Unit 4 Rational and Reciprocal Functions and Equations

General Outcome:

- *Develop algebraic reasoning and number sense.*
- *Develop algebraic and graphical reasoning through the study of relations.*

Specific Outcomes:

4.1 Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials).

4.2 Perform operations on rational expressions (limited to numerators and denominators that are monomials binomials, or trinomials).

4.3 Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials, or trinomials).

4.4 Graph and analyze reciprocal functions (limited to the reciprocal of linear and quadratic functions).

Topics

Unit 4 Rational and Reciprocal Functions and Equations

Rational Expressions:

Rational expressions are fractions that have a polynomial numerator and denominator.

Ex) $\frac{2x-7}{x^2-9}$ $\frac{3}{x+2}$ $\frac{2y-4x}{5y+3x}$

Any time a variable is found in a denominator the possibility of restrictions exist.

Ex) For each rational expression determine all non-permissible values.

a) $\frac{5t}{4st^2}$ b) $\frac{3x}{x(2x-3)}$ c) $\frac{2p-1}{p^2-p-12}$

Equivalent Rational Expressions:

Remember when the numerator and denominator of a fraction is multiplied or divided the same value the resulting fraction is equivalent.

Ex) $\frac{7x}{x-2}$

Simplifying Rational Expressions:

To simplify a rational expression you must first factor the numerator and denominator. Only like factors of the numerator and denominator will cancel each other out.

Ex) Simplify the following.

a) $\frac{x^3y}{x^2y^4}$

b) $\frac{3x-6}{2x^2+x-10}$

c) $\frac{x^2-4x-32}{x^2+9x+20}$

d) $\frac{1-t}{t^2-1}$

Ex) Consider the expression $\dfrac{16x^2 - 9y^2}{8x - 6y}$

a) What expression represents the non-permissible values of x?

a) Simplify the rational expression.

c) Evaluate the expression for $x = 2.6$ and $y = 1.2$

Simplifying Rational Expressions Assignment:

1) In each case below determine what the first rational expression must be multiplied by to obtain the second rational expression.

a) $\frac{3p}{q}, \quad \frac{3p^2q}{pq^2}$

b) $\frac{2}{x+4}, \quad \frac{2x-8}{x^2-16}$

c) $\frac{-4}{m+3}, \quad \frac{-4(m-3)}{m^2-9}$

d) $\frac{1}{y-1}, \quad \frac{y^2+y}{y^3-y}$

2) Determine the non-permissible values for each of the following.

a) $\frac{3a}{4-a}$

b) $\frac{2b+8}{b}$

c) $\frac{3(y+7)}{(y-4)(y+2)}$

d) $\frac{-7(x-1)}{(x-1)(x+3)}$

e) $\frac{2k+8}{k^2}$

f) $\frac{6x-8}{(3x-4)(2x+5)}$

3) Simplify the following rational expressions. State any restrictions.

a) $\dfrac{2a(a-5)}{3a(a-5)}$

b) $\dfrac{3x(2x+3)}{2x(3x+2)}$

c) $\dfrac{(x-7)(x+7)}{(2x-1)(x-7)}$

d) $\dfrac{5(a-3)(a+2)}{10(3-a)(a+2)}$

e) $\dfrac{6r^2p^3}{4rp^4}$

f) $\dfrac{3x-6}{10-5x}$

g) $\dfrac{a^2+2a-24}{2a^2-72}$

h) $\dfrac{10x^2+55x+75}{20x^2-10x-150}$

i) $\dfrac{x-4}{4-x}$

j) $\dfrac{5\left(x^2-y^2\right)}{x^2-2xy+y^2}$

4) Identify the error in the simplification shown below and provide the correct simplification.

$$\frac{x^2-4}{2x-4}=\frac{(x-2)(x+2)}{2(x-2)}$$

$$=\frac{x+2}{2}$$

$$=x+1$$

5) Create a rational expression that has non-permissible values of 1 and -2.

6) The area of right ΔPQR is $\left(x^2 - x - 6\right)$ square units, and the length of side *PQ* is $\left(x - 3\right)$ units. Side *PR* is the hypotenuse.

   a) Draw a diagram of this triangle.

   b) Write and expression for the length of side *QR*. Express your answer in simplest form.

   c) Identify the restrictions.

7) Express the following in simplest form. Identify any restrictions.

a) $\dfrac{(x+2)^2-(x+2)-20}{x^2-9}$

b) $\dfrac{4(x^2-9)^2-(x+3)^2}{x^2+6x+9}$

c) $\dfrac{(x^2-x)^2-8(x^2-x)+12}{(x^2-4)^2-(x-2)^2}$

d) $\dfrac{(x^2+4x+4)^2-10(x^2+4x+4)+9}{(2x+1)^2-(x+2)^2}$

Multiplying and Dividing Rational Expressions:

Multiplying:

- Factor all numerators and denominators
- Simplify by eliminating common factors (1 from a numerator and 1 from a denominator)
- State all restrictions

Ex) Simplify the following.

a) $\frac{x^2-3x-10}{x^2-x-56}\times\frac{x+7}{x-5}$

b) $\frac{3}{a}\times\frac{a^2}{b}$

c) $\frac{4x^2}{3y^3}\times\frac{9y^2}{8x^4}$

d) $\dfrac{a^2+5a+6}{a^2-6a+5}\times\dfrac{a^2+a-30}{a^2+9a+18}$

e) $\dfrac{x^2-x-12}{x^2-9}\times\dfrac{x^2-4x+3}{x^2-4x}$

Dividing:

- Factor all numerators and denominators
- Convert into a multiplication question

Ex) $\dfrac{2}{7}\div\dfrac{14}{5}=\dfrac{2}{7}\times\dfrac{5}{14}$

- Simplify by eliminating common factors
- State all restrictions

Ex) Simplify the following.

a) $\dfrac{x^2 - x - 20}{x^2 - 6x} \div \dfrac{x^2 + 9x + 20}{x^2 - 12x + 36}$

b) $\dfrac{a^2 - 3a - 28}{a^2 - 14a + 45} \div \dfrac{a^2 - 4a - 21}{a^2 - 2a - 15}$

c) $$\frac{\cancel{(x^2-4)}\;\cancel{(x+3)}}{\cancel{(2x-4)}\;\cancel{(x^2+2x-3)}}$$

d) $$\frac{2m^2-7m-15}{2m^2-10m} \div \frac{4m^2-9}{6} \times (3-2m)$$

Multiplying & Dividing Rational Expressions Assignment:

1) Simplify the following expressions. Identify any restrictions.

a) $\frac{12x^2y}{5zy} \times \frac{15z}{4x}$

b) $\frac{3(a-b)}{(a-1)(a+5)} \times \frac{(a-5)(a+5)}{15(a-b)}$

c) $\frac{(y-7)(y+3)}{(2y-3)(2y+3)} \times \frac{4(2y+3)}{(y+3)(y-1)}$

d) $\frac{2x-6}{x+3} \times \frac{x+3}{2}$

e) $\frac{d^2-100}{144} \times \frac{36}{d+10}$

f) $\frac{a+3}{a+1} \times \frac{a^2-1}{a^2-9}$

g) $\dfrac{4x^2-25}{2x^2-13x+20}\times\dfrac{x-4}{4x+10}$

h) $\dfrac{2a^2+5a-3}{2a-3}\times\dfrac{a^2-1}{6a-3}\times\dfrac{2a-3}{a^2+2a-3}$

2) Simplify the following rational expressions. Identify any restrictions.

a) $\dfrac{4x^2}{3y}\div\dfrac{2x}{y^2}$

b) $\dfrac{t^2-7t}{t^2-49}\div\dfrac{3t^2}{t+7}$

c) $\dfrac{5}{n+1} \div \dfrac{10}{n^2-1} \div (n-1)$

d) $\dfrac{y^2}{y^2-9} \div \dfrac{y}{y-3}$

e) $\dfrac{2w^2-w-6}{3w+6} \div \dfrac{2w+3}{w+2}$

f) $\dfrac{a-5}{a} \div \dfrac{a^2-2a-15}{a^2}$

g) $\dfrac{9x^2-1}{x+5} \div \dfrac{3x^2-5x-2}{2-x}$

h) $\dfrac{8y^2-2y-3}{y^2-1} \div \dfrac{2y^2-3y-2}{2y-2} \div \dfrac{3-4y}{y+1}$

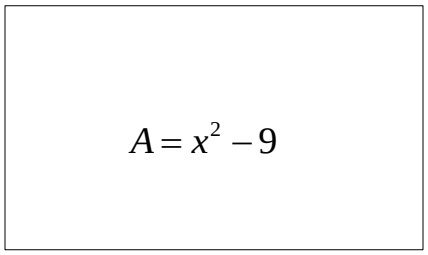
3) Identify the error in the simplification shown below and correct the simplification.

$$\frac{x^2-3}{2x} \div \frac{x+6}{8x^2} = \frac{2x}{(x-6)(x+6)} \times \frac{x+6}{(2x)(4x)}$$

$$= \frac{\cancel{2x}}{(x-6)\cancel{(x+6)}} \times \frac{\cancel{x+6}}{\cancel{(2x)}(4x)}$$

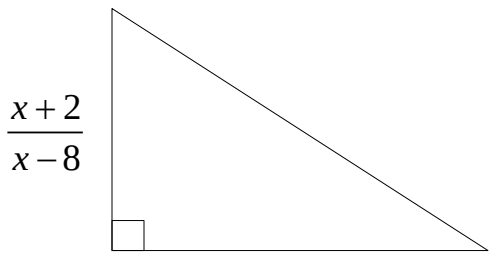
$$= \frac{1}{4x(x-6)}$$

4) Write an expression to represent the length of the rectangle shown below. Simplify your answer.



$\frac{x^2-2x-3}{x+1}$

5) Write an expression to represent the area of the triangle shown below. Simplify your answer.



$\frac{x+2}{x-8}$

$\frac{x^2-7x-8}{x^2-4}$

Adding and Subtracting Rational Expressions:

When adding or subtracting fractions we must have a common denominator.

Ex)

$$\frac{1}{2}+\frac{3}{10}=\frac{5}{10}+\frac{3}{10}$$

$$=\frac{8}{10}$$

$$=\frac{4}{5}$$

* 10 is the **lowest common multiple** of 2 & 10.

When finding the lowest common multiple (or lowest common denominator) we must consider the coefficients and the variables involved.

Ex) Find the lowest common multiple of the following.

a) 2, 6

b) 5, 3

c) a^2, a^5

d) $8t^2$, $12t^7$

Ex) Simplify the following.

a) $\frac{2}{3y}+\frac{4}{3y}$

b) $\frac{4t^2}{3}-\frac{6t^2}{3}+\frac{1}{3}$

c) $\frac{2a}{3}+\frac{5a}{4}$

d) $\frac{y-5}{9}-\frac{y-2}{6}$

e) $\frac{5}{2a}+\frac{2a}{3}$

f) $\frac{3x}{8x^4}-\frac{5}{6x^2}$

g) $\frac{6x+1}{5}+1-\frac{2x+4}{7}$

Remember when adding or subtracting rational expressions we must have a common denominator.

To find the lowest common denominator we must first factor each denominator.

$$\frac{x}{2x-4}+\frac{3}{3x-6}$$

Ex) Determine the lowest common denominator for each of the following.

a) $\frac{1}{xy}, \frac{2}{3x^2y}$

b) $\frac{5x}{2x^2-6x}, \frac{3}{5x^2+20}, \frac{4x}{x^2+x-12}$

Ex) Simplify the following.

a) $\frac{x+12}{4x-4}+\frac{2+x}{x-1}$

b) $\frac{x}{6x+6}+\frac{5}{4x-12}$

c) $\frac{4}{y^2+5y+6}-\frac{5}{y^2-y-12}$

d) $\frac{x+5}{x^2-3x-10}-\frac{x-1}{x^2-9x+20}$

e) $\dfrac{y^2-20}{y^2-4}+\dfrac{y-2}{y+2}$

f) $\dfrac{1+\frac{1}{x}}{x-\frac{1}{x}}$

g) $\dfrac{\frac{5}{x^2-16}+\frac{2x}{x^2+2x-24}}{\frac{7x}{x^2+10x+24}-\frac{3}{x-4}}$

Adding & Subtracting Rational Expressions Assignment:

1) Simplify the following expressions. Express all answers in simplest form and identify any non-permissible values.

a) $\frac{7}{x}+\frac{3}{x}$

b) $\frac{5t+3}{10}+\frac{3t+5}{10}$

c) $\frac{m^2}{m+1}-\frac{m}{m+1}$

d) $\frac{a^2}{a-4}-\frac{a}{a-4}-\frac{a-12}{a-4}$

e) $\frac{1}{3a}+\frac{2}{5a}$

f) $\frac{3}{2x}+\frac{1}{6}$

g) $4-\frac{6}{5x}$

h) $\frac{4z}{xy}-\frac{9x}{yz}$

i) $\frac{2x}{5y^2}+\frac{1}{10y}-\frac{6}{15y^3}$

j) $\frac{6xy}{a^2b}-\frac{2}{ab^2y}+1$

k) $\frac{8}{x^2-4}-\frac{5}{x+2}$

l) $\frac{1}{x^2-x-12}+\frac{3}{x+3}$

m) $\dfrac{3x}{x-2}-\dfrac{x}{2-x}$

n) $\dfrac{5}{y+1}-\dfrac{1}{y}-\dfrac{y-4}{y^2+y}$

o) $\dfrac{2h}{h^2-9}+\dfrac{h}{h^2+6h+9}-\dfrac{3}{h-3}$

p) $\dfrac{2}{x^2+x-6}+\dfrac{3}{x^3+2x^2-3x}$

q) $\dfrac{3x+15}{x^2-25}+\dfrac{4x^2-1}{2x^2+9x-5}$

r) $\dfrac{2x}{x^3+x^2-6x}-\dfrac{x-8}{x^2-5x-24}$

s) $\dfrac{n+3}{n^2-5n+6}+\dfrac{6}{n^2-7n+12}$

t) $\dfrac{2w}{w^2+5w+6}-\dfrac{w-6}{w^2+6w+8}$

2) Identify the error in the simplification shown below and provide the correct simplification.

$$\frac{6}{x-2}+\frac{4}{x^2-4}-\frac{7}{x+2}=\frac{6(x+2)+4-7(x-2)}{(x-2)(x+2)}$$

$$=\frac{6x+12+4-7x-14}{(x-2)(x+2)}$$

$$=\frac{-x+2}{(x-2)(x+2)}$$

3) Simplify the following. State and restrictions on the variable.

a) $\dfrac{2-\dfrac{6}{x}}{1-\dfrac{9}{x^2}}$

b) $\dfrac{\dfrac{3}{2}+\dfrac{3}{t}}{\dfrac{t}{t+6}-\dfrac{1}{t}}$

c) $\dfrac{\dfrac{3}{m}-\dfrac{3}{2m+3}}{\dfrac{3}{m^2}+\dfrac{1}{2m+3}}$

d) $\dfrac{\dfrac{1}{x+4}+\dfrac{1}{x-4}}{\dfrac{x}{x^2-16}+\dfrac{1}{x+4}}$

4) A right triangle has legs of length $\frac{x}{2}$ and $\frac{x-1}{4}$. If all measurements are in the same units, determine a simplified expression for the length of the hypotenuse.

5) Simplify the following. Identify all non-permissible values.

a) $\frac{x-2}{x+5}+\frac{x^2-2x-3}{x^2-x-6}\times\frac{x^2+2x}{x^2-4x}$

b) $\frac{2x^2-x}{x^2+3x}\times\frac{x^2-x-12}{2x^2-3x+1}-\frac{x-1}{x+2}$

c) $\dfrac{x-2}{x+5}-\dfrac{x^2-2x-3}{x^2-x-6}\times\dfrac{x^2+2x}{x^2-4x}$

d) $\dfrac{x+1}{x+6}-\dfrac{x^2-4}{x^2+2x}\div\dfrac{2x^2+7x+3}{2x^2+x}$

Solving Rational Equations:

- Factor all denominators
- Determine the lowest common denominator
- Multiply both sides of the equation by the lowest common denominator (this will eliminate all fractions)
- Solve as normal
- Check for extraneous roots (your answer cannot be a restriction)

Ex) Solve the following.

a) $\frac{x}{2}+3=2+\frac{3x}{4}$

b) $\frac{t+5}{8}=t+\frac{3}{2}$

c) $\frac{x-1}{2x}+\frac{1}{x}=2$

d) $\frac{4}{2x-1}=\frac{1}{x-2}$

e) $\frac{1}{m-2}=\frac{5}{m+4}$

f) $\frac{4}{x}+6=2$

g) $\frac{5}{2y}+\frac{11}{12}=\frac{2}{3y}$

h) $\frac{2}{x^2-4}+\frac{10}{6x+12}=\frac{1}{x-2}$

i) $\frac{9}{y-3}-\frac{4}{y-6}=\frac{18}{y^2-9y+18}$

j) $\dfrac{3}{x^2-x-2}+\dfrac{4}{x^2-4}=\dfrac{1}{x^2+3x+2}$

k) $\dfrac{4k-1}{k+2}-\dfrac{k+1}{k-2}=\dfrac{k^2-4k+24}{k^2-4}$

Solving Rational Equations Assignment:

1) Solve the following. Check your answers for extraneous roots.

a) $\frac{f+3}{2} - \frac{f-2}{3} = 2$

b) $\frac{3-y}{3y} + \frac{1}{4} = \frac{1}{2y}$

c) $\frac{9}{w-3} - \frac{4}{w-6} = \frac{18}{w^2-9w+18}$

d) $\frac{6}{t} + \frac{t}{2} = 4$

e) $\frac{6}{c-3}=\frac{c+3}{c^2-9}-5$

f) $\frac{d}{d+4}=\frac{2-d}{d^2+3d-4}+\frac{1}{d-1}$

g) $\frac{x^2+x+2}{x+1}-x=\frac{x^2-5}{x^2-1}$

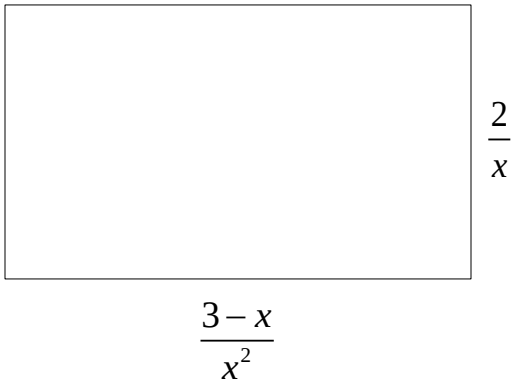
h) $\frac{-3y}{y-1}+6=\frac{6y-9}{y-1}$

i) $\frac{26}{b+5} = 1 + \frac{3}{b-2}$

j) $\frac{c}{c+2} - 3 = \frac{-6}{c^2 - 4}$

2) Experts claim that the golden rectangle is most pleasing to the eye. It has dimensions that satisfy the equation $\frac{l}{w} = \frac{l+w}{l}$, where w is the width and l is the length. According to this relationship, how long should a rectangular picture frame be if its width is 30 cm?

3) A rectangle has the dimensions shown in the diagram below.



a) Determine the difference between the length and the width of the rectangle.

b) Determine an expression that represents the area of the rectangle.

c) If the perimeter of the rectangle is 28 cm, determine the value of x.

Word Problems Involving Rational Equations:

- Identify the variable being used. This can be done with a let statement, a table, or a diagram
- Create an equation that describes the situation
- Solve for the variable
- Check to see that you have answered the question and that your answer makes sense
- Answer the question with a sentence

Ex) Find two consecutive numbers where half of the smaller is equal to 4 more than one third the larger.

Ex) The average life span of a woodland caribou is 5 years longer than half the average life span of a moose. The sum of their life spans is 35 years. What is the life span of a moose?

Ex) Two friends share a paper route. Sheena can deliver the papers in 40 min. Jeff can cover the same route in 50 min. How long, to the nearest minute, does the paper route take if they work together?

Ex) In a particular dog race from Pas to Flin Flon and back the total distance covered was 140 miles. Conditions were excellent on the way to Flin Flon. However, bad weather caused the winner's average speed to decrease by 6 mph on the return trip. The total time for the trip was $8\frac{1}{2}$ hours. What was the winning dog team's average speed on the way to Flin Flon?

Word Problems Involving Rational Equations Assignment:

1) The sum of two numbers is 25. The sum of their reciprocals is $\frac{1}{4}$. Determine the two numbers.

2) Two consecutive numbers are represented by x and $x+1$. If 6 is added to the first number and two is subtracted from the second number, the quotient of the new numbers is $\frac{9}{2}$. Determine the value of the original numbers.

3) A French club collected the same amount from each student going on a trip to Le Cercle Moliere in Winnipeg. When six students could not go, each of the remaining students was charged an extra $3. If the total cost was $540, how many students went on the trip.

4) A tub can be filled in 2 min. if only the cold tap is turned on. It fills in 3 min. if only the hot tap is turned on. How long will it take to fill the tub if both taps are turned on?

5) Two hoses together fill a pool in 2 hours. If only hose A is used, the pool fills in 3 hours. How long would it take to fill the pool if only hose B were used?

6) Two kayakers paddle 18 km downstream with the current in the same time it takes them to go 8 km upstream against the current. The rate of the current is 3 km/h. Determine the rate at which the kayakers are paddling.

7) Nikita lives in Kindersley, Saskatchewan. With her old combine, she can harvest her entire wheat crop in 72 hours. Her neighbor offers to help. His new combine can do the same job in 48 hours. How long would it take to harvest the wheat crop with both combines working together?

8) Two friends can paddles a canoe at a rate of 6 km/h in still water. It takes them 1 hour to paddle 2 km up a river and back again. Determine the speed of the current.

9) Suppose you have 21 days to read a 518-page novel. After finishing half of the book, you realize that you must read 12 pages more per day to finish the novel on time. What is your reading rate for the first half of the book? Use the table below to help solve the problem.

	Reading Rate in Pages per Day	**Number of Pages Read**	**Number of Days**
First Half			
Second Half			

10) If $b = \frac{1}{a}$ and $\frac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a} + \frac{1}{b}} = \frac{4}{5}$, solve for a.

Reciprocal Functions:

A reciprocal of a value is the "flip" of the fraction:

Ex) $\frac{2}{3} \xrightarrow{\text{reciprocal}} \frac{3}{2}$

$5 \xrightarrow{\text{reciprocal}} \frac{1}{5}$

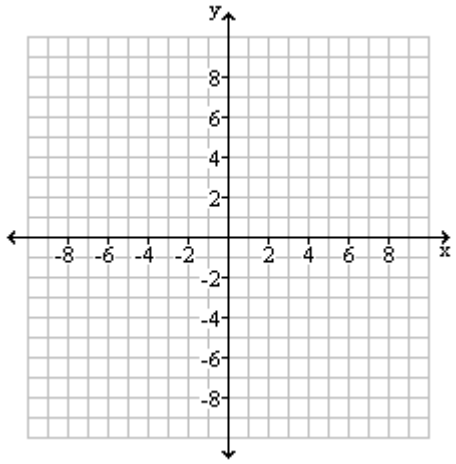
$\frac{-1}{x} \xrightarrow{\text{reciprocal}} -x$

A reciprocal of a function is found by dividing '1' by the function.

If $y = f(x)$, then $y = \frac{1}{f(x)}$ is its reciprocal.

Ex) Complete the table of values given below, then use this to sketch the graphs of $y = x$ and $y = \frac{1}{x}$.

x	$y = x$	$y = \frac{1}{x}$
–10		
–5		
–2		
–1		
$-1/2$		
$-1/5$		
0		
$1/5$		
$1/2$		
1		
2		
5		
10		



Characteristics:

Non-Permissible Values

Asymptotes

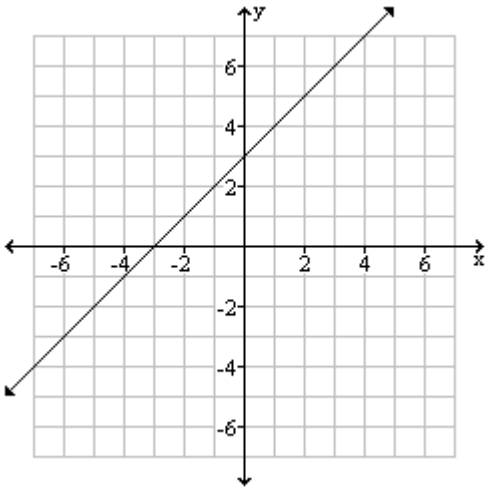
Invarient Points

Properties / Rules:

$y = f(x)$ (original)	$y = \frac{1}{f(x)}$ (reciprocal)
If $f(x) = 0$	then $\frac{1}{f(x)}$ is _____ and a _____ may exist.
If $f(x)$ is undefined and has a vertical asymptote.	then $\frac{1}{f(x)}$ = _____.
If $f(x) = 1$	then $\frac{1}{f(x)}$ =_____.
If $f(x) = -1$	then $\frac{1}{f(x)}$ =_____.
If $f(x)$ is positive	then $\frac{1}{f(x)}$ is _____.
If $f(x)$ is negative	then $\frac{1}{f(x)}$ is _____.
If $f(x)$ is increasing	then $\frac{1}{f(x)}$ is _____.
If $f(x)$ is decreasing	then $\frac{1}{f(x)}$ is _____.
If $f(x)$ approaches 0	then $\frac{1}{f(x)}$ approaches _____.
If $f(x)$ approaches $\pm\infty$ (∞ means infinity)	Then $\frac{1}{f(x)}$ approaches _____.

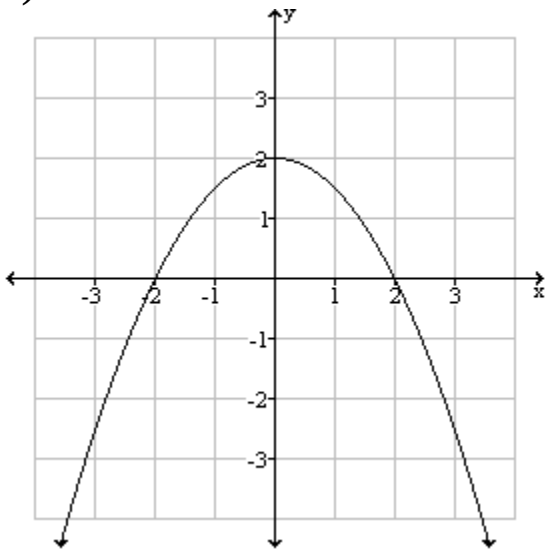
Ex) Given the graph of $y = f(x) = x + 3$, sketch the graph of $y = \dfrac{1}{f(x)}$.

Steps:

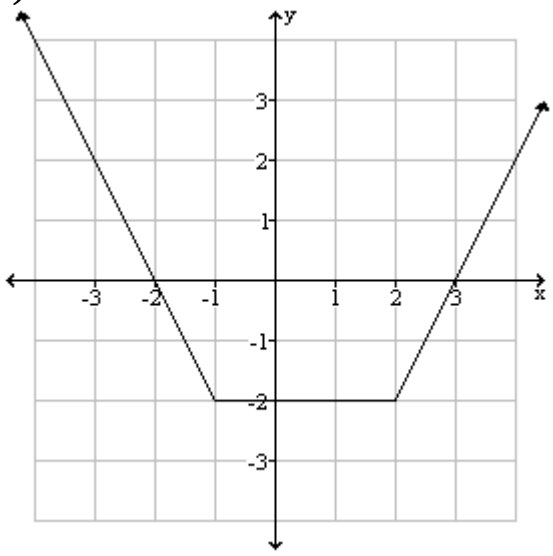


Ex) For each of the following, sketch the graph of $y = \dfrac{1}{f(x)}$ given the graph of $y = f(x)$.

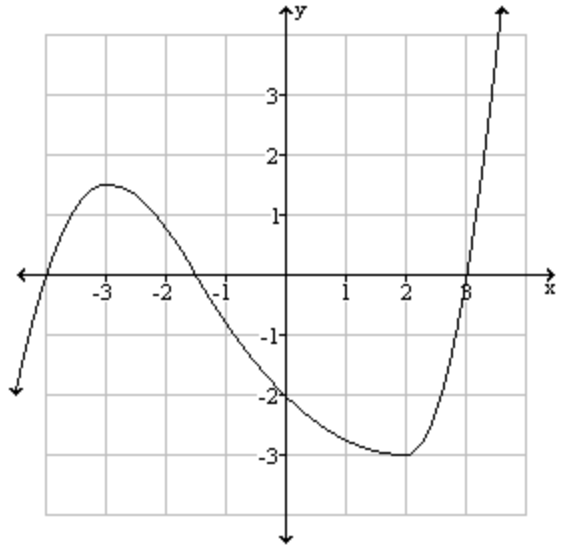
a)



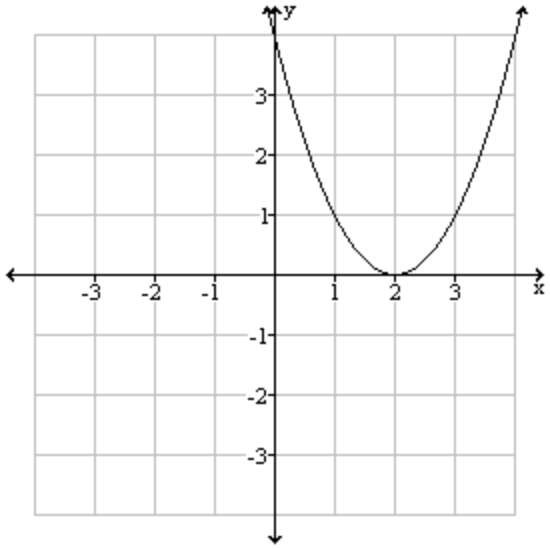
b)



c)



d)



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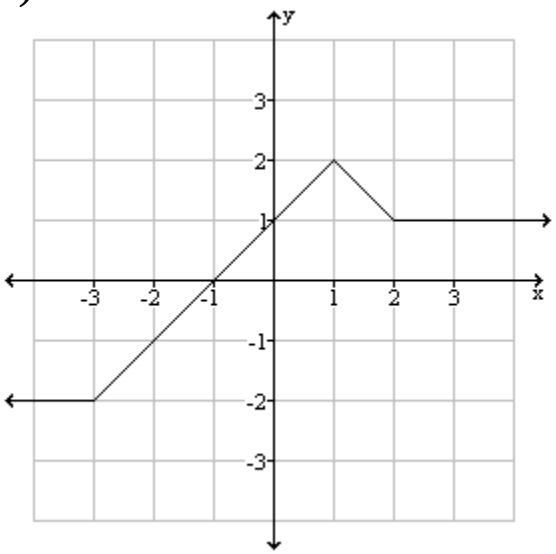


Reciprocal Functions Assignment:

1) For each function given write the reciprocal function and state its non-permissible values.

a) $f(x) = x + 5$

b) $g(x) = 2x + 1$

c) $h(x) = x^2 - 16$

d) $f(x) = x^2 + x - 12$

2) State the equation(s) of the vertical asymptote(s) for each function.

a) $f(x) = \dfrac{1}{5x - 10}$

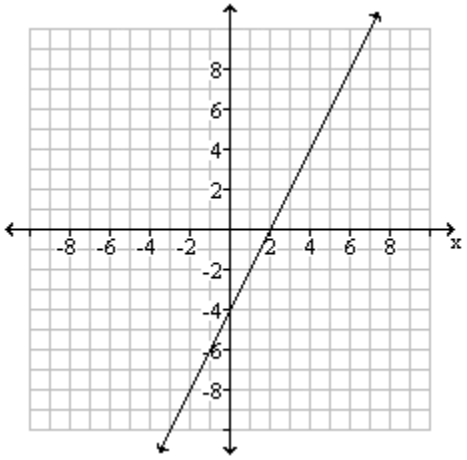
b) $f(x) = \dfrac{1}{3x + 7}$

c) $f(x) = \dfrac{1}{(x - 2)(x + 4)}$

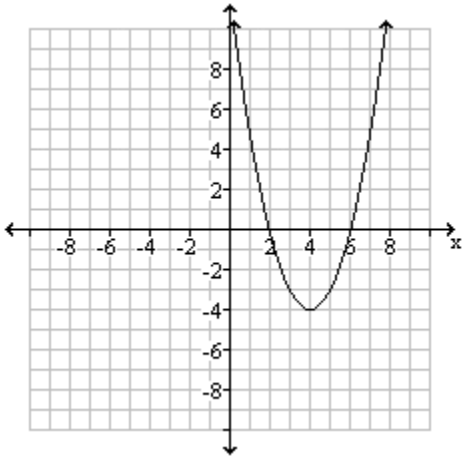
d) $f(x) = \dfrac{1}{x^2 - 9x + 20}$

3) Given the graph of $y = f(x)$, graph its reciprocal $y = \dfrac{1}{f(x)}$.

a)



b)



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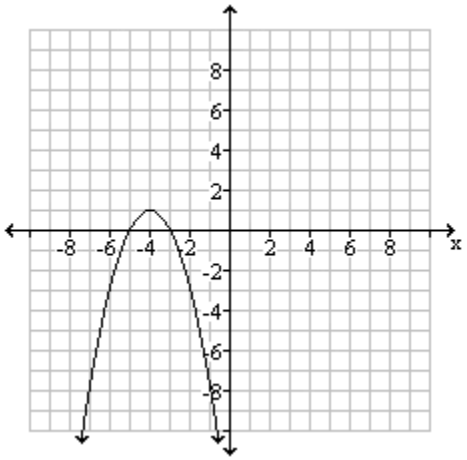

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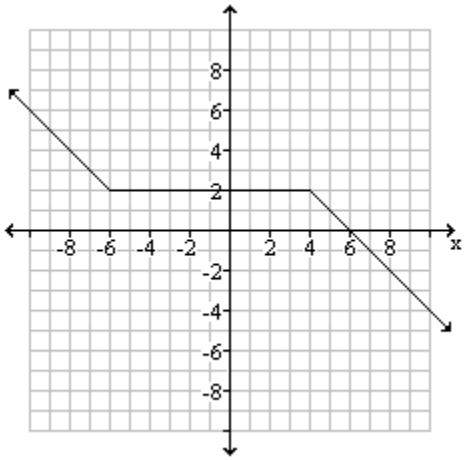
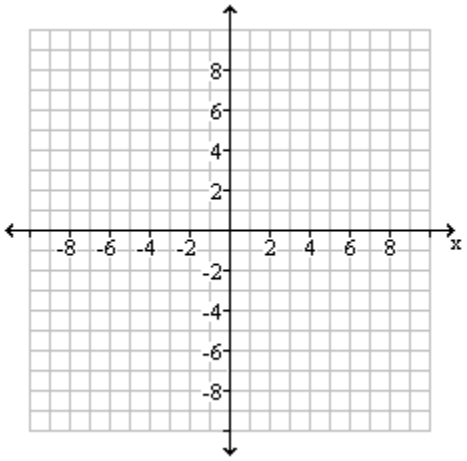
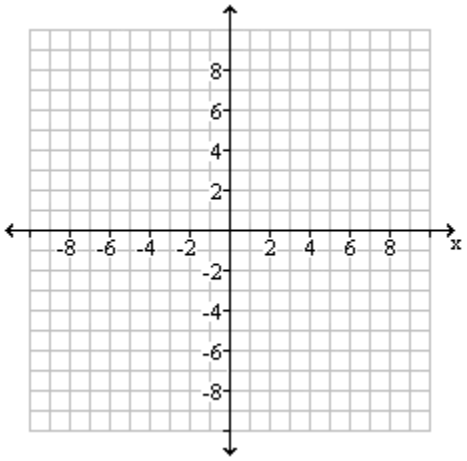


4) Sketch the graphs of $y = f(x)$ and $y = \dfrac{1}{f(x)}$ on the same set of axes. Identify the asymptotes and invariant points.

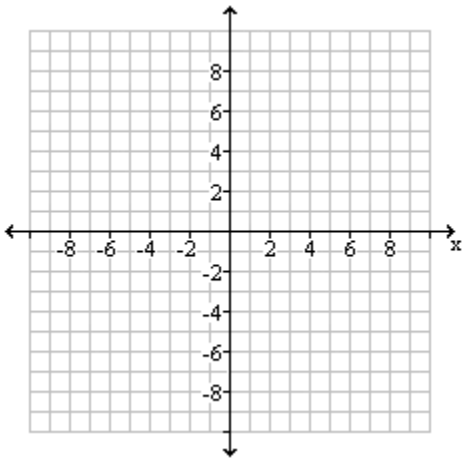
a) $f(x) = x - 5$



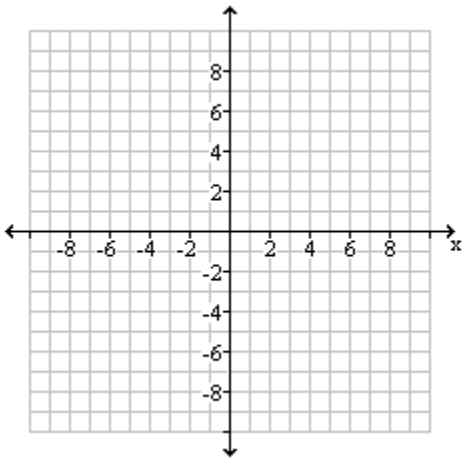
b) $f(x) = 2x + 4$



c) $f(x) = x^2 - 9$

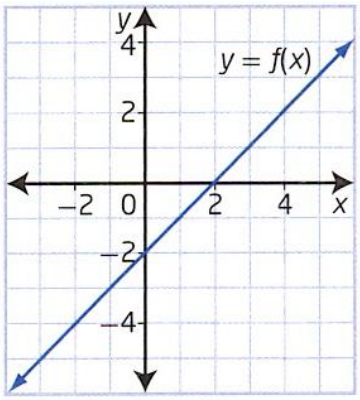


d) $f(x) = -(x + 5)^2 + 4$



5) Match the graph of the function with the graph of the reciprocal.

a)



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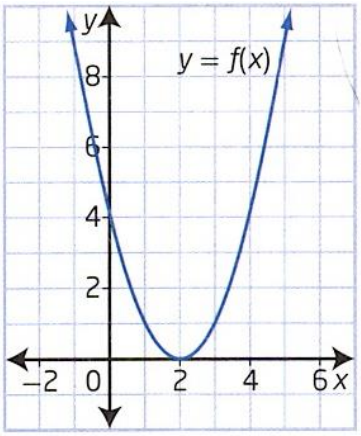

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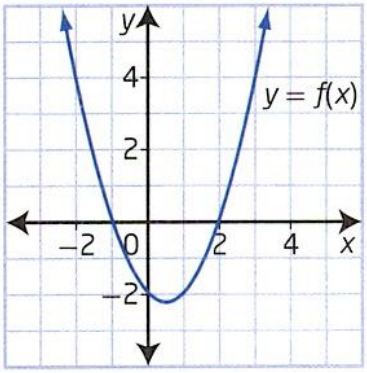

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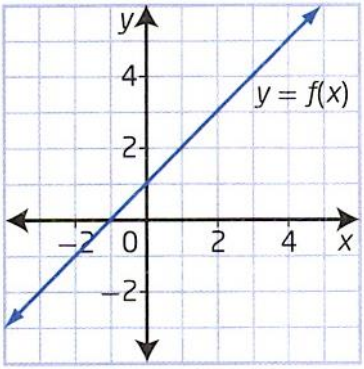
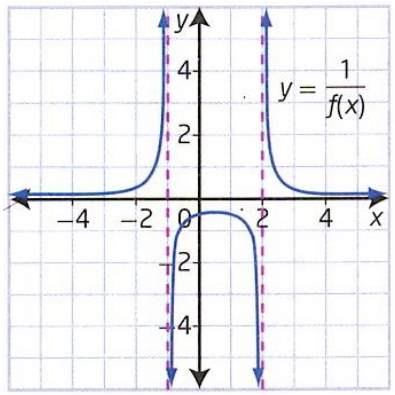


A



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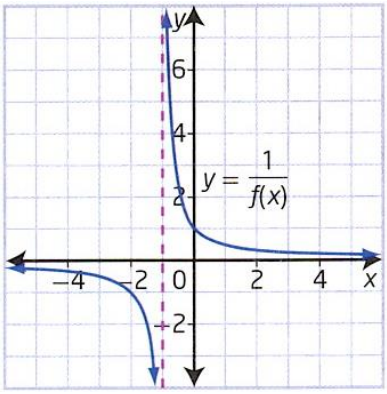
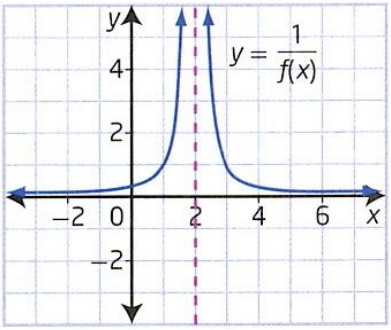
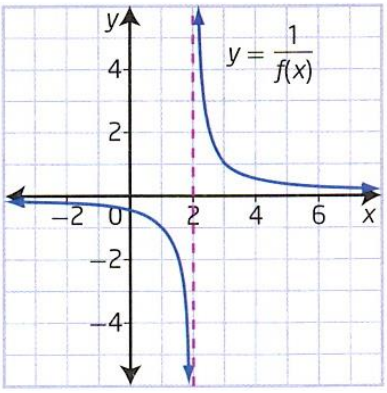


C



D



Answers

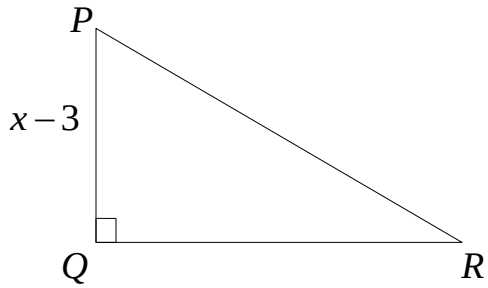
Reducing Rational Expressions Assignment:

1. a) $\frac{pq}{pq}$ b) $\frac{x-4}{x-4}$ c) $\frac{m-3}{m-3}$ d) $\frac{y(y+1)}{y(y+1)}$

2. a) $a \neq 4$ b) $b \neq 0$ c) $y \neq -2,\ 4$ d) $x \neq -3,\ 1$ e) $k \neq 0$
   f) $x \neq \frac{-5}{2},\ \frac{4}{3}$

3. a) $\frac{2}{3},\ a \neq 0,\ 5$ b) $\frac{3(2x+3)}{2(3x+2)},\ x \neq \frac{-2}{3},\ 0$ c) $\frac{(x+7)}{(2x-1)},\ x \neq \frac{1}{2},\ 7$
   d) $\frac{-1}{2},\ a \neq -2,\ 3$ e) $\frac{3r}{2p},\ r \neq 0,\ p \neq 0$ f) $\frac{-3}{5},\ x \neq 2$
   g) $\frac{(a-4)}{2(a-6)},\ a \neq \pm 6$ h) $\frac{(x+3)}{2(x-3)},\ x \neq \frac{-5}{2},\ 3$ i) $-1,\ x \neq 4$
   j) $\frac{5(x+y)}{(x-y)},\ x \neq y$

6. a)



   b) $QR = 2(x+2)$

   c) $x \neq -2$

7. a) $\frac{x+6}{x+3},\ x \neq \pm 3$ b) $(2x-7)(2x-5),\ x \neq -3$
   c) $\frac{(x-3)(x+2)}{(x-2)(x+3)},\ x \neq -3,\ -1,\ 2$ d) $\frac{(x+5)(x+3)}{3},\ x \neq \pm 1$

Multiplying & Dividing Rational Expressions Assignment:

1. a) $9x,\ x\neq 0,\ y\neq 0,\ z\neq 0$ b) $\dfrac{(a-5)}{5(a-1)},\ a\neq -5,\ 1,\ a\neq b$

   c) $\dfrac{4(y-7)}{(2y-3)(y-1)},\ y\neq -3,\ 1,\ \pm\dfrac{3}{2}$ d) $x-3,\ x\neq -3$

   e) $\dfrac{d-10}{4},\ d\neq -10$ f) $\dfrac{a-1}{a-3},\ a\neq -1,\ \pm 3$ g) $\dfrac{1}{2},\ x\neq \pm\dfrac{5}{2},\ 4$

   h) $\dfrac{(a+1)}{3}\ a\neq -3,\ \dfrac{1}{2},\ 1,\ \dfrac{3}{1}$

2. a) $\dfrac{2xy}{3},\ x\neq 0,\ y\neq 0$ b) $\dfrac{1}{3t},\ t\neq \pm 7,\ 0$ c) $\dfrac{1}{2},\ n\neq \pm 1$

   d) $\dfrac{y}{y+3},\ y\neq \pm 3$ e) $\dfrac{w-2}{3},\ w\neq \dfrac{-3}{2},\ -2$ f) $\dfrac{a}{a+3},\ a\neq -3,\ 0,\ 5$

   e) $\dfrac{-3x+1}{x+5},\ x\neq -5,\ \dfrac{-1}{3},\ 2$ h) $\dfrac{-2}{(y-2)},\ y\neq \pm 1,\ \dfrac{-1}{2},\ \dfrac{3}{4},\ 2$

4. $x+3$

5. $\dfrac{x+1}{2(x-2)}$

Adding & Subtracting Rational Expressions Assignment:

1. a) $\dfrac{10}{x},\ x\neq 0$ b) $\dfrac{4t+4}{5}$ c) $\dfrac{m^2-m}{m+1},\ m\neq 1$ d) $\dfrac{a^2-2a+12}{a-4},\ a\neq 4$

   e) $\dfrac{11a}{15a},\ a\neq 0$ f) $\dfrac{9+x}{6x},\ x\neq 0$ g) $\dfrac{20x-6}{5x},\ x\neq 0$

   h) $\dfrac{4z^2-9x^2}{xyz},\ x\neq 0,\ y\neq 0,\ z\neq 0$ i) $\dfrac{12xy+3y^2-12}{30y^3},\ y\neq 0$

   j) $\dfrac{6bxy^2-2a+a^2b^2y}{a^2b},\ a\neq 0,\ b\neq 0,\ y\neq 0$ k) $\dfrac{18-5x}{(x-2)(x+2)},\ x\neq \pm 2$

   l) $\dfrac{3x-11}{(x-4)(x+3)},\ x\neq -3,\ 4$ m) $\dfrac{4x}{x-2},\ x\neq 2$ n) $\dfrac{3}{y},\ y\neq -1,\ 0$

o) $\dfrac{-15h-27}{(h-3)(h+3)^2}$, $h \neq \pm 3$ p) $\dfrac{2x^2+x-6}{x(x+3)(x-2)(x-1)}$, $x \neq -3,\ 0,\ 1,\ 2$

q) $\dfrac{2x^2-6x+10}{(x-5)(x+5)}$, $x \neq \pm 5,\ \dfrac{1}{2}$ r) $\dfrac{4-x}{(x+3)(x-2)}$, $x \neq -3,\ 0,\ 2,\ 8$

s) $\dfrac{n+8}{(n-2)(n-4)}$, $n \neq 2,\ 3,\ 4$ t) $\dfrac{w+9}{(w+3)(w+4)}$, $w \neq -4,\ -3,\ -2$

3. a) $\dfrac{2x}{x+3}$, $x \neq \pm 3,\ 0$ b) $\dfrac{3(t+6)}{2(t-3)}$, $t \neq -6,\ -2,\ 0,\ 3$

c) $\dfrac{3m}{m+3}$, $m \neq -3,\ \dfrac{-3}{2},\ 0$ d) $\dfrac{x}{x-2}$, $x \neq \pm 4,\ 2$

4. $\dfrac{\sqrt{5x^2-2x+1}}{4}$

5. a) $\dfrac{2x^2+13}{(x+5)(x-4)}$, $x \neq -5,\ -2,\ 0,\ 3,\ 4$

b) $\dfrac{-9}{(x-1)(x+2)}$, $x \neq -3,\ -2,\ 0,\ \dfrac{1}{2},\ 1$

c) $\dfrac{-12x+3}{(x+5)(x-4)}$, $x \neq -5,\ -2,\ 0,\ 3,\ 4$

d) $\dfrac{15}{(x+6)(x+3)}$, $x \neq -6,\ -3,\ -2,\ \dfrac{-1}{2},\ 0$

Solving Rational Equations Assignment:

1. a) $f = -1$ b) $y = 6$ c) $w = 12$ d) $t = 2,\ 6$ e) $c = 2$

f) $d = -2,\ 3$ g) $x = 3$ h) no solution i) $b = 10 \pm \sqrt{43}$

j) $c = \dfrac{-1 \pm \sqrt{37}}{2}$

2. $l = 15\left(\sqrt{5}+1\right)$

3. a) $\dfrac{3-3x}{x^2}$ b) $\dfrac{6-2x}{x^3}$ c) $x = \dfrac{1}{2}$

Word Problems Involving Rational Equations Assignment:

1. 5 and 20 2. 3 and 4 3. 30 students 4. 1.2 min. 5. 6 h

6. 7.8 km/h 7. 28.8 h 8. 3.5 km/h 9. ≈ 20 pages per day 10. $a = \pm\frac{1}{3}$

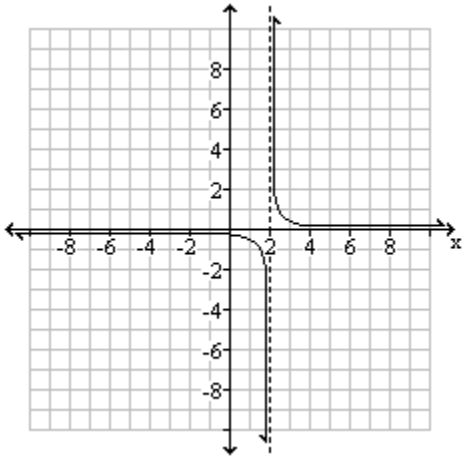
Reciprocal Functions Assignment:

1. a) $\frac{1}{f(x)} = \frac{1}{x+5},\ x \neq -5$ b) $\frac{1}{g(x)} = \frac{1}{2x+1},\ x \neq \frac{-1}{2}$

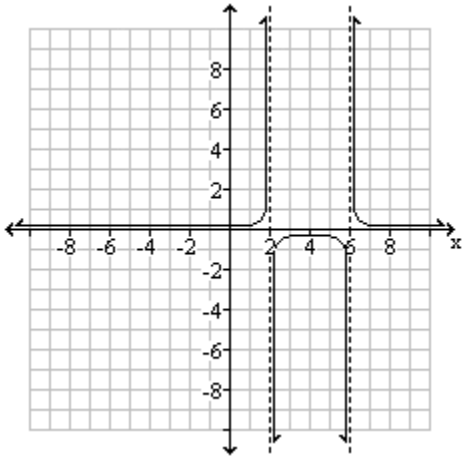
   c) $\frac{1}{h(x)} = \frac{1}{x^2-16},\ x \neq \pm 4$ d) $\frac{1}{f(x)} = \frac{1}{x^2+x-12},\ x \neq -4,\ 3$

2. a) $x = 2$ b) $x = \frac{-7}{3}$ c) $x = -4$ and $x = 2$ d) $x = 4$ and $x = 5$

3. a)



b)



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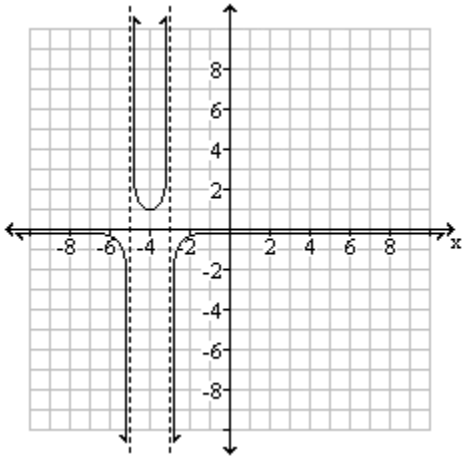

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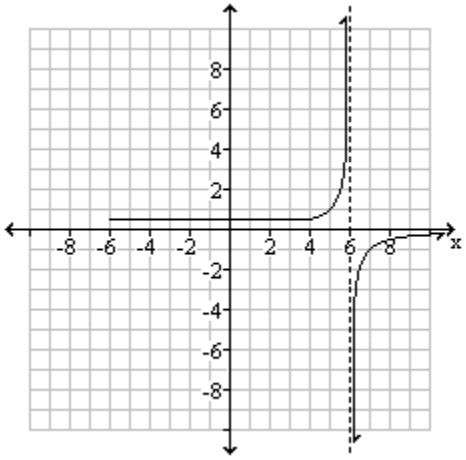
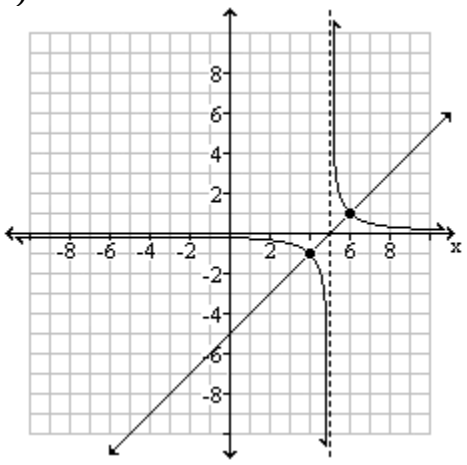


4. a)



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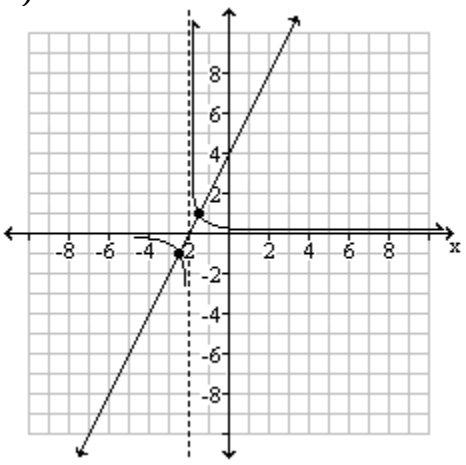

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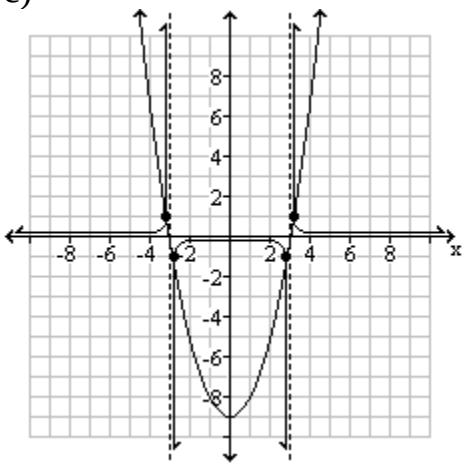
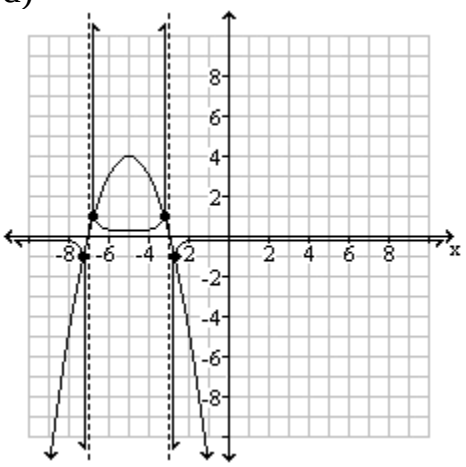


d)



5. a) D b) C c) A d) B