Unit 4, Lesson 12: Fractional Lengths

Lesson Goals

- Use multiplication and division to solve measurement problems involving fractional lengths.
- Recognize multiplicative comparison situations (i.e. questions such as "how many times as much as a is b?") as division problems.
- Use equations, diagrams, and division algorithm to reason about multiplicative comparison problems that involve fractions.

Required Materials

- geometry toolkits

12.1: Number Talk: Multiplication Strategies (5 minutes)

Setup: 2 minutes of quiet think time, followed by a whole-class discussion.

Student task statement

Find the product mentally.

$19 \cdot 14$

Possible responses

$19 \cdot 14 = 266$

Anticipated misconceptions

When multiplying $19 \cdot 14$, students may only multiply the tens digits and multiply the ones digits and add them to get 136. Ask these students to estimate an answer for the problem and consider whether their answer makes sense.

12.2: How Many Would It Take? (Part 1) (15 minutes)

Setup:

Students in groups of 2. 5 minutes of quiet work time, 5 minutes of partner collaboration and discussion. Access to geometry toolkits.

Student task statement

1. Jada was using square stickers with a side length of $\frac{3}{4}$ inch to decorate the spine of a photo album. The spine is $10\frac{1}{2}$ inches long. If she laid the stickers side by side without gaps or overlaps, how many stickers did she use to cover the length of the spine?

2. How many $\frac{5}{8}$-inch binder clips, laid side by side, make a length of $11\frac{1}{4}$ inches?

3. It takes exactly 26 paper clips laid end to end to make a length of $17\frac{7}{8}$ inches.

   a. Estimate the length of each paper clip.

   b. Calculate the length of each paper clip. Show your reasoning.

Possible responses

1. 14 stickers

2. 18 binder clips

3. a. Less than 1 inch and more than $\frac{1}{2}$ inch.

   b. $\frac{11}{16}$ inch

Anticipated misconceptions

If students struggle to represent the situations mathematically, suggest that they draw diagrams to represent the situations. They could start with sketches of the objects and then move toward other simpler or more abstract representations as they make better sense of the problems.

The last question requires students to divide a mixed number by a whole number. If students are unsure how to do so, remind them that, in an earlier lesson, we saw that dividing by a whole

number has the same outcome as multiplying by its reciprocal.

Are you ready for more?

Lin has a work of art that is 14 inches by 20 inches. She wants to frame it with large paper clips laid end to end.

1. If each paper clip is $1\frac{3}{4}$ inch long, how many paper clips would she need? Show your reasoning and be sure to think about potential gaps and overlaps. Consider making a sketch that shows how the paper clips could be arranged.

2. How many paper clips are needed if the paper clips are spaced $\frac{1}{4}$ inch apart? Describe the arrangement of the paper clips at the corners of the frame.

Possible Responses

Answers vary. Sample response:

1. 38 paper clips.
   - One side of the paper is 20 inches long. $20 \div 1\frac{3}{4} = 11\frac{3}{7}$, so Lin can fit 11 paper clips along the side with a gap of $\frac{3}{4}$ inch since $11 \cdot 1\frac{3}{4} = 19\frac{1}{4}$. If the paper clips are centered along the 20-inch length, there will be $\frac{3}{8}$ inch of gap on either side.
   - The other side is 14 inches long. $14 \div 1\frac{3}{4} = 8$, so Lin can fit 8 paper clips along the side with no gap at all.
   - At each corner of the paper, two paper clips will meet. If the paper clip has a width that is about $\frac{3}{8}$ inch (to fit in the $\frac{3}{8}$ gap left by the 11 paper clips along the longer side), then there will be no gap or overlap.

- Altogether, Lin will need 38 paper clips.
  $11 + 11 + 8 + 8 = 38$

2. 34 paper clips. If space is put between the paper clips, then fewer paper clips will be needed. If a gap of $\frac{1}{4}$ inch is between the paper clips, then each paper clip could have $\frac{1}{8}$ inch of space on either end so that the paper clip and its space takes up 2 inches. Then there are 7 paper clips along the sides of length 14 inches of the frame, and there are 10 paper clips along the sides of length 20 inches. There is a gap of $\frac{1}{8}$ inch between the end of the paper clip and the end of the frame.

12.3: How Many Times as Tall or as Far? (15 minutes)

Setup:

Students in groups of 4. 1–2 minutes of quiet time for each question in the first problem, 2 minutes to compare diagrams and equations in groups, followed by a brief whole-class discussion. 8–10 minutes to complete the activity, either independently or collaboratively with their group.

Student task statement

1. A second-grade student is 4 feet tall. Her teacher is $5\frac{2}{3}$ feet tall.
   a. How many times as tall as the student is the teacher?
   b. What fraction of the teacher's height is the student's height?

2. Find each quotient. Show your reasoning and check your answer.
   a. $9 \div \frac{3}{5}$
   b. $1\frac{7}{8} \div \frac{3}{4}$

3. Write a division expression that can help answer each of the following questions. Then answer the question. If you get stuck, draw a diagram.
   a. A runner ran $1\frac{4}{5}$ miles on Monday and $6\frac{3}{10}$ miles on Tuesday. How many times her Monday's distance was her Tuesday's distance?
   b. A cyclist planned to ride $9\frac{1}{2}$ miles but only managed to travel $3\frac{7}{8}$ miles. What fraction of his planned trip did he travel?

Possible responses

1. a. The teacher is $1\frac{5}{12}$ times as tall as the student.
   b. The student is $\frac{12}{17}$ as tall as the teacher.

2. a. 15
   b. $\frac{5}{2}$ or $2\frac{1}{2}$

3. a. Tuesday's distance is $3\frac{1}{2}$ times Monday's distance.
   b. The cyclist traveled $\frac{31}{76}$ of his planned trip.

Anticipated misconceptions

If students have trouble drawing and using a diagram to compare lengths, ask them to revisit the Fractions of Ropes activity (in Lesson 7) and use the diagrams there as examples. Suggest that they try drawing a diagram on graph paper, as the grid could support

them in drawing and making sense of the fractional lengths.

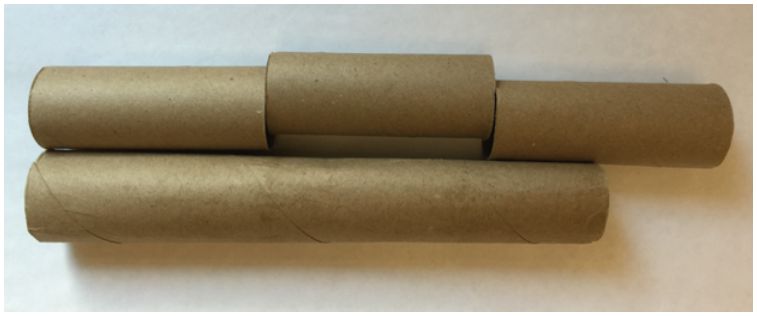
12.4: Comparing Paper Rolls (Optional, 15 minutes)

Setup:

Students in groups of 4. Ask students to keep their materials closed. Display the image of the paper rolls for all to see. 1–2 minutes to notice and wonder, followed by a brief whole-class discussion. 7–8 minutes of quiet work time.

Student task statement

The photo shows a situation that involves fractions.



1. Use the photo to help you complete the following statements. Explain or show your reasoning for the second statement.

   a. The length of the long paper roll is about _____ times the length of the short paper roll.

   b. The length of the short paper roll is about _____ times the length of the long paper roll.

2. If the length of the long paper roll is $11\frac{1}{4}$ inches, what is the length of each short paper roll?

   Use the information you have about the paper rolls to write a multiplication equation or a division equation for the question. Note that $11\frac{1}{4} = \frac{45}{4}$.

3. Answer the question. If you get stuck, draw a diagram.

Possible responses

1. a. About $\frac{5}{2}$ (or $2\frac{1}{2}$ or 2.5) times.

   b. About $\frac{2}{5}$ (or 0.4) times.

2. Answers vary. Possible equations:
   - $2\frac{1}{2} \cdot ? = 11\frac{1}{4}$
   - $\frac{5}{2} \cdot ? = \frac{45}{4}$
   - $11\frac{1}{4} \div 2\frac{1}{2} = ?$
   - $\frac{2}{5} \cdot 11\frac{1}{4} = ?$
   - $\frac{2}{5} \cdot \frac{45}{4} = ?$

3. $4\frac{1}{2}$ (or equivalent) inches. Reasoning varies.

Anticipated misconceptions

Students might estimate the relationships between the lengths of rolls by rounding too much. For example, they might say that the length

of the shorter roll is $\frac{1}{3}$ the length of the longer roll, or that the longer roll is twice as long as the shorter roll. If this happens, ask students to take a closer look and make a more precise estimate. Suggest that they divide the larger roll into smaller segments, each of which matches the length of the shorter rolls.

Lesson Synthesis (5 minutes)

How do we interpret and solve problems involving fractional lengths, such as: "How many $\frac{5}{8}$-inch paper clips, laid end to end, are in a length of $12\frac{1}{2}$ inches?" How do we compare two fractional lengths and find out how many times one is as long as the other?

12.5: Building A Fence (Cool-down, 5 minutes)

Setup: None.

Student task statement

A builder was building a fence. In the morning, he worked for $\frac{2}{5}$ of an hour. In the afternoon, he worked for $\frac{9}{10}$ of an hour. How many times as long as in the morning did he work in the afternoon?

Write a division equation to represent this situation, then answer the question. Show your reasoning. If you get stuck, you can draw a diagram.

Possible responses

$\frac{9}{10} \div \frac{2}{5} = ?$ (or $\frac{9}{10} \div ? = \frac{2}{5}$). In the afternoon, he worked $2\frac{1}{4}$ as long as he did in the morning.