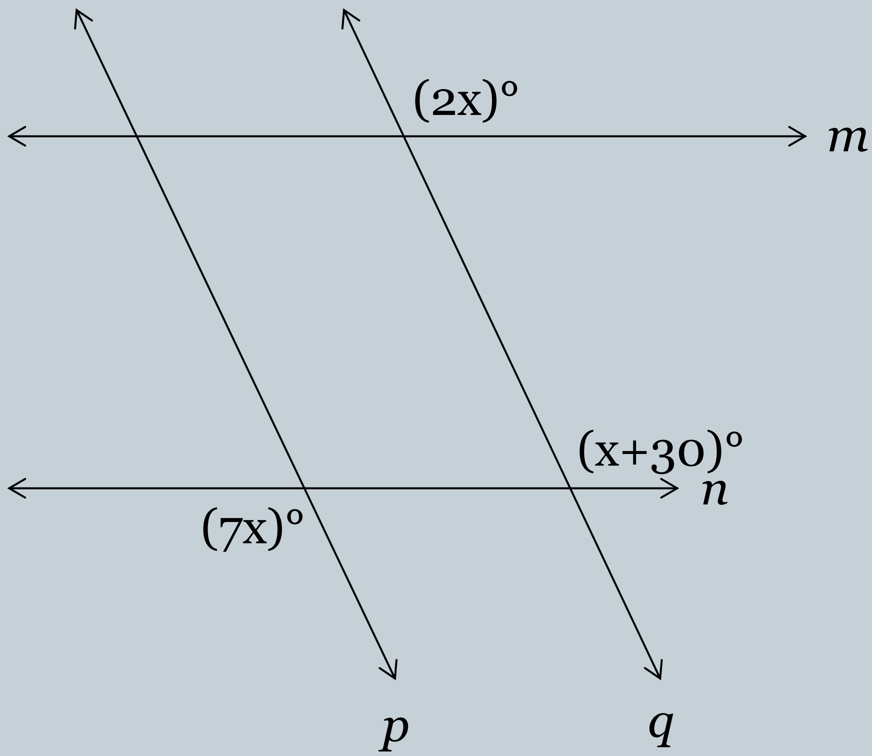
Example 3

- Find the value of x that makes *m*ll*n*.
- $2x+x+30=180$
- $3x+30=180$
- $3x=150$
- $x=50$



Example 4

- Find the value of x that makes *m*ll*n*.
- 2x=x+30
- x=30



Example 5

For each problem determine if there is enough information to state that *p*ll*q*.

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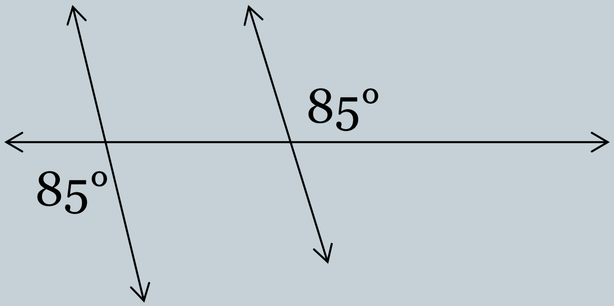

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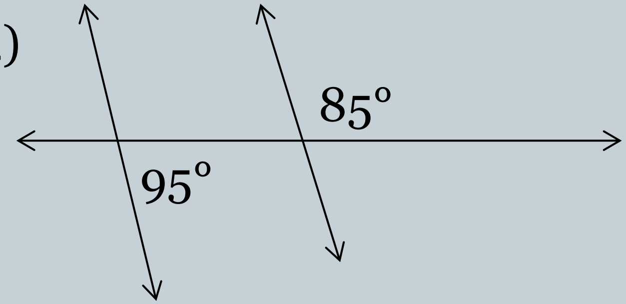

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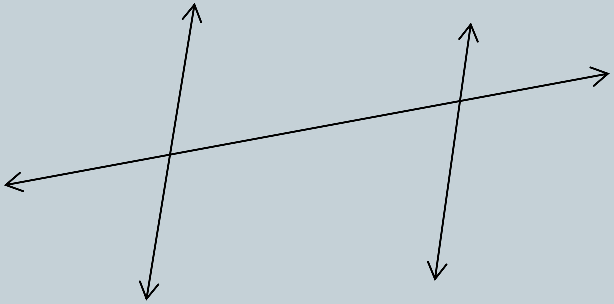

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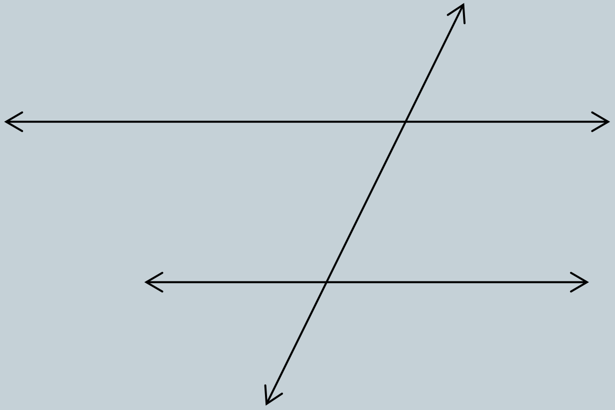
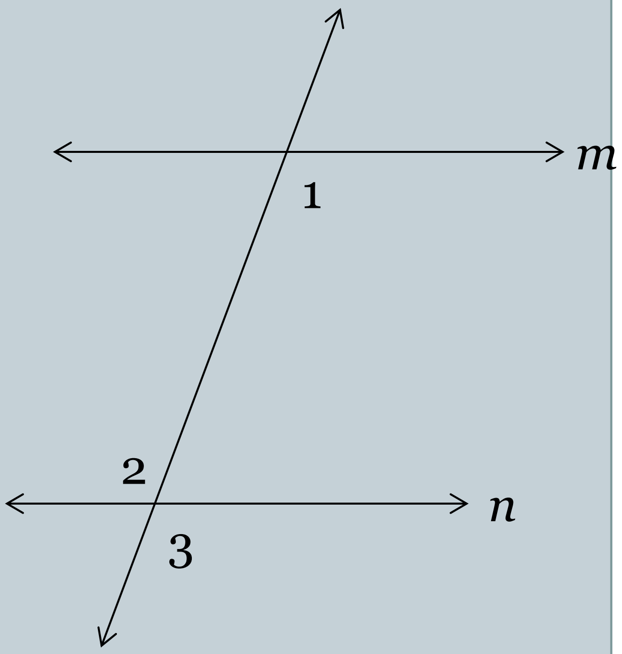


Example 6

- Prove the Alternate Interior Angles Converse Theorem.
- Given: $\angle 1 \cong \angle 2$
- Prove: $m \parallel n$



Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 3 \cong \angle 2$	2. Vertical Angles Congruence Theorem
3. $\angle 3 \cong \angle 1$	3. Transitive Property
4. $m \parallel n$	4. Corresponding Angles Converse Postulate

Example 6

- Prove the Alternate Interior Angles Converse Theorem. Use a Paragraph Proof.
- It is given that $\angle 1 \cong \angle 2$. From the diagram $\angle 3 \cong \angle 2$ due to the vertical angles congruence theorem. $\angle 3 \cong \angle 1$ due to the transitive property. Therefore $m \parallel n$ by the corresponding angles converse postulate.

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 3 \cong \angle 2$	2. Vertical Angles Congruence Theorem
3. $\angle 3 \cong \angle 1$	3. Transitive Property
4. $m \parallel n$	4. Corresponding Angles Converse Postulate

Find and Use Slopes of Lines

"A HERO IS NO BRAVER THAN AN ORDINARY MAN (OR WOMAN), BUT HE (/SHE) IS BRAVE FIVE MINUTES LONGER." –RALPH WALDO EMERSON

Goal

You will learn how to find the slope of the line and how the slope of parallel and perpendicular lines relate.

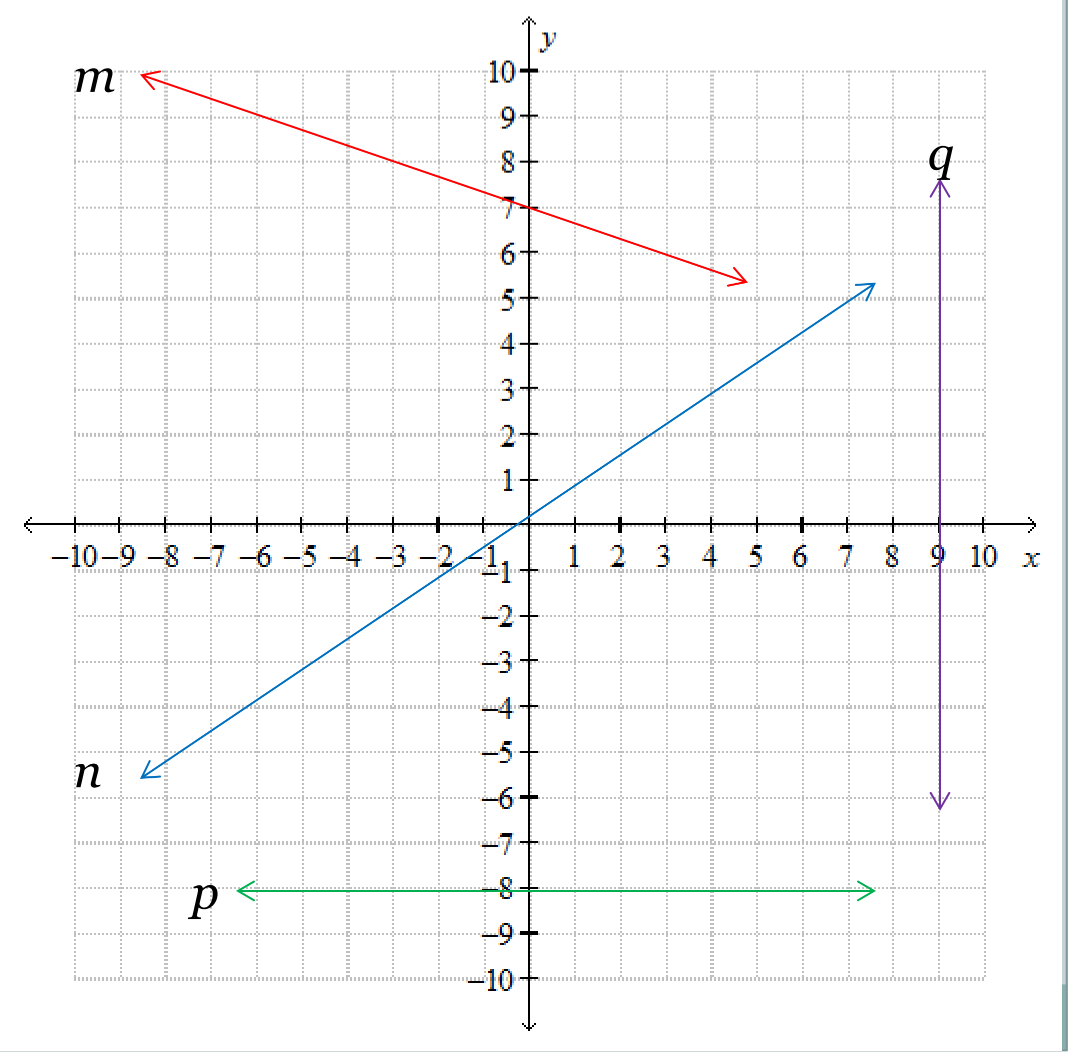
Slope of a line

- Slope can be thought of as the steepness of a line.
- Slope of a nonvertical line is the ratio of vertical change (rise) to horizontal change (run) between any two points on the line.

- $m = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$

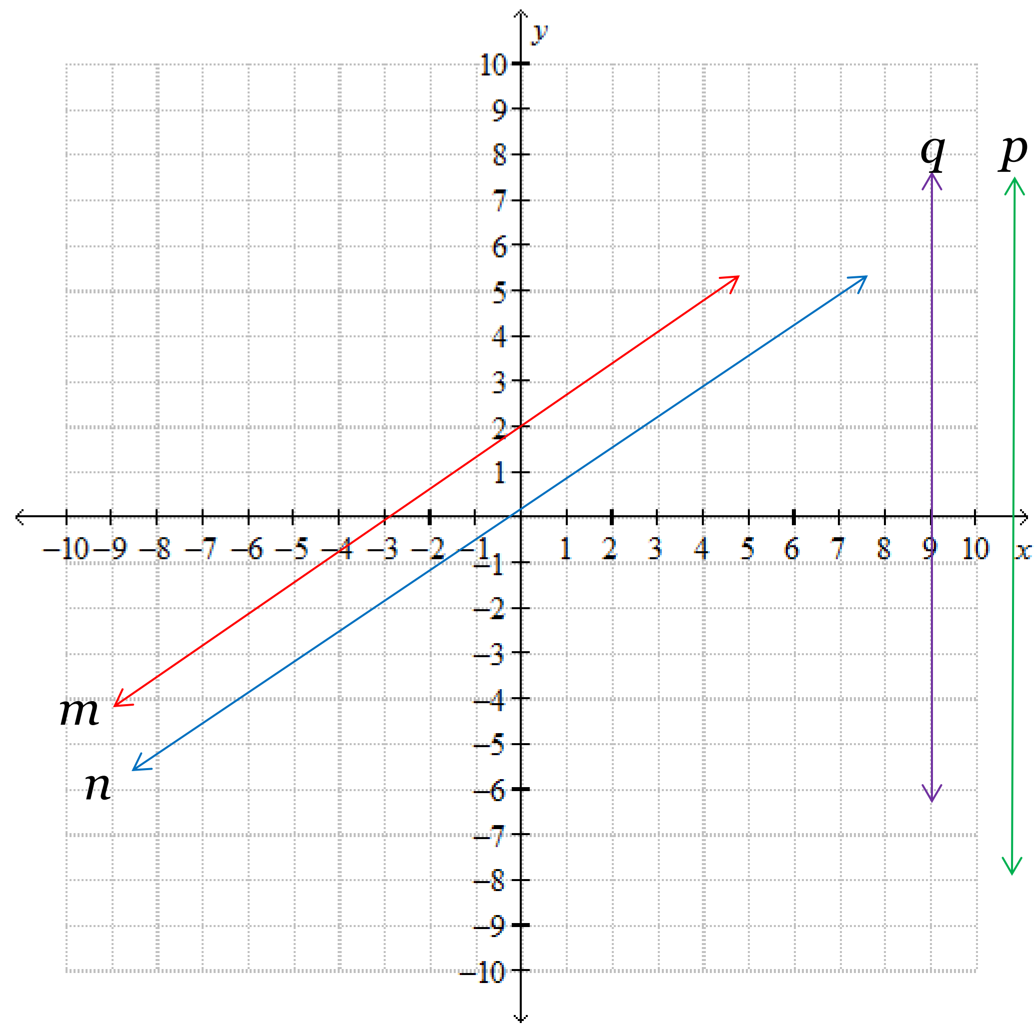
Types of slope

- m has negative slope
- n has positive slope
- p has zero slope
- q has undefined slope



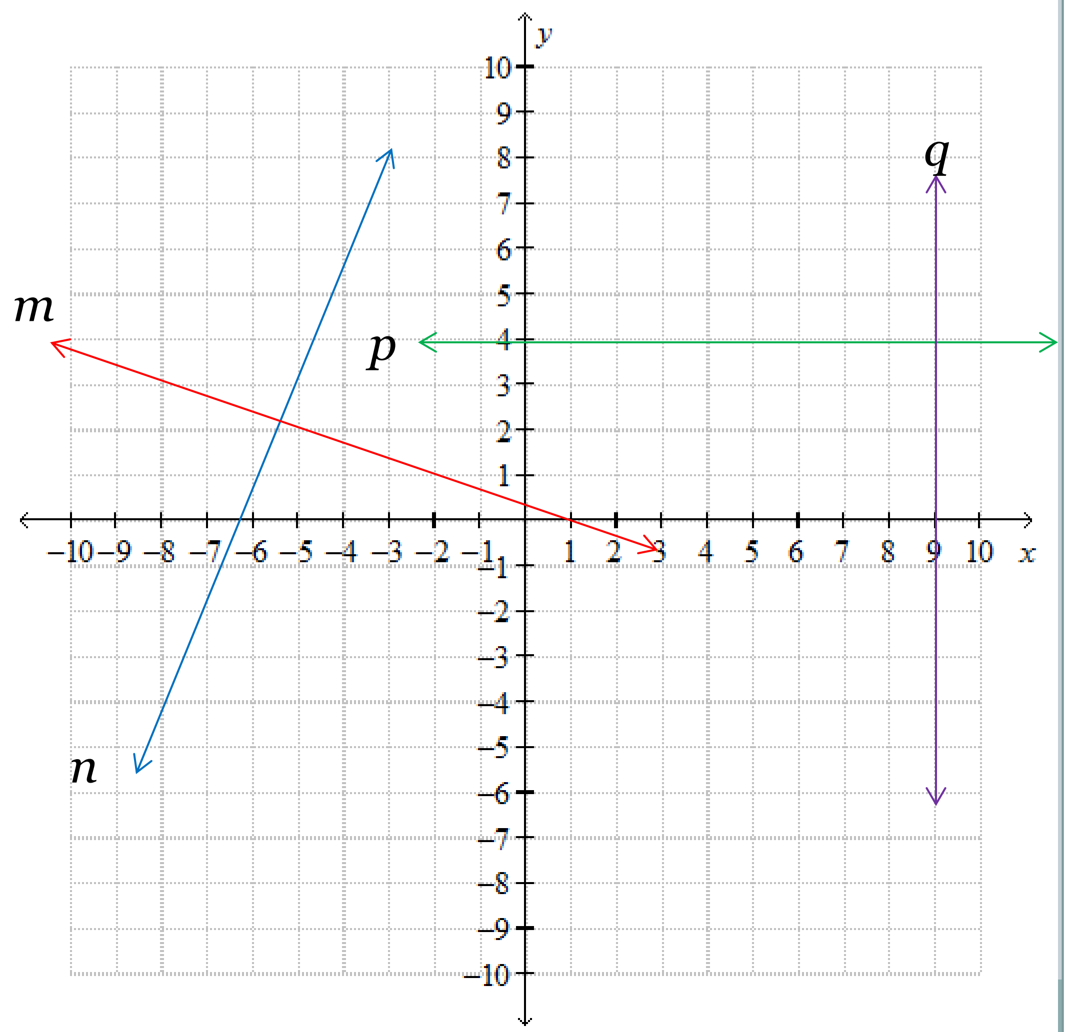
Slopes of Parallel Lines

- Two nonvertical lines are parallel iff they have the same slope.
- Any two vertical lines are parallel



Slopes of Perpendicular Lines

- Two nonvertical lines are perpendicular iff the product of their slope is -1.
- Horizontal and vertical lines are perpendicular.

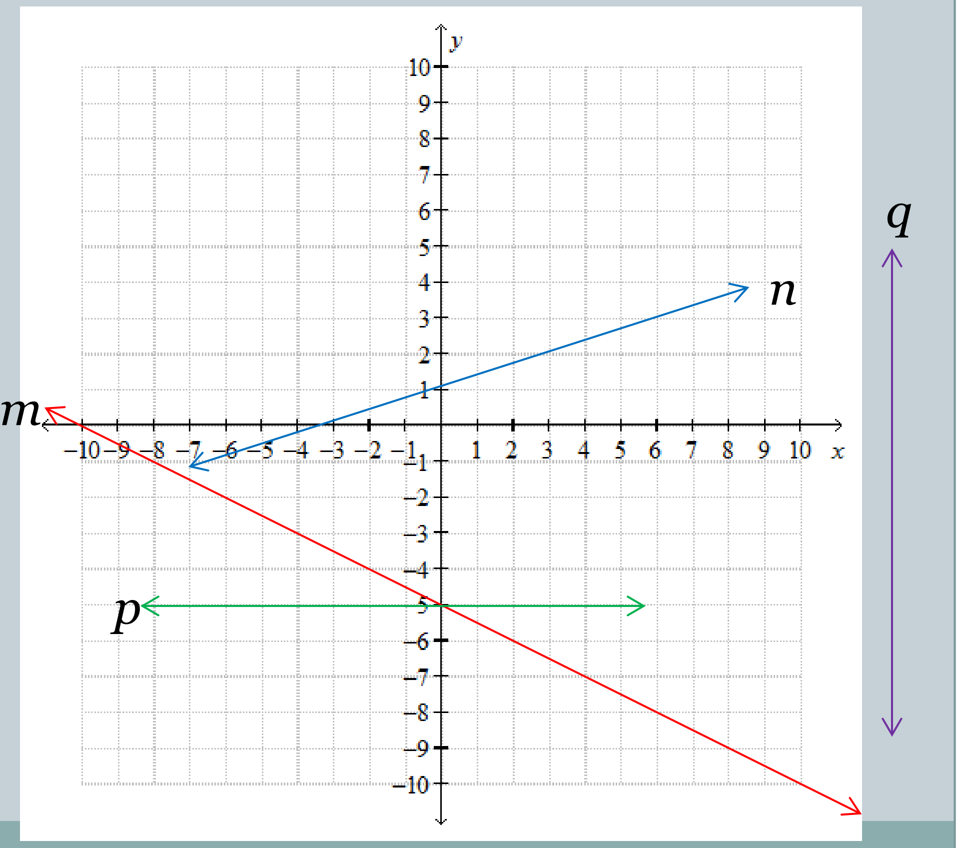


Summary

- You should be able to describe slope and identify slope of parallel and perpendicular lines.

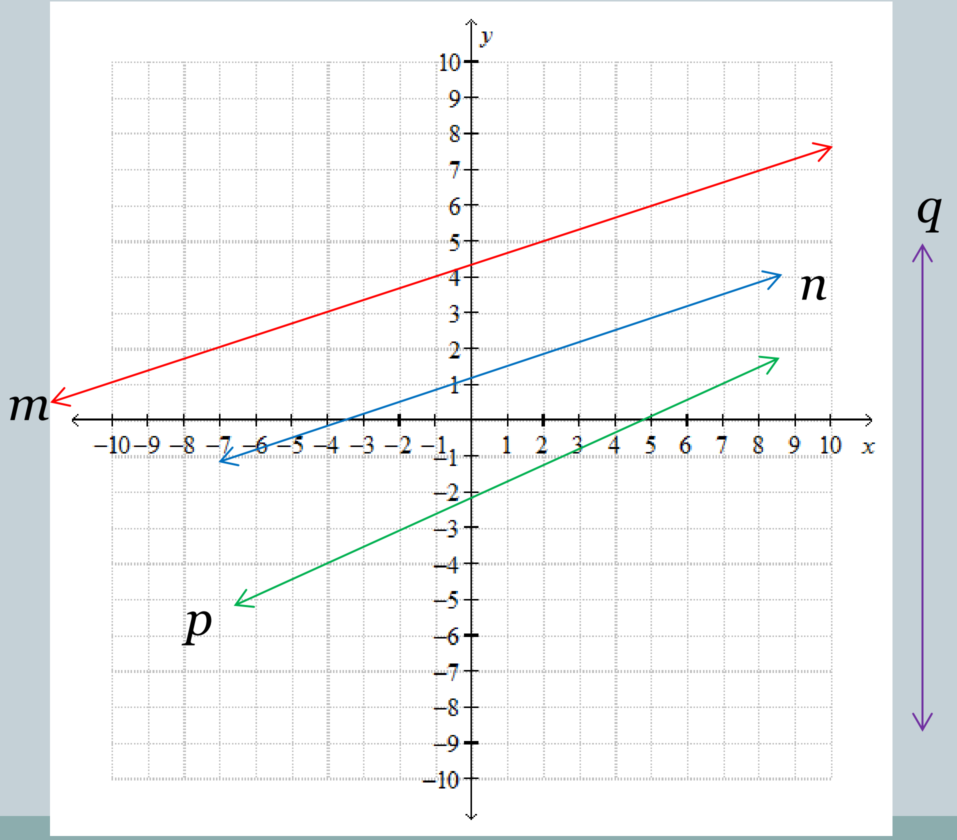
Example 1

- Find the slope of lines *m*, *p*, and *n*.



Example 2

- Find the slope of lines *m*, *p*, and *n*. Determine if any of the lines are parallel.



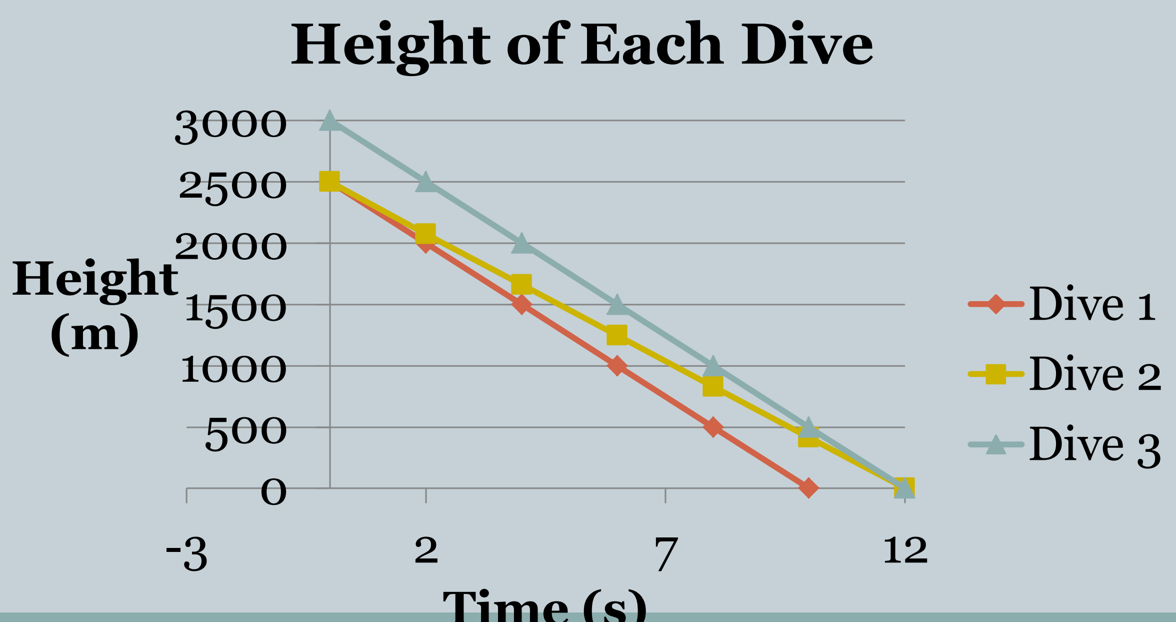
Example 3

- Find the slope of a line perpendicular to the line containing the points (5, -4) and (7, 0).

Example 4

- A skydiver made jumps with 3 different parachutes. The graph of his jumps are below. Which statement is true?

A) Dive 2 and Dive 3 started at the same height.
B) Dive 1 and Dive 2 lasted the same amount of time.
C) Dive 1 and Dive 3 were the same type of parachute.
D) Dive 2 had the parachute that had the slowest rate of decent.



Write and Graph Equations of Lines

"CERTAIN SIGNS PRECEDE CERTAIN EVENTS."

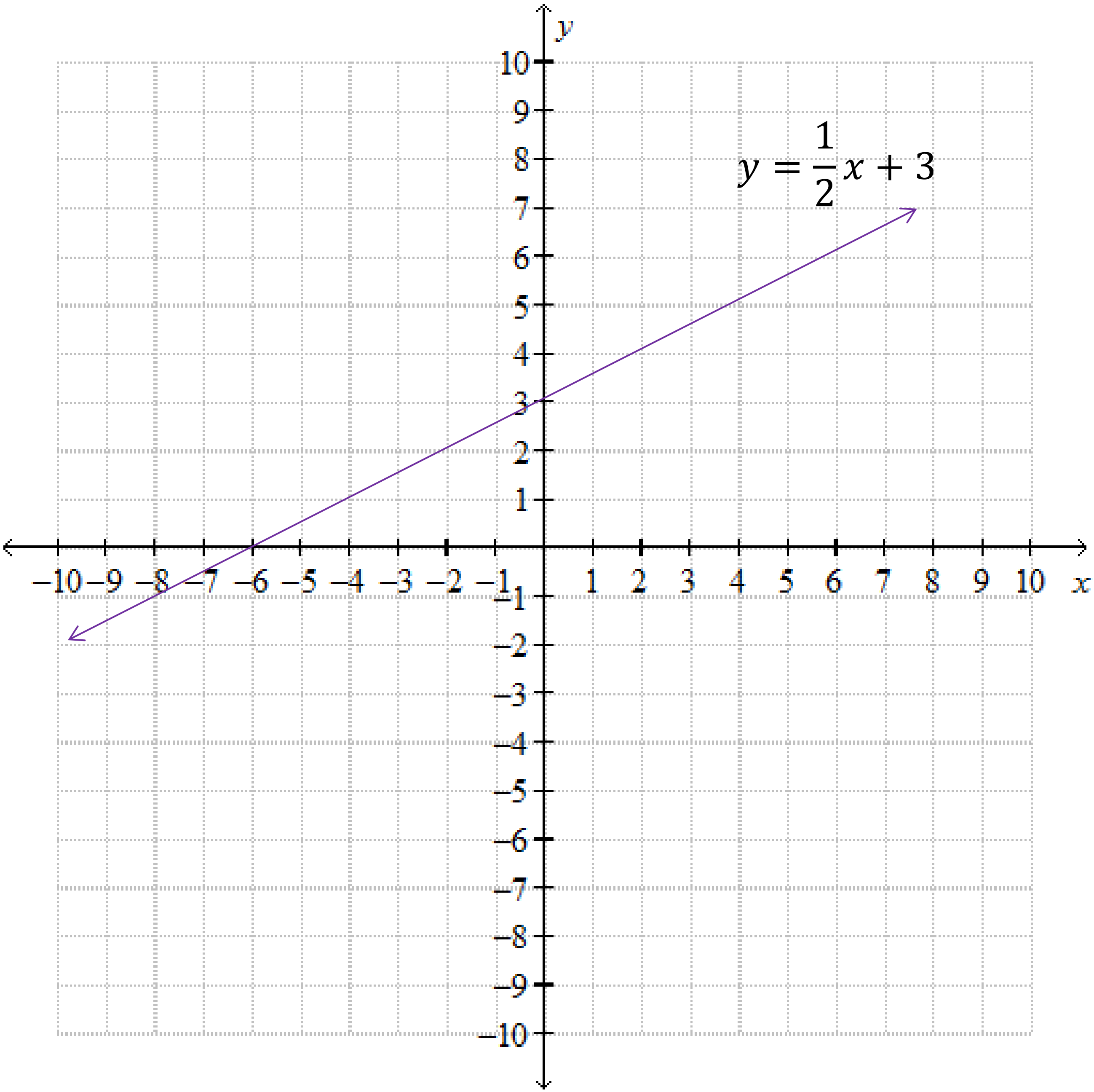
–CICERO

Goal

- Students will be able to write and graph equations of lines.

Slope-intercept Form

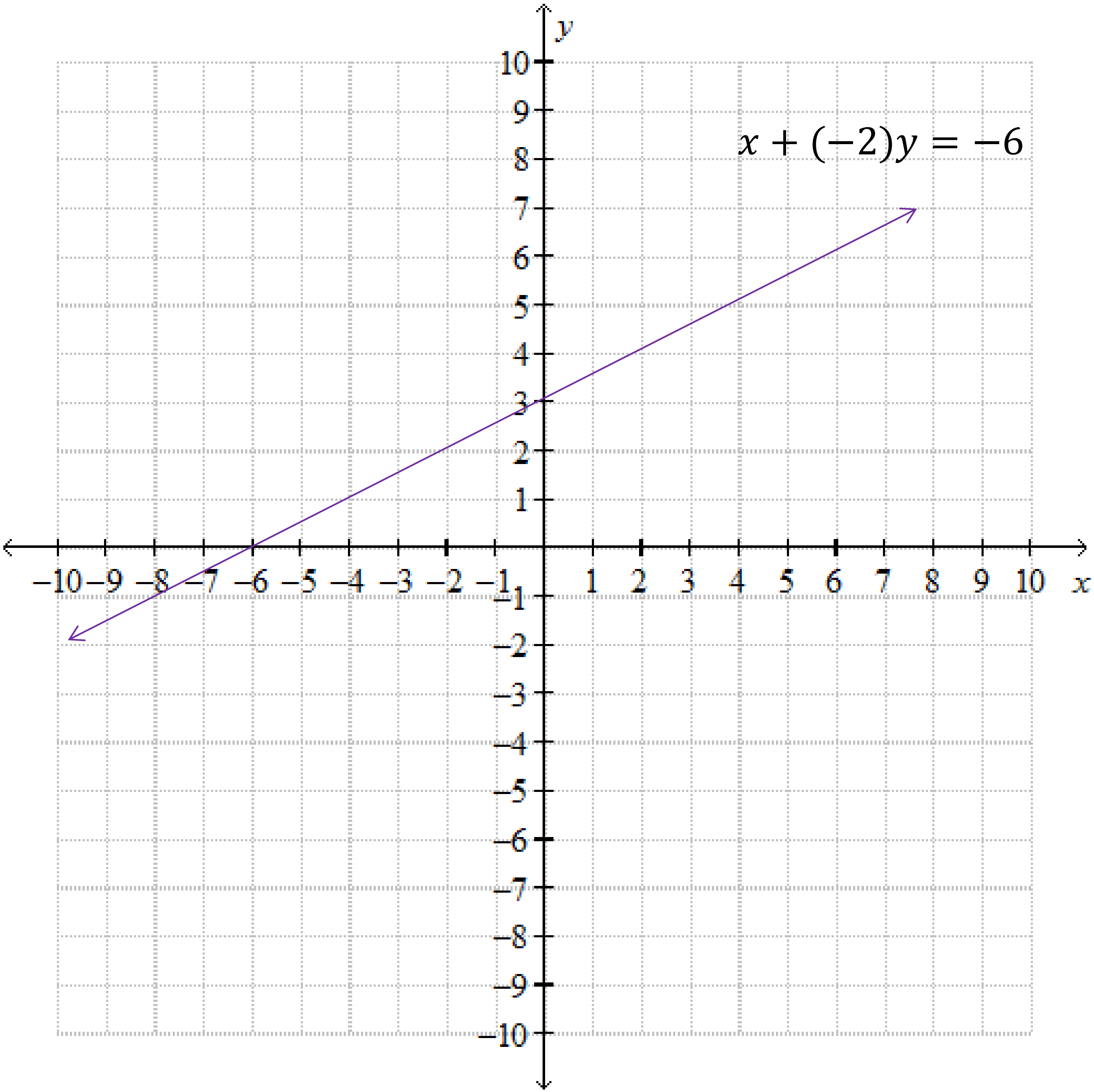
- $y=mx+b$
- M is the slope of the line.
- B is the y-intercept of the line (where the line crosses the y-axis).
- Y is y-coordinate of a point on the line.
- X is the x-coordinate of a point on the line.
- To write equation of line we need to find both b and m before writing the equation of the line.



$y = \frac{1}{2}x + 3$
y
x

Standard Form

- Ax+By=C
- X-intercept (where the graph crosses the x-axis) is $\frac{C}{A}$.
- Y-intercept (where the graph crosses the y-axis) is $\frac{C}{B}$.

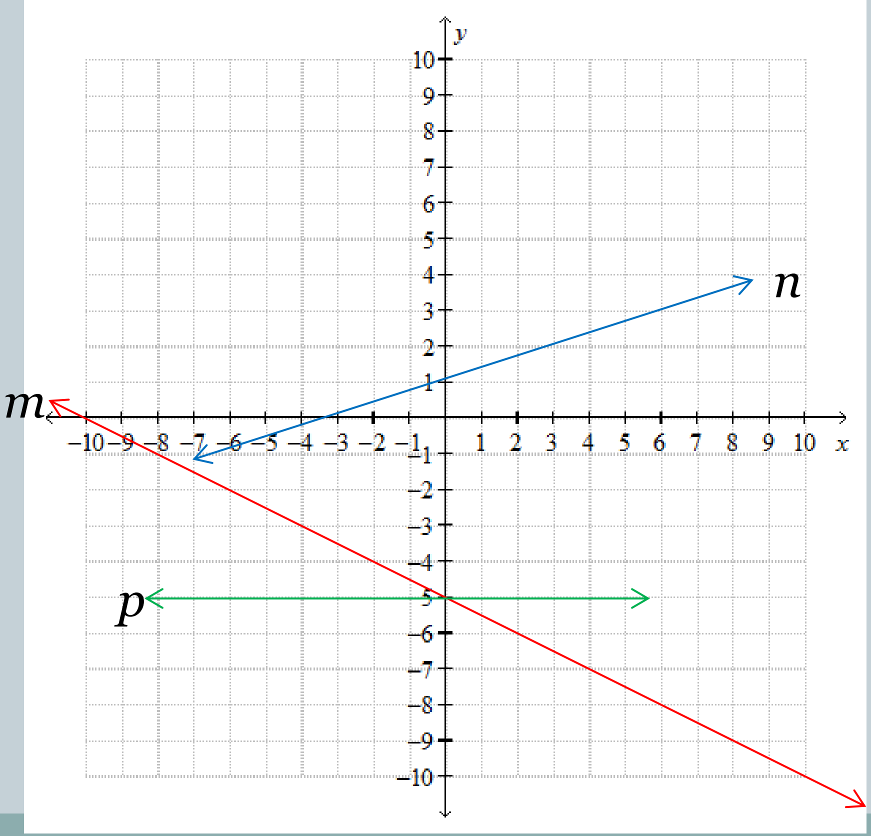

$x + (-2)y = -6$
y
x
10
9
8
7
6
5
4
3
2
1
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10

Summary

- You should be able to identify the different ways to write an equation of a line.

Example 1

- Write the equation of each line in slope-intercept form.



Example 2

- Graph the equation of the line.

1) $y=2x+7$
2) $5x+10y=20$
3) $y=-3x$
4) $y=8$
5) $x=1$

Example 3

- Write an equation of the line passing through points (2, 4) and is perpendicular to the line with the equation $y=2x+7$.

Example 4

- What is the slope and y-intercept of these lines.

1) 5x-10y=-20
    1) -10y=-5x-20
    2) $y = \frac{1}{2}x + 2$
2) 3x+y=6
    1) y=-3x+6

Example 5

- Write the equation of the line with the given information.

1) $m=4, b=-2$

$y=4x-2$

1) $m=-1, b=7$

$y=-x+7$

Prove Theorems about Perpendicular Lines

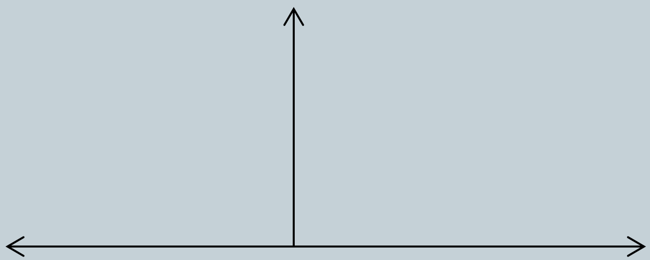
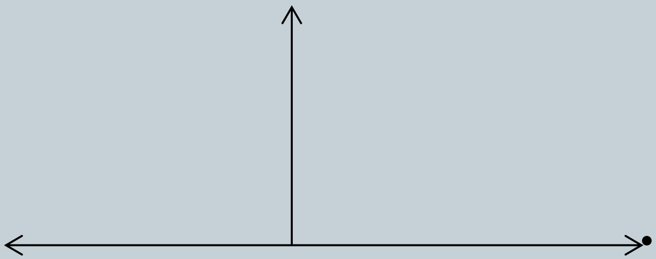
"ANXIETY IS FEAR OF ONE'S SELF." – WILHELM STEKEL

Goal

- You will learn how to prove statements about parallel and perpendicular lines.

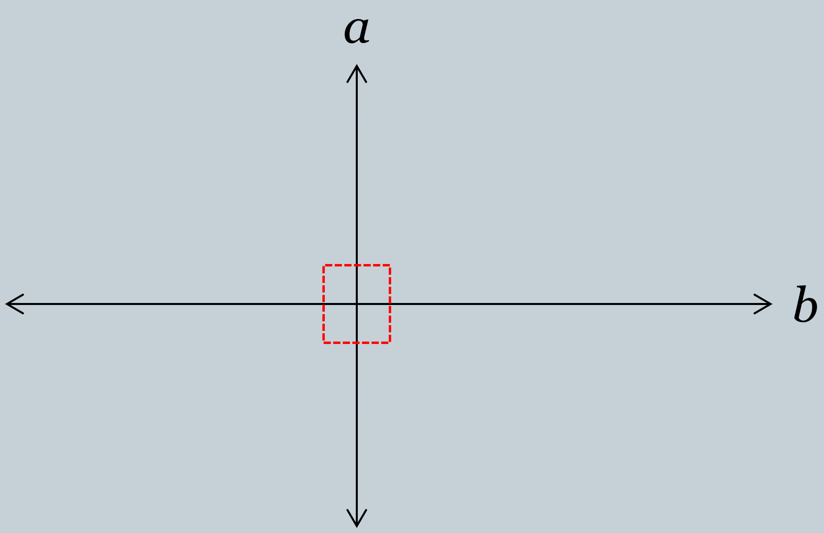
Theorem 3.8

- If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If  , then  .

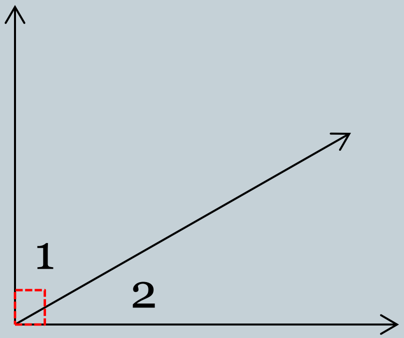
Theorem 3.9

- If two lines are perpendicular, then they intersect to form four right angles.



Theorem 3.10

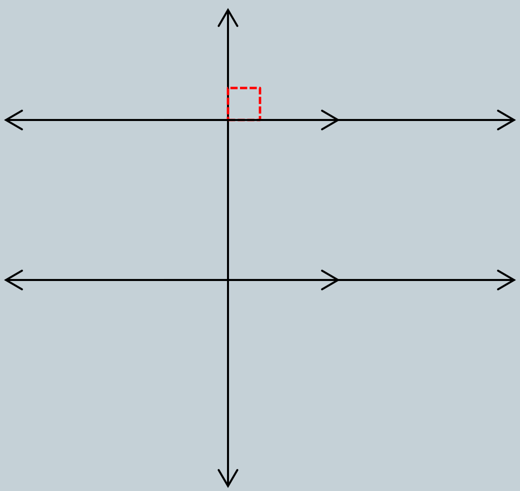
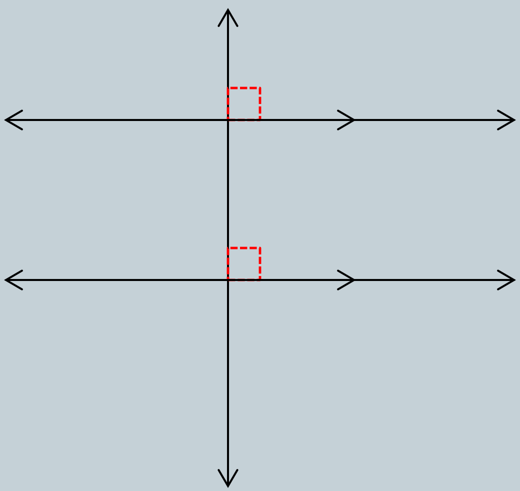
- If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

If 

, then ∠1 and ∠2 are complementary.

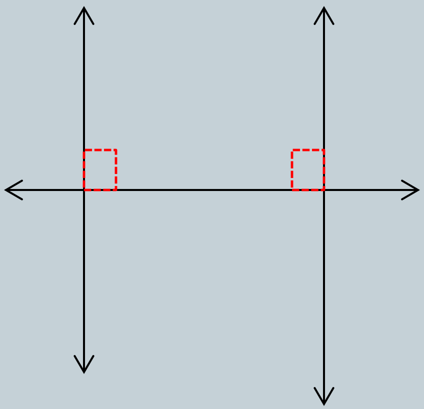
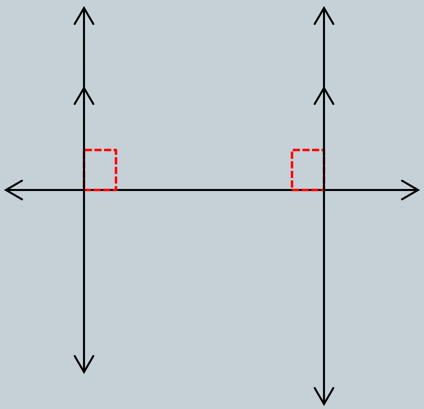
Perpendicular Transversal Theorem

- If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.

If  , then  .

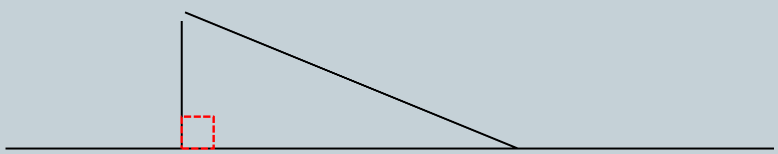
Lines Perpendicular to a Transversal Theorem

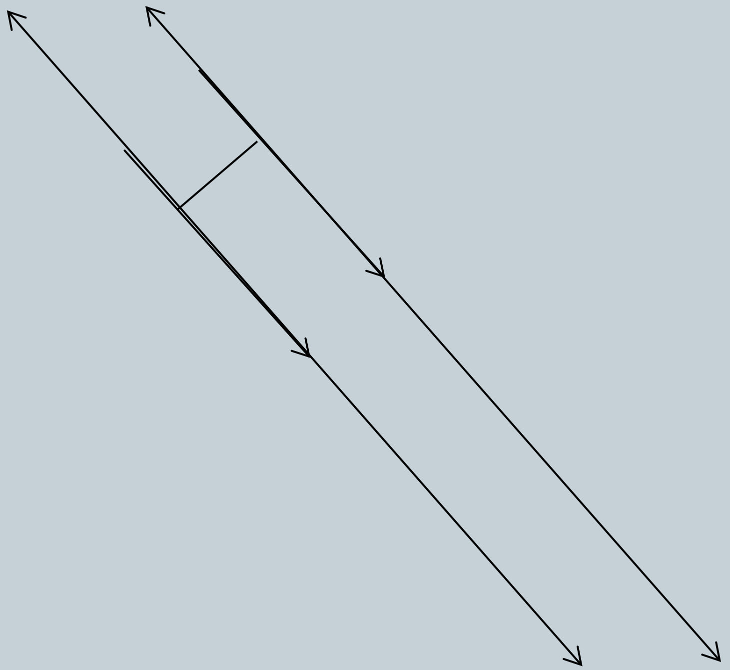
- In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

If  , then  .

Distance from a line.

- Distance from a point to a line is the length of the perpendicular segment from the point to the line.





Summary

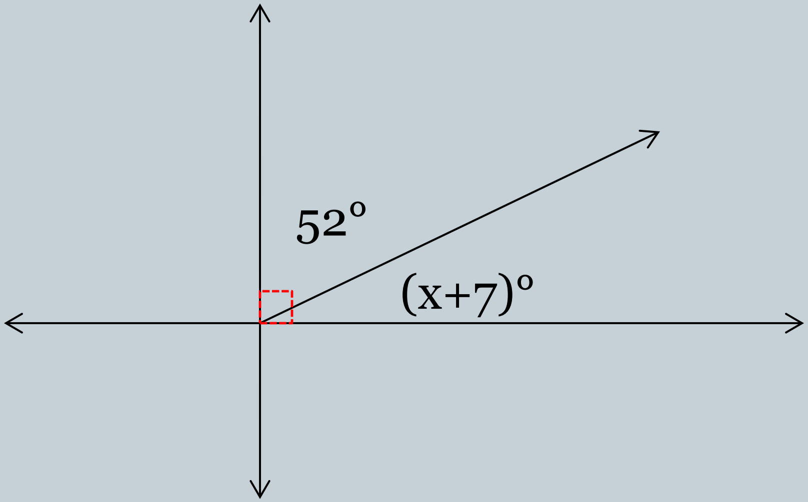
- You should be able to prove statements parallel and perpendicular lines.
- You should be able to determine what the distance from a point to a line is.

Example 1

- What is the value of x?

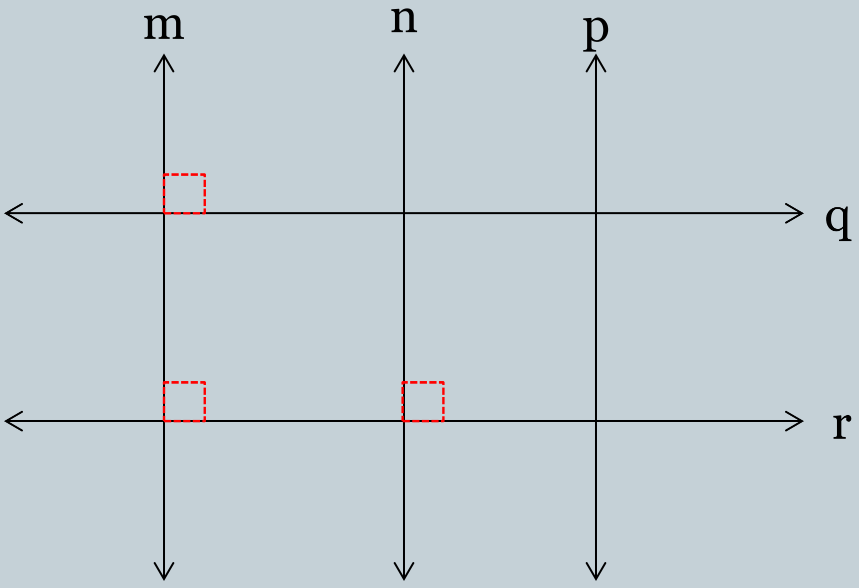
52=x+7

45=x



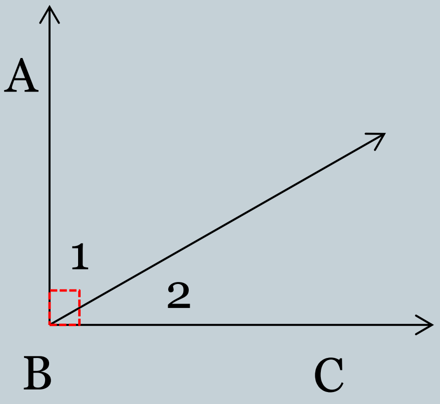
Example 2

- Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.



Example 3

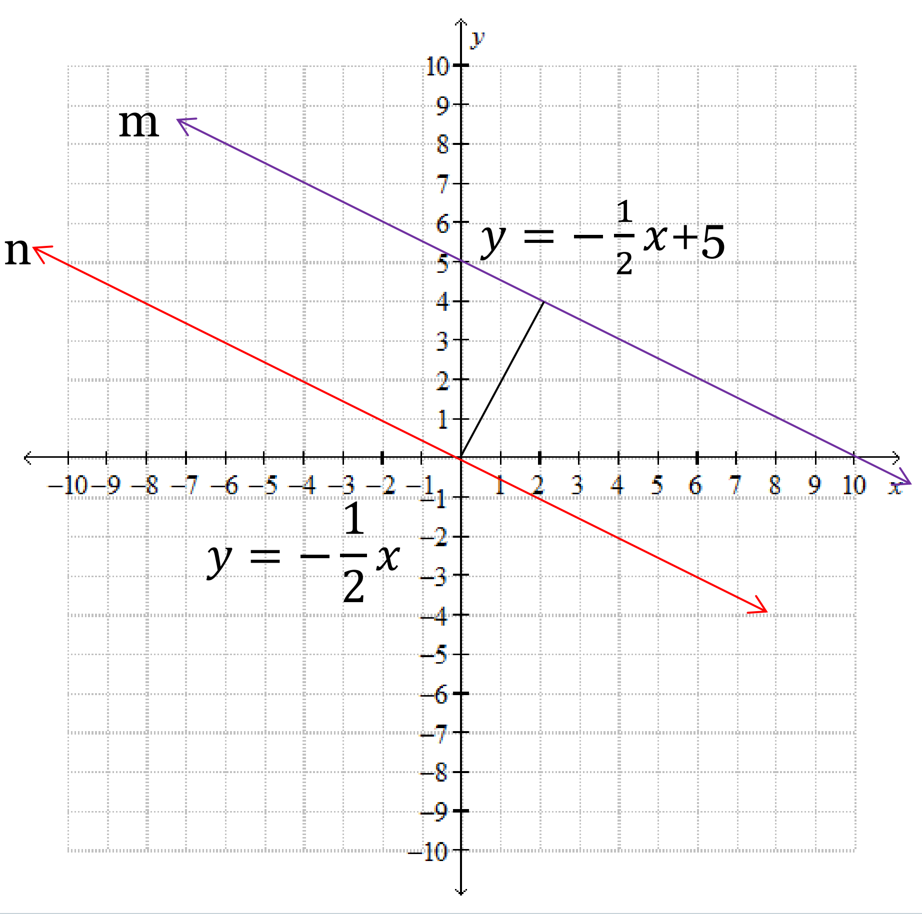
- Prove that if two sides of two adjacent angles are perpendicular, then the angles are complementary.
- Given: ray BA ⊥ ray BC
- Prove: ∠1 and ∠2 are complementary



Statements	Reasons
1. Ray BA ⊥ Ray BC	1. Given
2. ∠ABC is a right angle	2. ⊥ form 4 right angles
3. m∠ABC=90°	3. Definition of a right angle
4. m∠ABC=m∠1+m∠2	4. Angle Addition Postulate
5. 90°=m∠1+m∠2	5. Transitive Property
6. ∠1 and ∠2 are complementary	6. Definition of complementary angles

Example 4

- How far apart are lines m and n?
- Perpendicular slopes have a product of -1
- $(-\frac{1}{2})(\_)=-1$
- $(\_)=2$
- Use distance formula



Taxicab Geometry

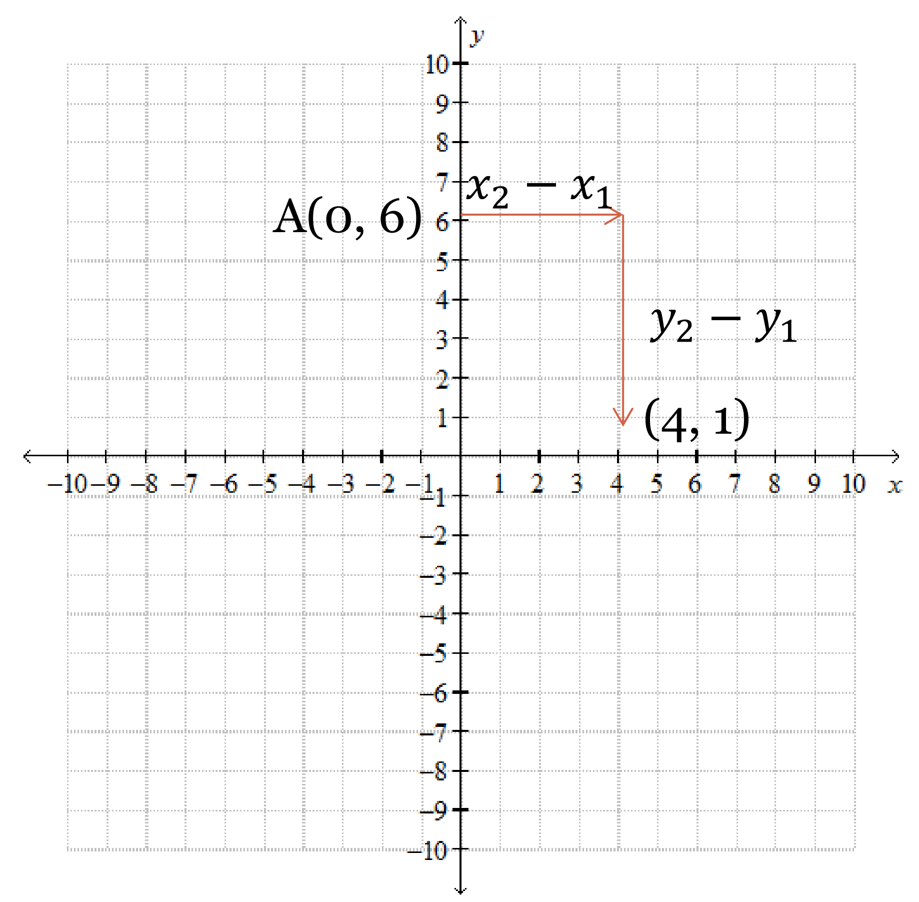
"REMEMBER THAT HAPPINESS IS A WAY OF TRAVEL—NOT A DESTINATION." –ROY M. GOODMAN

Goal

- To apply geometry to real world problems.

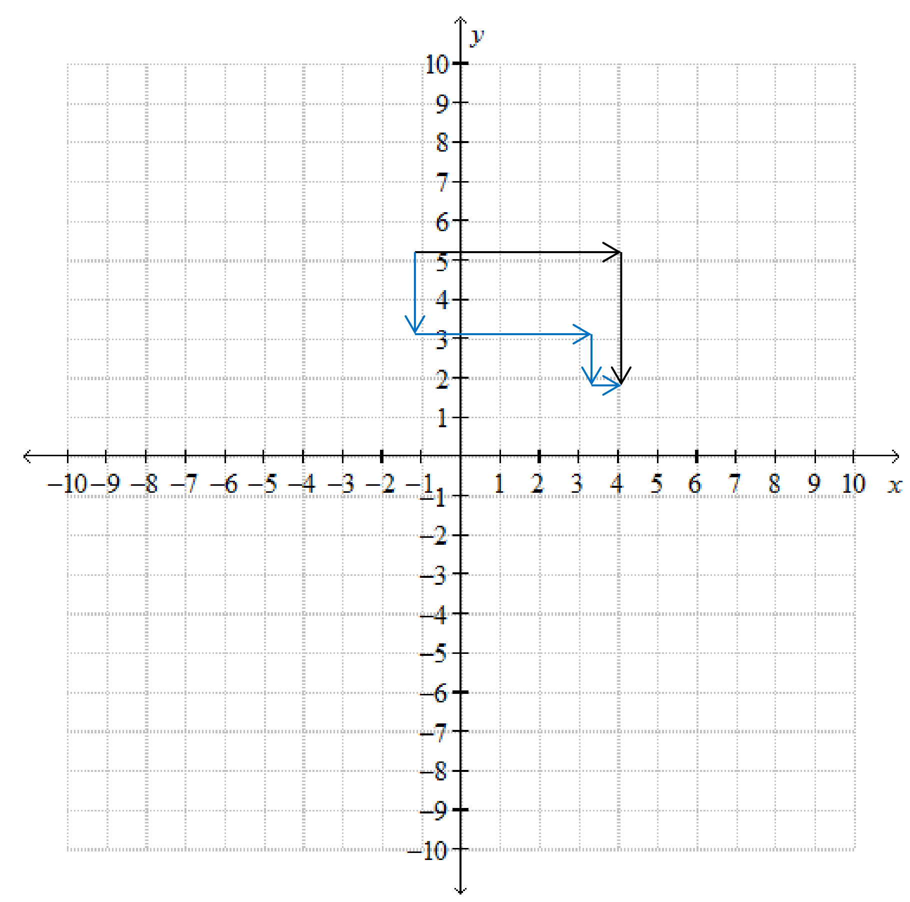
Taxicab Distance

- The distance between two points is the sum of the differences in their coordinates.
- AB=$|x_2 - x_1| + |y_2 - y_1|$
- $A(x_2, y_2)$, $B(x_1, y_1)$



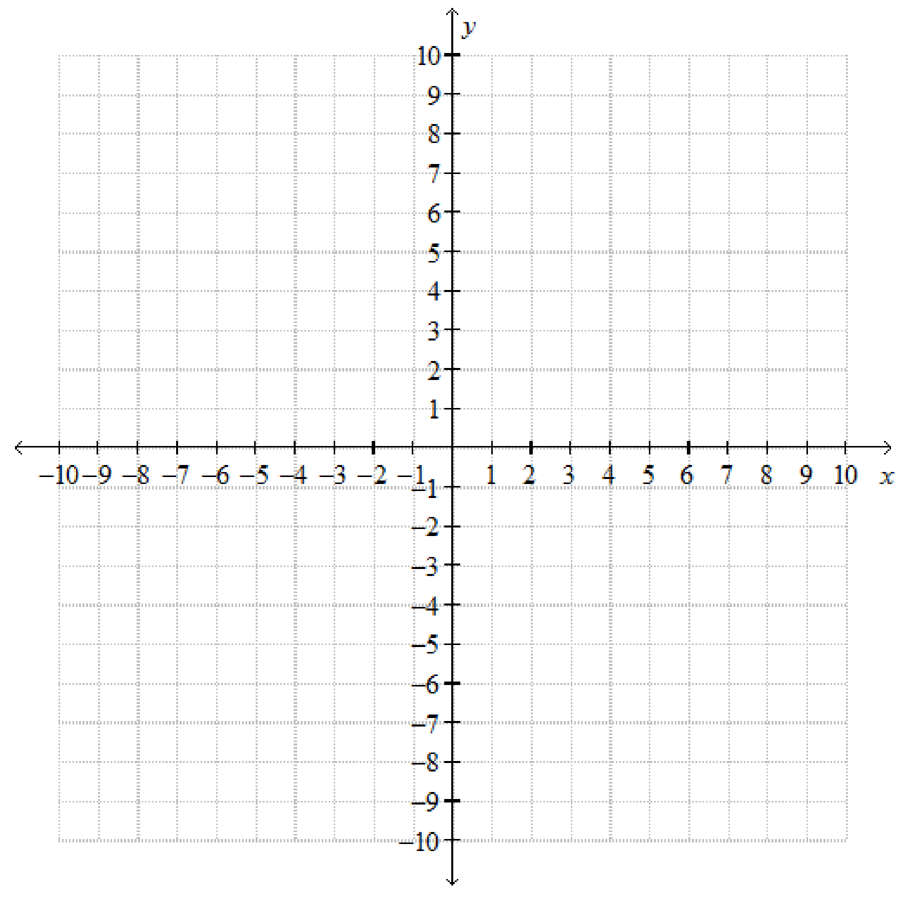
Example 1

- Find the taxicab distance from A(-1, 5) to B (4, 2). Draw two different shortest paths from A to B.
- |4--1|+|2-5|
- |5|+|-3|
- 5+3
- 8



Taxicab Circles

- These are circles that are the same distance from the center.
- Here is an example of a taxicab circle with radius of 3.



Summary

- You should be able to use taxicab geometry to find the “block” distance between two points.