Unit 3A: Factoring & Solving Quadratic Equations

After completion of this unit, you will be able to…

Learning Target #1: Factoring

- Factor the GCF out of a polynomial
- Factor a polynomial when a = 1
- Factor a polynomial when a ≠ 1
- Factor special products (difference of two squares)

Learning Target #2: Solving by Factoring Methods

- Solve a quadratic equation by factoring a GCF.
- Solve a quadratic equation by factoring when a is not 1.
- Create a quadratic equation given a graph or the zeros of a function.

Learning Target #3: Solving by Non Factoring Methods

- Solve a quadratic equation by finding square roots.
- Solve a quadratic equation by completing the square.
- Solve a quadratic equation by using the Quadratic Formula.

Learning Target #4: Solving Quadratic Equations

- Solve a quadratic equation by analyzing the equation and determining the best method for solving.
- Solve quadratic applications

Timeline for Unit 3A

Monday	Tuesday	Wednesday	Thursday	Friday
January 28th Day 1- Factoring Quadratic Expressions – GCF	**29th** Day 2 - Factoring Quadratic Trinomials, a = 1	**30th** Day 3 - Factoring Quadratic Trinomials, a ≠1	**31st** Day 4 - Factoring Special Products	**February 1st** Quiz over Days 1-4 **Quiz – Factoring Quadratics**
4th Day 5 – Solving Quadratics (GCF, a = 1, a ≠ 1)	**5th** Day 6 – More Practice Solving Quadratics (GCF, a = 1, a ≠ 1)	**6th** Day 7 – Solving by Square Roots	**7th** Day 8 - Solving by Completing the Square	**8th** Day 9 - Solving by Quadratic Formula
11th Quiz over Days 5 – 9 **Quiz – Solving Quadratics**	**12th** Day 10 – Applications of Quadratics	**13th** Day 11 – Determining the Best Method	**14th** Unit 3A Test Review	**15th** **Unit 3A Test**

Day 1 – Factor by GCF

Standard(s): ____

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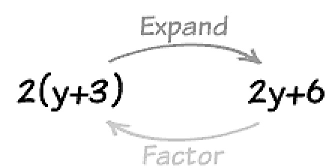
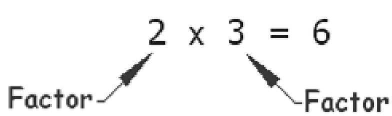
What is Factoring?

Factoring

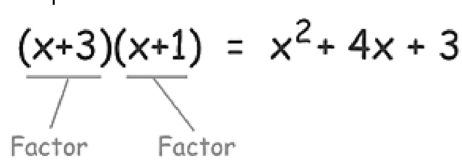
- Finding out which two expressions you multiplied together to get one single expression.
- Is like "splitting" an expression into a product of simpler expressions.
- The opposite of expanding or distributing.



Numbers have factors:



Expressions have factors too:



Review: Finding the GCF of Two Numbers

Common Factors

- Factors that are shared by two or more numbers are called common factors.

Greatest Common Factor (GCF)

- The greatest of the common factors is called the **Greatest Common Factor (GCF)**.
- To find the greatest common factor, you can make a factor tree and complete the prime factorization of both numbers. The GCF is the product of the common prime factors.
- You can also do a factor t-chart for each number and find the largest common factor
-

Example: Find the GCF of 56 and 104

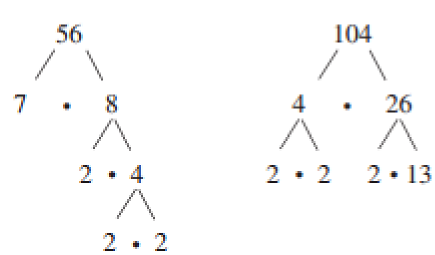
56

1	56
2	28
4	14
7	8

104

1	104
2	52
4	26
8	13

So, the GCF of 56 and 104 is 8.



$56 = 2 \cdot 2 \cdot 2 \cdot 7$

$104 = 2 \cdot 2 \cdot 2 \cdot 13$

So, the GCF of 56 and 104 is $2 \cdot 2 \cdot 2 = 8$.

Practice: Find the GCF of the following numbers.

a. 30, 45

b. 12, 54

Finding the GCF of Two Expressions

To find the GCF of two expressions, you will complete the prime factorization of the two numbers or factor chart of the two numbers AND expand the variables. Circle what is common to both.

Example: Find the GCF of $36x^2y$ and $16xy$

Practice: Find the GCF of the following pairs of expressions.

1) 100 and 60

2) $15x^3$ and $9x^2$

3) $9a^2b^2$, $6ab^3$, and $12b$

4) $8x^2$ and $7y^3$

Factoring by GCF

Steps for Factoring by GCF

1. Find the greatest common factor of all the terms.

2. The GCF of the terms goes on the outside of the expression and what is leftover goes in parenthesis after the GCF.

3. After "factoring out" the GCF, the only that number that divides into each term should be 1.

Practice: Factor each expression.

1. $x^2 + 5x$ GCF =

2. $x^2 - 8x$ GCF =

3. $x^2 - 3x$ GCF =

4. $28x - 63$ GCF =

5. $18x^2 - 6x$ GCF =

6. $4x^2 - 4x$ GCF =

7. $2m^2 - 8m$ GCF =

8. $-9a^2 - a$ GCF =

9. $35y^2 - 5y$ GCF =

10. $6x^3 - 9x^2 + 12x$ GCF =

11. $4x^3 + 6x^2 - 8x$ GCF =

12. $15x^3y^2 + 10x^2y^4$ GCF =

Day 2 – Factor Trinomials when a = 1

Standard(s): ____

____

____

____

Quadratic Trinomials

2nd Degree **(Quadratic)**

$$ax^2 + bx + c$$

3 Terms **(Trinomial)**

Factoring a trinomial means finding two binomials that when multiplied together produce the given **trinomial**.

Looking for Patterns

What do you observe in the following Area Models?

	3x	+4
x	$3x^2$	+4x
-3	-9x	-12

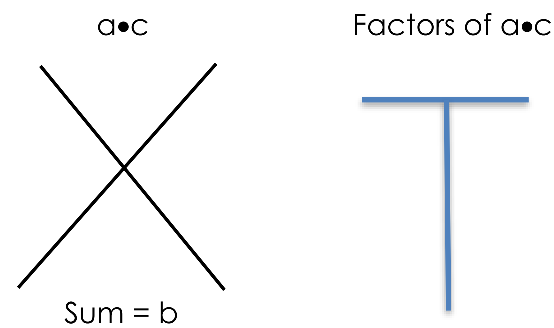
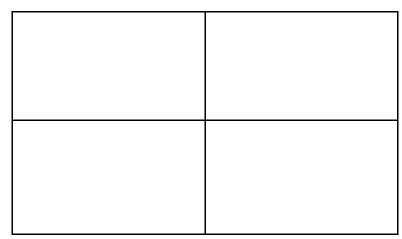
$(x-3)(3x+4) = 3x^2 - 5x - 12$

	4x	+3
2x	$8x^2$	+6x
+1	+4x	+3

$(2x+1)(4x+3) = 8x^2+10x+3$

Factoring using the Area Model

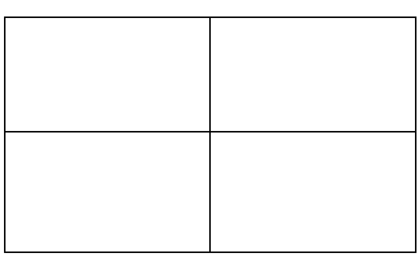
Factor: $x^2 - 4x - 32$

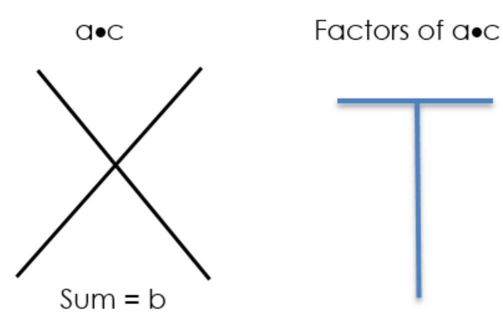
STEP 1: • ALWAYS check to see if you can factor out a GCF.	
STEP 2: • Multiply the coefficients of the "a" and "c" terms together and place that number in the bottom of the "number diamond" • Place the coefficient of the "b" term in the top. • Make a factor t-chart for the factors of "a.c" • Determine what two numbers can be multiplied to get your "a·c" term and added to get your "b" term.	
STEP 3: • Create a 2x2 Area Model and place your original "a" term in the top left box and "c" term in the bottom right box. • Fill the remaining two boxes with the two numbers you found in your number diamond and place an x after them.	
STEP 4: • Factor out a GCF from each row and column to create the binomials or factors you are looking for.	
STEP 5: • Check your factors on the outside by multiplying them together to make sure you get all the expressions in your box.	Factored Form: ______________________

Factor the following trinomials.

$x^2 + 6x + 8$

Factored Form: ______________________





Practice Factoring A = 1

Using the Area Model. Factor the following trinomials.

a. Factor $x^2 + 4x - 32$

b. Factor $x^2 + 5x + 6$

c. Factor $x^2 - 3x - 18$

d. Factor $x^2 - 14x + 48$

e. Factor $x^2 - 36$

f. Factor $x^2 + 10x + 50$

Look at these examples. **(HINT: Is there a GCF?)**

g. $2x^2 + 16x + 24$

h. $4x^3 + 12x^2 + 8x$

Remember…your factored form should always been equivalent to the polynomial you started with so you must always include the GCF on the outside of the factored form.

Day 3 – Factor Trinomials with a ≠1

Standard(s): ____

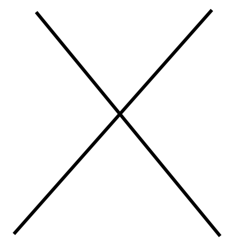
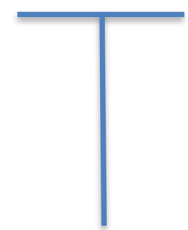
____

____

____

In the previous lesson, we factored polynomials for which the coefficient of the squared term, "a" was always 1. Today we will focus on examples for which a ≠ 1.

Factor: $2x^2 + 3x - 2$

<table>
<tr><td>STEP 1:
ALWAYS check to see if you can factor out a GCF.</td><td></td></tr>
<tr><td>STEP 2:
Multiply the coefficients of the "a" and "c" terms together and place that number in the top of the "number diamond"
Place the coefficient of the "b" term in the bottom.
Make a factor t-chart for the factors of "a.c"
Determine what two numbers can be multiplied to get your "a·c" term and added to get your "b" term.</td><td>a•c
Factors of a•c


Sum = b</td></tr>
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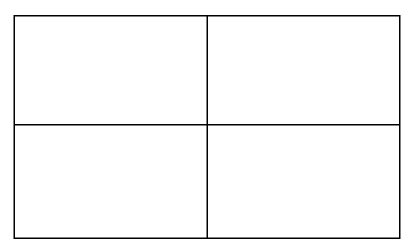
Create a 2x2 Area Model and place your original "a" term in the top left box and "c" term in the bottom right box.
Fill the remaining two boxes with the two numbers you found in your number diamond and place an x after them.</td><td rowspan="3">
Factored Form: ____________________</td></tr>
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Factor out a GCF from each row and column to create the binomials or factors you are looking for.</td></tr>
<tr><td>STEP 5:
Check your factors on the outside by multiplying them together to make sure you get all the expressions in your box.</td></tr>
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Factoring a ≠ 1

Using the Area Model. Factor the following trinomials.

1. $5x^2 + 14x - 3$ Factored Form: ______________________

2. $2x^2 - 5x + 3$ Factored Form: ______________________

3. $2x^2 - 17x - 30$ Factored Form: ______________________

Look at this example. **(HINT: Is there a GCF?)**

4. $6x^2 - 40x + 24$ Factored Form: ______________________

Practice: Take a look at the following trinomials and factor out the GCF, then use the Area Model to factor.

a. $12x^2 + 56x + 64$

b. $25x^2 + 210x - 400$

c. $10x^2 - 72x + 72$

d. $12x^2 + 30x + 30$

e. $8x^2 + 44x + 20$

Remember…your factored form should always been equivalent to the polynomial you started with so you must always include the GCF on the outside of the factored form.

Day 4 – Factor Special Products

Standard(s): ____________________

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Review: Factor the following expressions:

a. $x^2 - 49$ b. $x^2 - 25$ c. $x^2 - 81$

1. What do you notice about the "a" term?____________________

2. What do you notice about the "c" term? ____________________

3. What do you notice about the "b" term? ____________________

4. What do you notice about the factored form? ____________________

The above polynomials are a special pattern type of polynomials; this pattern is called a

Difference of Two Squares

$$a^2 - b^2 = (a - b)(a + b)$$

Always subtraction
Both terms are perfect squares
Always two terms

Can you apply the "Difference of Two Squares" to the following polynomials?

a. $9x^2 - 49$ b. $9x^2 - 100$ c. $4x^2 - 25$ d. $16x^2 - 1$

e. $x^2 + 25$ f. $25x^2 - 64$ g. $36x^2 - 81$ h. $49x^2 - 9$

Review: Factor the following expressions:

a. $x^2 + 8x + 16$ b. $x^2 - 2x + 1$ c. $x^2 - 10x + 25$

1. What do you notice about the "a" term? ______________________________

2. What do you notice about the "c" term? ______________________________

3. What do you notice about the "b" term? ______________________________

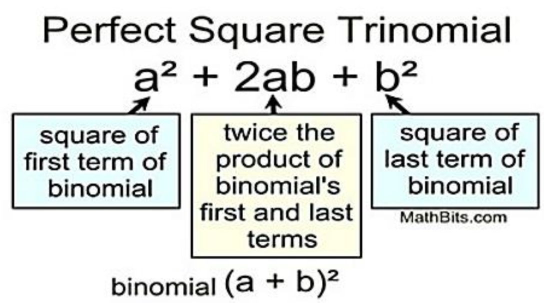
4. What do you notice about the factored form? ______________________________

The above polynomials are a second type of pattern; this pattern type is called a

Perfect Square Trinomials

$$\mathbf{a^2 + 2ab + b^2 = (a + b)^2}$$

$$\mathbf{a^2 - 2ab + b^2 = (a - b)^2}$$



Using the perfect square trinomial pattern, see if you can fill in the blanks below:

a. x^2 + ______ + 36 b. x^2 - ______ + 81 c. x^2 - ______ + 64

d. $x^2 + 4x$ + ______ e. $x^2 - 6x$ + ______ f. $x^2 + 20x$ + _______

Days 5 & 6 – Solving Quadratics (GCF, when a = 1, when a not 1)

Standard(s): ____

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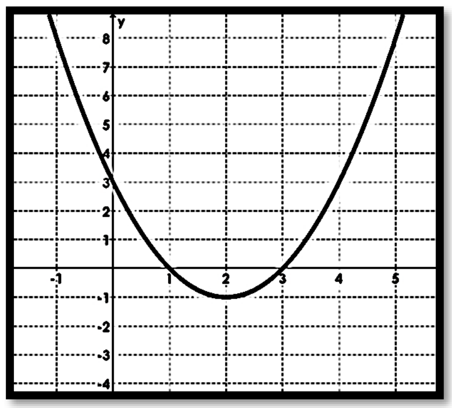
Take a look at the following graph. Do you know what type of graph it is? List some of the things you see:



The Main Characteristics of a Quadratic Function

- A quadratic function always has an exponent of ______. Therefore, a quadratic function always has the term of ______.
- The standard form of a quadratic equation is ____
- The U-shaped graph is called a ___________________________.
- The highest or lowest point on the graph is called the _______________________.
- The points where the graph crosses the x-axis are called the _______________________.
- The points where the graph crosses are also called the ________________________ to the quadratic equation. A quadratic equation can have ________, ________, or ________ solutions.

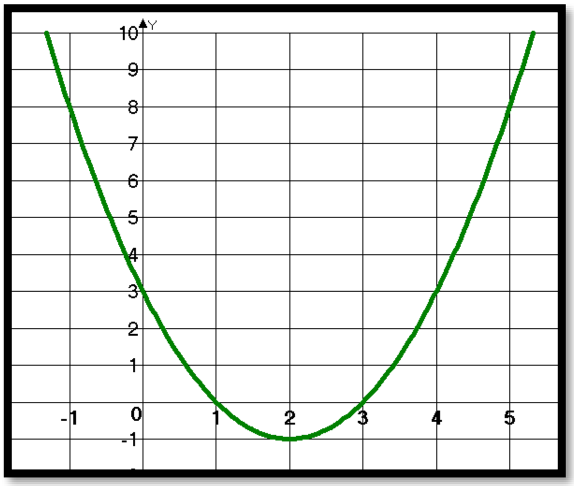
In this unit, we are going to explore how to **solve quadratic equations**. The solutions to the quadratic equations can look very different depending on what the graph of the quadratic equation looks like. We are going to explore how we can apply factoring to solving quadratic equations.

Exploration with Factoring and Quadratic Graphs

Exploration:

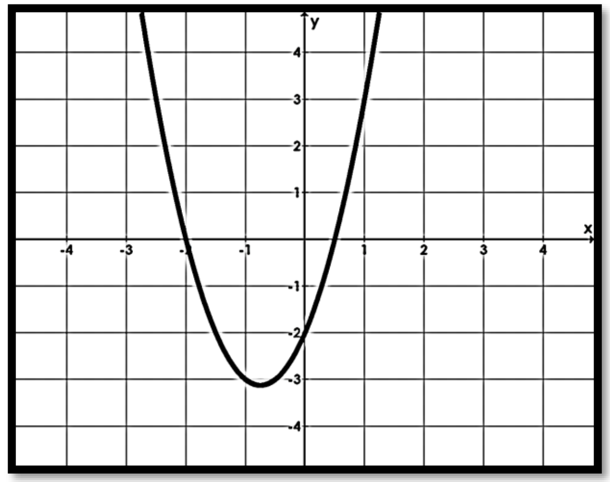
a. Given the equation and the graph, what are the zeros in the following graphs?

Graph 1: $y = x^2 - 4x + 3$



Zeros: ______________________

Graph 2: $y = 2x^2 + 3x - 2$



Zeros: ______________________

b. Were you able to accurately determine the zeros?

c. What if the equation was written in factored form? Can you accurately name the zeros?

Graph 1: $y = (x - 3)(x - 1)$

Graph 2: $y = (x + 2)(2x - 1)$

d. What is the value of y when the parabola crosses the x-axis for each graph?

Zero Product Property and Factored Form

A polynomial or function is in ***factored form*** if it is written as the product of two or more linear binomial factors.

Zero Product Property

- The ***zero product property*** is used to solve an equation when one side is zero and the other side is a product of binomial factors.
- The zero product property states that if $a \cdot b = 0$, then $a = 0$ or $b = 0$

Examples:

a. $(x - 2)(x + 4) = 0$

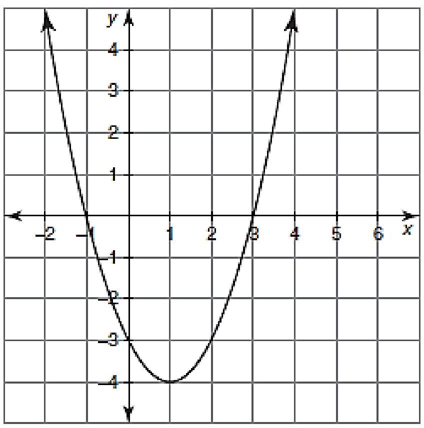
b. $x(x + 4) = 0$

c. $(x + 3)^2 = 0$

Practice: Identify the zeros of the functions:

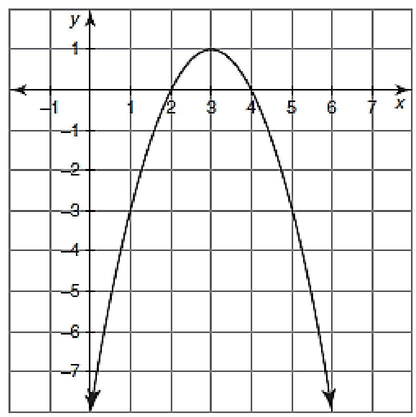
a. y = (x + 4)(x + 3) b. f(x) = (x – 7)(x + 5) c. y = x(x – 9) d. f(x) = 5(x – 4)(x + 8)

Practice: Create an equation to represent the following graphs:



Zeros: x = _____ & ______

y = ________________________



Zeros: x = _____ & _____

y = ________________________

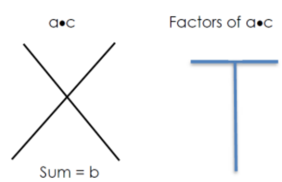
Solving a quadratic equation really means:

The place(s) where the graph crosses the x-axis has several names. They can be referred as:

____________________ ____________________ ____________________ ____________________

Review: Methods for Factoring

Before you factor any expression, you must always check for and factor out a **Greatest Common Factor (GCF)!**

	Looks Like	How to Factor	Examples
GCF (Two Terms)	$ax^2 - bx$	Factor out what is common to both terms (mentally or list method) $2m^2 \cdot 2\,m\,m$ $-8m \cdot -1\,2\,2\,2\,m$ $2m(m-4)$	$x^2 + 5x = x(x+5)$ $18x^2 - 6x = 6x(3x-1)$ $-9x^2 - x = -x(9x+1)$
A = 1	$x^2 + bx + c$	Think of what two numbers multiply to get the c term and add to get the b term (Think of the diamond). You also need to think about the signs: $x^2 + bx + c = (x + \#)(x + \#)$ $x^2 - bx + c = (x - \#)(x - \#)$ $x^2 - bx - c / x^2 + bx - c = (x + \#)(x - \#)$	$x^2 + 8x + 7 = (x+7)(x+1)$ $x^2 - 5x + 6 = (x-2)(x-3)$ $x^2 - x - 56 = (x+7)(x-8)$
A not 1	$ax^2 + bx + c$	**Area Model:** $3x^2 - 5x - 12$ $\begin{array}{c\|c\|c} & 3x & +4 \\ \hline x & 3x^2 & +4x \\ \hline -3 & -9x & -12 \end{array}$  **Factored Form :** $(x-3)(3x+4)$	$9x^2 - 11x + 2 = (9x-2)(x-1)$ $2x^2 + 15x + 7 = (2x+1)(x+7)$ $3x^2 - 5x - 28 = (2x+7)(x-4)$
Difference of Two Squares	$x^2 - c$	Both your "a"and "c" terms should be perfect squares and since there is no "b" term, it has a value of 0. You must also be subtracting the a and c terms. Your binomials will be the exact same except for opposite signs. **Difference of Squares** $a^2 - b^2 = (a+b)(a-b)$	$x^2 - 9 = (x+3)(x-3)$ $x^2 - 100 = (x+10)(x-10)$ $4x^2 - 25 = (2x+5)(2x-5)$
Perfect Square Trinomials	$x^2 + bx + c$ "c" is a perfect square "b" is double the square root of c	Factor like you would for when a = 1	$x^2 - 6x + 9 = (x-3)(x-3)$ $= (x-3)^2$ $x^2 + 16x + 64 = (x+8)(x+8)$ $= (x+8)^2$

Solving Quadratic Equations by Factoring & Using the Zero Product Property

Factor and solve: $2x^2 + 3x = 2$

1. Rewrite the equation so it is set equal to 0.	
2. Check for any GCF's. Then factor.	
3. Using the Zero Product Property, set each factor equal to 0. Solve each equation.	

1: Factoring & Solving Quadratic Equations - GCF

Solve the following quadratic equations by factoring (GCF) and using the Zero Product Property.
Practice: Solve the following equations by factoring out the GCF.

1. $2x^2 - 6x = 0$

Factored Form: ___________________

Zeros: __________________________

2. $x^2 + x = 0$

Factored Form: ___________________

Zeros: __________________________

3. $-3x^2 - 12x = 0$

Factored Form: ___________________

Zeros: __________________________

4. $3x^2 = 18x$

Factored Form: ___________________

Zeros: __________________________

2: Factoring & Solving Quadratic Equations when a =1

Solve the following quadratic equations by factoring and using the Zero Product Property.

1. $x^2 + 6x + 8 = 0$

Factored Form: ________________

Zeros: ______________________

2. $y = x^2 - 6x + 9$

Factored Form: ________________

Zeros: ______________________

3. $x^2 + 4x = 32$

Factored Form: ________________

Zeroes: ______________________

4. $5x = x^2 - 6$

Factored Form: ________________

Zeroes: ______________________

5. $-x^2 = 2x + 1$

Factored Form: ________________

Zeroes: ______________________

6. $y = x^2 - 9$

Factored Form: ________________

Zeroes: ______________________

3: Factoring & Solving Quadratic Equations when a not 1

Solve the following quadratic equations by factoring and using the Zero Product Property.

1. $y = 5x^2 + 14x - 3$

Factored Form: ____________________

Zeroes: ____________________

2. $2x^2 - 8x - 42 = 0$

Factored Form: ____________________

Zeroes: ____________________

3. $7x^2 - 16x = -9$

Factored Form: ____________________

Zeroes: ____________________

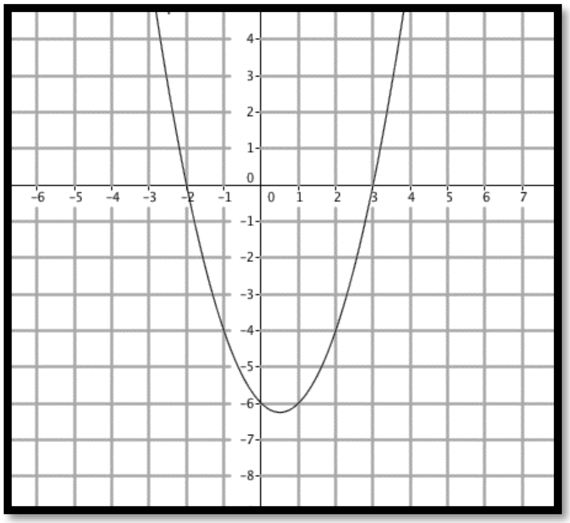
4. $6x^2 + 3 = 11x$

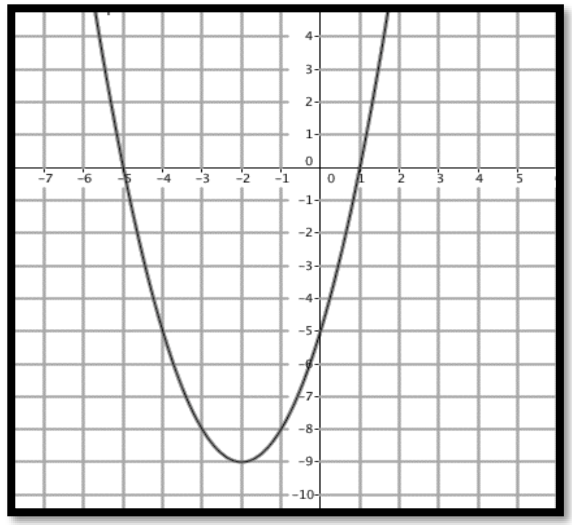
Factored Form: ____________________

Zeroes: ____________________

Putting It All Together

Can you find the factored AND standard form equations for these graphs?
(Remember – standard form is $y = ax^2 + bx + c$)





Factored Form: ______________________________ **Factored Form:** ______________________________

Standard Form: ______________________________ **Standard Form:** ______________________________

How do you transform an equation from factored form TO standard form?

How do you transform an equation from standard form TO factored form?

Day 7 – Solving by Finding Square Roots

Standard(s): ____

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Review: If possible, simplify the following radicals completely.

a. $\sqrt{25}$ b. $\sqrt{125}$ c. $\sqrt{24}$

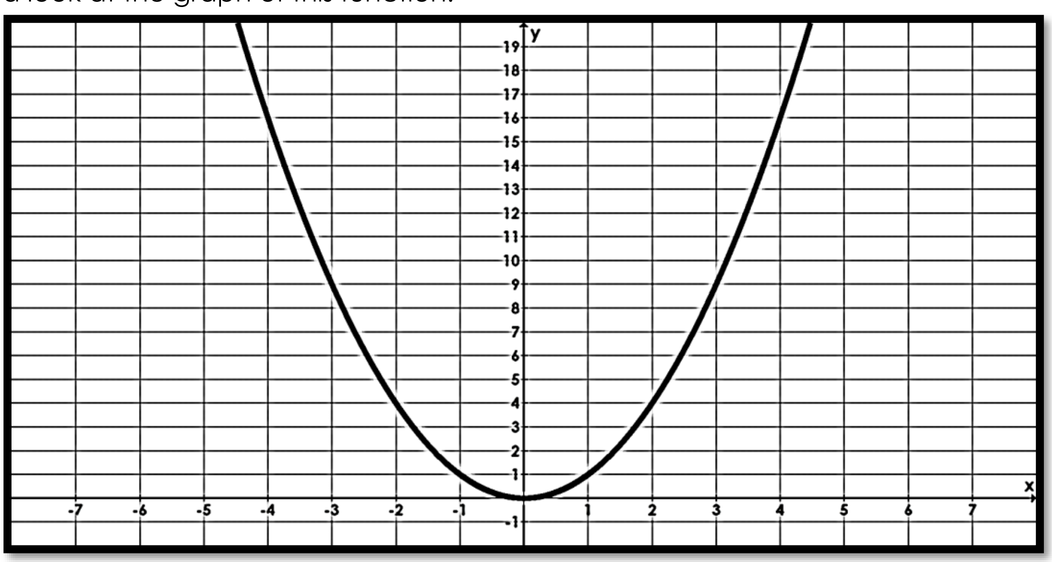
Explore: Solve the following equations for x:

a. $x^2 = 16$ b. $x^2 = 4$ c. $x^2 = 9$ d. $x^2 = 1$

What operation did you perform to solve for x?

How many of you only had one number as an answer for each equation?

Well, let's take a look at the graph of this function.



After looking at the graph, what values of x produce a y value of 1, 4, 9, and 16?

What would be your new answers for the previous equations?

a. $x^2 = 16$ b. $x^2 = 4$ c. $x^2 = 9$ d. $x^2 = 1$

Solving by Taking Square Roots without Parentheses

<u>Steps for Solving Quadratics by Finding Square Roots</u>

1. Add or Subtract any constants that are on the same side of x^2.
2. Multiply or Divide any constants from x^2 terms. "Get x^2 by itself"
3. Take square root of both sides and set equal to positive and negative roots (±).

Ex: $x^2 = 25$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

$x = +5$ and $x = -5$

REMEMBER WHEN SOLVING FOR X YOU GET A ____________ AND ____________ ANSWER!

Solve the following for x:

1) $x^2 = 49$

2) $x^2 = 20$

3) $x^2 = 7$

4) $3x^2 = 108$

5) $2x^2 = 128$

6) $x^2 - 11 = 14$

7) $7x^2 - 6 = 57$

8) $2x^2 + 8 = 170$

9) $x^2 = 0$

10) $10x^2 + 9 = 499$

11) $4x^2 - 6 = 74$

12) $3x^2 + 7 = 301$

Solving by Finding Square Roots (More Complicated)

<u>Steps for Solving Quadratics by Finding Square Roots with Parentheses</u>

1. Add or Subtract any constants outside of any parenthesis.
2. Multiply or Divide any constants around parenthesis/squared term. "Get ($)^2$ by itself"
3. Take square root of both sides and set your expression equal to BOTH the positive and negative root (±). Ex: $(x + 4)^2 = 25$

$$\sqrt{(x+4)^2} = \sqrt{25}$$
$$(x + 4) = \pm 5$$
$$x + 4 = +5 \text{ and } x + 4 = -5$$
$$x = 1 \text{ and } x = -9$$

4. Add, subtract, multiply, or divide any remaining numbers to isolate x.

REMEMBER WHEN SOLVING FOR X YOU GET A POSITIVE AND NEGATIVE ANSWER!

Solve the following for x:

1) $(x-4)^2 = 81$

2) $(p-4)^2 = 16$

3) $10(x-7)^2 = 440$

4) $\frac{1}{2}(x+8)^2 = 14$

5) $-2(x+3)^2 - 16 = -48$

6) $3(x-4)^2 + 7 = 67$

Day 8 – Solving by Completing the Square

Standard(s): ____

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Some trinomials form special patterns that can easily allow you to factor the quadratic equation. We will look at two special cases:

Review: Factor the following trinomials.

1. $x^2 - 6x + 9$	2. $x^2 + 10x + 25$	3. $x^2 - 16x + 64$

(a) How does the constant term in the binomial relate to the b term in the trinomial?

(b) How does the constant term in the binomial relate to the c term in the trinomial?

Problems 1-3 are called **Perfect Square Trinomials**. These trinomials are called perfect square trinomials because when they are in their factored form, they are a binomial squared.

An example would be $x^2 + 12x + 36$. Its factored form is $(x + 6)^2$, which is a binomial squared.

But what if you were not given the c term of a trinomial? How could we find it?

Complete the square to form a perfect square trinomial and then factor.

a. $x^2 + 12x + \square$ b. $z^2 - 4z + \square$ c. $x^2 - 18x + \square$

Solving equations by "COMPLETING THE SQUARE" $\boxed{}$

The Equation:	$x^2 + 6x + 2 = 0$
STEP 1: Write the equation in the form $x^2 + bx + \boxed{} = c + \boxed{}$ (Bring the constant to the other side)	$x^2 + 6x + \boxed{} = -2 + \boxed{}$
STEP 2: Make the left hand side a perfect square trinomial by adding $\left(\frac{b}{2}\right)^2$ to **both** sides	$x^2 + 6x + \boxed{(3)^2} = -2 + \boxed{(3)^2}$
STEP 3: Factor the left side, simplify the right side	$(x+3)^2 = 7$
STEP 4: Solve by taking square roots on both sides	$x+3 = \sqrt{7}$ and $x+3 = -\sqrt{7}$ $x = \sqrt{7} - 3$ and $x = -\sqrt{7} - 3$

Group Practice: Solve for x by "Completing the Square".

1. $x^2 - 6x - 72 = 0$

X = ______________________

2. $x^2 + 80 = 18x$

X = ______________________

3. $x^2 - 14x - 59 = -20$

X = ______________________

4. $2x^2 - 36x + 10 = 0$

X = ______________________

Day 9- Solving by Quadratic Formula

Standard(s): ____

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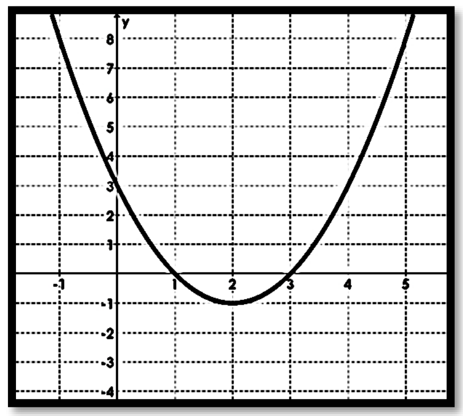
Exploring the Nature of Roots

In this task you will investigate the number of real solutions to a quadratic equation.

1. $f(x) = x^2 - 4x + 3$

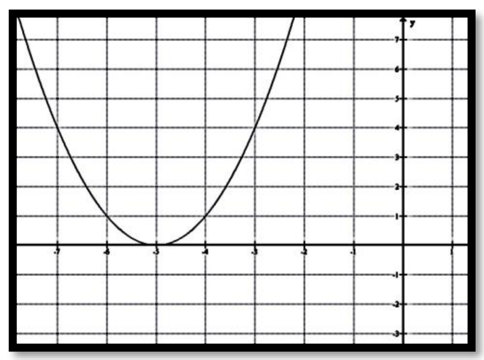


a) How many x-intercepts does the function have?

b) Label and state the x-intercept(s), if any.

c) Solve the quadratic function by factoring, if possible.

2. $f(x) = x^2 + 10x + 25$

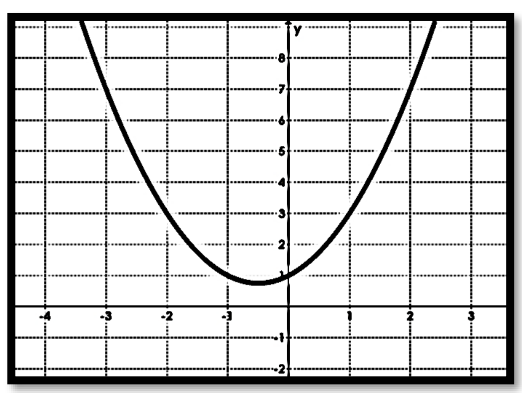


a) How many x-intercepts does the function have?

b) Label and state the x-intercept(s), if any.

c) Solve the quadratic function by factoring, if possible.

3. $f(x) = x^2 + x + 1$



a) How many x-intercepts does the function have?

b) Label and state the x-intercept(s), if any.

c) Solve the quadratic function by factoring, if possible.

4. a) What is true about quadratics (and their graphs) with two real solutions?
   b) What is true about quadratics (and their graphs) with only one real solution?
   c) What is true about quadratics (and their graphs) with no real solutions?

The Discriminant

Instead of observing a quadratic function's graph and/or solving it by factoring, there is an alternative way to determine the number of real solutions called the **discriminant**.

Given a quadratic function in standard form:
$ax^2 + bx + c = 0, \text{ where } a \neq 0$,

The **discriminant** is found by using: ***$b^2 - 4ac$***

This value is used to determine the number of real solutions/zeros/roots/x-intercepts that exist for a quadratic equation.

Interpretation of the Discriminant ($b^2 - 4ac$)

- If $b^2 - 4ac$ is positive:
- If $b^2 - 4ac$ is zero:
- If $b^2 - 4ac$ is negative:

Practice: Find the discriminant for the previous three functions:

a.) $f(x) = x^2 - 4x + 3$

a = ______ b = ______ c = ______ Discriminant: ________________________

#. of real solutions: __________________

b.) $f(x) = x^2 + 10x + 25$

a = ______ b = ______ c = ______ Discriminant: ________________________

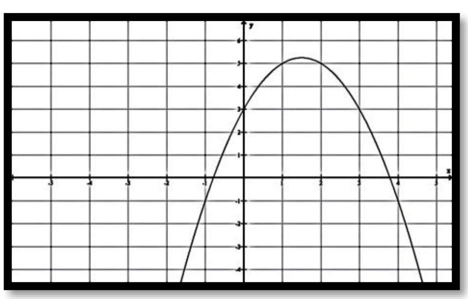
# of real zeros: _____________________

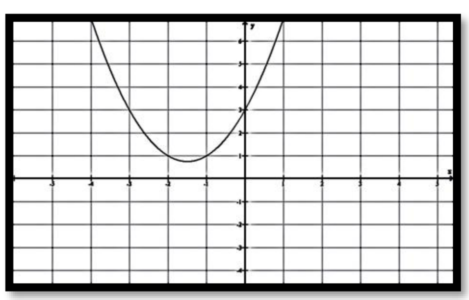
c.) $f(x) = x^2 + x + 1$

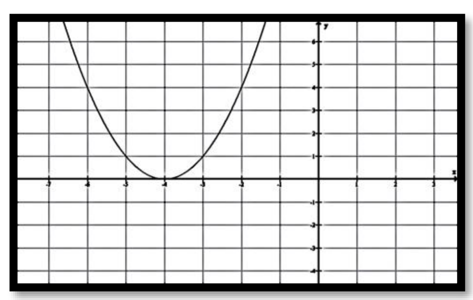
a = ______ b = ______ c = ______ Discriminant: ________________________

# of real roots: _____________________

Practice: Determine whether the discriminant would be greater than, less than, or equal to zero.







The Quadratic Formula

We have learned **three methods** for solving quadratics:

- **Factoring** *(Only works if the equation is factorable)*
- **Taking the Square Roots** *(Only works when equations are not in Standard Form)*
- **Completing the Square** *(Only works when a is 1 and b is even)*

What method do you use when your equations are not factorable, but are in standard form, and a may not be 1 and b may not be even?

The Quadratic Formula

for equations in standard form: $y = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

x represents the zeros and $b^2 - 4ac$ is the discriminant

Practice with the Quadratic Formula

For the quadratic equations below, use the quadratic formula to find the solutions. Write your answer in simplest radical form.

1) $4x^2 - 13x + 3 = 0$ a = ____ b = ____ c = _____

2) $9x^2 + 6x + 1 = 0$ a = ____ b = ____ c = _____

Discriminant: ________________

Solutions: ___________________

Approx: ____________________

Discriminant: ________________

Zeros: _______________________

Approx: _____________________

3) $7x^2 + 8x + 3 = 0$ a = ____ b = ____ c = _____

4) $-3x^2 + 2x = -8$ a = ____ b = ____ c = _____

Discriminant: ____________

X = _____________________

Approx: _____________________

Discriminant: ____________

Roots: __________________

Approx: ______________________

Day 10 – Applications of Quadratics

Standard(s): ____

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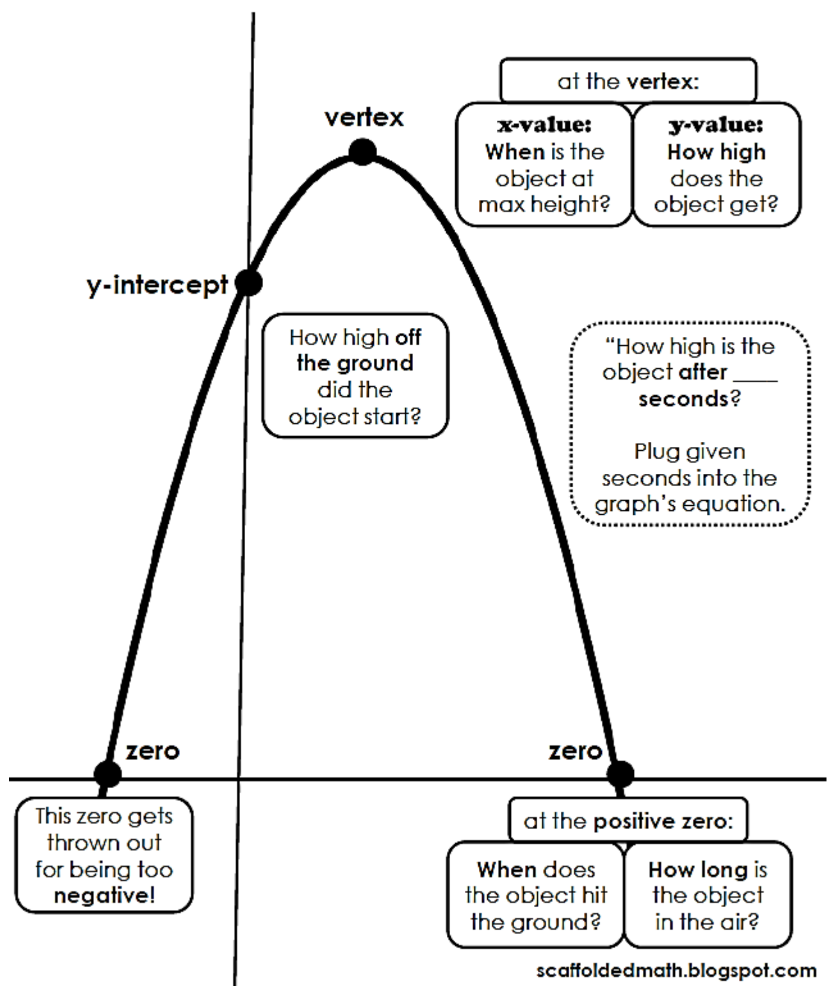
If you are solving for the vertex:	If you are solving for the zeros:
-Maximum/Minimum (height, cost, etc) -Greatest/Least Value -Maximize/Minimize -Highest/Lowest	-How long did it take to reach the ground? -How long is an object in the air? -How wide is an object? -Finding a specific measurement/dimension

Vertical Motion Models

Falling Objects: $h = -16t^2 + h_0$ h_0 = starting height, h = ending height

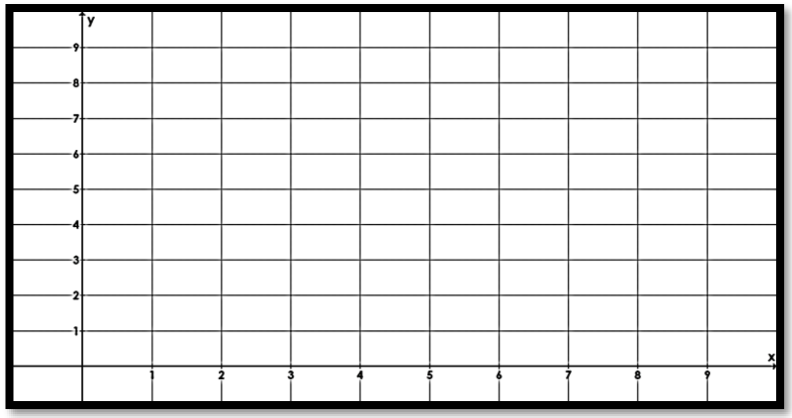
Thrown Object: $h = -16t^2 + vt + h_0$ h_0 = starting height, h = ending height, v = velocity (going up (+) or down (-))

Quadratic Keywords



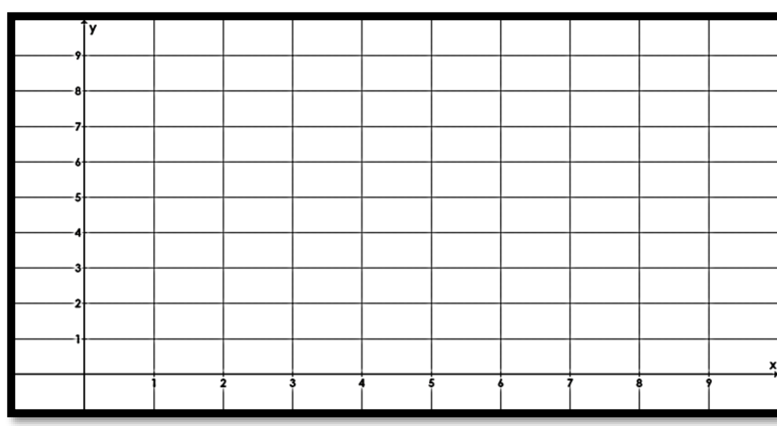
Scenario 1. *Suppose the flight of a launched bottle rocket can be modeled by the equation $y = -x^2 + 6x$, where y measures the rocket's height above the ground in meters and x represents the rocket's horizontal distance in meters from the launching spot at $x = 0$.*

a. How far has the bottle rocket traveled horizontally when it reaches it maximum height? What is the maximum height the bottle rocket reaches?



b. When is the bottle rocket on the ground? How far does the bottle rocket travel in the horizontal direction from launch to landing?

Scenario 2. *A frog is about to hop from the bank of a creek. The path of the jump can be modeled by the equation $h(x) = -x^2 + 4x + 1$, where h(x) is the frog's height above the water and x is the number of seconds since the frog jumped. A fly is cruising at a height of 5 feet above the water. Is it possible for the frog to catch the fly, given the equation of the frog's jump?*



b. When does the frog land back in the water?

Scenario 3. *A school is planning to host a dance with all profits going to charity. The amount of profit is found by subtracting the total costs from the total income. The income from ticket sales can be expressed as $200x - 10x^2$, where x is the cost of a ticket. The costs of putting on the dance can be expressed as $500 + 20x$.*

a. What are the ticket prices that will result in the dance breaking even?

b. What are ticket prices that will result in a profit of \$200?

Applications of Solving by Square Roots

Falling Objects: **$h = -16t^2 + h_0$** **h_0 = starting height, h = ending height**

Scenario 4. The tallest building in the USA is in Chicago, Illinois. It is 1450 ft tall. How long would it take a penny to drop from the top of the building to the ground?

Scenario 5. When an object is dropped from a height of 72 feet, how long does it take the object to hit the ground?

Day 11 – Determining the Best Method

Standard(s): ____

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Non Factorable Methods		
Completing the Square	**Finding Square Roots**	**Quadratic Formula**
$ax^2 + bx + c = 0$, *when $a = 1$ and b is an even #* **Examples** $x^2 – 6x + 11 = 0$ $x^2 – 2x - 20 = 0$	*$ax^2 - c = 0$* Parenthesis in equation **Examples** $2x^2 + 5 = 9$ $5(x + 3)^2 – 5 = 20$ $x^2 – 36 = 0$	*$ax^2 + bx + c = 0$* Any equation in standard form Large coefficients **Examples** $3x^2 + 9x – 1 = 0$ $20x^2 + 36x – 17 = 0$

Factorable Methods	
A = 1 & A Not 1 (Factor into 2 Binomials)	**GCF**
$ax^2 + bx + c = 0$, when $a = 1$ *$ax^2 \pm bx \pm c = 0$, when $a > 1$* *$x^2 - c = 0$* **Examples** $3x^2 – 20x – 7 = 0$ $x^2 – 3x + 2 = 0$ $x^2 + 5x = -6$ $x^2 – 25 = 0$	*$ax^2 + bx = 0$* **Examples** $5x^2 + 20x = 0$ $x^2 – 6x = 8x$

Determine the best method for solving. Explain why.

1. $6x^2 - 11x + 3 = 0$

2. $x^2 + 6x – 45 = 0$

3. $x^2 – 7x = 8$

4. $8x^2 + 24x = 0$

5. $2x^2 - 11x + 5 = 0$

6. $x^2 - 9x = -20$

7. $x^2 - 9 = 0$

8. $x^2 + 4x + 17 = 0$

9. $2x^2 + 6x - 37 = 0$

10. $4(x + 4)^2 = 16$

11. $x^2 - 15x + 36 = 0$

12. $18x^2 + 100x = 63$