UNIT 4: CIRCLES AND VOLUME

This unit investigates the properties of circles and addresses finding the volume of solids. Properties of circles are used to solve problems involving arcs, angles, sectors, chords, tangents, and secants. Volume formulas are derived and used to calculate the volumes of cylinders, pyramids, cones, and spheres.

Understand and Apply Theorems about Circles

MGSE9-12.G.C.1 Understand that all circles are similar.

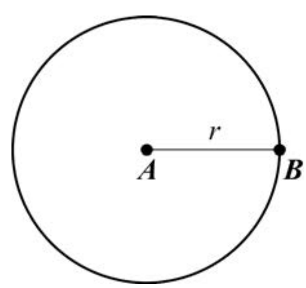
MGSE9-12.G.C.2 Identify and describe relationships among inscribed angles, radii, chords, tangents, and secants. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

MGSE9-12.G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

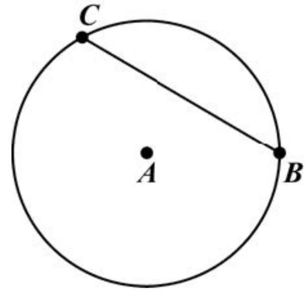
MGSE9-12.G.C.4 Construct a tangent line from a point outside a given circle to the circle.

KEY IDEAS

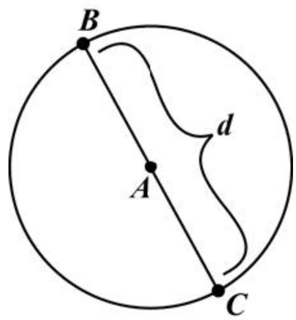
1. A ***circle*** is the set of points in a plane equidistant from a given point, which is the center of the circle. All circles are similar.
2. A ***radius*** is a line segment from the center of a circle to any point on the circle. The word radius is also used to describe the length, r, of the segment. $\overline{AB}$ is a radius of circle A.



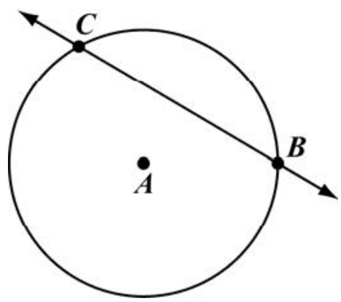
3. A ***chord*** is a line segment whose endpoints are on a circle. $\overline{BC}$ is a chord of circle A.



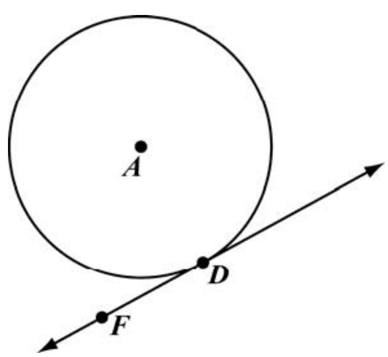
4. A ***diameter*** is a chord that passes through the center of a circle. The word diameter is also used to describe the length, *d,* of the segment. $\overline{BC}$ is a diameter of circle *A*.



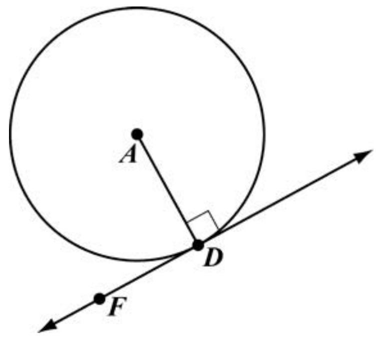
5. A ***secant line*** is a line that is in the plane of a circle and intersects the circle at exactly two points. Every chord lies on a secant line. $\overleftrightarrow{BC}$ is a secant line of circle *A*.



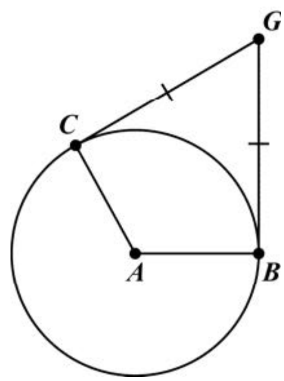
6. A ***tangent line*** is a line that is in the plane of a circle and intersects the circle at only one point, the ***point of tangency***. $\overleftrightarrow{DF}$ is tangent to circle *A* at the point of tangency, point *D.*



7. If a line is tangent to a circle, the line is perpendicular to the radius drawn to the point of tangency. $\overleftrightarrow{DF}$ is tangent to circle A at point D, so $\overline{AD} \perp \overline{DF}$.

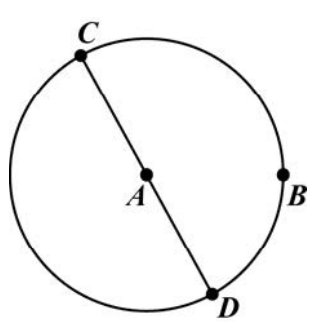


8. Tangent segments drawn from the same point are congruent. In circle A, $\overline{CG} \cong \overline{BG}$.

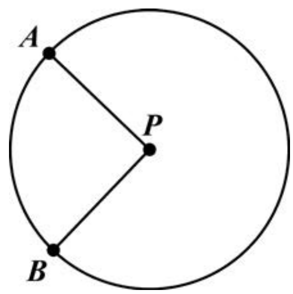


9. ***Circumference*** is the distance around a circle. The formula for circumference C of a circle is $C = \pi d$, where d is the diameter of the circle. The formula is also written as $C = 2\pi r$, where r is the length of the radius of the circle. π is the ratio of circumference to diameter of any circle.

10. An ***arc*** is a part of the circumference of a circle. A ***minor arc*** has a measure less than 180°. Minor arcs are written using two points on a circle. A ***semicircle*** is an arc that measures exactly 180°. Semicircles are written using three points on a circle. This is done to show which half of the circle is being described. A ***major arc*** has a measure greater than 180°. Major arcs are written with three points to distinguish them from the corresponding minor arc. In circle A, $\overset{\frown}{CB}$ is a minor arc, $\overset{\frown}{CBD}$ is a semicircle, and $\overset{\frown}{CDB}$ is a major arc.

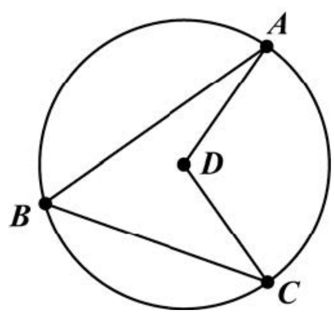


11. A ***central angle*** is an angle whose vertex is at the center of a circle and whose sides are radii of the circle. The measure of a central angle of a circle is equal to the measure of the intercepted arc. $\angle APB$ is a central angle for circle P, and $\overset{\frown}{AB}$ is the intercepted arc.



$$m\angle APB = m\overset{\frown}{AB}$$

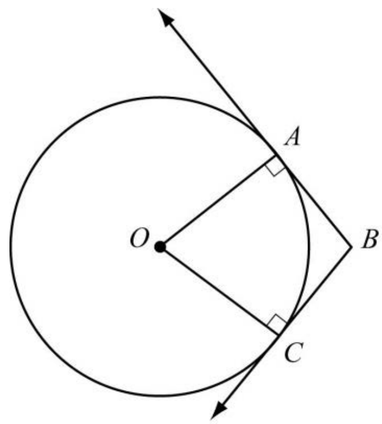
12. An ***inscribed angle*** is an angle whose vertex is on a circle and whose sides are chords of the circle. The measure of an angle inscribed in a circle is half the measure of the intercepted arc. For circle D, $\angle ABC$ is an inscribed angle, and $\overset{\frown}{AC}$ is the intercepted arc.



$$m\angle ABC = \frac{1}{2}m\overset{\frown}{AC} = \frac{1}{2}\ m\angle ADC$$

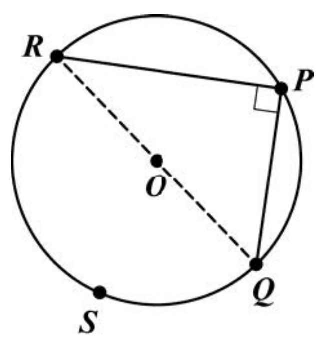
$$m\angle ADC = m\overset{\frown}{AC} = 2(m\angle ABC)$$

13. A ***circumscribed angle*** is an angle formed by two rays that are each tangent to a circle. These rays are perpendicular to radii of the circle. In circle *O*, the measure of the circumscribed angle is equal to 180° minus the measure of the central angle that forms the intercepted arc. The measure of the circumscribed angle can also be found by using the measures of two intercepted arcs [see Key Idea 18].



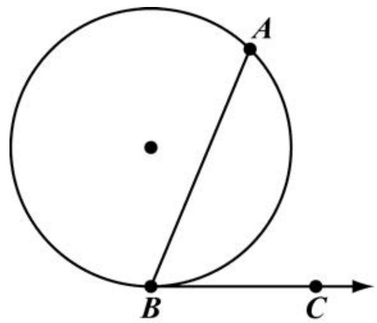
$$m\angle ABC = 180° - m\angle AOC$$

14. When an inscribed angle intercepts a semicircle, the inscribed angle has a measure of 90°. For circle *O*, $\angle RPQ$ intercepts semicircle *RSQ* as shown.



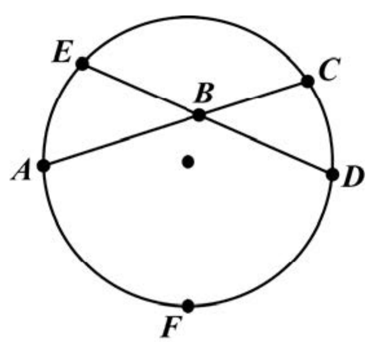
$$m\angle RPQ = \frac{1}{2}\left(m\widehat{RSQ}\right) = \frac{1}{2}(180°) = 90°$$

15. The measure of an angle formed by a tangent and a chord with its vertex on the circle is half the measure of the intercepted arc. $\overline{AB}$ is a chord for the circle, and $\overrightarrow{BC}$ is tangent to the circle at point B. So, $\angle ABC$ is formed by a tangent and a chord.



$$m\angle ABC = \frac{1}{2}\left(m\widehat{AB}\right)$$

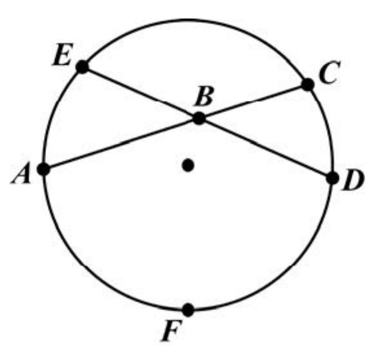
16. When two chords intersect inside a circle, two pairs of vertical angles are formed. The measure of any one of the angles is half the sum of the measures of the arcs intercepted by the pair of vertical angles.



$$m\angle ABE = \frac{1}{2}\left(m\widehat{AE} + m\widehat{CD}\right)$$

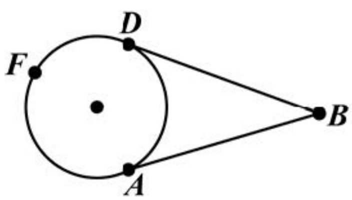
$$m\angle ABD = \frac{1}{2}\left(m\widehat{AFD} + m\widehat{EC}\right)$$

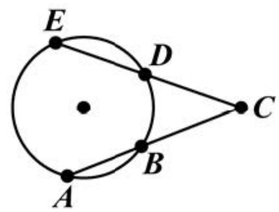
17. When two chords intersect inside a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

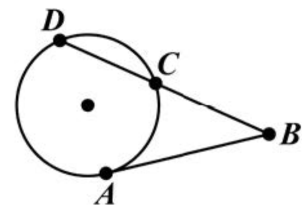


$$AB \bullet BC = EB \bullet BD$$

18. Angles outside a circle can be formed by the intersection of two tangents (circumscribed angle), two secants, or a secant and a tangent. For all three situations, the measure of the angle is half the difference of the measure of the larger intercepted arc and the measure of the smaller intercepted arc.

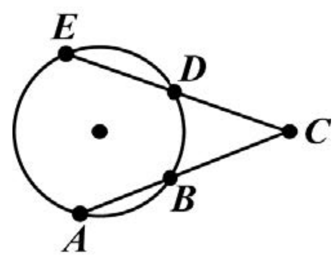






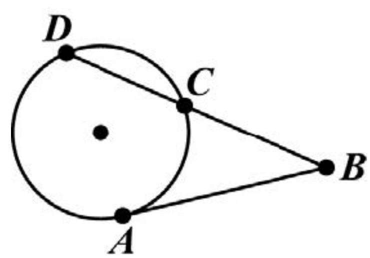
$m\angle ABD = \frac{1}{2}\left(m\widehat{AFD} - m\widehat{AD}\right)$ $m\angle ACE = \frac{1}{2}\left(m\widehat{AE} - m\widehat{BD}\right)$ $m\angle ABD = \frac{1}{2}\left(m\widehat{AD} - m\widehat{AC}\right)$

19. When two secant segments intersect outside a circle, part of each secant segment is a segment formed outside the circle. The product of the length of one secant segment and the length of the segment formed outside the circle is equal to the product of the length of the other secant segment and the length of the segment formed outside the circle.



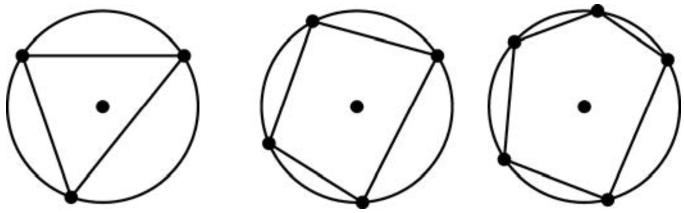
$EC \bullet DC = AC \bullet BC$

20. When a secant segment and a tangent segment intersect outside a circle, the product of the length of the secant segment and the length of the segment formed outside the circle is equal to the square of the length of the tangent segment.

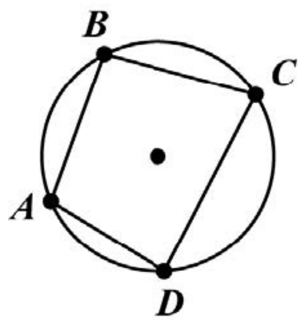


$DB \bullet CB = AB^2$

21. An ***inscribed polygon*** is a polygon whose vertices all lie on a circle. This diagram shows a triangle, a quadrilateral, and a pentagon each inscribed in a circle.



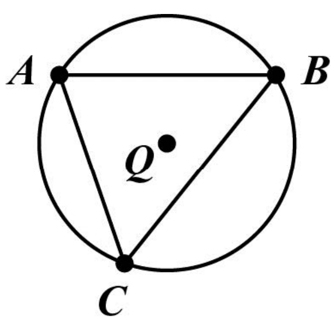
22. In a quadrilateral inscribed in a circle, the opposite angles are supplementary.



$m\angle ABC + m\angle ADC = 180°$

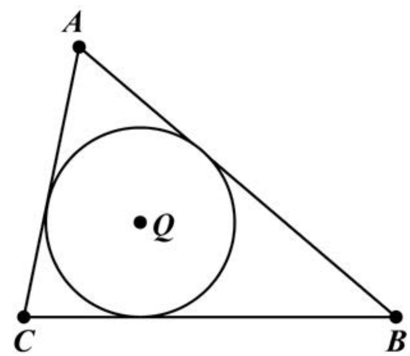
$m\angle BCD + m\angle BAD = 180°$

23. When a triangle is inscribed in a circle, the center of the circle is the ***circumcenter*** of the triangle. The circumcenter is equidistant from the vertices of the triangle. Triangle *ABC* is inscribed in circle *Q*, and point *Q* is the circumcenter of the triangle.



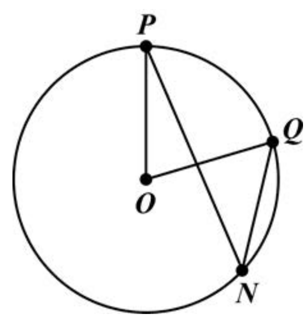
$AQ = BQ = CQ$

24. An ***inscribed circle*** is a circle enclosed in a polygon, where every side of the polygon is tangent to the circle. Specifically, when a circle is inscribed in a triangle, the center of the circle is the ***incenter*** of the triangle. The incenter is equidistant from the sides of the triangle. Circle *Q* is inscribed in triangle *ABC*, and point *Q* is the incenter of the triangle. Notice also that the sides of the triangle form circumscribed angles with the circle.



REVIEW EXAMPLES

1. $\angle PNQ$ is inscribed in circle *O* and $m\overset{\frown}{PQ} = 70°$.



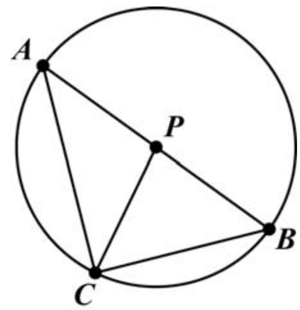
a. What is the measure of $\angle POQ$?
b. What is the relationship between $\angle POQ$ and $\angle PNQ$?
c. What is the measure of $\angle PNQ$?

Solution:

a. The measure of a central angle is equal to the measure of the intercepted arc. $m\angle POQ = m\overset{\frown}{PQ} = 70°$.
b. $\angle POQ$ is a central angle that intercepts $\overset{\frown}{PQ}$. $\angle PNQ$ is an inscribed angle that intercepts $\overset{\frown}{PQ}$. The measure of the central angle is equal to the measure of the intercepted arc. The measure of the inscribed angle is equal to one-half the measure of the intercepted arc. So $m\angle POQ = m\overset{\frown}{PQ}$ and $m\angle PNQ = \frac{1}{2}m\overset{\frown}{PQ}$, so $m\angle POQ = 2m\angle PNQ$.
c. From part (b), $m\angle POQ = 2m\angle PNQ$

Substitute: $70° = 2m\angle PNQ$
Divide: $35° = m\angle PNQ$

2. In circle P below, $\overline{AB}$ is a diameter.



If $m\angle APC = 100°$, find the following:

a. $m\angle BPC$
b. $m\angle BAC$
c. $m\overset{\frown}{BC}$
d. $m\overset{\frown}{AC}$

Solution:

a. $\angle APC$ and $\angle BPC$ are supplementary, so $m\angle BPC = 180° - m\angle APC$, so $m\angle BPC = 180° - 100° = 80°$.
b. $\angle BAC$ is an angle in $\triangle APC$. The sum of the measures of the angles of a triangle is 180°.

For $\triangle APC$: $m\angle APC + m\angle BAC + m\angle ACP = 180°$

You are given that $m\angle APC = 100°$.

Substitute: $100° + m\angle BAC + m\angle ACP = 180°$

Subtract 100° from both sides: $m\angle BAC + m\angle ACP = 80°$

Because two sides of $\triangle APC$ are radii of the circle, $\triangle APC$ is an isosceles triangle. This means that the two base angles are congruent, so $m\angle BAC = m\angle ACP$.

Substitute: $m\angle BAC$ for $m\angle ACP$: $m\angle BAC + m\angle BAC = 80°$

Add: $2m\angle BAC = 80°$

Divide: $m\angle BAC = 40°$

You could also use the answer from part (a) to solve for $m\angle BAC$. Part (a) shows $m\angle BPC = 80°$.

Because the central angle measure is equal to the measure of the intercepted arc, $m\angle BPC = m\overset{\frown}{BC} = 80°$.

Because an inscribed angle is equal to one-half the measure of the intercepted arc, $m\angle BAC = \frac{1}{2}m\overset{\frown}{BC}$.

By substitution: $m\angle BAC = \frac{1}{2}(80°)$

Therefore, $m\angle BAC = 40°$.

c. $\angle BAC$ is an inscribed angle intercepting $\overset{\frown}{BC}$. The intercepted arc is twice the measure of the inscribed angle.

$m\overset{\frown}{BC} = 2m\angle BAC$

From part (b), $m\angle BAC = 40°$.

Substitute: $m\overset{\frown}{BC} = 2 \cdot 40°$

$m\overset{\frown}{BC} = 80°$

You could also use the answer from part (a) to solve. Part (a) shows $m\angle BPC = 80°$. Because $\angle BPC$ is a central angle that intercepts $\overset{\frown}{BC}$, $m\angle BPC = m\overset{\frown}{BC} = 80°$.

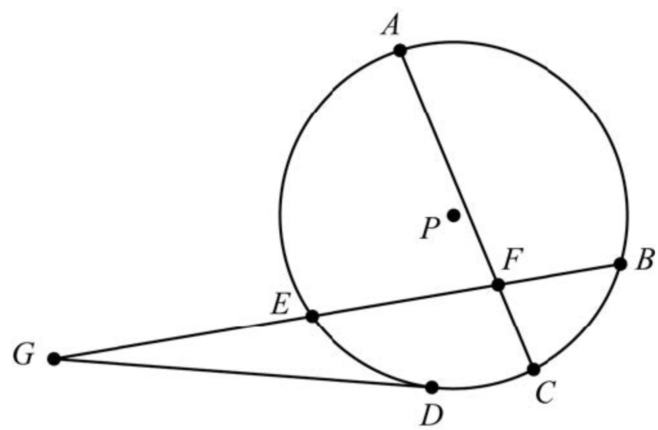
d. $\angle APC$ is a central angle intercepting $\overset{\frown}{AC}$. The measure of the intercepted arc is equal to the measure of the central angle.

$m\overset{\frown}{AC} = m\angle APC$

You are given $m\angle APC = 100°$.

Substitute: $m\overset{\frown}{AC} = 100°$

3. In circle P below, $\overline{DG}$ is a tangent. $AF = 8$, $EF = 6$, $BF = 4$, and $EG = 8$.



Find CF and DG.

Solution:

First, find CF. Use the fact that $\overline{CF}$ is part of a pair of intersecting chords.

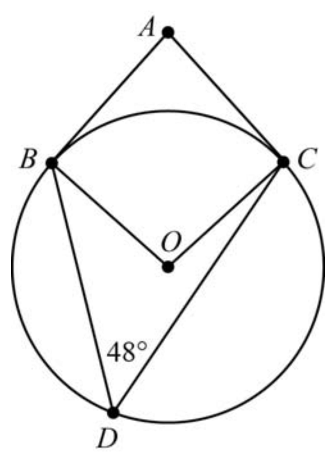
$$AF \cdot CF = EF \cdot BF$$
$$8 \cdot CF = 6 \cdot 4$$
$$8 \cdot CF = 24$$
$$CF = 3$$

Next, find DG. Use the fact that $\overline{DG}$ is tangent to the circle.

$$\begin{aligned} EG \cdot BG &= DG^2 \\ 8 \cdot (8+6+4) &= DG^2 \\ 8 \cdot 18 &= DG^2 \\ 144 &= DG^2 \\ \pm 12 &= DG \\ 12 &= DG \text{ (since length cannot be negative)} \end{aligned}$$

$CF = 3$ and $DG = 12$.

4. In this circle, $\overline{AB}$ is tangent to the circle at point B, $\overline{AC}$ is tangent to the circle at point C, and point D lies on the circle. What is $m\angle BAC$?



Solution:

Method 1

First, find the measure of angle BOC. Angle BDC is an inscribed angle, and angle BOC is a central angle.

$$\begin{aligned} m\angle BOC &= 2 \bullet m\angle BDC \\ &= 2 \bullet 48° \\ &= 96° \end{aligned}$$

Angle BAC is a circumscribed angle. Use the measure of angle BOC to find the measure of angle BAC.

$$\begin{aligned} m\angle BAC &= 180° - m\angle BOC \\ &= 180° - 96° \\ &= 84° \end{aligned}$$

Method 2

Angle *BDC* is an inscribed angle. First, find the measures of $\overparen{BC}$ and $\overparen{BDC}$.

$$m\angle BDC = \frac{1}{2} \bullet m\overparen{BC}$$
$$48^\circ = \frac{1}{2} \bullet m\overparen{BC}$$
$$2 \bullet 48^\circ = m\overparen{BC}$$
$$96^\circ = m\overparen{BC}$$

$$\begin{aligned} m\overparen{BDC} &= 360^\circ - m\overparen{BC} \\ &= 360^\circ - 96^\circ \\ &= 264^\circ \end{aligned}$$

Angle *BAC* is a circumscribed angle. Use the measures of $\overparen{BC}$ and $\overparen{BDC}$ to find the measure of angle *BAC*.

$$\begin{aligned} m\angle BAC &= \frac{1}{2}\left(m\overparen{BDC} - m\overparen{BC}\right) \\ &= \frac{1}{2}\left(264^\circ - 96^\circ\right) \\ &= \frac{1}{2}\left(168^\circ\right) \\ &= 84^\circ \end{aligned}$$

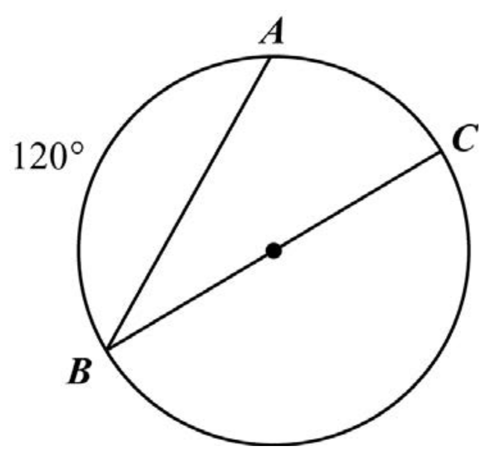
SAMPLE ITEMS

1. Circle *P* is dilated to form circle *P′*. Which statement is ALWAYS true?

A. The radius of circle *P* is equal to the radius of circle *P′*.
B. The length of any chord in circle *P* is greater than the length of any chord in circle *P′*.
C. The diameter of circle *P* is greater than the diameter of circle *P′*.
D. The ratio of the diameter to the circumference is the same for both circles.

Correct Answer: D

2. In the circle shown, $\overline{BC}$ is a diameter and $m\overset{\frown}{AB} = 120°$.



What is the measure of ∠*ABC*?

A. 15°
B. 30°
C. 60°
D. 120°

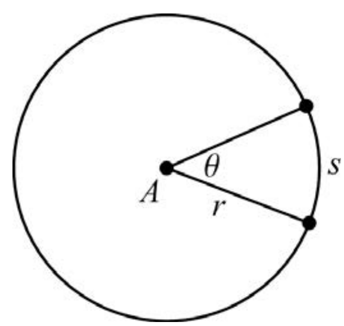
Correct Answer: B

Find Arc Lengths and Areas of Sectors of Circles

MGSE9-12.G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

KEY IDEAS

1. ***Circumference*** is the distance around a circle. The formula for the circumference, C, of a circle is $C = 2\pi r$, where r is the length of the radius of the circle.
2. ***Area*** is a measure of the amount of space a circle covers. The formula for the area, A, of a circle is $A = \pi r^2$, where r is the length of the radius of the circle.
3. ***Arc length*** is a portion of the circumference of a circle. To find the length of an arc, divide the number of degrees in the central angle of the arc by 360, and then multiply that amount by the circumference of the circle. The formula for the arc length, s, is $s = \frac{2\pi r\theta}{360}$, where θ is the degree measure of the central angle and r is the radius of the circle.



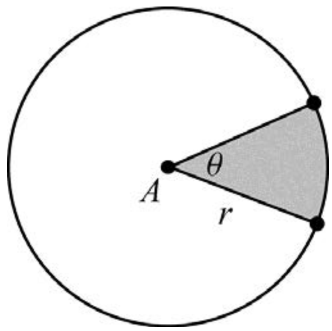
Important Tip

✍ Do not confuse arc length with the measure of the arc in degrees. Arc length depends on the size of the circle because it is part of the circumference of the circle. The measure of the arc is independent of (does not depend on) the size of the circle.

One way to remember the formula for arc length:

arc length = fraction of the circle × circumference = $s = \frac{2\pi r\theta}{360}$.

4. A ***sector*** of a circle is the region bounded by two radii of a circle and the resulting arc between them. To find the area of a sector, divide the number of degrees in the central angle of the arc by 360, and then multiply that amount by the area of the circle. The formula for area of sector = $\frac{\pi r^2\theta}{360}$, where θ is the degree measure of the central angle and r is the radius of the circle.

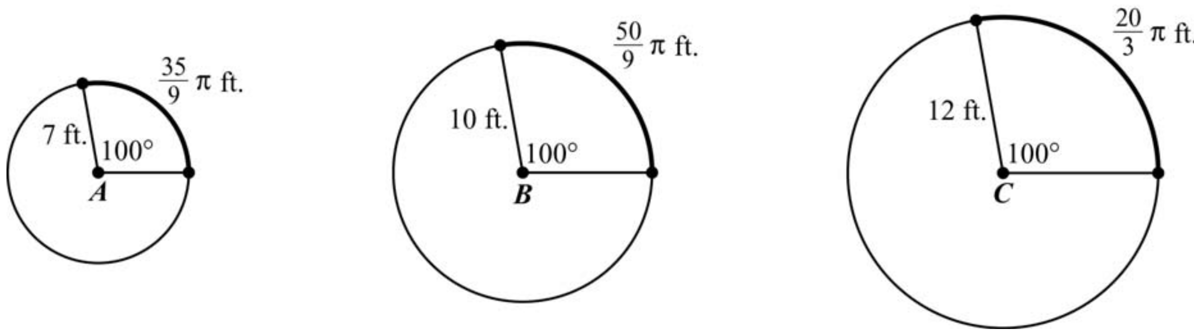


Important Tip

✍ One way to remember the formula for area of a sector:
area of a sector = fraction of the circle × area = $\frac{\theta}{360}\left(\pi r^2\right)$.

REVIEW EXAMPLES

1. Circles *A*, *B*, and *C* have a central angle measuring 100°. The length of each radius and the length of each intercepted arc are shown.



   a. What is the ratio of the radius of circle *B* to the radius of circle *A*?
   b. What is the ratio of the length of the intercepted arc of circle *B* to the length of the intercepted arc of circle *A*?
   c. Compare the ratios in parts (a) and (b).
   d. What is the ratio of the radius of circle *C* to the radius of circle *B*?
   e. What is the ratio of the length of the intercepted arc of circle *C* to the length of the intercepted arc of circle *B*?
   f. Compare the ratios in parts (d) and (e).
   g. Based on your observations of circles *A*, *B*, and *C*, what conjecture can you make about the length of the arc intercepted by a central angle and the radius?
   h. What is the ratio of arc length to radius for each circle?

Solution:

a. Divide the radius of circle *B* by the radius of circle *A*: $\frac{\text{circle B}}{\text{circle A}} = \frac{10}{7}$

b. Divide the length of the intercepted arc of circle *B* by the length of the intercepted arc of circle *A*:

$$\frac{\frac{50}{9}\pi}{\frac{35}{9}\pi} = \frac{50\pi}{9} \cdot \frac{9}{35\pi} = \frac{10}{7}$$

c. The ratios are the same.

d. Divide the radius of circle *C* by the radius of circle *B*: $\frac{\text{circle C}}{\text{circle B}} = \frac{12}{10} = \frac{6}{5}$

e. Divide the length of the intercepted arc of circle *C* by the length of the intercepted arc of circle *B*: $\frac{\frac{20}{3}\pi}{\frac{50}{9}\pi} = \frac{20\pi}{3} \cdot \frac{9}{50\pi} = \frac{6}{5}$

f. The ratios are the same.

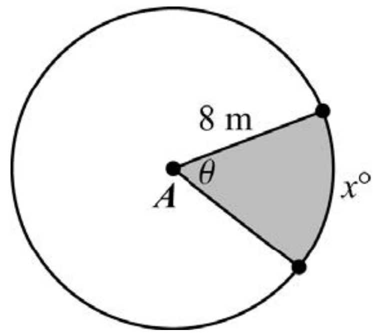
g. When circles, such as circles *A*, *B*, and *C*, have the same central angle measure, the ratio of the lengths of the intercepted arcs is the same as the ratio of the radii.

h. Circle *A*: $\frac{\frac{35}{9}\pi}{7} = \frac{35}{63}\pi = \frac{5}{9}\pi$

Circle *B*: $\frac{\frac{50}{9}\pi}{10} = \frac{50}{90}\pi = \frac{5}{9}\pi$

Circle *C*: $\frac{\frac{20}{3}\pi}{12} = \frac{20}{36}\pi = \frac{5}{9}\pi$

2. Circle A is shown.



If $x = 50$, what is the area of the shaded sector of circle A?

Solution:

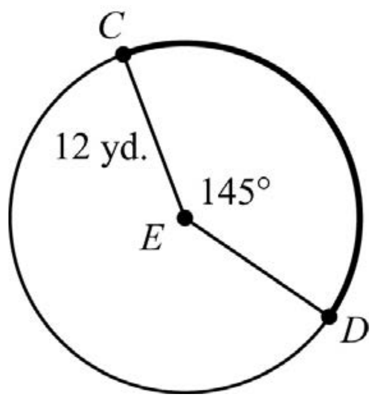
To find the area of the sector, divide the measure of the central angle of the arc in degrees by 360, and then multiply that amount by the area of the circle. The arc measure, x, is equal to the measure of the central angle, θ. The formula for the area of a circle is $A = \pi r^2$.

$A_{\text{sector}} = \dfrac{\pi r^2 \theta}{360}$	Area of sector of a circle with radius r and central angle θ in degrees
$A_{\text{sector}} = \dfrac{50\pi(8)^2}{360}$	Substitute 50 for θ and 8 for r.
$A_{\text{sector}} = \dfrac{5\pi(64)}{36}$	Rewrite the fraction and the power.
$A_{\text{sector}} = \dfrac{320\pi}{36}$	Multiply.
$A_{\text{sector}} = \dfrac{80\pi}{9}$	Rewrite.

The area of the sector is $\dfrac{80}{9}\pi$ square meters.

SAMPLE ITEMS

1. Circle *E* is shown.



What is the length of $\overset{\frown}{CD}$?

A. $\frac{29}{72}\pi$ yd.

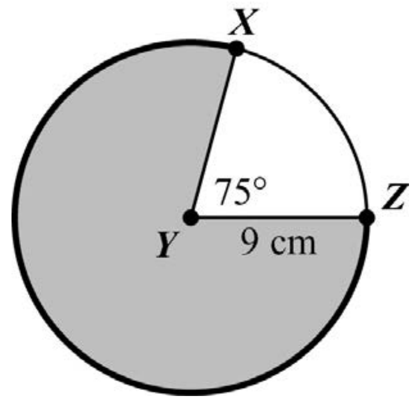
B. $\frac{29}{6}\pi$ yd.

C. $\frac{29}{3}\pi$ yd.

D. $\frac{29}{2}\pi$ yd.

Correct Answer: C

2. Circle *Y* is shown.



What is the area of the shaded part of the circle?

A. $\frac{57}{4}\pi$ cm^2

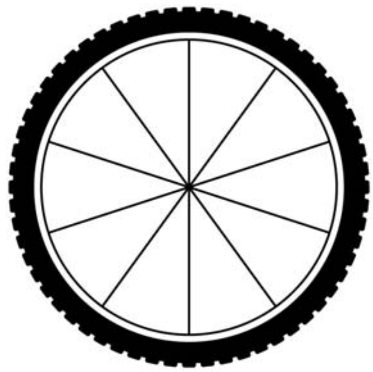
B. $\frac{135}{8}\pi$ cm^2

C. $\frac{405}{8}\pi$ cm^2

D. $\frac{513}{8}\pi$ cm^2

Correct Answer: D

3. **The spokes of a bicycle wheel form 10 congruent central angles. The diameter of the circle formed by the outer edge of the wheel is 18 inches.**



What is the length, to the nearest 0.1 inch, of the outer edge of the wheel between two consecutive spokes?

A. 1.8 inches
B. 5.7 inches
C. 11.3 inches
D. 25.4 inches

Correct Answer: B

Explain Volume Formulas and Use Them to Solve Problems

MGSE9-12.G.GMD.1 Give informal arguments for geometric formulas.

a. Give informal arguments for the formulas of the circumference of a circle and area of a circle using dissection arguments and informal limit arguments.

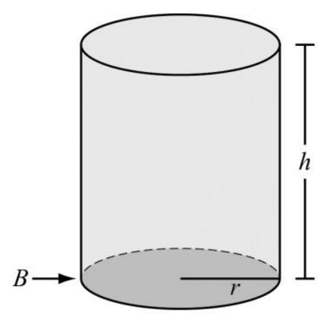
b. Give informal arguments for the formula of the volume of a cylinder, pyramid, and cone using Cavalieri's principle.

MGSE9-12.G.GMD.2 Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

MGSE9-12.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

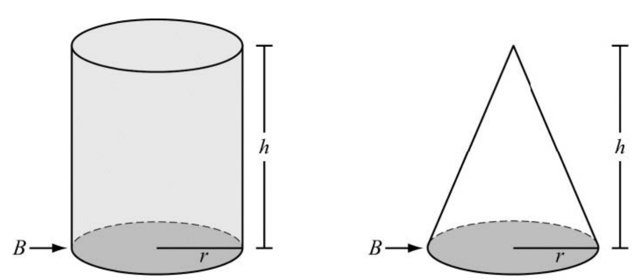
KEY IDEAS

1. The ***volume*** of a figure is a measure of how much space it takes up. Volume is a measure of capacity.
2. The formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height. The volume formula can also be given as $V = Bh$, where B is the area of the base. In a cylinder, the base is a circle and the area of a circle is given by $A = \pi r^2$. Therefore, $V = Bh = \pi r^2 h$.

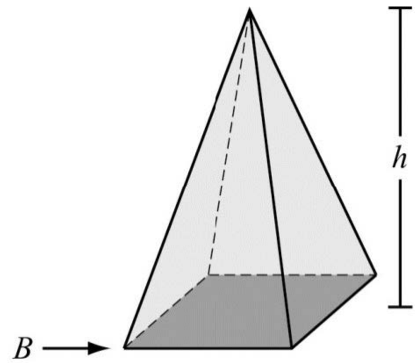


3. When a cylinder and a cone have congruent bases and equal heights, the volume of exactly three cones will fit into the cylinder. So, for a cone and cylinder that have the same radius r and height h, the volume of the cone is one-third of the volume of the cylinder.

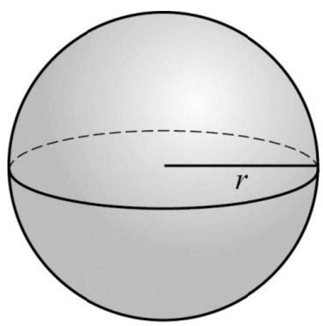
   The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height.



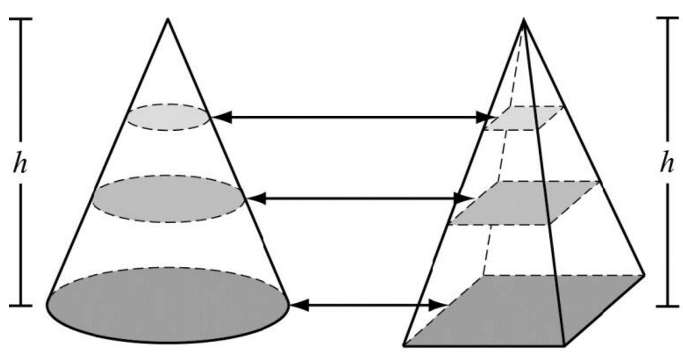
4. The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.



5. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius.

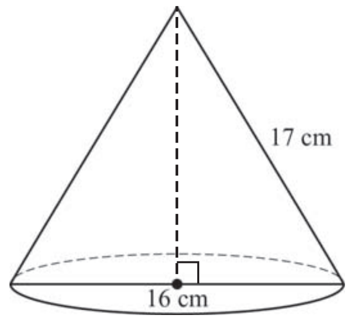


6. Cavalieri's principle states that if two solids are between parallel planes and all cross sections at equal distances from their bases have equal areas, the solids have equal volumes. For example, this cone and this pyramid have the same height and the cross sections have the same area, so they have equal volumes.



REVIEW EXAMPLES

1. What is the volume of the cone shown below?



Solution:

The diameter of the cone is 16 cm. So the radius is 16 cm ÷ 2 = 8 cm. Use the Pythagorean theorem, $a^2 + b^2 = c^2$, to find the height of the cone. Substitute 8 for b and 17 for c and solve for a:

$$a^2 + 8^2 = 17^2$$
$$a^2 + 64 = 289$$
$$a^2 = 225$$
$$a = 15$$

The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Substitute 8 for r and 15 for h:

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2(15)$$

The volume is 320π cm^3.

2. A sphere has a radius of 3 feet. What is the volume of the sphere?

Solution:

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Substitute 3 for r and solve.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(3)^3$$
$$V = \frac{4}{3}\pi(27)$$
$$V = 36\pi \text{ ft}^3$$

3. A cylinder has a radius of 10 cm and a height of 9 cm. A cone has a radius of 10 cm and a height of 9 cm. Show that the volume of the cylinder is three times the volume of the cone.

Solution:

The formula for the volume of a cylinder is $V = \pi r^2 h$. Substitute 10 for r and 9 for h:

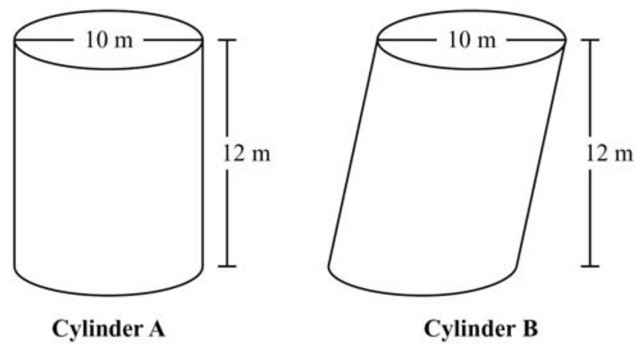
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(10)^2(9) \\ &= \pi(100)(9) \\ &= 900\pi \text{ cm}^3 \end{aligned}$$

The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Substitute 10 for r and 9 for h:

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi (10)^2(9) \\ &= \frac{1}{3}\pi (100)(9) \\ &= 300\pi \text{ cm}^3 \end{aligned}$$

Divide: $900\pi \div 300\pi = 3$

4. Cylinder A and Cylinder B are shown below. What is the volume of each cylinder?

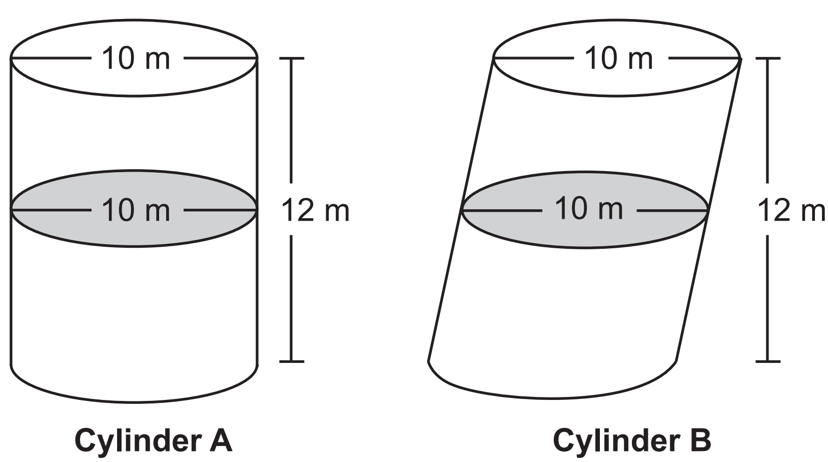


Solution:

To find the volume of Cylinder A, use the formula for the volume of a cylinder, which is $V = \pi r^2 h$. Divide the diameter by 2 to find the radius: $10 \div 2 = 5$. Substitute 5 for r and 12 for h:

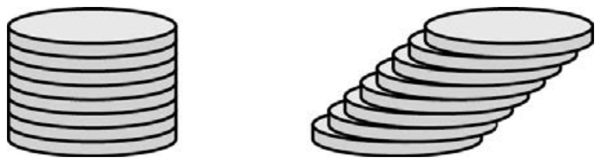
$$\begin{aligned} V_{\text{Cylinder A}} &= \pi r^2 h \\ &= \pi (5)^2 (12) \\ &= \pi (25)(12) \\ &= 300\pi \text{ m}^3 \\ &\approx 942 \text{ m}^3 \end{aligned}$$

Notice that Cylinder B has the same height and the same radius as Cylinder A. The only difference is that Cylinder B is slanted. For both cylinders, the cross section at every plane parallel to the bases is a circle with the same area. By Cavalieri's principle, the cylinders have the same volume; therefore, the volume of Cylinder B is 300π m^3, or about 942 m^3.



SAMPLE ITEMS

1. **Jason constructed two cylinders using solid metal washers. The cylinders have the same height, but one of the cylinders is slanted as shown.**



Which statement is true about Jason's cylinders?

A. The cylinders have different volumes because they have different radii.
B. The cylinders have different volumes because they have different surface areas.
C. The cylinders have the same volume because each of the washers has the same height.
D. The cylinders have the same volume because they have the same cross-sectional area at every plane parallel to the bases.

Correct Answer: D

2. **What is the volume of a cylinder with a radius of 3 in. and a height of $\frac{9}{2}$ in.?**

A. $\frac{81}{2}\pi$ in.^{3}
B. $\frac{27}{4}\pi$ in.^{3}
C. $\frac{27}{8}\pi$ in.^{3}
D. $\frac{9}{4}\pi$ in.^{3}

Correct Answer: A